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Exchange Rates and Fundamentals: A General Equilibrium Exploration

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Abstract

Engel and West (2005) show that the observed near random-walk behavior of nominal exchange rates is an equilibrium outcome of a partial equilibrium asset approach when economic fundamentals follow exogenous first-order integrated processes and the discount factor approaches one. In this paper, I argue that the unit market discount factor creates a theoretical trade-off within a two-country general equilibrium model. The unit discount factor generates near random-walk nominal exchange rates, but it counterfactually implies perfect consumption risk sharing and flat money demand. Bayesian posterior simulation exercises based on post-Bretton Woods data from Canada and the United States reveal difficulties in reconciling the equilibrium random-walk proposition within the canonical model; in particular, the market discount factor is identified as being much smaller than one. A relative money demand shock is identified as the main driver of nominal exchange rates.

Key Words: Exchange rate; Present-value model; Economic fundamental; Random walk; Two-country model; Incomplete market; Cointegrated TFPs; Perfect risk sharing.
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1. Introduction

Few equilibrium models for nominal exchange rates systematically beat a naïve random-walk counterpart in terms of out-of-sample forecast performance. Since the study of Meese and Rogoff (1983), this robust empirical property of nominal exchange rate fluctuations has stubbornly resisted theoretical attempts to understand the behavior of nominal exchange rates as an equilibrium outcome of observed fundamentals. Many open-economy dynamic stochastic general equilibrium (DSGE) models also suffer from this problem. They fail to generate random-walk nominal exchange rates along an equilibrium path because their exchange rate forecasts are closely related to observed macroeconomic fundamentals.[1]

Engel and West (2005, hereafter EW) shed a new light on this vexing empirical property of nominal exchange rates. Using a partial equilibrium asset approach, in which nominal exchange rates reflect the present discounted values of expected future economic fundamentals, they show that if economic fundamentals are integrated of order one (hereafter I(1)) and the discount factor approaches one, then a nominal exchange rate follows a near random-walk process in equilibrium. This equilibrium random-walk property is attributable to the fact that only the Beveridge-Nelson trend components of the I(1) economic fundamentals are reflected in the present-value calculation at the limit of the unit discount factor.

The I(1) property is likely to hold in actual data of economic fundamentals, particularly monetary fundamentals, which are identified in neoclassical open-economy monetary models as the most important driver of nominal exchange rates.[2] Subsequent studies, thus, have focused on the empirical validity of the requirement that the discount factor be close to one. Based on the monetary fundamentals, Sarno and Sojli (2009) and Balke et al. (2013, hereafter BMW) identify a discount factor using partial equilibrium asset models similar to that of EW, examine data on different currencies and sample periods, and estimate the discount factor to be distributed near to one. In

[1] Engel (2014) provides a recent survey of studies on nominal exchange rates.

[2] For example, the reduced-form regression exercises of Mark (1995), Rapach and Wohar (2002), Mark and Sul (2001), and Cerra and Saxena (2010) show that the monetary fundamentals contain economically significant predictive components of nominal exchange rates. The model of this paper would offer a general equilibrium foundation for these empirical results.

particular, the Bayesian unobservable component (UC) model of BMW estimates relative money demand shocks to be a dominant underlying driver of a long sample of the British pound/U.S. dollar rate. This empirical finding supports the conjecture of EW that persistent unobservable economic fundamentals and a discount factor near one play significant roles in near random-walk nominal exchange rates.

Nason and Rogers (2008, hereafter NR) examine EW's proposition in a neoclassical two-country monetary dynamic stochastic general equilibrium (DSGE) model. NR rely only on a subset of the first-order necessary conditions (FONCs) of the two-country model and estimate Bayesian posteriors using a restricted UC model with post-Bretton Woods data of Canada and the United States. Characterizing the steady-state market discount factor as the relevant discount factor for nominal exchange rates, they confirm EW's proposition, finding that the market discount factor is close to one. NR identify a shock to I(1) relative consumption to be the main driver of the nominal exchange rate.

In this paper, I use a neoclassical two-country monetary DSGE model to explore EW's proposition that unobserved, integrated economic fundamentals, and a discount factor near one, generate random-walk behavior in nominal exchange rates. My focus on the model is motivated by the past studies that find the monetary fundamentals crucial for nominal exchange rates. In particular, BMW and NR infer the unit discount factor depending on neoclassical open-economy monetary models. To identify relative money demand shocks, which are estimated by BMW to be the dominant unobservable persistent economic fundamental for nominal exchange rates, I also scrutinize an open-economy general equilibrium model with explicit money demand.[3]

The general equilibrium approach in this paper indicates that a unit discount factor has

[3]In contrast, the recent literature of exchange rates such as Chari et al. (2002), Benigno (2004), Steinsson (2008), Iversen and Söderström (2014), and Engel (2014, 2018) commonly adopt open-economy new Keynesian (NK) models with the Taylor type monetary policy as the data generating process of real and nominal exchange rates. The NK framework typically determines the equilibrium dynamics of the real exchange rate, based on three equations: the NK Phillips curve, the risk sharing condition/the real uncover interest parity (RUIP) condition, and the Taylor rule. As shown by Benigno (2004) and Engel (2014, 2018), what matter for the endogenous persistence of real exchange rates are the degrees of price stickiness and monetary policy inertia. Common in the recent literature of NK models, the market discount factor is calibrated to a higher value around 0.99, but not estimated, because the restrictions imposed on exchange rate data by the NK models identify weakly the underlying subjective discount factor.

implications for the joint dynamics of monetary fundamentals, such as consumption, money supply, and interest rates, that do not sit easily with observed data. The unit discount factor is sufficient, but not necessary to generate near random-walk behavior in the nominal exchange rate. In general equilibrium, a unit market discount factor generates near random-walk nominal exchange rates but, when the consumption Euler equations are incorporated as a FONC, implies an implausibly high correlation between relative consumption and the real exchange rate (Backus and Smith 1993). A near unit discount factor also implies inelastic money demand, which requires implausibly large money supply and demand shocks. To reconcile the joint behavior of exchange rates and monetary fundamentals, estimation in general equilibrium environment with a full set of FONCs may inform on the relative roles of integrated, near random-walk fundamentals and a discount factor near one. Because BMW and NR do not impose the consumption Euler equation on their UC models, the full consideration of the FONCs explored by the general equilibrium approach in this paper might yield different inferences about the unit discount factor and the main drivers of nominal exchange rates.

