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# Envy-free Pricing for Impure Public Good*

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**Abstract**

In this paper, we study optimal public good provision with congestion and user fees to exclude some agents under lump-sum tax/transfer, constrained by the condition of reduction of envy. We adopt the $\lambda$ envy-free constraint proposed by Diamantaras and Thomson (1990), and employ the exclusion technique used in Hellwig (2005), *i.e.*, the policymaker decides the level of provision and user fee paid by people accessing a public good, as well as a uniform level of tax/transfer. We characterize the optimal public sector pricing rule that depends on utilitarian distributive concerns and envy reduction concerns, which are in conflict with each other. We show that if the social welfare function is strictly increasing and strictly concave and the government is not concerned with reducing envy, the user fee is greater than the marginal congestion cost. Additionally, we show that if the government reflects the notion of equality of opportunity under the reduction of envy, the user fee is lower than the marginal congestion cost. These results imply that the two fairness concerns are countervailing with regard to the surcharge fee.

**JEL Classification: D61, D63, H21, H41, H44**

**Keywords:** Public sector pricing, $\lambda$ envy-free, public good, excludability, congestion

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## 1. Introduction

Throughout the world, income inequality is a big contributor to social disorder, and government redistribution policies offer a potential solution to this problem. Several methods exist for the redistribution of wealth from the rich to the poor. For instance, one tool is to levy taxes on incomes and redistribute the wealth that is gathered from doing this. The other expedient is for the government to provide public services that benefit everyone but are especially beneficial to which those with low incomes who would have more limited access to such services if they were not provided by government. With decreasing the number of complaints by members of society, policymakers set an optimal policy for income redistribution and implement public projects as appropriate.

Policymakers seek to reduce inequalities or envy for two different reasons. First, to realize widespread social justice as in the Scandinavian countries. Second, to eliminate social disorder. Like Sweden, the other Scandinavian countries are welfare states where social justice is founded upon egalitarianism. Recently, the social systems in these states have undergone partial reform, but societal norms based on egalitarianism remain ingrained. For the latter, as Bös and Tillmann (1985) noted:

> *the economic rationale for a minimization or reduction of envy by taxation is the following. Excessive envy in a society is an element of social disorder. Reducing envy in a society is a step towards increasing social harmony.* (p. 34)

However, this is not only a normative concept, but also an important issue that the whole world confronts. As seen in reality, Brexit or other electoral outcomes like the election of Donald Trump as president of the United States reveal the strength of anti-globalism, and some specialists claim that one driver of such events is a desire by the poor to deal with their envy of the rich. Recently, World Economic Forum (2017) has reported that the income gap, a major source of envy, is a major spearhead of polarized political outcomes. Representing another perspective, citizens with extensive complaints about government policies have demonstrated against those policies. Some such actions have been volatile, and participants have become violent and destroyed cars, stores and other important urban facilities. Such disruptive activities have caused extensive damage, necessitating enormous repair costs. To avoid the associated expenses, governments should seek to implement policies that prevent excessive envy. The envy-free constraint thus can be seen as economic incentive for policymakers.

In an economic model where initial wealth differs among different agents, several ethical reasons exist to consider redistribution, having backgrounds for mitigating inequality. One reason is envy. An agent envies another agent if he prefers the other's commodity bundle to his own. Envy-free allocation describes the situation where no agent experiences envy. While it is difficult to apply the original envy-free constraint presented by Foley (1967), Kolm (1972), and Varian (1974), we use the weaker and cardinal criterion proposed by Diamantaras and Thomson (1990) to evaluate the intensity of envy, called $\lambda$ envy-free, and examine the optimal policy schedule under both the reduction of envy constraint and the resource constraint. In the literature on fair allocation, conventional setups are based on heterogeneous preferences. In our model, people have the same quasilinear utility, but different tastes for public goods.Therefore, our setups do not differ markedly from those used in other related studies.

Here, we describe the theoretical and conceptual differences between a weaker criterion of no-envy and the maximin criterion (Rawls (1971)), where the latter means that the allocation is one that maximizes the utility of agent at the bottom. Theoretically, Nishimura (2003a) shows that, in a setup of Mirrleesian optimal nonlinear income taxation, the Diamantaras-Thomson allocation does not necessarily coincide with a Rawlsian type allocation. This implies that a conflict may exist between envy reduction and compensation of low-skilled individuals. Therefore, utilitarian distributive concerns arising from income inequality may differ from envy-reduction concerns. Conceptually, an intuitive appeal of envy-free allocation as an equity criterion is that it does not require interpersonal comparability of utilities (e.g., Varian (1974)). Indeed, since the equity criterion allows individuals to judge fair allocations based on their own preferences, this notion is likely to be accepted as an equity criterion in economies where knowing the preferences of others is impractical.

In this paper, we analyze optimal public good provision with both congestion and use exclusion by a government and a surcharge fee under lump-sum tax/transfer when individuals have additive and separable preferences and differ in both preferences for public goods and initial wealth or income. The basic framework of our model follows that of Hellwig (2005), but differs in two main ways. First, we introduce heterogeneity in initial wealth for agents and the $\lambda$ envy-free constraint. If agents have identical initial wealth then no envy exists in our model. In this case, our model conceptually coincides with Hellwig (2005), and from this point of view, heterogeneity in initial wealth decisively influences whether the optimal pricing rule reflects the reduction of envy. Second, we consider that public goods are subject to congestion. Increasing user numbers lead to efficiency loss arising from congestion. Thanks to these supplements, we find that the consideration of $\lambda$ envy-free affects not only the amount of public goods but also the level of user fees. Particularly, the optimal public sector pricing rule depends on conflicting utilitarian distributive and envy reduction concerns. On the one hand, the former concerns call for redistribution via the private goods of individuals who benefit from public goods to individuals who are excluded. In other words, utilitarian distributive concerns based on inequalities related to the use of public goods require an increase in user fees to compensate for differences in benefits arising from the provision of public goods. This is consistent with Hellwig (2005). On the other hand, the latter concerns based on income inequalities pursue reduced surcharges as a means to reduce envy. We show that if the social welfare function is strictly increasing and strictly concave and the government is not concerned with reducing envy, user fees exceed the marginal congestion cost. However, the equality of opportunity literature argues that income inequalities arising from non-responsibility factors such as innate skills should be eliminated and those arising from responsibility factors such as preferences should be respected. Following this notion, utilitarian distributive concerns disappear because utilitarian distributive concerns are based on the responsibility factor while envy reduction concerns are based on the compensation factor. Thus, we show that if people are responsible for their tastes because of the equality of opportunity criterion under the reduction of envy, the user fee is lower than the marginal congestion cost. This implies that the two different equity concerns are countervailing with regard to the surcharge fee.

