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Abstract

This paper examines the problem of the indexing dilemma in the context of an ordinal interpersonal comparison of individual situations and proposes a new class of equity criteria based on the dominance principle and individual preferences. First, we show that an interpersonal comparison ordering that satisfies the dominance principle and the monotonicity condition must be a consensus ranking, which requires that individual i's situation with consumption bundle x should be better than j's situation with y whenever all individuals strictly prefer x to y. Second, we propose a new class of equity criteria based on the worst evaluation of each person's situation, which is a class of ordering extensions of interpersonal rankings that respect the consensus condition. In addition, we show its representation theorem and characterize maximin orderings based on our criteria. Third, possibility and impossibility results between the Pareto principle and dominance methods are obtained. Then, we propose a class of median

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rules as another equity criterion that satisfies the weak Pareto and dominance principles.

JEL codes: D60, D63, I30, I31, I32

1 Introduction

This paper examines the problem of the indexing dilemma in the context of an ordinal interpersonal comparison of individual situations and proposes a new class of equity criteria based on the consensus approach *à la* Suprimont (2012). As shown in the well-known results (Gibbard 1979; Brun and Tungodden 2004; Fleurbaey 2007; Pattanaik and Xu 2007; Weymark 2017), respecting individual preferences and dominance relations on consumption bundles is not compatible with constructing *acyclic* interpersonal comparison rankings. Due to this indexing dilemma problem, two methods are usually used for an ordinal interpersonal comparison of an individual's well-being. One is the equivalent approach based on Pazner-Schmeidler functions (Pazner and Schmeidler 1978; Pazner 1979; Fleurbaey 2005), ray utility, and money-metric utility, making it possible to build a ranking that reflects individual preferences for an ordinal interpersonal comparison of individual well-being instead of abandoning the dominance principle on consumption bundles[1].

The other approach is the compound index approach based on the human development index (HDI) and the multi-dimensional poverty index (MPI), which evaluates human well-being by seeing some weighted sum of components which seem to be ingredients of a good human life[2]. This approach can build a ranking that respects the dominance principle on consumption bundles.

However, the latter approach often ignores individual preferences. For example, let's assume that $x = (4,\ 1,\ 0.8)$ and $y = (2.5,\ 2,\ 1)$; each vector means a three-dimensional consumption bundle (food expenditure, educational expenditure, and health level). Then, if the weight to each item is equal to 1, x will be better

[1] Recently, a series of studies compared individuals' well-being by using information on intersections or unions of lower contour sets and upper contour sets (Fleurbaey and Maniquet 2017a; 2017b; 2018). However, these studies constructed welfare orderings in terms of the equivalent approach. In addition, Pivato (2015) provided a unique framework of interpersonal comparison methods that focus on individuals' welfare gains. In the context of the social choice problem under uncertainty, Miyagishima (2018) developed an approach that focuses on the intersection of upper contour sets.

[2] The most influential MPI method is the Alkire-Foster index (Alkire and Foster 2011). A practical guide to this index is given by Alkire et al. (2015).

than y in the HDI approach. If all individuals place importance on their education expenditure and health level relative to food expenditure, then all consumers would prefer consumption bundle y to x. Thus, the HDI approach completely ignores even unanimous preferences.

In order to avoid such a situation, this paper considers the monotonicity condition that requires interpersonal comparison ranking should respect individual judgments and should not reverse them at least whenever all individual preferences are the same over some ordered pairs. When this monotonicity condition and the dominance principle are imposed on interpersonal comparison rankings, then the rankings must satisfy a consensus condition that respects individual *strict* preferences if the strict preference relations are unanimous over some pairs. However, because the consensus condition itself can be interpreted as the weak Pareto axiom for interpersonal comparison settings, if we impose independence of irrelevant alternatives (IIA) as a condition on informational efficiency, the interpersonal comparison ranking must be dictatorship,; that is, the ranking must completely correspond to a single person's preference relation.

Therefore, this paper proposes a class of reasonable interpersonal rankings as a method for a non-dictatorial interpersonal comparison that satisfies the consensus condition and then provides a framework to compare resource allocations. In particular, one of the interpersonal comparison methods on which this paper focuses can be interpreted as a variation of the concept of the Pazner-Schmeidler function (hereafter, the PS function). Usually, in the PS function, individual i's well-being with consumption bundle x is measured by magnification λ_i with given reference bundle r, where $\lambda_i r$ is indifferent from x in terms of i's preference relation. That is, given r, i's well-being with x is measured by λ_i where $u_i(x) = u_i(\lambda_i r)$. On the contrary, in the method proposed ub this paper, we measure i's well-being with x as the smallest value among magnifications λ_i of reference r which is indifferent from x in terms of *everyone's* preference relations. That is, given reference bundle r, i's well-being with x is measured by the minimum value of λ_j where $u_j(x) = u_j(\lambda_j r)$ among all individuals[3].

[3] In addition, we can define a similar equity criterion using the concept of money-metric utility.

Next, this paper extends interpersonal comparison rankings based on our equity criteria to social orderings in the framework of *universal social orderings* (Fleurbaey and Tadenuma 2014), which evaluates pairs of resource allocations and preference relations of variable population groups. Then, we show a simple representation theorem based on our equity criteria and provide universal social orderings that follow the maximin principle.