Bayesian posterior simulation exercises with a restricted UC model, based on post-Bretton Woods data from Canada and the United States, reconcile the equilibrium random-walk exchange rate behavior and the observed behavior of relative consumption and interest rates with a discount factor estimated to be much smaller than one with the posterior mean of 0.537, and identify a persistent unobserved, money demand differential as the main source of persistence in the nominal exchange rate.[4] The estimated low value of the discount factor stems from the model's failure in explaining the data of the monetary fundamentals with the discount factor near one. The dominant role for money demand shocks echoes the findings of EW, BMW, and Sarno and Schmelling (2014) — economic fundamentals of near random-walk nominal exchange rates should be unobservable

[4]In the literature of the open-economy NK models such as Kollmann (2002) and Itsukhoki and Mukuhin (2017), additive exogenous disturbances to the UIP condition are frequently identified as the dominant driver of real and nominal exchange rates. BMW observe that exogenous risk premium shocks play only a minor role in nominal exchange rate fluctuations once relative money demand shocks are correctly identified. This finding also echoes the results of the likelihood-based inferences of the open-economy NK models by Bergin (2006) and Lubik and Schorfheide (2006).

and nominal, such as money demand shocks.[5]

The posterior inference of such a low discount factor suggests significant misspecification in this canonical open economy model. The empirical result of this paper, hence, uncovers difficulties that the literature needs to overcome in explaining data variations in nominal exchange rates and the corresponding macroeconomic fundamentals, jointly and consistently, through the lens of open-economy general equilibrium models.[6]

Potential approaches to reconciling the finding of this paper should generate a weak association between the real exchange rate and relative consumption at the unit discount factor. A promising future extension of the canonical monetary DSGE model in this paper, thus, allows for home bias in preference with a high degree of trade elasticity, as shown by Corsetti et al.(2008) as a resolution for Backus and Smith's (1993) observation.

The remainder of this paper is organized as follows. In the next section, I introduce the two-country incomplete market model employed in this paper. Section 3 then derives and discusses the equilibrium random-walk property of nominal exchange rates and the Backus and Smith puzzle of the perfect correlation between relative consumption and the real exchange rate at the limit of the unit market discount factor. After reporting the main results of the Bayesian exercises in section 4, I conclude in section 5.

2. A two-country incomplete markets model

2.1. The model

In this paper, I investigate a canonical incomplete markets model with two countries, home (h) and foreign (f). Each country is populated by a representative household whose objective is

[5] It is worth noting that the standard open-economy NK models with the Taylor rule are still absent from unobserved, near I(1) nominal economic fundamentals. I will discuss this point in the conclusion.

[6] As related research to this paper, Kano and Morita (2015) apply the model of this paper to post-Plaza Accord data of the Japanese yen/U.S. dollar to understand the anecdotal "Soros chart" (i.e., the observed high correlation between the near random-walk Japanese yen/U.S. dollar rate and the two-country differential in the monetary base). Modeling the reserve markets and the money creation processes of the two countries, their Bayesian posterior simulation exercise finds a better match of the model to the data: the posterior mean of the subjective discount factor is 0.96.

the lifetime utility $\sum_{j=0}^{\infty} \beta^j E_t \left\{ \ln C_{i,t+j} + \phi_{i,t+j} \ln \left(\frac{M_{i,t+j}}{P_{i,t+j}} \right) \right\}$ for $i = h, f$, where β, $C_{i,t}$, $M_{i,t}$, and $P_{i,t}$ represent the subjective discount factor, the ith country's consumption, money stock, and price index, respectively. As in NR, money-in-the utility delivers a tractable money demand function that characterizes an equilibrium nominal interest rate. I assume that the utility value of money is subject to a persistent money demand shock $\phi_{i,t}$.

The representative home and foreign households maximize their lifetime utility subject to the home budget constraint $B^h_{h,t} + S_t B^f_{h,t} + P_{h,t} C_{h,t} + M_{h,t} = (1 + r^h_{h,t-1}) B^h_{h,t-1} + S_t (1 + r^f_{h,t-1}) B^f_{h,t-1} + M_{h,t-1} + P_{h,t} Y_{h,t} + T_{h,t}$, and its foreign counterpart $\frac{B^h_{f,t}}{S_t} + B^f_{f,t} + P_{f,t} C_{f,t} + M_{f,t} = (1 + r^h_{f,t-1}) \frac{B^h_{f,t-1}}{S_t} + (1 + r^f_{f,t-1}) B^f_{f,t-1} + M_{f,t-1} + P_{f,t} Y_{f,t} + T_{f,t}$ respectively, where $B^l_{i,t}$, $r^l_{i,t}$, $Y_{i,t}$, $T_{i,t}$, and S_t denote the ith country's holdings of the lth country's nominal bonds at the end of time t, the ith county's returns on the lth country's bonds, the ith country's output level, the ith country's government transfers, and the level of the bilateral nominal exchange rate, respectively. Each country's output $Y_{i,t}$ is given as an exogenous endowment following a stochastic process $Y_{i,t} = y_{i,t} A_{i,t}$, where $y_{i,t}$ is the transitory component and $A_{i,t}$ is the permanent component. I interpret the permanent component $A_{i,t}$ as the TFP in the underlying production technology.

Each country's government transfers the seigniorage to the household as a lump-sum. Hence, the government's budget constraint is $M_{i,t} - M_{i,t-1} = T_{i,t}$, for $i = h, f$. The money supply $M_{i,t}$ is specified to be driven by permanent and transitory components, respectively $M^\tau_{i,t}$ and $m_{i,t}$: $M_{i,t} \equiv m_{i,t} M^\tau_{i,t}$ for $i = h, f$.

To close the model, I allow for a debt-elastic risk premium in the interest rates faced only by the home country: $r^l_{h,t} = r^l_{w,t}[1 + \psi\{\exp(-B^l_{h,t}/M^\tau_{l,t} + \bar{d}) - 1\}]$ with $\bar{d} \leq 0$ and $\psi > 0$ for $l = h, f$, where $r^l_{w,t}$ is the equilibrium risk-free interest rate of the lth country's bond. The debt-elastic risk premium prevents the home household from running up infinite debt, and addresses the non-stationarity associated with an incomplete international financial market.[7] The risk premium is

[7] As characterized by Schmitt-Grohé and Uribe (2003) in a small open-economy model and Boileu and Normandin (2008) in a two-country international business cycle model, a debt-elastic risk premium has served as a popular instrument to induce the stationarity of the net foreign asset distribution. A non-exhaustive list of studies that adopt a debt-elastic risk premium as a device to avoid the non-stationarity problem in open-economy DSGE models includes Nason and Rogers (2006), Adolfson et al. (2007), Kano (2009), Justiniano and Preston (2010), García-Cicco

given as an externality: the household does not internalize the effect of the debt position on the risk premium when maximizing the lifetime utility function. The foreign country's interest rate are $r^l_{f,t} = r^l_{w,t}$ for $l = h, f$.[8]

Following EW and BMW, I assume that purchasing power parity (PPP) holds only up to a persistent PPP deviation shock $\ln q_t$: $S_t P_{f,t} = P_{h,t} q_t$, where the PPP deviation q_t follows an exogenous stochastic process. The market-clearing conditions of the two countries' bond markets are $B^h_{h,t} + B^h_{f,t} = 0$ and $B^f_{h,t} + B^f_{f,t} = 0$. Along the equilibrium path, the world net supply of nominal bonds is zero on a period-by-period basis.