## Related Literatures

We list relevant literature categorized according to whether it deals with optimal policy under envy reduction and public good provision with use exclusion and congestion. With regard to optimal policy for envy reduction, Nishimura (2003b) studies optimal nonlinear income taxation under constraints on envy reduction, following the two-type model developed by Stern (1982) and Stiglitz (1982). Nishimura (2003b) shows that the marginal tax rate increases only if leisure is a luxury. Also, Nishimura (2003a) examines optimal commodity taxation for reduction of envy. Both papers adopt the envy-free notion suggested by Diamantaras and Thomson (1990), and we follow this same idea but focus on government provision of public goods with associated user fees, and exclude endogenous labor supply but exogenous different incomes. Obara and Tsugawa (2019) incorporates endogenous labor supply like Nishimura's two works, and makes a novel contribution by considering pure public good provision by the policymaker. In that paper, we show that such an envy-free constraint distorts the provision level proposed by Boadway and Keen (1993), and employ numerical simulation to find that the intensity of envy affects the amount of provision.[1]

With regard to the relevant literature on public good provision with use exclusion and congestion, several articles examine utilitarian analyses of equity-efficiency tradeoffs dealing with distributional issues arising from individuals' heterogeneity. User charges are well known to correct inefficiently high consumption of public goods under congestion (e.g., Oakland (1987)). For example, capacity limitations are relevant for museums, highways, and university education. Birulin (2006) considers public good provision with congestion in the context of mechanism design problem, and shows that it is possible to construct an incentive compatible mechanism that always produces a good at an efficient level, balances the budget and satisfies voluntary participation constraints given limited capacity of that good.[2]

[1]Many papers investigate the mechanism design problem with envy-free constraint in the fields of computer science as well as economics. For instance, Guruswami et al. (2005) and Devanur et al. (2015) study the prior-free mechanism design problem, and characterize envy-free outcomes that maximize designer revenue.

[2]From efficiency considerations, Huber and Runkel (2009) study the role of user charges on excludable public goods under tax competition. Imposing user charges on a public good enables the government to obtain additional

Imposing user fees on excludable public goods is justified not only by efficiency considerations but also by equity considerations. Hellwig (2005) allows a policymaker to exclude agents with heterogeneous preferences for a given public good who value that public good less than the surcharge set by the policymaker. This study shows that a utilitarian government will set the optimum surcharge to zero because the social welfare benefits arising from decreasing the surcharge exceed the revenues raised from its increase, and that the revenue effect strengthens when the government is more risk-averse because it is better to redistribute additional surcharge fees.[3] Hellwig (2004) extends the model by allowing for not only heterogeneity of public goods preferences but also of earning abilities. However, user fees do not reflect utilitarian distributive concerns that relate to differences in earning abilities. This means that user fees continue to compensate only for differences in public goods preferences from utilitarian distributive concerns, even if heterogeneity in earning abilities is introduced. Blomquist and Christiansen (2005) investigate optimal admission fees for excludable public goods when heterogeneity stems from earning abilities. They show that a necessary condition for a positive user fee is that the marginal valuation of the public good is increasing in leisure. This means that the optimal user fee is zero if leisure is weakly separable from market goods. These papers offer the distributive implications of the heterogeneity of public goods preferences or earning abilities, but do not consider government concerns about the reduction of envy which is a different fairness concern. Our paper presents the implications of envy reduction for the setting of user fees on excludable public goods, and finds that such fees must reflect envy reduction concerns related to income inequalities. This implies that envy reduction concerns, not utilitarian distributive concerns, are the main reason that income inequalities should be reflected in public sector pricing. These findings are related to Nishimura (2003a) who states that compensation to low skilled individuals does not necessarily reduce envy.

Few papers deal with Ramsey pricing for public utility concerned with fairness requirement, with Baumol's works published in the 1980s being one exception. For instance, Baumol (1980) examines the equity issues raised in the use of pricing to keep limited resources and introduces the concept of "superfair" distribution, which means that each class of participants prefers its own share to the shares received by other groups, and Baumol and Fischer (1987) also study classical peak-load pricing incorporating the equity concept. Biggar (2010) identifies stylized facts about public attitudes to fairness in utility pricing, and suggests that the notion of fairness does not conflict with the conventional concept of economic efficiency. Lloret-Batlle and Jayakrishnan (2016) and Lloret-Batlle and Jayakrishnan (2017) study an optimal pricing scheme for a traffic system that addresses

revenue. Moreover, additional revenue obtained from such sources allows a decrease in the capital tax rate, enabling the government to attract more capital from overseas.

[3] Schmitz (1997) and Norman (2004) also study public good provision with use exclusion via surcharges. According to Hellwig (2005), surcharges are redundant if lump-sum transfers are available, following the critique of Atkinson and Stiglitz (1976). However, in Hellwig (2005) and our model, user fees play a key role in equity considerations even under uniform transfers.

fairness by employing a weaker envy-free constraint. However, these papers do not characterize the optimal pricing rule under the reduction of envy stemming from income inequality and clarify how fairness should be reflected in the rule. Therefore, our paper is the first to examine Ramsey pricing for a public utility concerned with mitigating envy among agents with different incomes, and where a trade-off must exist between equity and efficiency.

In the optimal taxation literature, several theoretical studies have explored the effect of status or relative consumption (or income); that is, individuals' utilities depend not only on their own consumption of goods but also on their relative social standing (e.g., Boskin and Sheshinski (1978), Oswald (1983), Seidman (1987), Persson (1995), Ireland (2001), Corneo (2002), Aronsson and Johansson-Stenman (2008), Balestrino (2009), Micheletto (2011), Kanbur and Tuomala (2013), Bruce and Peng (2018)). However, these studies on social comparisons that have been analyzed extensively in the optimal taxation literature do not consider that the government must take equitable allocation into account, although individuals care about their relative positions owing to the Veblen effect. In other words, because the government does not care about fairness in distribution, the model allows the government to implement unfair distribution in the sense of violating an equity criterion for allocations. In contrast, this paper considers a situation in which the government is constrained by the fairness requirement to promote social harmony when agents lack preferences regarding social comparisons with others. Note that the government's concern for envy in the allocation does not stem from utility interdependence. Our perspective is that the government's intervention is justified by equity concerns when the concept of envy-free is considered as an equity criterion for allocation. To clarify how status effects should be reflected in the optimal provision rule of public goods and in the optimal public sector pricing rule, we will extend the model to that where their preferences regard social comparisons in future research, as with Velez (2016).

Unlike the standard welfarist approach in which a government fully respects all aspects of individual preferences, several studies incorporate non-welfarist principles in policy evaluation. In a non-welfarist framework, the government is suspected to have a paternalistic motive for policy implementation stemming from differences between social and private preferences. In the context of optimal taxation and other redistributive policy, various works adopt the viewpoint of non-welfarism. First, poverty reduction is one of the non-welfarist concerns, and papers on the topic have explored this point (e.g., Besley and Kanbur (1988), Besley and Coate (1992, 1995), Kanbur et al. (1994), Pirttilä and Tuomala (2004), Kanbur et al. (2018)). Instead of social welfare maximization, the government seeks to minimize poverty, which is defined as deprivation of individual consumption relative to some desired level and measured using the Gini-based index. Second, a strand of literature on merit goods and sin taxes is considered non-welfarist (e.g., Sandmo (1983), Besley (1988), Racionero (2001), Schroyen (2005), O'Donoghue and Rabin (2003, 2006)). Individuals with self-control problems may disregard the beneficial impact of the consumption of goods such as education and health or may consume harmful goods such as alcohol and drugs in excess. Such individuals might benefit if the government employs tax and subsidy policies to induce individuals to behave as if they had perfect self-control. Thus, to correct these faulty choices, a paternalistic government ensures that its preferences reflect positive or negative effects that individuals do not care about. This leads to subsidization of merit goods to encourage costly but beneficial consumption and taxation of sin goods to discourage harmful consumption. Third, relative consumption is related to both welfarist and non-welfarist literature. Harsanyi (1982) argues that the government should not respect antisocial preferences such as envy. Following Harsanyi, the non-welfarist literature on relative consumption considers that the government excludes such preferences from the social objective function even if individuals care about social comparisons. For example, Micheletto (2011) and Aronsson and Johansson-Stenman (2018) investigate optimal nonlinear income tax policies under the welfarist and paternalist cases. Finally, non-welfarist approaches have also been used in a framework with multi-dimensional heterogeneity. Boadway et al. (2002) consider that individuals differ in their abilities and their preferences for leisure, and examine the properties of the optimal nonlinear income tax when different weights can be assigned to individuals with different preferences for leisure. Fleurbaey and Maniquet (2006) derive the optimal income tax schedule in settings where the social planner maximizes the social index to satisfy several axioms for fairness and inequality aversion. In their framework, weights are determined by fairness principles, a weak version of the Pigou-Dalton transfer principle and a condition precluding redistribution when all agents have the same skills. According to Kanbur et al. (2006), the approach adopted by these papers resembles the non-welfarist approach because the government determined weights do not necessarily coincide with individual preferences. Moreover, Schokkaert et al. (2004) employs the concept of a reference preference for leisure through the advantage function. As a paternalistic criterion, the social planner evaluates individual preferences for leisure as social preferences reflecting socially desirable effort levels. Our paper may relate to the literature on non-welfarist public economics in the sense that the government cares about envy-free allocations despite individuals lacking preferences regarding social comparisons (or ”envy”). However, this paper adopts the stance of introducing the concept of envy-free as an equity criterion for allocations when the government fully respects all aspects of individual preferences. This implies that our paper belongs to the literature on welfarist public economics.