Furthermore, the paper shows that continuous universal social orderings that satisfy the separability, dominance and weak Pareto principles must follow each single-domain dictatorship for each profile. Then, we propose a class of median consensus methods as one of the appealing solutions for ordinal interpersonal comparisons of individual well-being.

Based on the results of this paper, ordinal interpersonal comparison rankings can be divided into three categories. The first class is the equivalent approach which completely respects each individual preference for evaluating each individual's situation but cannot satisfy the dominance principle. The second class is the compound index approach which can satisfy the dominance principle but completely ignores individual preferences. The third class is the consensus approach which compares individual situations in a way that satisfies the dominance principle and monotonicity.

Although a number of empirical studies have been analyzed in terms of the compound index approach thus far (Alkire et al. 2015), no theoretical research has been conducted on the specific counting method of the consensus approach. As a result, there is little accumulation of empirical research in the consensus approach. However, the compound index approach cannot reflect individual preferences in the sense that it could not satisfy the monotonicity condition. Therefore, this paper sheds new light on the consensus approach and motivates additional empirical research.

The rest of this paper is organized as follows. Section 2 gives the basic notations and definitions used in this paper. Section 3 shows theoretical results arising from

The equity criteria proposed in this paper essentially evaluate individuals' well-being based on the union of all individuals' lower contour sets.

the problem of indexing dilemmas and proposes a class of ordinal interpersonal rankings which respects the dominance principle and the monotony condition. Section 4 extends a concept of interpersonal rankings to the framework of universal social orderings proposed by Fleurbaey and Tadenuma (2014) and derives a simple representation theorem based on the consensus approach, and characterizes the maximin principle respecting the consensus condition. Section 5 shows the impossibility and possibility results between the dominance methods and the Pareto principle. Section 6 gives a summary of this paper and final remarks.

2 Basic Notations and Definitions

We consider n-individuals and l-goods (or characteristics, functionings) in the canonical division economy. For all natural numbers m, let $\mathbb{R}_+^m$ (resp. $\mathbb{R}_{++}^m$) be the non-negative (resp. positive) m-dimensional Euclidean space. A society consists of n individuals. Let the set of individuals be denoted by $N = \{1, ..., n\}$. An allocation is a vector $x = (x_1, x_2, ..., x_n) \in \mathbb{R}_+^{nl}$ where each $x_i = (x_{i1}, ..., x_{il}) \in \mathbb{R}_+^m$ is a consumption bundle of individual $i \in N$. Then, we write $X = \mathbb{R}_+^l$ as the set of consumption bundles. Given two vectors x and y in $\mathbb{R}^m$, $x \geqq y$ if and only if $x_k \geqq y_k$ for all k in $\{1, 2, ..., m\}$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_k > y_k$ for all k in $\{1, 2, ..., m\}$. Each individual $i \in N$ has his/her preference ordering[4] R_i on X that satisfies continuity and strict monotonicity in the usual sense[5]. Then, P_i and I_i represent asymmetric and symmetric factors of R_i, respectively. Let $\mathscr{R}$ be the set of preference orderings on X.

An interpersonal comparison ranking on $X \times \mathscr{R}$ is a mapping $\succsim$ which assigns a non-empty subset of $(X \times \mathscr{R})^2$ to each profile $R_N \in \mathscr{R}^N$. Then, $\forall i, j \in N$, $\forall (x, R_i), (y, R_j) \in X \times \mathscr{R}$, $(x, R_i) \succsim (y, R_j)$ is interpreted that individual i' s situ-

[4]An ordering is a binary relation R that satisfies completeness and transitivity. Completeness requires that, for all alternatives x, $y \in X$, xRy or yRx. Transitivity demands that, for all alternatives x, y, $z \in X$, xRy and yRz imply xRz.

[5]Continuity requires that, for all $x \in X$, both $\{y|yRx\}$ and $\{y|xRy\}$ are closed. Strict monotonicity says that, for all x, $y \in X$, $x \geqq y$ implies xRy and $x \geq y$ implies xPy, where P is an asymmetric factor of R.

ation where his/her consumption bundle and preference relation are given by x and R_i is at least as good as j' s situation where his/her consumption bundle and preference relation are given by y and R_j . Let $\succ$ and $\sim$ denote asymmetric and symmetric factors of an interpersonal comparison ranking $\succsim$, respectively. We assume that interpersonal comparison rankings are orderings and satisfy the following continuity property.

Conditional Continuity (Pattanaik and Xu 2007)
$\forall i \in N, \forall (x, R_i) \in X \times \mathscr{R}$, both $\{(y, R_i) | (y, R_i) \succsim (x, R_i)\}$ and $\{(y, R_i) | (x, R_i) \succsim (y, R_i)\}$ are closed.

This continuity condition simply requires that any interpersonal comparison ranking must be continuous with respect to consumption bundles given a fixed preference relation.

Our purpose is to construct some reasonable interpersonal comparison rankings and extend them to the universal comparison settings. In the next section, we investigate the problem of the indexing dilemma and show a class of interpersonal comparison rankings that respect the dominance principle and the monotonicity condition.

3 Results in an Interpersonal Ranking Setting

In this section, we examine whether reasonable interpersonal orderings could be constructed considering the indexing dilemma problem. First, we show and examine the properties of the classical indexing dilemma. The indexing dilemma problem is derived from combining the following two axioms for interpersonal comparison rankings.