I assume that the logarithms of the total factor productivity (TFP) and the permanent component of the money supply, respectively $\ln A_{i,t}$ and $\ln M^\tau_{i,t}$, are I(1) for $i = h, f$, and the cross-country differential in the permanent component of money supply, $\ln M^\tau_{h,t} - \ln M^\tau_{f,t}$, is also I(1). I specify each country's monetary growth rate $\Delta \ln M^\tau_{i,t}$ to be an independent AR(1) process: $\Delta \ln M^\tau_{i,t} = (1 - \rho_M) \ln \gamma_M + \rho_M \Delta \ln M^\tau_{i,t-1} + \epsilon^i_{M,t}$ for $i = h, f$, where $\ln \gamma_M$ and ρ_M are the mean and AR root, respectively, of the money supply growth rate common to the two countries.

To guarantee the balanced growth path of the general equilibrium model, I assume that the cross-country TFP differential, $\ln a_t \equiv \ln A_{h,t} - \ln A_{f,t}$, is integrated of order zero (I(0)).[9] The stationary TFP differential implies that the TFP of the home country must be cointegrated with that of the foreign country. $\ln A_{h,t}$ and $\ln A_{f,t}$ are cointegrated with the cointegrated vector $[1, -1]$ and have the error correction models: $\Delta \ln A_{h,t} = \ln \gamma_A - \frac{\lambda}{2}(\ln A_{h,t-1} - \ln A_{f,t-1}) + \epsilon^h_{A,t}$ and $\Delta \ln A_{f,t} = \ln \gamma_A + \frac{\lambda}{2}(\ln A_{h,t-1} - \ln A_{f,t-1}) + \epsilon^f_{A,t}$, where $\gamma_A > 1$ is the common drift term and $\lambda \in [0, 1)$ is the adjustment speed of the error correction mechanism.

The stochastic process of the logarithm of the transitory output component for each country, $\ln y_{i,t}$, is specified as the following AR(1) process: $\ln y_{i,t} = (1 - \rho_y) \ln y_i + \rho_y \ln y_{i,t-1} + \epsilon^i_{y,t}$, for $i = h, f$. Similarly, the stochastic process of the logarithm of the transitory money supply component for each

et al. (2010), and Bodenstein (2011).

[8]A one-sided debt-elastic risk premium is sufficient to close the model.

[9]I allow the stochastic trends of the two countries, which are interpreted as their TFPs, to be cointegrated, as emphasized in recent papers by Mandelman et al. (2011), Rabanal et al. (2011), and Ireland (2013) in the context of international business cycles. In this case, because the TFP differential is stationary in population, a balanced growth path is guaranteed to exist in equilibrium.

country, $\ln m_{i,t}$, is specified as the following AR(1) process: $\ln m_{i,t} = (1-\rho_m)\ln m_i + \rho_m \ln m_{i,t-1} + \epsilon^i_{m,t}$, for $i = h, f$. The three other structural shocks, the home and foreign money demand shocks $\phi_{h,t}$ and $\phi_{f,t}$, respectively, and the PPP shock q_t, follow persistent stationary processes. Specifically, they are characterized by AR(1) processes as follows: $\ln \phi_{i,t} = (1-\rho_\phi)\ln\phi + \rho_\phi \ln \phi_{i,t-1} + \epsilon^i_{\phi,t}$, for $i = h, f$ and $\ln q_t = \rho_q \ln q_{t-1} + \epsilon_{q,t}$. I assume that all structural shocks are distributed independently.

2.2. The log-linear approximation of the stochastically de-trended system

Define stochastically de-trended variables as $c_{i,t} \equiv C_{i,t}/A_{i,t}$, $p_{i,t} \equiv P_{i,t}A_{i,t}/M^\tau_{i,t}$, $b^l_{i,t} \equiv B^l_{i,t}/M^\tau_{l,t}$, $\gamma^i_{A,t} \equiv A_{i,t}/A_{i,t-1}$, $\gamma^i_{M,t} \equiv M^\tau_{i,t}/M^\tau_{i,t-1}$, and $s_t \equiv S_t M^\tau_{f,t}/M^\tau_{h,t}$. I construct the stochastically de-trended system of the FONCs, as reported in the accompanying online appendix. The resulting ten equations determine the ten endogenous variables $c_{h,t}$, $c_{f,t}$, $p_{h,t}$, s_t, $b^h_{h,t}$, $b^f_{h,t}$, $r^h_{h,t}$, $r^f_{h,t}$, $r^h_{w,t}$, and $r^f_{w,t}$, given nine exogenous variables $\gamma^h_{M,t}$, $\gamma^f_{M,t}$, $\gamma^h_{A,t}$, $\gamma^f_{A,t}$, a_t, $m_{h,t}$, $m_{f,t}$, $y_{h,t}$, and $y_{f,t}$.

Let $\hat{x}$ denote a percentage deviation of any variable x_t from its deterministic steady state value x^*, $\hat{x} \equiv \ln x_t - \ln x^*$. Additionally, let $\tilde{x}$ denote a deviation of x from its deterministic steady state, $\tilde{x} = x - x^*$. The online appendix reports the results of the ten equations of the log-linearized FONCs. Under the assumption of the symmetric two-countries with $\bar{d} = 0$ (i.e., the deterministic steady state value of bond holdings is assumed to be zero), the resulting linear rational expectations (LRE) system is further simplified as a three-equation representation with two-country relative variables. To see this, let c_t, y_t, m_t, and ϕ_t denote the de-trended consumption ratio, the de-trended output ratio, the transitory money supply ratio, the money demand shock ratio between the two countries, $c_t \equiv c_{h,t}/c_{f,t}$, $y_t \equiv y_{h,t}/y_{f,t}$, $m_t \equiv m_{h,t}/m_{f,t}$, and $\phi_t \equiv \phi_{h,t}/\phi_{f,t}$, respectively. Furthermore, let M^τ_t denote the ratio of the permanent money supplies of the home and foreign countries $M^\tau_{h,t}/M^\tau_{f,t}$; let M_t denote the ratio of money supplies of the home to the foreign countries $M_{h,t}/M_{f,t} \equiv m_t M^\tau_t$; let $\gamma_{M,t}$ denote the ratio of the permanent money supply growth rate $\gamma_{M,t} \equiv \gamma^h_{M,t}/\gamma^f_{M,t}$; let C_t denote the ratio of the consumptions of the home and foreign countries $C_{h,t}/C_{f,t}$. The log-linearized FONCs then reduce to the following three expectational difference equations with respect to the three endogenous variables $\hat{s}_t$, $\hat{c}_t$, and $\tilde{b}_t$, given the six exogenous variables $\hat{\gamma}_{M,t}$, $\hat{m}_t$, $\hat{a}_t$, $\hat{y}_t$, $\hat{\phi}_t$, and $\hat{q}_t$:

$$\begin{aligned}
&\hat{s}_t = \kappa E_t \hat{s}_{t+1} - (1-\kappa)\hat{c}_t + (1-\kappa)(\hat{m}_t - \hat{\phi}_t + \hat{q}_t - \hat{a}_t) + \kappa E_t \hat{\gamma}_{M,t+1} - \psi\kappa(1-\kappa)\tilde{b}_t, \\
&\hat{s}_t + \hat{c}_t - \hat{q}_t + \hat{a}_t = \kappa E_t(\hat{s}_{t+1} + \hat{c}_{t+1} - \hat{q}_{t+1} + \hat{a}_{t+1}) + (1-\kappa)(\hat{m}_t - \hat{\phi}_t) + \kappa E_t \hat{\gamma}_{M,t+1}, \\
&\tilde{b}_t = \beta^{-1}\tilde{b}_{t-1} + p_h^* y^*(\hat{y}_t - \hat{c}_t), \qquad (1)
\end{aligned}$$

where $y^* = y/4$ and $y = y_h = y_f$ under the assumption of the symmetric two countries.

The first equation of LRE system (1) represents the stochastically de-trended uncovered interest rate rarity (UIP) condition; the second equation is the cross-country difference in the Euler equation; and the third equation is the law of motion of the net foreign asset position, after solving for the interest rate differential through the money demand functions of the two countries. As shown by the first equation above, in this model the steady-state nominal market discount factor, $\kappa \equiv 1/(1+r^*) = \beta/\gamma_M$, is the discount factor that governs the stochastic process of the de-trended nominal exchange rate $\hat{s}_t$.

3. A general equilibrium analysis of random-walk exchange rates

3.1. Equilibrium random-walk property of nominal exchange rates

I will now explore how the equilibrium random-walk property of the exchange rate holds in this two-country model. Rewriting the first equation of LRE system (1) by restoring stochastic trends and solving the resulting expectational difference equation of the nominal exchange rate $\ln S_t$ by forward iterations under a suitable transversality condition provides the PVM:

$$\ln S_t = (1-\kappa)\sum_{j=0}^{\infty}\kappa^j E_t\left(\ln M_{t+j} - \ln C_{t+j} - \psi\kappa\tilde{b}_{t+j} - \ln\phi_{t+j} + \ln q_{t+j}\right). \qquad (2)$$

If the sum of economic fundamentals in the right-hand side of PVM (2) is I(1), so is the exchange rate. The online appendix shows that, after rearranging PVM (2) in several steps, the currency return is

$$\Delta \ln S_t = \frac{1-\kappa}{\kappa}\left(\ln S_{t-1} - \ln M_{t-1} + \ln C_{t-1} + \ln \phi_{t-1} - \ln q_{t-1}\right) + \psi(1-\kappa)\tilde{b}_{t-1} + u_{s,t}, \qquad (3)$$

where $u_{s,t}$ is an i.i.d., rational expectations error. NR's argument that PVM (2) implies an error-correction representation of the currency return $\Delta \ln S_t$, in which $\Delta \ln S_t$ depends on the lagged error correction term $\ln S_{t-1} - \ln M_{t-1} + \ln C_{t-1}$, holds in this model.

Rewriting the second equation of LRE system (1) by restoring stochastic trends and solving the resulting expectational difference equation by forward iterations under a suitable transversality condition provides the following cross-equation restriction (CER):

$$\ln S_t - \ln M_t + \ln C_t - \ln q_t = \frac{\kappa\rho_M}{1-\kappa\rho_M}\gamma_{M,t} - \frac{\kappa(1-\rho_m)}{1-\kappa\rho_m}\ln m_t - \frac{1-\kappa}{1-\kappa\rho_\phi}\ln \phi_t. \qquad (4)$$

Imposing CER (4) on error-correction process (3) provides the equilibrium currency return

$$\Delta \ln S_t = \psi(1-\kappa)\tilde{b}_{t-1} + \frac{(1-\kappa)\rho_M}{1-\kappa\rho_M}\gamma_{M,t-1} + \frac{(1-\kappa)(1-\rho_\phi)}{1-\kappa\rho_\phi}\ln \phi_{t-1} - \frac{(1-\kappa)(1-\rho_m)}{1-\kappa\rho_m}\ln m_{t-1} + u_{s,t}. \qquad (5)$$

Equation (5) shows that any dependence of the currency return on past information emerges through the persistence of the net foreign asset position, the money supply growth differential, the transitory money demand shock differential, and the transitory money supply differential.

The important implication of equilibrium currency return (5) is that the logarithm of the nominal exchange rate follows a Martingale difference sequence at the limit of $\kappa \to 1$ because $\lim_{\kappa\to 1} E_t \Delta \ln S_{t+1} = 0$. Therefore, in this model, the exchange rate behaves like a random walk when the market discount factor approaches one along the equilibrium path of the two-country model. Equation (5) exhibits no dependence of the equilibrium currency return on past information in this case. Hence, the equilibrium random-walk property of the exchange rate, as found in EW and NR, is also present in this model.

In the limiting case of a unit market discount factor, the equilibrium currency return is dominated by the i.i.d. rational expectations error $u_{s,t} = (E_t - E_{t-1})\Delta \ln S_t = (E_t - E_{t-1})\hat{s}_t + \epsilon_{M,t}$ where $\epsilon_{M,t} \equiv \epsilon^h_{M,t} - \epsilon^f_{M,t}$ denotes the relative permanent money supply shock. The online appendix shows that, in the special case of two symmetric countries, assuming $\bar{d} = 0$ and $y_h = y_f$, the equilibrium de-trended exchange rate is a linear function of $\tilde{b}_{t-1}$, $\ln a_t$, $\ln m_t$, $\ln \phi_t$, $\ln y_t$, $\ln q_t$, and $\hat{\gamma}_{M,t}$:

$$\hat{s}_t = \frac{\beta\eta - 1}{\beta p_h^* y^*}\tilde{b}_{t-1} - \frac{1-\beta\eta}{1-\beta\eta(1-\lambda)}\ln a_t + \frac{1-\kappa}{1-\kappa\rho_m}\ln m_t - \frac{1-\kappa}{1-\kappa\rho_\phi}\ln\phi_t - \frac{1-\beta\eta}{1-\beta\eta\rho_y}\ln y_t + \frac{1-\beta\eta}{1-\beta\eta\rho_q}\ln q_t + \frac{\kappa\rho_M}{1-\kappa\rho_M}\gamma_{M,t}, \quad (6)$$