This remainder of this paper is organized as follows. Section 2 analyzes the optimal provision rule for pure public goods and the optimal pricing rule for user fees under the reduction of envy. Section 3 offers concluding remarks.

## 2. The model

We consider a two-class economy in which each agent ($i = H, L$) possesses initial wealth (exogenous income) $Y_i$, where $Y_H > Y_L > 0$. The population of each agent is equal to $\pi_i$ where $\pi_L + \pi_H = 1$.

The preferences for a public good, denoted by $\theta$, is distributed over $[\underline{\theta}, \overline{\theta}]$ according to cumulative distributive function $F(\theta)$ with the strictly positive and continuously differentiable density function $f(\theta)$ where $0 = \underline{\theta} < \overline{\theta} < \infty$. For simplicity, we assume that the preferences for a public good and initial wealth are independently distributed. We allow the government to provide public goods with use exclusion by imposing user fees on those who enjoy public goods. Because of the excludability, individuals are divided into two groups indexed by $j = B, NB$. Indicator $B$ represents individuals who obtain the benefits from a public good and indicator $NB$ represents agents excluded from a public good. With those indicators, the utility function of individuals $i$ in group $j$ is described by

$$U_i^j = \mathbf{1}(j) \cdot \theta G + c_i^j. \tag{1}$$

where $G \in \mathbb{R}_+$ denotes a public good, and $c_i^j \in \mathbb{R}_+$ represents the private consumption of individuals $i$ in group $j$, and $\mathbf{1}(j)$ is the characteristic function, as follows:

$$\mathbf{1}(j) = \begin{cases} 1 & \text{if } j = B \\ 0 & \text{if } j = NB \end{cases}$$

For all goods except public goods, subscript $i$ denotes that a good is enjoyed by agent $i$.

Because the initial wealth is private information, the government cannot implement nonlinear taxes on their income. In any case, high administrative costs make nonlinear income taxes unattractive. Thus, the government can only levy uniform income tax/transfer denoted by $T \in \mathbb{R}$. Additionally, the government imposes admission fees denoted by $p \in \mathbb{R}_+$ on those who access a public good. Consequently, the budget constraints of individuals for each group are written as $c_i^j = Y_i - \mathbf{1}(j) \cdot p - T$.

## 2.1 Extensive Margin

Individuals decide whether to access a public good. Individuals with type $(\theta, Y_i)$ obtain utility $\theta G + c_i^B$ from access to a public good, and utility $c_i^{NB}$ if they are excluded. Therefore, they choose access to a public good if and only if

$$\theta \geq \frac{c_i^{NB} - c_i^B}{G} = \frac{p}{G} \equiv \hat{\theta} \tag{2}$$

where $\hat{\theta}$ is considered the net gain from being excluded from a public good. We derive the equality in equation (2) using individual budget constraints. Equation (2) means that, if the public goods preferences of individuals are greater (lower) than the threshold $\hat{\theta}$, they (do not) choose access to a public good. Moreover, the equation is rewritten as:

$$p = \hat{\theta} G \tag{3}$$

That is, type $i$ individuals will pay admission fee $p$ to access a public good if the benefit $\theta G$ they draw from the enjoyment of that public good exceeds $p$.

## 2.2 The Government

The budget constraint of the government takes the following form:

$$\begin{aligned} \int_{\hat{\theta}}^{\bar{\theta}} pf(\theta)d\theta + T &= \phi(G,N) \\ \Leftrightarrow \quad (1-F(\hat{\theta}))\hat{\theta}G + T &= \phi(G,N) \end{aligned} \tag{4}$$

The first term is the aggregate revenue from admission fees. The second term represents the aggregate revenue from uniform income taxes. The government compensates for the public expenditure required for public good provision through collecting taxes and imposing surcharges. In providing a public good policymakers incur associated production cost $\phi(G,N)$ with strictly increasing, strictly convex, and differentiable function with respect to $G$ given any $N \geq 0$ and $G > 0$ and with respect to $N$ given any $G > 0$, where $N \equiv 1 - F(\hat{\theta})$ denotes the population who benefit from the public good. That is, the cost function captures both provision and congestion costs.[4] Moreover, to prevent corner solutions, we assume that both $\phi = 0$ and $\phi_G = 0$ hold at $G = 0$, given any $N$.

Following Hellwig (2005), we use a Bergson-Samuelson function with the following criteria:

$$\begin{aligned} \mathcal{W} &\equiv \sum_{i=H,L} \pi_i \Big[ \int_{\hat{\theta}}^{\bar{\theta}} W(\theta G + c_i^B) f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(c_i^{NB}) f(\theta)d\theta \Big] \\ &= \sum_{i=H,L} \pi_i \Big[ \int_{\hat{\theta}}^{\bar{\theta}} W(\theta G - \hat{\theta}G + Y_i - T) f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(Y_i - T) f(\theta)d\theta \Big] \end{aligned} \tag{5}$$

where $W$ is a strictly increasing and concave function, that is, $W' > 0$ and $W'' < 0$. The equality holds using the budget constraint of individuals and equation (3).

Finally, we impose an ethical constraint to reduce envy. The equity concept of no-envy faces a difficulty in the second-best situation, because the low-skilled agent always envies the high-skilled one, whereas the reverse never occurs. As a less-demanding criterion of envy reduction, we introduce the concept of Diamantaras and Thomson (1990) as a cardinal measure of intensity of envy, which means that the government must implement an allocation satisfying the $\lambda$ envy-free constraint.[5] Without loss of generality, we assign weights to responsibility for choices about public good, not to income. Indeed, the $\lambda$ envy-free constraints between groups are satisfied if equation

[4]To describe the congestion effect, we adopt the functional form of McGuire (1974) such that the number of users causes the production effect. This reflects the maintenance costs associated with utilization. See Obara (2019) for more details.

[5]Cardinal concepts are employed because, according to Bös and Tillmann (1985), ordinal concepts are not useful because an invariant hierarchy of envy exists in the second-best analysis. Also, note that the Lagrangian expression of the optimization problem with the $\lambda$ envy-free constraint (equation (8)) resembles the social objective of Varian (1976), who incorporates degrees of envy, not constraint, into the social objective. However, $\lambda$ envy-free is better in the sense that it is independent of the comparability and cardinality of utility functions (see Nishimura (2003b) for details).