Consumer Sovereignty (Fleurbaey and Tadenuma 2014)
$\forall i \in N$, $\forall (x, R_i), (y, R_i) \in X \times \mathscr{R}$, $(x, R_i) \succsim (y, R_i)$ iff $x R_i y$.

Consumer sovereignty requires that a society should respect each individual's preference relation when it evaluates only his/her well-being. This axiom is so appealing for interpersonal comparison rankings that many studies focus on this axiom and derive the equity criteria that satisfy consumer sovereignty such as the PS function.

Dominance Principle

$\forall i, j \in N$, $\forall (x, R_i), (y, R_j) \in X \times \mathscr{R}$, $x \geqq y \Longrightarrow (x, R_i) \succsim (y, R_j)$. Moreover, $x \geq y \Longrightarrow (x, R_i) \succ (y, R_j)$.

The dominance principle requires that a society should respect a dominance relation on consumption bundles. This axiom is appealing in the cases that a consumption space equals a functionings space *à la* Sen (1985), or that the ability to transform consumption bundles into functionings is equal among all individuals.

Then, the classical indexing dilemma can be formulated as follows.

Proposition 1 (Gibbard 1979; Brun and Tungodden 2004; Fleurbaey 2007): *There exists no interpersonal comparison ranking satisfying consumer sovereignty, the dominance principle, and acyclicity.*

Brun and Tungodden (2004) show that there exist interpersonal comparison rankings that satisfy the properties above if and only if all individuals have an identical preference.

Fleurbaey (2007) proves a similar result to Brun and Tungodden (2004)[6]. In addition, he shows that any interpersonal comparison ranking that satisfy consumer sovereignty must be a class of rankings that satisfy the minimal version of the dominance principle which applies for only the set of consumption bundles where any element is bigger or smaller than other elements[7].

These results mean that if a social planner wants to respect the dominance principle, then any interpersonal comparison ranking must depend on consumption bundles only.

Proposition 2 (Brun and Tungodden 2004; Fleurbaey 2007; Pattanaik

[6]Precisely speaking, Fleurbaey (2007) shows that all individuals' preferences are ordinal equivalent whenever an interpersonal comparison ranking satisfies the dominance principle.

[7]Fleurbaey (2007) calls such a set "*thin*". Given a set $A \subset X$, A is *thin* if $\forall a, b \in A$, $a \geqq b$ or $b \geqq a$. Then, he proves that if an interpersonal comparison ranking satisfies consumer sovereignty and the dominance principle applied to a specified set, then the set must be thin.

and Xu 2007): *If an interpersonal comparison ranking* $\succsim$ *satisfies the dominance principle, then there exists an ordering* R^* *on* X *such that* $\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$, $(x, R_i) \succsim (y, R_j) \Longleftrightarrow xR^*y$.

Proposition 2 is easily proved by the two facts that the dominance principle implies $(x, R_i) \sim (x, R_j)$ for all x, R_i, and R_j and that there exist no alternatives (x, R_i), (y, R_i), (x, R_j), and (y, R_j) such that $(x, R_i) \succ (y, R_i)$ and $(y, R_j) \succ (x, R_j)$ from the results in Brun and Tungodden (2004), Fleurbaey (2007), and Pattanaik and Xu (2007). Thus, from Proposition 2, if a society respects the dominance principle, then the informational basis of the interpersonal comparison rankings must shrink to the set of consumption bundles.

However, as shown in the simple example in the Introduction, the compound index approach, which can satisfy the dominance principle, often violates unanimous preferences. Therefore, we need to consider the following condition.

Monotonicity for Interpersonal Comparison
$\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$, if $\forall i \in N$, xR_iy, then $\forall j, k \in N$, $(x, R_j) \succsim (y, R_k)$.

Monotonicity for interpersonal comparison is interpreted as the minimal condition for respecting individual preferences because it simply requires that a society should not reverse unanimous preferences. Another condition of respecting individual preferences is the following consensus for interpersonal comparison, which requires that a society should respect strict unanimous preferences.

Consensus for Interpersonal Comparison (Sprumont 2012)
$\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$, if $\forall i \in N$, xP_iy, then $\forall j, k \in N$, $(x, R_j) \succ (y, R_k)$.

Then, if a social planner wants to respect monotonicity for interpersonal comparison and the dominance principle, then an interpersonal comparison ranking also satisfies consensus for interpersonal comparison[8].

[8]Sprumont (2012) axiomatically characterizes a maximin social ordering based on a continuous interpersonal comparison ordering that satisfies the consensus condition. However, it is not determined

Proposition 3: *If an interpersonal comparison ranking $\succsim$ satisfies the dominance principle and monotonicity for interpersonal comparison, then it satisfies consensus for interpersonal comparison.*

[Proof] Suppose that $\succsim$ satisfies the dominance principle and monotonicity for interpersonal comparison. Consider a profile $R_N \in \mathscr{R}^N$ such that for some $x, y \in X$, $\forall i \in N$, xP_iy. Then, monotonicity for interpersonal comparison implies $\forall j, k \in N$, $(x, R_j) \succsim (y, R_k)$. If it does not hold $\forall j, k \in N$, $(x, R_j) \succ (y, R_k)$, then $\exists j', k' \in N$ s.t. $(x, R_{j'}) \sim (y, R_{k'})$. Because of the continuity of individual preferences, we can choose x' from the neighborhood of x such that $x > x'$ and $\forall i \in N$, $x'P_iy$. By the dominance principle, we have $(x, R_{j'}) \succ (x', R_{j'})$. By monotonicity for interpersonal comparison, we have $(x', R_{j'}) \succsim (y, R_{k'})$. Then, transitivity of interpersonal comparison rankings implies $(x, R_{j'}) \succ (x', R_{j'}) \succsim (y, R_{k'}) \sim (x, R_{j'})$. This is a contradiction. Thus, $\forall j, k \in N$, $(x, R_j) \succ (y, R_k)$ holds true. $\|$