where the constant η, which is less than one, approaches one at the limit of $\kappa \to 1$.[10] The rational expectations error, $u_{s,t}$, is then given as a linear function of the structural shocks: $u_{s,t} = \frac{1}{1-\kappa\rho_M}\epsilon_{M,t} - \frac{1-\beta\eta}{1-\beta\eta(1-\lambda)}\epsilon_{A,t} + \frac{1-\kappa}{1-\kappa\rho_m}\epsilon_{m,t} - \frac{1-\kappa}{1-\kappa\rho_\phi}\epsilon_{\phi,t} - \frac{1-\beta\eta}{1-\beta\eta\rho_y}\epsilon_{y,t} + \frac{1-\beta\eta}{1-\beta\eta\rho_q}\epsilon_{q,t}$, where $\epsilon_{A,t} \equiv \epsilon^h_{A,t} - \epsilon^f_{A,t}$, $\epsilon_{m,t} \equiv \epsilon^h_{m,t} - \epsilon^f_{m,t}$, $\epsilon_{\phi,t} \equiv (\epsilon^h_{\phi,t} - \epsilon^f_{\phi,t})$, and $\epsilon_{y,t} \equiv \epsilon^h_{y,t} - \epsilon^f_{y,t}$ denote shocks to the relative TFP, the relative transitory money supply, the relative money demand shock, and the relative transitory income.

The steady-state gross growth rate of money is empirically close to one, $\gamma_M \approx 1$.[11] Therefore, the limit of $\kappa \equiv \beta/\gamma_M \to 1$, by construction, implies that the subjective discount factor, β, must also be close to one. In this limiting case, the shock to the monetary supply growth differential, $\epsilon_{M,t}$, dominates the rational expectations error, $u_{s,t}$, and, as a result, the random walk behavior of the exchange rate: $\lim_{\kappa\to 1}\Delta \ln S_t = \lim_{\kappa,\beta,\eta\to 1} u_{s,t} = \frac{1}{1-\rho_M}\epsilon_{M,t}$. Therefore, no transitory shock matters for the random-walk behavior of the nominal exchange rate.

Equation (5), also, reveals that a market discount factor close to one is sufficient, but not necessary, for the random-walk exchange rate.[12] Regardless of the value of κ, if the permanent money supply differential, the transitory money supply differential, or the money demand shock differential follows a near random-walk process, so does the nominal exchange rate. In that case, the rational expectations error $u_{s,t}$ implies $\lim_{\rho_M\to 0,\ \rho_\phi,\ \rho_m\to 1}\Delta\ln S_t = \epsilon_{M,t} + \epsilon_m - \epsilon_\phi - \frac{1-\beta\eta}{1-\beta\eta(1-\lambda)}\epsilon_{A,t} - \frac{1-\beta\eta}{1-\beta\eta\rho_y}\epsilon_{y,t} + \frac{1-\beta\eta}{1-\beta\eta\rho_q}\epsilon_{q,t}$. A lower market discount factor allows the model to generate the near random-walk nominal exchange rate by a richer and more flexible impulse structure, with multiple structural

[10] As defined in the online appendix, the constant η is one of the two roots of the expectational difference equation of the de-trended net foreign asset position $\tilde{b}_t$. A simple calculation shows that the equilibrium currency return (5) can be derived directly from the CER once the approximated relation $\hat{s}_t \approx \ln S_t - \ln M^\tau_t$ is recognized.

[11] The sample mean of the M1 money supply's gross growth rate is 1.016 for Canada and 1.014 for the United States.

[12] I appreciate the editor, Kenneth West, for pointing out this implication of the model.

shocks. Near random-walk nominal exchange rate behavior could be driven by all the exogenous economic fundamentals in this model.

3.2. Backus and Smith's puzzle at the limit

This model has an unrealistic implication for the equilibrium dynamics of consumption differential, $\ln C_t$, when the discount factor approaches one. At the limit of the unit discount factor, the perfect correlation between relative consumption and the real exchange rate (i.e., the one-to-one co-movement in consumption in terms of a common currency between the two countries) emerges, even under incomplete international financial markets. To observe this property, taking the first difference of CER (4), substituting equilibrium currency return (5) into the result, and exploiting rational expectations error $u_{s,t}$ yields the following equilibrium consumption differential dynamics:

$$\Delta \ln C_t = \Delta \ln q_t - \psi(1-\kappa)\tilde{b}_{t-1} + \frac{1-\beta\eta}{1-\beta\eta(1-\lambda)}\epsilon_{A,t} + \frac{1-\beta\eta}{1-\beta\eta\rho_y}\epsilon_{y,t} - \frac{1-\beta\eta}{1-\beta\eta\rho_q}\epsilon_{q,t}. \tag{7}$$

Except through the net foreign asset position, no monetary shock directly matters for the change in the equilibrium consumption differential. As in the standard international business cycle model, only real shocks to the endowments and the PPP deviation affect the equilibrium consumption allocation between the two countries.

Taking the limit of equation (7) above when $\kappa \to 1$ and $\gamma_M \approx 1$ (which imply that $\eta \to 1$ and $\beta \approx 1$) results in $\lim_{\kappa,\beta,\eta\to 1} \Delta \ln C_t = \Delta \ln q_t$. The consumption differential becomes unrelated to any shocks to the endowments of the two countries, but is perfectly correlated with the exogenous real exchange rate. The intuition behind this result is as follows. In this incomplete markets model, consumption in each country is determined by splitting the global aggregate endowment between the two countries in each period. The portion of the global aggregate endowment allocated to one country is simply given as the present discounted values of the expected future relative endowments of this country to the other. Because the endowment differential is stationary, due to the balanced growth restriction, the unit discount factor and $\gamma_M \approx 1$ make the portion converge to a constant, which is one-half in the case of two symmetric countries. Consumption in both countries, hence, responds to any endowment shocks in the same fashion. As a result, when the discount factor is close to one, relative consumption depends neither on permanent nor transitory endowment shocks. The only shock that affects relative consumption is the real exchange rate q_t.

3.3. Inelastic money demand

The two-country model of this paper also characterizes the analytical closed-form solutions of the nominal interest rates along the equilibrium path. The online appendix shows that the equilibrium interest rate differential $(1 + r_t) \equiv (1 + \hat{r}^h_{h,t}) - (1 + \hat{r}^f_{f,t})$ is

$$(1 + r_t) = (1 - \kappa) \left(\frac{\rho_M}{1 - \kappa \rho_M} \gamma_{M,t} - \frac{1 - \rho_m}{1 - \kappa \rho_m} \ln m_t + \frac{1 - \rho_\phi}{1 - \kappa \rho_\phi} \ln \phi_t \right). \tag{8}$$

If the AR root of the money supply growth differential, ρ_M, is close to zero, then the main determinants of the nominal interest rates are the money demand shock differential and the transitory money supply differential, $\ln \phi_t$ and $\ln m_t$.