(7) hold (see Appendix C). Thus, we consider that the government is constrained by a given $\lambda$ envy-free requirement within each group:

$$\begin{aligned} & c_L^{NB} \geq \lambda c_H^{NB}, \quad \forall \theta \leq \hat{\theta} \\ \Leftrightarrow \quad & Y_L - T \geq \lambda(Y_H - T) \end{aligned} \tag{6}$$

for $NB$ group, and

$$\begin{aligned} & \theta G + c_L^B \geq \theta G + \lambda c_H^B, \quad \forall \theta \geq \hat{\theta} \\ \Leftrightarrow \quad & Y_L - p - T \geq \lambda(Y_H - p - T) \end{aligned} \tag{7}$$

for group $B$. Obviously, the $\lambda$ envy-free constraint for the high class is satisfied. Thus, we focus on the $\lambda$ envy-free constraint for the low class. Because the inequality (6) holds if (7) is true, it is enough to check inequality (7) for this ethical requirement. Furthermore, rearranging (7),

$$Y_L - \lambda Y_H \geq (1 - \lambda)(T + p).$$

If $\lambda \geq \frac{Y_L}{Y_H}$, (7) cannot hold vacuously because the right side of the equation is non-negative, so we assume that $\lambda < \frac{Y_L}{Y_H}$. To consider a case in which the $\lambda$ envy-free constraint can be binding, we restrict $\lambda$ to the range $(\frac{Y_L - \hat{p} - \hat{T}}{Y_H - \hat{p} - \hat{T}}, \frac{Y_L}{Y_H})$, where $\hat{p}$ and $\hat{T}$ indicate the optimal value given the $\lambda$ envy-free constraint with $\lambda = 0$.

To sum up, the government chooses the policy $\{T, \hat{\theta}, G\}$ to maximize the social welfare function (5) subject to its budget constraint (4) and the $\lambda$ envy-free constraint (7). The corresponding Lagrangian is

$$\begin{aligned} \mathcal{L}(T, G, \hat{\theta}; \gamma, \eta) = & \mathcal{W} \\ & + \gamma\{(1 - F(\hat{\theta}))\hat{\theta} G + T - \phi(G, N)\} \\ & + \eta\{Y_L - p - T - \lambda(Y_H - p - T)\} \end{aligned} \tag{8}$$

where $\gamma$ and $\eta$ are Lagrangian multipliers corresponding to individual constraints.

Before proposing our main results, we show that both $\hat{\theta}$ and $G$ are not corner solutions. First, whether government intervention is optimal is unclear because the first-order conditions (B.2) and (B.3) are always satisfied at $\hat{\theta} = \overline{\theta}$ and $G = 0$, which implies that $T = 0$ from government's budget constraint. This means that no government intervention, i.e., $\hat{\theta} = \overline{\theta}$, $G = 0$, and $T = 0$, may be desirable. However, as shown in Appendix A, it is not optimal and we have the following lemma:

**Lemma 1.** *At the optimum, $\hat{\theta}$ is bounded away from $\overline{\theta}$ and $G$ is bounded away from zero.*

Second, while Lemma 1 argues that the government should provide public goods subject to congestion, it does not imply that it should impose positive admission fees. Indeed, the government can induce all individuals to access the public good by setting $p = 0$, i.e., $\hat{\theta} = 0$. However, this case does not occur in our model in the presence of congestion, as shown in Appendix A.

**Lemma 2.** *At the optimum, $\hat{\theta}$ is bounded away from zero.*

To sum up, we can obtain the following proposition:

**Proposition 1.** *At the optimum, both $\hat{\theta}$ and G are interior solutions.*

## 2.3 Main results

Using the first-order conditions with respect to the Lagrangian, we characterize the optimal provision rule for impure public good and the optimal public sector pricing rule under the reduction of envy (Appendix B).

**Proposition 2.** *The optimal provision rule and the optimal public sector pricing rule considering the reduction of envy are characterized by:*

$$\frac{\Omega_A}{\Omega_B} + (1 - F(\hat{\theta}))\hat{\theta} - \frac{\eta}{\gamma}\frac{\Omega_A}{\Omega_B}(1-\lambda) - \frac{\eta}{\gamma}\hat{\theta}(1-\lambda) = \phi_G \tag{9}$$

$$\frac{p - \phi_N}{p} = \delta^{-1}\Big(F(\hat{\theta}) \sum_{i=L,H} \pi_i (g_i^{NB} - g_i^{B}) - \frac{F(\hat{\theta})}{1 - F(\hat{\theta})}\frac{\eta}{\gamma}(1-\lambda)\Big) \tag{10}$$

*where*

$$\Omega_A \equiv \sum_{i=L,H} \pi_i \int_{\hat{\theta}}^{\bar{\theta}} (\theta - \hat{\theta}) W'(\theta G - \hat{\theta} G + Y_i - T) f(\theta) d\theta$$

$$\Omega_B \equiv \sum_{i=H,L} \pi_i \Big[\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta} G + Y_i - T) f(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(Y_i - T) f(\theta) d\theta\Big]$$

$$\delta \equiv \frac{\hat{\theta} f(\hat{\theta})}{1 - F(\hat{\theta})} = -\frac{\partial 1 - F(\frac{p}{G})}{\partial p}\frac{p}{1 - F(\frac{p}{G})}$$

$$g_i^{NB} \equiv \frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(Y_i - T) f(\theta) d\theta}{\gamma F(\hat{\theta})} \quad \textit{and} \quad g_i^{B} \equiv \frac{\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta} G + Y_i - T) f(\theta) d\theta}{\gamma (1 - F(\hat{\theta}))}.$$

Equation (9) is the optimal provision rule for an impure public good. On the left side of the equation, the first term is the sum of marginal rate of substitution between income and public good, while the second term represents the marginal benefit due to the increase in revenue from user fees proposed by Hellwig (2005), and the other terms express the marginal loss proposed to result from envy associated with the increase in user fees. The third term is consistent with Obara and Tsugawa (2019), showing that the provision rule undergoes downwards distortion when the utility function is additively separable. The term reflects the indirect effect in the sense that decreasing a quality of public good allows the government to enhance income distribution, which causes reduction of envy. Particularly, the fourth term is novel and expresses direct effect. An increase in the amount of public good increases user fees and so strengthens envy from equation (7). Thus, this term exerts a downward effect on the provision rule to mitigate envy. Overall, it is unclear whether the original Samuelson rule for public good and the provision level experience upward or downward distortion.

Equation (10) indicates the Ramsey inverse elasticity rule in terms of public sector pricing. Four main terms determine the amount of user fees charged. First, the term in the numerator of the left side $\phi_N$ exhibits a congestion effect. To mitigate efficiency loss stemming from the congestion effect, the government imposes positive user fees, which is consistent with Oakland (1987). Second, the elasticity of demand for surcharge fees $\delta$ in the denominator of the right side represents distortions, that is, the imposition of user fees decreases the number of individuals accessing the public good. If $\delta$ is highly inelastic, the user fees tend to increase. Before explaining the interpretation of the first term in brackets on the right side, note that $g_i^j$ measures the relative value of the government that gives an additional 1$ to individuals with $Y_i$ in group $j$. Thus, if the government has redistributive tastes, $g_i^j$ is decreasing in $\theta$, which allows the imposition of positive user fees. According to the statement, the term indicates the net welfare gains from redistribution between groups. Because $g_i^{NB}$ is greater than $g_i^B$ because of the assumption on the curvature of $W$, the government prefers to redistribute from group $B$ to $NB$. Thus, it is desirable that the user fees exceed the marginal congestion cost to increase revenues and raise consumption levels. The second term in brackets on the right side expresses equity loss from increased envy arising from an increase in user fees. The increase in user fees forces low-type individuals to envy high-type individuals from equation (7). To reduce this envy, it is recommended decrease user fees to below the optimally efficient level.