Propositions 1-3 show theoretical properties and limits of interpersonal comparison methods that respect consumer sovereignty or the dominance principle. Given the indexing dilemma problem, we seem to have only three feasible options for ordinal interpersonal comparison of individual well-being. The first option is the equivalent approach based on the PS function, ray utility, and money-metric utility, which can respect consumer sovereignty but violates the dominance principle. The second is the compound index approach based on the HDI, the MPI, the quality of adjusted life years (QALY), and the happiness index which can respect the dominance principle but violates consumer sovereignty and monotonicity for interpersonal comparison. The third is the consensus approach we focus on, which can respect the dominance principle and monotonicity for interpersonal comparison but violates consumer sovereignty.

Now we introduce some reasonable classes of the consensus approach.

As a preliminary step, we consider theoretical properties of interpersonal compar-

what consensus-type interpersonal comparison rankings should be chosen. In this paper, we specify classes of interpersonal comparison rankings that satisfy the consensus condition.

ison rankings that satisfy consensus for interpersonal comparison. In the setting of interpersonal comparison rankings, the consensus condition can be interpreted as the weak Pareto condition for the ordinal social choice problem[9]. Thus, by Arrow's theorem on economic domain (Arrow 1951, 1963; Bordes and Le Breton 1989), if a social planner requires the informational efficiency property (such as the IIA) for an interpersonal comparison ranking, then it must be dictatorial. In fact, Fleurbaey, Suzumura, and Tadenuma (2005) show that even a very weak informational efficiency requirement fails to construct a Paretian social ordering except for Hansson's (1973) independence condition[10].

Therefore, we must violate informational efficiency requirements to avoid dictatorship for interpersonal comparison rankings that satisfy the consensus condition. Then, we propose that non-dictatorial interpersonal comparison orderings can be constructed by the following simple aggregation methods.

Example 1: An interpersonal comparison ranking $\succsim$ is the *minimum consensus for interpersonal comparison* iff, given a reference bundle $r \in X$, $\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$, $\forall i, j \in N$, $(x, R_i) \succsim (y, R_j) \Longleftrightarrow min_{h\in N}\{\lambda_h | x I_i \lambda_h r\} \geqq min_{h\in N}\{\lambda_h | y I_i \lambda_h r\}$.

The minimum consensus for interpersonal comparison evaluates individual i's well-being with consumption bundle x by seeing the minimum value λ_h such as $x I_h \lambda_h r$ among all individuals given reference bundle r. Then, social orderings based on this interpersonal ranking guarantee at least a utility level when consuming $\lambda_h r$ for all individuals. In addition to this ranking, we can consider various rankings that have the same spirit as the minimum consensus for interpersonal comparison. For example, consider the following interpersonal ranking.

[9] According to Proposition 2, an interpersonal comparison ranking that satisfies the dominance principle is equivalent to an ordering defined on the set of consumption bundles. Therefore, the consensus condition can be interpreted as an unanimity condition on X, so it can be also seen as the weak Pareto principle in the context of interpersonal comparison rankings.

[10] Fleurbaey, Suzumura, and Tadenuma (2005) show that various Arrovian impossibility theorems reemerge even after weakening IIA in an economic environment such as a condition focusing on marginal rates of substitutions. Although more information is used in economic environments, there still exist so-called *free triples*. Thus, Arrovian impossibility theorems can survive except for the case where social orderings satisfy Hansson's independence.

Example 2: An interpersonal comparison ranking $\succsim$ is a *minimum money-metric consensus for interpersonal comparison* iff, given a price vector $p \in \mathbb{R}^l_{++}$ with $\Sigma_{i=1,\ldots,l} p_i = 1$, $\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$, $\forall i, j \in N$, $(x, R_i) \succsim (y, R_j) \iff min_{h \in N} min_{x' \in X}\{p \cdot x' | x' R_h x\} \geqq min_{h \in N} min_{y' \in X}\{p \cdot y' | y' R_h y\}$.

The two interpersonal comparison rankings above have common properties in the sense that both rankings focus on *the worst* value of each individual's situation among *all* individual preferences. In short, our economic equity criteria are to set a lower bound based on all individuals' preferences and require everyone's well-being to be higher than this bound. Thus, these methods satisfy the following condition[11].

Lower Contour Monotonicity for Interpersonal Comparison (LCM)
$\forall x \in X, \forall R_N, R'_N \in \mathscr{R}^N, \bigcap_{i \in N} LC(x, R_i) \subseteq \bigcap_{i \in N} LC(x, R'_i) \Longrightarrow \forall i \in N, \{(y, R_i) | (x, R_i) \succsim (y, R_i)\} \subseteq \{(y, R_i) | (x, R_i) \succsim' (y, R_i)\}$, where $\forall x \in X, \forall R_i \in \mathscr{R}, LC(x, R_i) = \{x' | x R_i x'\}$.