At the limit of the unit market discount factor, the interest rate differential becomes insensitive to domestic monetary shocks because money demand functions are perfectly flat. In fact, under the presumption of $\rho_M = 0$, taking the limit of equation (8) gives $(1 + r_t) \approx \lim_{\kappa \to 1} (1 - \kappa)(\ln \phi_t - \ln m_t)$. The interest rate differential, hence, becomes dependent only on the transitory money supply differential $\ln m_t$ and the money demand shock differential $\ln \phi_t$ with very small sensitivities. Inelastic money demand means that the two monetary shocks would have to be extremely volatile to explain the actual data variations in the nominal interest rates.

4. A Bayesian unobserved component approach

This section empirically explores the question of how the unrealistic restrictions implied by the unit market discount factor affect posterior inferences about the two-country model. I utilize a Bayesian UC approach to estimate the two-country model, using data of the nominal exchange rate, the consumption differential, the money supply differential, the output differential, and the interest rate differential. The online appendix describes the Bayesian UC approach used in this paper, in detail.

4.1. Data and prior construction

The two countries that I empirically examine in this paper are Canada and the United States, as the model's home and foreign countries, respectively. I examine post-Bretton Woods quarterly data for these two countries because these data are likely to satisfy the model assumption of a flexible exchange rate regime under nearly perfect international capital movements between the two countries. The data span the period from Q1:1973 to Q4:2007.[13]

Table 1 reports the prior distributions of the structural parameters of the two-country model. Since an important goal of this paper's empirical investigation is to draw a posterior inference on the market discount factor $\kappa \equiv \beta/\gamma_M$, I use a uniform prior distribution of κ and let the data dictate the posterior position of κ, given the identification of the restricted UC model. The prior distribution of the mean gross monetary growth rate, γ_M, is intended to tightly cover its sample counterparts in both countries through the Gamma distribution, with a mean of 1.015 and standard deviation of 0.005. In contrast, the prior distribution of the subjective discount factor, β, is uniformly distributed between zero and one. As a result, the prior distribution of the market discount factor κ is well approximated as the uniform distribution spread over the support of the unit interval.

To guarantee the stationarity of the de-trended net foreign asset position, $\tilde{b}_t$, the debt elasticity of the home risk premium, ψ, should be positive. I therefore set the prior distribution of ψ to the Gamma distribution, with a mean of 0.010 and standard deviation of 0.001. Closing the model also requires the technological diffusion speed, λ, to be positive but less than one. This necessary condition for the equilibrium-balanced growth path elicits the prior distribution of λ as the Beta distribution, with a mean of 0.010 and standard deviation of 0.001. The slow technological diffusion, which the prior mean of λ implies, is intended to capture the slow-moving time-series properties observed in the actual consumption and output differentials between Canada and the United States. The prior distribution of the mean monetary demand shock ϕ follows the Gamma distribution, with a mean of 1.000 and small standard deviation of 0.010. The small standard deviation assumes, *a*

[13]The appendix below provides a detailed description of the source and construction of the data examined in this paper.

priori, that the monetary demand shock has no effect on the deterministic steady state.

I admit a small persistence of the permanent money growth rate by setting the prior distribution of the AR(1) coefficient ρ_M to the Beta distribution, with a mean of 0.100 and standard deviation of 0.010. The PPP deviation shock (i.e., the real exchange rate shock) is presumed to be very persistent, as observed by many past empirical studies on the real exchange rate. The AR(1) coefficient of the real exchange rate, ρ_q, is then accompanied by the Beta prior distribution, with a mean of 0.850 and standard deviation of 0.100. This prior distribution mimics fairly well the posterior distribution of the same structural parameter reported in Figure 3 of BMW, who used a long annual sample of data from the United Kingdom and the United States.[14] On the other hand, there is no robust empirical consensus on the extent of the persistence of the money demand shock. Hence, I allow the prior distribution of the AR(1) coefficient of the money demand shock, ρ_ϕ, to be distributed around 0.850 following the Beta distribution, with a mean of 0.850 and a large standard distribution of 0.100. The prior distribution's 95% coverage is [0.607 0.983], which also covers the corresponding posterior distribution displayed within Figure 3 of BMW. To better identify the permanent components of the money supplies and TFPs of both countries, while avoiding overprametarizing the model, I assume that the corresponding transitory components are white noise by setting the prior mass points of the AR(1) coefficients ρ_m and ρ_y to zero. Following NR, I also allow for the deterministic time trend in the exchange rate, γ_S, with the normal prior distribution with the zero mean and the large standard deviation of 1.500. Finally, the prior standard deviations of all the structural shocks are assumed to share the identical inverse-Gamma distribution, with a mean of 0.010 and standard deviation of 0.010. Below, I refer to this prior specification as the Benchmark model.

4.2. Main Results

The second, third, and fourth columns of Table 2 describe the posterior distributions of the structural parameters of the Benchmark model. The most striking posterior inference conveyed by

[14] In fact, the 95% interval of [0.607 0.983] includes the most inferences on real exchange rate persistence established in major past studies (see, e.g., Rogoff 1996 and Lothian and Taylor 2000).

these columns is that the market discount factor κ is identified as being far below one. As displayed in the first row, the data pins down the location of κ very tightly around the posterior mean of 0.537, with the standard deviation of 0.041. This posterior distribution of the market discount factor is too low to guarantee the second necessary condition of the equilibrium random-walk exchange rate established by EW and NR (i.e., that the market discount factor is sufficiently close to one). The other significant posterior inference of the Benchmark model is found on the stochastic process of the money demand shock differential, ρ_ϕ and σ_ϕ. The data show a more persistent and volatile money demand shock differential compared to the prior specification of the Benchmark model. Notice that the posterior mean of ρ_ϕ is 0.997 and almost 10% larger than its prior mean value; the posterior mean of σ_ϕ is 0.027 and 17% larger than its prior mean value. The very persistent money demand differential shock provides evidence that such a structural shock could play a significant role in actual exchange rate movements.

Does this lower market discount factor impair the model's fit to the actual exchange rate movements? The answer is clearly no. The estimated Benchmark model is indeed successful in explaining the historical trajectory of the exchange rate. Figure 1(a) plots the actual Canadian dollar against the U.S. dollar as the solid black line. The figure also displays the posterior mean of the in-sample prediction of the exchange rate by the Benchmark model as the dot-dashed block line accompanied by the corresponding 95% Bayesian highest probability density (HPD) interval as the gray area. Observe almost no visual discrepancy between the actual outcome and the prediction of the exchange rate. The HPD interval is very narrow and includes the actual exchange rate in almost all the sample periods. The Benchmark model, hence, tracks the actual exchange rate closely.