## 2.4 Special Cases for the Ramsey Inverse Elasticity Formula

If $W$ is a strictly increasing and strictly concave function and the $\lambda$ envy-free constraint is binding, equation (10) does not explicitly tell us how to optimize the pricing of user charges. This is because the first term in brackets on the right side of equation (10) is positive and the second term in brackets on the right side of equation (10) is negative under this situation. To examine how equation (10) is characterized in relaxing these assumptions, we present three special cases: two cases of non-binding $\lambda$ envy-free constraints and one case of the binding constraint.

### 2.4.1 Non-binding $\lambda$ envy-free constraint

When the $\lambda$ envy-free constraint is slack, $\eta = 0$ holds. Then, equation (10) reduces to:

$$\frac{p - \phi_N}{p} = \delta^{-1} F(\hat{\theta}) \sum_{i=L,H} \pi_i (g_i^{NB} - g_i^B) > 0 \tag{11}$$

This result is analogous to Hellwig (2005), which implies that utilitarian distributive concerns demand redistribution from group $B$ to $NB$. Thus, $p$ exceeds $\phi_N$.

To clarify the determinants of redistribution stemming from utilitarian distributive concerns, we consider a weighted utilitarian social objective with type-specific weights denoted by $\beta_{Y_i}(\theta)$. Here, the social welfare function expressed by equation (5) can be rewritten as

$$\begin{aligned}\hat{\mathcal{W}} \equiv \pi_H\Big[\int_{\hat{\theta}}^{\overline{\theta}} \beta_{Y_H}(\theta)\{\theta G - \hat{\theta}G + c_H^{NB}\}f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \beta_{Y_H}(\theta)\{c_H^{NB}\}f(\theta)d\theta\Big] \\ + \pi_L\Big[\int_{\hat{\theta}}^{\overline{\theta}} \beta_{Y_L}(\theta)\{\theta G - \hat{\theta}G + c_L^{NB}\}f(\theta)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \beta_{Y_L}(\theta)\{c_L^{NB}\}f(\theta)d\theta\Big]\end{aligned} \tag{12}$$

In this case, $g_i^{NB} \equiv \frac{\int_{\underline{\theta}}^{\hat{\theta}} \beta_{Y_i}(\theta)f(\theta)d\theta}{\gamma F(\hat{\theta})}$ and $g_i^B \equiv \frac{\int_{\hat{\theta}}^{\overline{\theta}} \beta_{Y_i}(\theta)f(\theta)d\theta}{\gamma(1-F(\hat{\theta}))}$. First of all, we consider a case where $\beta_{Y_i}(\theta)$ is a strictly decreasing function with respect to both $\theta$ and $Y_i$. This means that the weighted utilitarian social objective is structurally identical to the Bergson-Samuelson criterion. Thus, the conclusion under the Bergson-Samuelson criterion remains. Also, this result holds if the type-specific weights are strictly decreasing in $\theta$ and constant with $Y_i$. However, if the type-specific weights are constant with $\theta$ and strictly decreasing in $Y_i$, $g_i^{NB}$ equals $g_i^B$ for any $i$. This means that equation (10) reduces to:

$$\frac{p - \phi_N}{p} = 0 \tag{13}$$

This implies that the government's motivation for inter-group redistribution stems from inequalities related to the use of a public good, not from income inequality itself. In particular, the social objective with type-specific weights depending only on $Y_i$ reflects the notion of equality of opportunity. The equality of opportunity literature argues that income inequalities arising from non-responsibility factors (compensation factors) such as innate skills should be eliminated and inequalities arising from responsibility factors such as preferences should be respected. Following the notion, if the type-specific weights are strictly decreasing in $Y_i$ and constant with $\theta$, only efficiency considerations can justify the imposition of surcharges on access to public goods.

**Corollary 1.** *Consider that the $\lambda$ envy-free constraint is not binding. If the type-specific weights are strictly decreasing in $\theta$, it is desirable that user fees exceed the marginal congestion cost. However, the type-specific weight is constant with respect to $\theta$, it is optimal that user fees equal the marginal congestion cost.*

### 2.4.2 Binding $\lambda$ envy-free constraint

Here, we assume that individuals are responsible for their preferences in relation to public goods but not for their innate skills. Thus, the first term in brackets on the right side of equation (10) vanishes. Under $\eta > 0$ (the $\lambda$ envy-free constraint is binding), equation (10) reduces to:

$$\frac{p - \phi_N}{p} = -\delta^{-1} \frac{F(\hat{\theta})}{1 - F(\hat{\theta})} \frac{\eta}{\gamma}(1 - \lambda) < 0 \tag{14}$$

The findings suggest that envy reduction concerns necessitate lower surcharges. Thus, utilitarian distributive concerns and envy reduction concerns simultaneously push the pricing of user fees in two different directions. Also, compared with utilitarian distributive concerns, envy-free concerns are based on income inequalities (compensation factor), not responsibility factors such as preferences. Indeed, if $Y_L = Y_H$ holds, the term related to envy-free vanishes. The arguments are summarized as follows:

**Corollary 2.** *Consider the equality of opportunity criterion. If the $\lambda$ envy-free constraint is binding, the lump-sum transfer should optimally be less than the marginal congestion cost.*

Finally, we present two objective functions satisfying the notion of equality of opportunity:

$$\mathcal{W}^R \equiv \int_0^{\bar{\theta}} \min[\text{oper}_{\delta_H(\theta)}\{\theta G + c_H^B, c_H^{NB}\}, \text{oper}_{\delta_L(\theta)}\{\theta G + c_L^B, c_L^{NB}\}] f(\theta) d\theta \tag{15}$$

$$\mathcal{W}^V \equiv \min[\int_0^{\bar{\theta}} \text{oper}_{\delta_H(\theta)}\{\theta G + c_H^B, c_H^{NB}\} f(\theta) d\theta, \int_0^{\bar{\theta}} \text{oper}_{\delta_L(\theta)}\{\theta G + c_L^B, c_L^{NB}\} f(\theta) d\theta] \tag{16}$$

where, $\delta_i(\theta) = 1$ ($\delta_i(\theta) = 0$) if type $i$ individuals with $\theta$ benefits (are excluded) from a public good, and $\text{oper}_{\delta_i(\theta)}\{a, b\} = a$ if $\delta_i(\theta) = 1$ and $\text{oper}_{\delta_i(\theta)}\{a, b\} = b$ if $\delta_i(\theta) = 0$.

Following Jacquet and Van de gaer (2011), the former corresponds to the objective function of Roemer (1998) and the latter corresponds to that of Van de gaer (1993). Under Roemer's objective function, the government maximizes the sum (over $\theta$) of minimal utility levels corresponding to the smallest utility across income levels for each $\theta$. Also, under Van de gaer's objective function, the government intends to maximize the size of the smallest opportunity set across income levels. Each objective function is shown to reduce to the following form:

$$\mathcal{W}^R = \mathcal{W}^V = \int_{\hat{\theta}}^{\bar{\theta}} \{\theta G + c_L^B\} f(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} c_L^{NB} f(\theta) d\theta \tag{17}$$

Note that Roemer's criterion coincides with that of Van de gaer. Also, these approaches reduce to utilitarianism because of a special case of the Bergson-Samuelson criterion (equation (12)) in that type-specific weights are zero (one) if individuals are high-skilled (low-skilled) and the weights are indifferent in $\theta$. This implies that these criteria satisfy the notion of equality of opportunity, namely that the first term of brackets on the right side of equation (10) vanishes. Therefore, we summarize the following:

**Corollary 3.** *Under objective functions proposed by Roemer and Van de gaer, if the $\lambda$ envy-free constraint is (not) binding, it is optimal that user fees are lower than (equal to) the marginal congestion cost.*

## 3. Conclusion

In this paper, we analyze optimal public good provision with congestion under both lump-sum tax/transfer and user fees for exclusion purposes when the government is concerned with both utilitarian distributional issues and envy reduction. We adopt the $\lambda$ envy-free constraint borrowed from Diamantaras and Thomson (1990), and derive the optimal provision rule for an impure public good and the optimal public sector pricing rule when the government applies a Bergson-Samuelson social welfare function under constraints on envy reduction for low income agents. Our model employs a similar technique to Hellwig (2005) for realizing user exclusion, so we do not consider endogenous labor incomes, but heterogeneous initial wealth. However, our approach differs from Hellwig (2005) in that we introduce the concept of envy-free as an equity criterion for allocation.