Lower contour monotonicity for interpersonal comparison requires that if an intersection of all individuals' lower contour sets expands, then the lower contour set of the corresponding interpersonal comparison ranking should also expand. In other words, if the relative position of x gets better in everyone's preference relation, then the relative position of x should also get better in the interpersonal comparison ranking.

In addition to examples 1 and 2, we can easily construct interpersonal comparison rankings that satisfy the consensus condition. The next two rankings are variations of Example 1, but their spirits are far different from the minimum consensus rankings[12].

Example 3: An interpersonal comparison ranking $\succsim$ is a *utilitarian consensus for interpersonal comparison* iff, given a reference bundle $r \in X$, $\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$,

[11] Lower contour monotonicity is similar to preference monotonicity in the universal social ordering setting in Fleurbaey and Tadenuma (2014). In addition, lower contour monotonicity has quite similar properties to Hannson's independence in the sense that both conditions relate to multi-profile independence.

[12] Using similar methods as in Examples 3 and 4, the money-metric utility versions of utilitarian and first-boys consensus can be constructed.

$\forall i, j \in N$, $(x, R_i) \succsim (y, R_j) \Longleftrightarrow \Sigma_{h\in N}\{\lambda_h | x I_h \lambda_h r\} \geqq \Sigma_{i\in N}\{\lambda_h | y I_i \lambda_h r\}$.

Example 4: An interpersonal comparison ranking $\succsim$ is a *first-boys consensus for interpersonal comparison* iff, given a reference bundle $r \in X$, $\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$, $\forall i, j \in N$, $(x, R_i) \succsim (y, R_j) \Longleftrightarrow max_{h\in N}\{\lambda_h | x I_h \lambda_h r\} \geqq max_{h\in N}\{\lambda_h | y I_h \lambda_h r\}$.

In the next section, we extend interpersonal comparison rankings based on the consensus approach to universal social orderings.

4 Results in the Universal Social Ordering Setting

Fleurbaey and Tadenuma (2014) propose a new framework of social choice theory that can make us compare variable situations including inter-society comparisons and interpersonal comparisons. Formally, a universal social ordering (hereafter, USO) is an ordering $\succsim$ on $\bigcup_{S\in P(N)\setminus\{\emptyset\}}(X^S \times \mathscr{R}^S)$. We define a USO as a mapping from $\mathscr{R}^N$ into the subset of $[\bigcup_{S\in P(N)\setminus\{\emptyset\}}(X^S \times \mathscr{R}^S)]^2$. Then, Fleurbaey and Tadenuma (2014) show that a USO that satisfies consumer sovereignty and some reasonable axioms (separability, individual continuity,[13] and preference monotonicity) must be a class of rankings based on the equivalent approach. In contrast to their results, we propose a USO based on the consensus approach. Then, all axioms defined in the previous sections can be translated to the setting of USOs from that of interpersonal comparison rankings.

In the framework of universal social orderings, once a profile R_N is specified, then all alternatives (x_S, R'_S) and (x_T, R''_T) are evaluated based on this profile. In this sense, USOs that satisfy the dominance principle completely ignore any differences between individual preferences and focus on the current preference R_N. Therefore, since any influence of preference changes vanishes, these USOs can satisfy the following separability condition. Then, let us introduce the separability axiom that requires a USO to be independent from individuals who have the same situation between alternatives in order to show our representation theorem.

[13]Individual continuity is equivalent to conditional continuity.

Separability

$\forall x_N, y_N \in X^N$, $\forall R_N, R'_N \in \mathscr{R}^N$, if $x_i = y_i$ and $R_i = R'_i$ for some $i \in N$, then for all $S \in P(N) \setminus \{\emptyset\}$, $(x_S, R_S) \succsim (y_S, R_S) \Longleftrightarrow (x_{S\cup\{i\}}, R_{S\cup\{i\}}) \succsim (y_{S\cup\{i\}}, R_{S\cup\{i\}})$.

The following simple representation theorem is obtained by combining consensus, separability and lower contour monotonicity conditions defined in the previous section.

Theorem 1: *If a universal social ordering $\succsim$ satisfies the dominance principle, monotonicity for interpersonal comparison, separability, and lower contour monotonicity for interpersonal comparison, then there exists an ordering R^{**} such that* $[\forall(x_S, R'_S), (y_T, R''_T), (U(\bigcap_{i\in N} LC(x_j, R_i)))_{j\in S} R^{**} (U(\bigcap_{i\in N} LC(y_j, R_i)))_{j\in T} \Longleftrightarrow (x_S, R'_S) \succsim (y_T, R''_T)]$, *where U is a real valued function such that* $[\forall x, y \in X,\ \forall R_N \in \mathscr{R}^N$, $\bigcap_{i\in N} LC(x, R_i) \supseteq \bigcap_{i\in N} LC(y, R_i) \Longrightarrow U(\bigcap_{i\in N} LC(x, R_i)) \geqq U(\bigcap_{i\in N} LC(y, R_i))$ *and* $inte \bigcap_{i\in N} LC(x, R_i) \supset \bigcap_{i\in N} LC(y, R_i) \Longrightarrow U(\bigcap_{i\in N} LC(x, R_i)) > U(\bigcap_{i\in N} LC(y, R_i))]$[14].