A natural but crucial question is as follows: Why does the Benchmark model fit the seemingly random-walk exchange rate fairly well even with such a low market discount factor? To answer this question, it is worth remembering the theoretical implication of the equilibrium currency return equation (5): if the three exogenous stochastic impulses — the permanent money supply differential, the transitory money supply differential, and the money demand shock differential — follow near random-walk processes, so does the exchange rate. Note that the transitory money supply differential is assumed to be white noise with $\rho_m = 0$ for identification purposes. As shown in Table 2, the data require in the Benchmark model that the permanent money supply differential and the money demand shock differential follow near random walks; the posterior mean of ρ_M is 0.088 and that of ρ_ϕ is 0.997. Therefore, even with such a low market discount factor, the Benchmark model replicates the near random-walk property of the exchange rate data.

To verify the above conjecture graphically, Figure 1(b) displays the in-sample prediction of the exchange rate simulated by the Benchmark model *but fixing ρ_ϕ and ρ_M to their prior values of 0.850 and 0.100.* The figure plots the posterior mean of the in-sample prediction of the exchange rate as the dot-dashed black line and the corresponding 95% HPD interval as the gray area accompanied by the actual exchange rate data, as the solid black line. Observe that the mean in-sample prediction is far above the actual exchange rate. The 95% HPD interval is very wide and does not contain the actual exchange rate over the whole sample period. The near random-walk processes of the money demand shock differential and permanent money supply differential, therefore, result in the successful in-sample prediction of the exchange rate of the Benchmark model even with such a low discount factor.

Table 3 reports the forecast error variance decompositions (FEVDs), estimated in the Benchmark model, of the three endogenous variables, the exchange rate, the consumption differential, and the interest rate differential, into the six structural shocks to the permanent money supply differential shock $\epsilon_{M,t}$, the TFP differential shock $\epsilon_{A,t}$, the transitory money supply differential shock $\epsilon_{m,t}$, the transitory output differential shock $\epsilon_{y,t}$, the PPP deviation shock $\epsilon_{q,t}$, and the money demand differential shock $\epsilon_{\phi,t}$, over the forecast horizons of the impact, one year, three years, five years, and ten years. Window (a) corresponds to the exchange rate; window (b) the consumption differential; and window (c) the interest rate differential.

Window (a) in Table 3 clearly shows that the most important structural shock for the near-random-walk exchange rate between Canada and the United States is the persistent money demand differential shock in conjunction with the permanent money supply differential shock; about 65% of the FEVD of the exchange rate is attributed to the former shock, and about 30% to the latter shock over all the forecast horizons. This inference about the main drivers of nominal exchange rates echoes the findings of the past studies by EW, BMW, and Sarno and Schmelling (2014): economic fundamentals of near random-walk exchange rates should be unobservable and nominal, such as a money demand differential shock.

Windows (b) and (c) of Table 3 display the FEVDs of the consumption differential and the interest rate differential in the Benchmark model. Observe the dominant roles of the TFP differential shock in the consumption differential and the transitory money supply differential shock in the interest rate differential. The former result fully reflects the data aspect that the consumption differential shares the same stochastic trend as the output differential. The latter result is consistent with the theoretical implication of the Benchmark model for the equilibrium interest rate differential, given a highly persistent money demand differential shock.

4.3. Understanding the lower discount factor: The High Discount Factor model

Why does the Benchmark model draw the sharp inference of such a lower discount factor? To dig deeper into this question, in the online appendix, I derive two posterior predictive densities of the one-period-ahead forecast errors implied by the following two models. The first posterior predictive density simply exploits the posterior distributions of the structural parameters of the Benchmark model. The second one, on the other hand, uses the same posterior distributions of the structural parameters of the Benchmark model *except that I draw the subjective discount factor β from a new prior distribution that is the Beta distribution with the mean of 0.999 and standard deviation of 0.001.* I refer to this new specification as the High Discount Factor (HDF) model.[15] Comparing the two posterior predictive densities of the forecast errors uncovers the data aspects in which the high discount factor of 0.999 deteriorates the fit of the Benchmark model significantly.

The HDF model yields much larger forecast errors for the consumption differential and the money supply differential than does the Benchmark model. The large forecast error of the HDF model in the consumption differential results from the model's implying tight positive correlation

[15]Note that this prior construction of the subjective discount factor is almost the same exercise as calibrating the market discount factor to the unconditional mean of the nominal interest rates observed in actual data.

between the consumption differential and the PPP deviation shock. It is the high dependence of the consumption difference on the PPP deviation shock that leads to the significant failure in the HDF model to explain the actual data of the consumption differential. The data, as a result, forces the subjective discount factor to be much lower to increase the empirical fit as in the Benchmark model.

Understanding the difficulty of the HDF model in explaining the money supply differential requires several steps. When the discount factor κ approaches one, the interest rate differential becomes dependent only on the transitory money supply differential and the money demand shock differential with very small sensitivities. Because of the flat money demand function, the forecast of the money demand shock differential should be very volatile to explain the actual data of the interest rate differential. Transitory exchange rate equation (6), then, predicts that under the HDF model the forecast of the transitory exchange rate should be dominated by the volatile forecast of the money demand shock differential. To keep the fit of the model to the actual $\ln S_t$ good, the volatile forecast of $\ln s_t$ should be offset by a volatile forecast of $\ln M_t^{\tau}$. The forecast of the money supply differential $\ln M_t$, therefore, should be counterfactually dominated only by the volatile forecast of the permanent money supply differential $\ln M_t^{\tau}$.[16]

In sum, the high discount factor, as in the HDF model, imposes counterfactual restrictions on the consumption differential and the money supply differential. In the Benchmark model, the two aspects of the data require β to be much lower than the prior value of the HDF model. These restrictions, indeed, are so strictly binding that they jointly pin down the low value of β with a high degree of precision, as shown by the corresponding small posterior standard deviation reported in Table 2.

Finally, I estimate the HDF model. The fifth, sixth, and seventh columns of Table 2 correspond to the posterior distributions of the structural parameters under the HDF model. Observe that the resulting posterior distributions of both the market and subjective discount factors are much closer to one, with posterior means of 0.950 and 0.998, respectively. Crucial changes in the

[16]The online appendix explains why the HDF model fails in relative money supply in more detail.

posterior distributions of the structural parameters from the Benchmark model are found in (i) significant increases in the posterior means of the standard deviations of the three monetary shocks, σ_M, σ_m, and σ_ϕ, and (ii) a large increase in the posterior mean of the AR(1) coefficient of the PPP deviation shock, ρ_q.