The pricing of user fees is determined by a trade-off between efficiency loss and equity gain. Note that this resembles Hellwig (2005) except for the novel features associated with the $\lambda$ envy-free constraint. Utilitarian distributional concerns stemming from inequalities related to the use of public goods increase the surcharge fee to redistribute income from those who access a public good to those who do not; however, the term reflecting the responsibility factor, such as preferences, vanishes when the government maintains equality of opportunity for all agents. The $\lambda$ envy-free constraint that aims to mitigate income inequalities arising from compensation factors, such as innate skills, pushes the price downward because the government has an incentive to decrease prices to loosen constraints. Restated, as the user fee increases, the $\lambda$-equitability constraint tightens. Thus, the novel finding of this study is that the envy-free constraint decreases public sector pricing contrary to utilitarian distributive concerns. Additionally, we find that the envy-free constraint distorts the provision rule for public good downward.

In our model, the $\lambda$ envy-free constraint is imposed. Although all individuals share the same quasi-linear utility function, their tastes for public goods differ. Studies on fair allocation or envy-free allocation usually assume heterogeneous preferences for each agent, meaning our setups are consistent from the perspective of heterogeneity. Another interpretation of the $\lambda$ envy-free constraint is that it allows "admissible income inequalities under equal treatment for choice". The constraint requires that agents with low income must not enjoy private goods $\lambda$ times more than those with high income among the same group. Therefore, we can reinterpret that the policymaker decreases the price to avoid widening the gap among group $B$.

Our model has two policy implications. First of all, in paying attention to envy reduction, a policymaker with the equality of opportunity criterion will set the lower fee to mitigate envy. Second, even if she considers different notions of fairness, in other words, distributional concerns and envy reduction to keep the harmony in a society, such policy tools create different driving forces because these targets are different.

For future research, the comparative statics of provision level and user fee due to the change of $\lambda$, generalizing utility function such as $u(g(\theta, G), c)$, and allowing labor income to be endogenously determined by innate skills and labor supply may be good directions. Additionally, following the spirit of Velez (2016), we will incorporate individual preferences regarding social comparisons into the present model. Such adjustments will enable interesting comparisons of the optimal policies of welfarist and paternalist (or non-welfarist) governments taking account of fair distribution.

## Appendix A

## Proof of Lemma 1

Consider the optimal allocation to be $\{G^*, \hat{\theta}^*, T^*\}$. If $G^*$ is positive, $\hat{\theta}^* < \overline{\theta}$ is optimal. This is because, at $\hat{\theta} = \overline{\theta}$, nobody obtains the benefits from public good even if the provision is financed by lump-sum taxes forcing individuals' utilities to decrease. Therefore, it is sufficient to show that $G^*$ is positive. To show this, we assume that $G^*$ is zero, resulting in $\hat{\theta}^* = \overline{\theta}$ and $T^* = 0$. From the assumption on the curvature of $\phi$, we have $\phi_G < \max_\theta \theta(1 - F(\theta))$ at $G = 0$. Thus, there exists an allocation $\{G^0, \hat{\theta}^0, T^0\}$ such that $(1 - F(\hat{\theta}^0))\hat{\theta}^0 G^0 > \phi(G^0, 1 - F(\hat{\theta}^0))$, $T^0 < 0$, and both the government's budget constraint and the $\lambda$ envy-free constraint are satisfied. Here, we define the social welfare function achieved by the allocation $\{G^h, \hat{\theta}^h, T^h\}$, $h = *, 0$, as

$$\hat{\mathcal{W}}^h \equiv \sum_{i=H,L} \pi_i \Big[ \int_{\hat{\theta}}^{\overline{\theta}} W(\theta G^h - \hat{\theta}^h G^h + Y_i - T^h) f(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(Y_i - T^h) f(\theta) d\theta \Big]$$

Obviously, we have $\hat{\mathcal{W}}^* \geq \hat{\mathcal{W}}^0$. Also, because $\theta G^0 - \hat{\theta}^0 G^0 + Y_i - T^0 > Y_i - T^0$ holds for any $\theta \geq \hat{\theta}^0$ and $W$ is a strictly increasing function, it follows that

$$\tilde{\mathcal{W}}^0 \equiv \sum_{i=H,L} \pi_i W(Y_i - T^0) < \hat{\mathcal{W}}^0$$

That is, $\hat{\mathcal{W}}^* > \tilde{\mathcal{W}}^0$. Here, notice that we have

$$\mathcal{W} < \sum_{i=H,L} \pi_i W(\overline{\theta} G - \hat{\theta} G + Y_i - T)$$

for all $G$, $\hat{\theta}$, and $T$. Combining these inequalities, we have $\tilde{\mathcal{W}}^0 < \sum_{i=H,L} \pi_i W(\overline{\theta} G^* - \hat{\theta}^* G^* + Y_i - T^*) = \sum_{i=H,L} \pi_i W(Y_i - T^*)$. From the fact that $G^* = 0$ and $T^* = 0$, $\tilde{\mathcal{W}}^0 < \sum_{i=H,L} \pi_i W(Y_i)$ holds. However, because $T^0$ is negative, $Y_i - T^0$ exceeds $Y_i$. This is inconsistent with the inequality and so $G^*$ is positive. □

## Proof of Lemma 2

Suppose that $\hat{\theta}$ is zero at the optimum. Given Lemma 1, the first-order conditions of equation (8) with respect to $T$ and $\hat{\theta}$ should be

$$\frac{\partial \mathcal{L}}{\partial T} = -\sum_{i=H,L} \pi_i \int_0^{\bar{\theta}} W'(\theta G + Y_i - T) f(\theta) d\theta + \gamma + \eta(\lambda - 1) = 0 \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}} = -G \sum_{i=L,H} \pi_i \int_0^{\bar{\theta}} W'(\theta G + Y_i - T) f(\theta) d\theta + \gamma G + \gamma \phi_N f(0) + G\eta(\lambda - 1) \leq 0 \tag{A.2}$$

From equation (A.1), we have $\gamma = \sum_{i=H,L} \pi_i \int_0^{\bar{\theta}} W'(\theta G + Y_i - T) f(\theta) d\theta - \eta(\lambda - 1) > 0$. Substituting it into equation (A.2) yields:

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}} = \gamma \phi_N f(0) \leq 0 \tag{A.3}$$

However, since $\phi_N$ and $f(0)$ is positive, $\frac{\partial \mathcal{L}}{\partial \hat{\theta}}$ must be positive. Therefore, since equation (A.3) contradicts with the fact, $\hat{\theta}$ is not zero. □