[Proof] By Proposition 2, we have a continuous ordering R^* on X^N. Thus, we have a real-valued function U such that for all $x, y \in X$, $u(x) \geqq u(y)$ iff xR^*y. Then, separability implies there exists an ordering R^{**} such that $\forall(x_S, R'_S), (y_T, R''_T)$, $(x_S, R'_S) \succsim (y_T, R''_T) \Longleftrightarrow (u(x_i))_{i\in S} R^{**} (u(y_i))_{i\in T}$. Next, by lower contour monotonicity for interpersonal comparison, we can find a function U such that $u(x) \geqq u(y) \Longleftrightarrow U(\bigcap_{i\in N} LC(x, R_i)) \geqq U(\bigcap_{i\in N} LC(y, R_i))$ for all $x, y \in X$. Moreover, lower contour monotonicity for interpersonal comparison and consensus implies that $\bigcap_{i\in N} LC(x, R_i) \supseteq \bigcap_{i\in N} LC(y, R_i) \Longrightarrow U(\bigcap_{i\in N} LC(x, R_i)) \geqq U(\bigcap_{i\in N} LC(y, R_i))$ and $inte \bigcap_{i\in N} LC(x, R_i) \supset \bigcap_{i\in N} LC(y, R_i) \Longrightarrow U(\bigcap_{i\in N} LC(x, R_i)) > U(\bigcap_{i\in N} LC(y, R_i))]$. Thus, we have function U and ordering R^{**} in the statement of Theorem 1. $\|$

Note that the USO above cannot satisfy even the weak Pareto principle for USO frameworks. The latter section shows that Paretian USOs that satisfy the dominance principle must be a single-domain dictatorship. Thus, we must face a trade-off between

[14]Given a set A, $inteA$ is an interior of set A.

a single-domain dictatorship and the Pareto principle in the setting of universal social orderings whenever a society respects the dominance principle for ordinal interpersonal comparisons.

Next, we introduce the Pigou-Dalton transfer principle for dominance relations and provide a set of sufficient conditions for maximin USOs respecting a class of the minimum consensus for interpersonal comparisons.

Pigou-Dalton Transfer Principle for Dominance Relations
$\forall x_N, y_N \in X^N$, $\forall R_N, R'_N \in \mathscr{R}^N$, if $\exists i, j \in N$, $y_i \geq x_i \geqq x_j \geq y_j$ and $\forall k \in N \setminus \{i, j\}$, $x_k = y_k$, then $(x_S, R_S) \succsim (y_S, R'_S)$ for all S including individuals i and j.

Since the axioms above have essentially the same effects as the axioms of Sprumont's (2012) Proposition 1, we have the following result.

Theorem 2: *If a universal social ordering $\succsim$ satisfies the dominance principle, monotonicity for interpersonal comparison, separability, lower contour monotonicity for interpersonal comparison, and the Pigou-Dalton transfer principle for dominance relations, then $\forall (x_S, R'_S), (y_T, R''_T)$ with $|S| = |T|$, $min_{j \in S}(U(\bigcap_{i \in N} LC(x_j, R_i)))_{j \in S} \geq min_{j \in T}(U(\bigcap_{i \in N} LC(y_j, R_i)))_{j \in T} \Longrightarrow (x_S, R'_S) \succ (y_T, R''_T)$, where function U is defined in Theorem 1.*

Moreover, these results can be easily extended in the framework of social choice with variable population by adding the similar axioms of Blackorby, Bossert, and Donaldson (2005). In the setting of variable population size, we can compare extended alternatives (x_S, R'_S) and (y_T, R''_T) where $|S| \neq |T|$. However, social choice theory for variable population has some ethical difficulties; e.g., social ordering functions proposed by the literature arbitrarily set a threshold and prefer a social state without some people to one with them whenever their well-being is below the threshold. Such a comparison provokes sensitive and serious concerns in the context of the meaning of human existence. Therefore, we abbreviate the results and discussions of social choice problems with variable population size[15].

5 Possibility and Impossibility Results of Pareto Principle

In this section, we investigate the logical consistency problem between the Pareto and dominance principles. As many studies show that all social welfare functions that satisfy the dominance principle cannot reflect individual preferences (see Weymark 2017), there are many variations of impossibility results respecting the dominance principle and preference monotonicity conditions. This paper considers two versions of the Pareto principle which are well-known and usual ones in social choice theory.

Pareto Principle

$\forall x_N, y_N \in X^N$, $\forall R_N \in \mathscr{R}^N$, if $\forall i \in N, x_i R_i y_i$, then $(x_N, R_N) \succsim (y_N, R_N)$. Moreover, if $\forall i \in N, x_i R_i y_i$ and $\exists j \in N, x_j P_j y_j$, then $(x_N, R_N) \succ (y_N, R_N)$.

Weak Pareto Principle

$\forall x_N, y_N \in X^N$, $\forall R_N \in \mathscr{R}^N$, if $\forall i \in N, x_i P_i y_i$, then $(x_N, R_N) \succ (y_N, R_N)$.

Note that the Pareto principle contains the so-called Pareto indifference principle and the weak Pareto principle. The following result is obvious but strong for constructing social welfare ordering that respects the dominance principle.