## Appendix B

Given Proposition 1, the first-order conditions of equation (8) with respect to $T$, $\hat{\theta}$, and $G$ can be written as

$$\frac{\partial \mathcal{L}}{\partial T} = -\sum_{i=H,L} \pi_i \Big[\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta} G + Y_i - T) f(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(Y_i - T) f(\theta) d\theta\Big] + \gamma + \eta(\lambda - 1) = 0 \tag{B.1}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}} = -G \sum_{i=L,H} \pi_i \int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta} G + Y_i - T) f(\theta) d\theta + \gamma G[(1 - F(\hat{\theta})) - \hat{\theta} f(\hat{\theta})] + \gamma \phi_N f(\hat{\theta}) + G\eta(\lambda - 1) = 0 \tag{B.2}$$

$$\frac{\partial \mathcal{L}}{\partial G} = \sum_{i=L,H} \pi_i \int_{\hat{\theta}}^{\bar{\theta}} (\theta - \hat{\theta}) W'(\theta G - \hat{\theta} G + Y_i - T) f(\theta) d\theta + \gamma(1 - F(\hat{\theta}))\hat{\theta} - \gamma \phi_G + \eta \hat{\theta}(\lambda - 1) = 0 \tag{B.3}$$

First, we derive the optimal provision rule for public goods. Rearranging (B.1) yields:

$$\gamma = \sum_{i=H,L} \pi_i \Big[\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta} G + Y_i - T) f(\theta) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(Y_i - T) f(\theta) d\theta\Big] + \eta(1 - \lambda) \tag{B.4}$$

Using the definition of $\Omega_A$ and $\Omega_B$ and dividing by $\gamma$, we can rewrite (B.3) as follows:

$$\frac{\Omega_A}{\Omega_B + \eta(1 - \lambda)} + (1 - F(\hat{\theta}))\hat{\theta} - \frac{\eta}{\gamma} \hat{\theta}(1 - \lambda) = \phi_G \tag{B.5}$$

Moreover, the first term on the left side can be rewritten as follows:

$$\frac{\Omega_A}{\Omega_B + \eta(1-\lambda)} = \frac{\Omega_A}{\Omega_B} - \frac{\eta}{\gamma}\frac{\Omega_A}{\Omega_B}(1-\lambda) \tag{B.6}$$

Substituting (B.6) into (B.5), we can obtain (9). Next, we derive the optimal pricing rule for excludable public goods. We transform (B.2) using $p = \hat{\theta}G$, as follows:

$$\gamma\frac{p-\phi_N}{p}\hat{\theta}f(\hat{\theta}) = \gamma(1-F(\hat{\theta})) - \sum_{i=L,H}\pi_i\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta}G + Y_i - T)f(\theta)d\theta - \eta(1-\lambda) \tag{B.7}$$

Substituting (B.4) into (B.7) and dividing by $\gamma(1-F(\hat{\theta}))$, we can obtain the following equation:

$$\begin{aligned}\frac{p-\phi_N}{p}\frac{\hat{\theta}f(\hat{\theta})}{1-F(\hat{\theta})} &= \frac{1}{1-F(\hat{\theta})}\Bigg[(1-F(\hat{\theta}))\sum_{i=L,H}\pi_i\frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(Y_i-T)f(\theta)d\theta}{\gamma} \\ &\quad - F(\hat{\theta})\sum_{i=L,H}\pi_i\frac{\int_{\hat{\theta}}^{\bar{\theta}} W'(\theta G - \hat{\theta}G + Y_i - T)f(\theta)d\theta}{\gamma}\Bigg] - \frac{F(\hat{\theta})}{1-F(\hat{\theta})}\frac{\eta}{\gamma}(1-\lambda)\end{aligned} \tag{B.8}$$

From the definition of $g_i^j$ and $\delta$, we can rewrite (B.8) as follows:

$$\frac{p-\phi_N}{p}\delta = \frac{1}{1-F(\hat{\theta})}\Bigg[F(\hat{\theta})(1-F(\hat{\theta}))\sum_{i=L,H}\pi_i g_i^{NB} - F(\hat{\theta})(1-F(\hat{\theta}))\sum_{i=L,H}\pi_i g_i^{B}\Bigg] - \frac{F(\hat{\theta})}{1-F(\hat{\theta})}\frac{\eta}{\gamma}(1-\lambda) \tag{B.9}$$

Rearranging (B.9), we can obtain (10). □

## Appendix C

The $\lambda$ envy-free constraints between groups can be written as

$$\begin{aligned} & c_L^{NB} \geq \theta G + \lambda c_H^B, \quad \forall\theta \leq \hat{\theta} \\ \Leftrightarrow \quad & Y_L - T \geq \theta G + \lambda(Y_H - p - T)\end{aligned} \tag{C.1}$$

and

$$\begin{aligned} & \theta G + c_L^B \geq \lambda c_H^{NB}, \quad \forall\theta \geq \hat{\theta} \\ \Leftrightarrow \quad & \theta G + Y_L - p - T \geq \lambda(Y_H - T)\end{aligned} \tag{C.2}$$

Now, we show that equations (C.1) and (C.2) are satisfied if equation (7) hold. First, equation (7) allows us to get the following inequalities:

$$\begin{aligned} & Y_L - p - T \geq \lambda(Y_H - p - T) \\ \Leftrightarrow \quad & \theta G + Y_L - p - T \geq \theta G + \lambda(Y_H - p - T) \quad \forall\theta \leq \hat{\theta} \\ \Leftrightarrow \quad & Y_L - T \geq \theta G + \lambda(Y_H - p - T) \quad \forall\theta \leq \hat{\theta}\end{aligned} \tag{C.3}$$

Thus, equation (C.1) holds. Second, we can obtain the following inequalities from equation (7):

$$\theta G + c_L^B \geq \theta G + \lambda c_H^B > \theta G - p + \lambda(Y_H - T) \geq \lambda(Y_H - T), \quad \forall\theta \geq \hat{\theta} \tag{C.4}$$

This means that equation (C.2) holds.

□

## References


Aronsson, T., Johansson-Stenman, O., 2008. When the Joneses' consumption hurts: optimal public good provision and nonlinear income taxation. *Journal of Public Economics* 92, 986–997.

Aronsson, T., Johansson-Stenman, O., 2018. Paternalism against Veblen: optimal taxation and non-respected preferences for social comparisons. *American Economic Journal: Economic Policy* 10, 39–76.

Atkinson, A. B., Stiglitz, J. E., 1976. The design of tax structure: direct versus indirect taxation. *Journal of Public Economics* 6, 55–75.

Balestrino, A., 2009. Tax avoidance, endogenous social norms, and the comparison income effect, CHILD Working Paper No.15.

Baumol, W. J., 1980. Theory of equity in pricing for resource conservation. *Journal of Environmental Economics and Management* 7, 308–320.

Baumol, W. J., Fischer, D., 1987. Peak pricing, congestion, and fairness. In: Arrow and the Foundations of the Theory of Economic Policy. pp. 382–409.

Besley, T., 1988. A simple model for merit good arguments. *Journal of Public economics* 35, 371–383.

Besley, T., Coate, S., 1992. Workfare versus welfare: incentive arguments for work requirements in poverty-alleviation programs. *American Economic Review* 82, 249–261.

Besley, T., Coate, S., 1995. The design of income maintenance programmes. *Review of Economic Studies* 62, 187–221.

Besley, T., Kanbur, R., 1988. Food subsidies and poverty alleviation. *Economic Journal* 98, 701–719.

Biggar, D., 2010. Fairness in public-utility regulation: a theory. *Agenda: A Journal of Policy Analysis and Reform* 17, 5–29.

Birulin, O., 2006. Public goods with congestion. *Journal of Economic Theory* 129, 289–299.

Blomquist, S., Christiansen, V., 2005. The role of prices for excludable public goods. *International Tax and Public Finance* 12, 61–79.

Boadway, R., Keen, M., 1993. Public goods, self-selection and optimal income taxation. *International Economic Review* 34, 463–478.

Boadway, R., Marchand, M., Pestieau, P., Racionero, M., 2002. Optimal redistribution with heterogenous preference for leisure. *Journal of Public Economic Theory* 4, 475–498.

Bös, D., Tillmann, G., 1985. An envy tax: theoretical principles and applications to the German surcharge on the rich. *Public Finance* 40, 35–63.

Boskin, M. J., Sheshinski, E., 1978. Optimal redistributive taxation when individual welfare depends upon relative income. *Quarterly Journal of Economics* 92, 589–601.

Bruce, D., Peng, L., 2018. Optimal taxation in the presence of income-dependent relative income effects. *Social Choice and Welfare* 51, 1–23.

Corneo, G., 2002. The efficient side of progressive income taxation. *European Economic Review* 46, 1359–1368.

Devanur, N. R., Hartline, J. D., Yan, Q., 2015. Envy freedom and prior-free mechanism design. *Journal of Economic Theory* 156, 103–143.

Diamantaras, D., Thomson, W., 1990. A refinement and extension of the no-envy concept. *Economics Letters* 33, 217–222.

Fleurbaey, M., Maniquet, F., 2006. Fair income tax. *Review of Economic Studies* 73, 55–83.

Foley, D. K., 1967. Resource allocation and the public sector. *Yale Economic Essays* 7.

Guruswami, V., Hartline, J. D., Karlin, A. R., Kempe, D., Kenyon, C., McSherry, F., 2005. On profit-maximizing envy-free pricing. In: Proceedings of the 16th ACM-SIAM Symposium on Discrete Algorithms.

Harsanyi, J., 1982. Morality and the theory of rational behavior. In A. Sen and B. Williams (eds.), *Utilitarianism and Beyond*. Cambridge: Cambridge University Press.

Hellwig, M. F., 2004. Optimal income taxation, public-goods provision and public-sector pricing: a contribution to the foundations of public economics, Preprint 04/14 Max Planck Institute for Research on Collective Goods, Bonn.

Hellwig, M. F., 2005. A utilitarian approach to the provision and pricing of excludable public goods. *Journal of Public Economics* 89, 1981–2003.

Huber, B., Runkel, M., 2009. Tax competition, excludable public goods, and user charges. *International Tax and Public Finance* 16, 321–336.

Ireland, N. J., 2001. Optimal income tax in the presence of status effects. *Journal of Public Economics* 81, 193–212.

Jacquet, L., Van de gaer, D., 2011. A comparison of optimal tax policies when compensation or responsibility matter. *Journal of Public Economics* 95, 1248–1262.

Kanbur, R., Keen, M., Tuomala, M., 1994. Optimal non-linear income taxation for the alleviation of income-poverty. *European Economic Review* 38, 1613–1632.

Kanbur, R., Paukkeri, T., Pirttilä, J., Tuomala, M., 2018. Optimal taxation and public provision for poverty reduction. *International Tax and Public Finance* 25, 64–98.

Kanbur, R., Pirttilä, J., Tuomala, M., 2006. Non-welfarist optimal taxation and behavioral public economics. *Journal of Economic Surveys* 20, 849–868.

Kanbur, R., Tuomala, M., 2013. Relativity, inequality, and optimal nonlinear income taxation. *International Economic Review* 54, 1199–1217.

Kolm, S., 1972. Justice et équité, Editions du centre national de la recherche scientifique. Paris.

Lloret-Batlle, R., Jayakrishnan, R., 2016. Envy-minimizing pareto efficient intersection control with brokered utility exchanges under user heterogeneity. *Transportation Research Part B: Methodological* 94, 22–42.

Lloret-Batlle, R., Jayakrishnan, R., 2017. Envy-free pricing for collaborative consumption of supply in transportation systems. *Transportation Research Procedia* 23, 772–789.

McGuire, M., 1974. Group segregation and optimal jurisdictions. *Journal of Political Economy* 82, 112–132.

Micheletto, L., 2011. Optimal nonlinear redistributive taxation and public good provision in an economy with Veblen effects. *Journal of Public Economic Theory* 13, 71–96.

Nishimura, Y., 2003a. Optimal commodity taxation for reduction of envy. *Social Choice and Welfare* 21, 501–527.

Nishimura, Y., 2003b. Optimal non-linear income taxation for reduction of envy. *Journal of Public Economics* 87, 363–386.

Norman, P., 2004. Efficient mechanisms for public goods with use exclusions. *The Review of Economic Studies* 71, 1163–1188.

Oakland, W. H., 1987. Theory of Public Goods, in Auerbach, A. J. and Feldstein, M. (eds.), *Handbook of Public Economics*, Amsterdam, North-Holland, Elsevier, Vol.2, pp.485–536.

Obara, T., 2019. Differential income taxation and Tiebout sorting. *FinanzArchiv* (forthcoming).

Obara, T., Tsugawa, S., 2019. Public good provision financed by nonlinear income tax under reduction of envy. *The B.E. Journal of Economic Analysis and Policy* (forthcoming).

O'Donoghue, T., Rabin, M., 2003. Studying optimal paternalism, illustrated by a model of sin taxes. *American Economic Review* 93, 186–191.

O'Donoghue, T., Rabin, M., 2006. Optimal sin taxes. *Journal of Public Economics* 90, 1825–1849.

Oswald, A. J., 1983. Altruism, jealousy and the theory of optimal non-linear taxation. *Journal of Public Economics* 20, 77–87.

Persson, M., 1995. Why are taxes so high in egalitarian societies? *Scandinavian Journal of Economics* 97, 569–580.

Pirttilä, J., Tuomala, M., 2004. Poverty alleviation and tax policy. *European Economic Review* 48, 1075–1090.

Racionero, M., 2001. Optimal tax mix with merit goods. *Oxford Economic Papers* 53, 628–641.

Rawls, J., 1971. A theory of justice. Cambridge, Harvard University Press.

Roemer, J., 1998. Equality of Opportunity, Harvard University Press, Cambridge.

Sandmo, A., 1983. Ex post welfare economics and the theory of merit goods. *Economica* 50, 19–33.

Schmitz, P. W., 1997. Monopolistic provision of excludable public goods under private information. *Public Finance* 52, 89–101.

Schokkaert, E., Van de gaer, D., Vandenbroucke, F., Luttens, R. I., 2004. Responsibility sensitive egalitarianism and optimal linear income taxation. *Mathematical social sciences* 48, 151–182.

Schroyen, F., 2005. An alternative way to model merit good arguments. *Journal of Public Economics* 89, 957–966.

Seidman, L. S., 1987. Relativity and efficient taxation. *Southern Economic Journal* 54, 463–474.

Stern, N., 1982. Optimum taxation with errors in administration. *Journal of Public Economics* 17, 181–211.

Stiglitz, J. E., 1982. Self-selection and Pareto efficient taxation. *Journal of Public Economics* 17, 213–240.

Van de gaer, D., 1993. Equality of opportunity and investment in human capital. *PhD Thesis*, *KULeuven*, Leuven, Belgium.

Varian, H., 1974. Equity, envy, and efficiency. *Journal of Economic Theory* 9, 63–91.

Varian, H., 1976. Two problems in the theory of fairness. *Journal of Public Economics* 5, 249–260.

Velez, R. A., 2016. Fairness and externalities. *Theoretical Economics* 11, 381–410.

World Economic Forum, 2017. The global risks report 2017 12th edition.