Proposition 4: *If a universal social ordering $\succsim$ satisfies continuity, separability, the dominance principle and the weak Pareto principle, then $\forall R_N \in \mathscr{R}^N$, $\exists i^*(R_N) \in N$, $\forall x, y \in X$, $\forall R', R'' \in \mathscr{R}$, $x R_{i^*(R_N)} y \Leftrightarrow (x, R') \succsim (y, R'')$.*

[Proof] Consider USO $\succsim$ that satisfies the axioms above. Then, from Proposition 2, any interpersonal comparison ranking must be based on judgments of one ordering

[15] As in Fleurbaey and Tadenuma (2014), basic results can be obtained in a social choice model with variable population size. That is, given a threshold for interpersonal comparisons of well-being, if some people's well-being is below the threshold, then a society evaluates that a state without these persons should be strictly better than a state with them.

R^* on X. If this ordering is different from any individuals' rankings, then we can find pairs x and y with xP^*y while every person strictly prefers y to x. By the weak Pareto principle, we have $(y_N, R_N) \succ (x_N, R_N)$ where $\forall j \in N$, $y_j = y$ and $x_j = x$. Consider a sequence $\{y_N^t\}$ such that for any number t, $y_N^t = (y_i, y_{-i}^t)$, $\forall j \in N$, $y_j^t P_j x_j$ and $\lim_{t\to\infty}(y_j^t) = x_j$. Continuity implies that $((y_i, x_{-i}), R_N) \succsim (x_N, R_N)$. By separability, we have $(y_i, R_i) \succsim (x_i, R_i)$ iff $((y_i, x_{-i}), R_N) \succsim (x_N, R_N)$. This obviously contradicts the fact that xP^*y iff $(x_i, R_i) \succ (y_i, R_i)$. $\|$

We can construct anonymous USOs that satisfy the dominance and weak Pareto principles as follows:

For all R_N, $\exists i^*(R_N)$ such that for all (x_S, R'_S) and (y_T, R''_T), $(x_S, R'_S) \succsim^{min} (y_T, R''_T)$ iff $x^* R_{i^*(R_N)} y^*$ where $x^* \in \{x_j | \forall x_k \in x_S, x_k R_{i^*(R_N)} x_j\}$ and $y^* \in \{y_j | \forall y_k \in y_T, y_k R_{i^*(R_N)} y_j\}$.

If individual $i^*(R_N)$ is the same person for all profile $R_\pi(N)$ with some bijection π on N, then this USO obviously satisfies anonymity, the dominance and weak Pareto principles[16].

In particular, it is interesting to consider the following interpersonal comparison ranking based on median rules because it can satisfy several appealing properties[17].

Example 5: An interpersonal comparison ranking $\succsim_{med}$ is a *median money-metric consensus for interpersonal comparison* iff, given a reference bundle $r \in X$, $\forall R_N \in \mathscr{R}^N$, let $i_{med}(R_N)$ be an individual whose money-metric utility at r is just median among all individual's utilities at r. Then, $\forall x, y \in X$, $\forall R_N \in \mathscr{R}^N$, $\forall i, j \in N$, $(x, R_i) \succsim_{med} (y, R_j) \Longleftrightarrow x R_{i_{med}(R_N)} y$.

By using the median consensus method above, it is easy to construct a maximin USO based on this single-domain dictator's ranking. Note that both equivalent and

[16]The maximin ordering proposed in Sen (1970, Ch. 9 *) is well-defined by using a single extended ordering. However, since he discusses a problem of evaluation methods defined on *a finite set of extended alternatives* $(X \times N)$ (moreover, X is the set of *general* social states), the indexing dilemma –the problem of incompatibility among the dominance principle, consumer sovereignty, and acyclicity– will not occur. Sen imposes the identity axiom on his single extended ordering, which makes an *intrapersonal* comparison ordering of well-being perfectly consistent with each individual preference.

[17]In addition to this median ranking, we can construct various median rankings by using the concept of ray utilities or the PS function.

consensus approaches focusing on individual preferences may tend to be vulnerable to the participation of extremely irrational individual preferences. That is, there is a possibility that the social evaluation could be greatly distorted against preference changes of bizarre individuals. However, the median rules above seem to have a kind of reasonableness because they should be less sensitive to the problem of bizarre preferences. Thus, the median consensus method can be a desirable candidate as one of the social evaluation methods, because it can satisfy not only the weak Pareto and dominance principles but also stability against quite strange preference changes.

Finally, we refer to impossibility results between the principles of Pareto and dominance. As any Paretian interpersonal rankings that satisfy the dominance principle must equal one of some person's rankings, we cannot design dominant interpersonal comparison rankings that satisfy lower contour monotonicity and the weak Pareto principle.

Proposition 5: *There is no universal social ordering function that satisfies continuity, separability, the dominance Principle, the weak Pareto principle and lower contour monotonicity for interpersonal comparison.*

If separability is required for USOs, then the Pareto principle implies consumer sovereignty. This fact goes back to the classical indexing dilemma for our USO framework. Separability induces the following:

$\forall R_N \in \mathscr{R}^N$, $\forall i \in N$, $\forall x_N, y_N \in X^N$ with $\forall j \neq i, x_j = y_j$, $(x_i, R_i) \succsim (y_i, R_i) \Leftrightarrow (x_N, R_N) \succsim (y_N, R_N)$.

If this USO satisfies the Pareto principle, then we have $x_i R_i y_i \Leftrightarrow (x_N, R_N) \succsim (y_N, R_N)$. Thus, an interpersonal comparison ranking between (x_i, R_i) and (y_i, R_i) which is induced by the USO above must equal i's ranking R_i. This means that combining the Pareto principle and separability implies consumer sovereignty.

Thus, Proposition 1 immediately induces the following impossibility result.

Proposition 6: *There is no universal social ordering function that satisfies the dominance principle, separability and the Pareto principle.*

As Weymark (2017) rightly pointed out, there is a conflict between the dominance principle and individual preferences. If we would like to respect the weak version of the Pareto principle, then we must choose a single-domain dictator for each profile. Of course, if we add some consistency conditions or informational efficiency requirements for the inter-profile social choice, then one of the single-domain dictators must be a global-domain dictator. As long as we do not impose any extra requirements of informational efficiency on USOs, we may get along well with each single-domain dictator for each profile (e.g., a variant of the median rules proposed in this paper). However, the evaluation methods proposed in the previous section provide non-Paretian comparisons over extended alternatives. It could be a question of USOs satisfying the dominance principle whether to use the method for respecting only one preference of a single-domain dictator for the weak Pareto principle, or to use the method for respecting all individual preferences while sacrificing the weak Pareto principle.

6 Concluding Remarks

In this paper, we examine the possibility of the consensus approach as a method of ordinal interpersonal comparisons respecting the dominance principle and individual preferences and proposes the minimum consensus and median consensus methods. Then, we show a representation theorem that extends an interpersonal comparison ranking based on the minimum consensus methods to an universal social ordering and get a maximin principle ranking based on these consensus methods. We note some remarks about these methods.

First, the problem of the indexing dilemma holds in the environment of *complete* individual preferences, but the dilemma also holds in that of *incomplete* individual preferences. Thus, our results are easily extended to the problems of incomplete interpersonal comparison[18]. Moreover, although all results obtained in this paper are shown in the space of goods or functionings, these results can be replicated in the space of opportunity sets. Therefore, the results of this study also hold true for Amartya Sen's (1985) capability approach. In the capability approach, if we want to respect the dominance principle applied for the family of opportunity sets, interpersonal comparison rankings for capability sets must follow the class of the consensus approach proposed in this paper.

Second, further consideration is needed for a trade-off between the dominance principle and Pareto efficiency in the framework of ordinal interpersonal comparison rankings. We might consider alternative methods of single-domain dictatorships for each profile. If we want to place great emphasis on Pareto efficiency, there will be various discussions about who should be a single-domain dictator.

Third, regardless of which aggregation methods are used for universal social orderings, note that all alternatives are evaluated by a single profile R_N whenever a society respects the dominance principle for interpersonal comparison. That is, once preference relations are given R_N, all allocations with different preferences are evaluated by only one profile R_N. This might be too restrictive for evaluating situations with different preferences. Thus, if we weaken the dominance principle for multi-profile framework and respect a dominance relation for each single profile, we will have various escape ways.

Fourth, logical relationships between the consensus method and various concepts of fairness should be considered further. In general, neither the equivalent approach nor the consensus approach yields a subset of fair (no-envy and Pareto efficient) allocations. As well-known results show, no-envy and Pareto efficient allocation does not always exist, and there is a substantial conflict among no-envy, Pareto efficiency, and collective rationality (Suzumura 1981; Tadenuma 2002; Sakamoto 2013). However, if the

[18] Similarly, the results can be obtained in the context of *intrapersonal* well-being comparisons with variable preferences. These results could give a normative basis for behavioral welfare economics.

concept of no-envy is weakened and changed to the non-dominated diversity (NDD) condition (van Parijs 1995)[19], while the equivalent approach still cannot choose the NDD allocation, the consensus approach could yield NDD allocations. In this sense, the consensus approach has the advantage in terms of the concepts of fairness.

Fifth, we must testify that the equivalence approach faces the preference adaptation problem even though this approach contains ordinal transferable goods and external non-transferable goods, such as health and education levels. For example, health equivalent incomes for low-income households tend to equal their actual incomes, because a low-income family cannot afford to pay their health expenditure (Mori and Sakamoto 2018). One's health equivalence income equals to income under perfect health status that is indifferent from his/her actual situation. Generally, health equivalent incomes satisfy consumer sovereignty, thus, they violate the dominance principle by the indexing dilemma. Then, in the context of interpersonal well-being comparisons for poor countries, it seems to be quite strange that unhealthy families and healthy families with the same low income are at the same well-being level. Thus, we have a good reason for considering the dominance principle for interpersonal well-being comparisons.

Finally, in the compound index approach, it is important to try to reflect individual preferences for the interpersonal comparison problem even if they cannot get full reflections of individual preferences. Although various discussions have been held about which components should be considered and which weights should be appreciated in the context of studies of happiness, quality of adjusted life years, and multidimensional poverty index studies, it seems there has been little accumulation of research in terms of reflecting individual preferences. From the viewpoint of theoretical and empirical studies, inquiries into the problem of ignoring preferences in the compound index approach are needed.

[19] The non-dominated diversity condition is formally defined as follows:
An allocation x is non-dominated diversity if and only if, for all individuals $i, j \in N$, there exists an individual k such that $x_i R_k x_j$.
Clearly, the non-dominated diversity condition is weaker than the no-envy condition.

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