The HDF model, indeed, displays a great decline in overall fit. The last row of Table 2 reports the logarithms of the estimated marginal likelihood for each model.[17] The HDF model yields a smaller log marginal likelihood (1871.309) than that of the Benchmark model (2148.572). The difference in the log marginal likelihoods of the two models indicates that forcing the discount factor to be close to one makes the HDF model's overall fit to the data significantly worse than that of the Benchmark model.[18]

Figure 1(c) depicts the in-sample predictive density of the exchange rate implied by the HDF model. The figure confirms that the HDF model tracks the actual near random-walk exchange rate to almost the same degree as the Benchmark model. Table 4 exhibits the HDF model's results for the FEVDs of the three endogenous variables, as in Table 3. The FEVDs convey three properties of the HDF model: (i) the permanent money supply differential shock and the persistent money demand differential shock jointly and dominantly explain actual exchange rate movements; (ii) the PPP deviation shock, not the TFP shock as in the Benchmark model, is the dominant driver of the consumption differential; and (iii) not only the transitory money supply differential shock but also the persistent money demand differential shock explains the interest rate differential. The second property represents the clear drawback of the HDF model.

5. Conclusion

In this paper, I explore the random-walk property of nominal exchange rates with a neoclassical two-country monetary model. A general equilibrium environment allows the joint estimation

[17]This paper estimates marginal likelihoods by using Geweke's (1999) modified harmonic mean estimator. A marginal likelihood is the probability of data conditional on an underlying model. In general, the higher the marginal likelihood is, the better the underlying model's overall fit to the data.

[18]The ratio of the marginal likelihood of the Benchmark model to that of the HDF model is 3.152.

of equilibrium dynamics of nominal exchange rates and endogenous economic fundamentals — relative consumption and interest rates. The equilibrium random-walk behavior of nominal exchange rates can be driven by either the steady-state market discount factor near one or by the exogenous structural shocks following near random-walk processes. A market discount factor close to one, however, implies unrealistic restrictions that do not easily sit on the observed data. A unit discount factor implies a perfect correlation between relative consumption and real exchange rate, which is not seen in the data (Backus and Smith, 1993), and counterfactually large volatility in the near random-walk money demand shock differential due to flat money demand.

Bayesian posterior simulation exercises of the implied restricted UC model, based on post-Bretton Woods data from Canada and the United States, rejects a market discount factor near one and identifies the main driver of near random-walk nominal exchange rate behavior as unobservable near random-walk behavior in relative money demand. This paper's posterior inference of the low discount factor around 0.537 suggests significant misspecifications in this canonical open economy model. The empirical result of this paper, hence, uncovers difficulties that the literature needs to overcome in explaining data variations in nominal exchange rates and the corresponding macroeconomic fundamentals, jointly and consistently, through the lens of open-economy general equilibrium.

There are many potential extensions of the simple canonical model in this paper such as price stickiness, monetary policy, non-tradable goods, home bias, distributional margins, and endogenous risk premium.[19] The real exchange rate is not determined endogenously in this model, as in the two-country models of Benigno (2004), Benigno and Thoenissen (2008), and Corsetti et al. (2008). For example, Corsetti et al. (2008) argue that the Backus-Smith observation could be replicated in an international real business cycle model with a high degree of trade elasticity when each country's productivity shock is nearly permanent. An interesting question is whether the same result would hold at the limit of the unit discount factor, when cointegration is incorporated into the home and foreign productivity processes of their model. Together with sticky nominal prices, the Taylor type

[19] Among them, the online appendix extends the argument of this paper by introducing non-tradable goods into the Benchmark model.

monetary policy rule is investigated by Benigno (2004), Steinsson (2008), and Engel (2014, 2018) as a primary source of persistent and volatile moments in real exchange rates. Incorporating such a framework into the model employed here would change its CERs significantly. Very persistent trend inflation, as investigated by Cogley and Sbordone (2008), Coibion and Gorodnichenko (2011), and Ascari and Sbordone (2014), might be a plausible candidate for an unobserved I(1) nominal economic fundamental, and interpret relative money demand shocks in a better way. Finally, the log-linear approximation of the FONCs automatically eliminates any potential relationship between the interest rate differential and endogenous time-varying risk and liquidity premium, which is emphasized by Alvarez et al. (2007, 2009) and Engel (2016). I leave these challenging questions as avenues for future research.

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Appendix. Data description and construction

All data for the United States are distributed by Federal Reserve Economic Data (FRED), operated by the Federal Reserve Bank of St. Louis (http://http://research.stlouisfed.org/fred2/). The consumption data are constructed as the sum of the real personal consumption expenditures on non-durables and services. FRED, however, distributes only the nominal values of the two categories of personal consumption expenditures as *Personal Consumption Expenditure on Non-Durables (PCND)* and *Personal Consumption Expenditure on Services (PCESV)*. To construct the real total personal consumption expenditure $C_{us,t}$, I first calculate the share of the two nominal consumption categories in the nominal total personal consumption expenditure *Personal Consumption Expenditure* and then multiply the real total personal consumption expenditures, *Real Personal Consumption Expenditures at Chained 2005 Dollars (PCECC96)*, by the calculated share. Following NR, I adopt the M1 money stock, *M1SL*, as the aggregate money supply $M_{us,t}$. The nominal interest rate $r_{us,t}$ is provided by *three-month Treasury Bill (TB3MS)*. All the variables except for the nominal interest rate are seasonally adjusted at annual rates and converted to the corresponding per capita terms by *Total Population (POP)*.

All Canadian data are distributed by Statistics Canada (CANSIM) (http://www5.statcan.gc.ca/cansim/). The real consumption data $C_{can,t}$ are constructed as the sum of *Personal Expenditure on Non-Durables at Chained 2002 Dollars*, *Personal Expenditure on Semi-Durables at Chained 2002 Dollars*, and *Personal Expenditure on Services at Chained 2002 Dollars*. I use the M1 money stock as the money supply $M_{can,t}$. The nominal interest rate $r_{can,t}$ is provided by *three-Month Treasury Bills*. All the variables except for the nominal interest rate are seasonally adjusted at annual rates and converted to the corresponding per capita terms by *Estimate of Total Population*.

The output measures for Canada and the United States, $Y_{can,t}$ and $Y_{us,t}$, are constructed in a model-consistent way. In this two-country endowment economy model, a country's output is given by the sum of consumption and the trade balance. To measure the bilateral trade balance between Canada and the United States, TB_t, I use the Canadian goods trade balance for the United States included in CANSIM's balance of international payments data (CANSIM Table 376-0005). The Canadian output $Y_{can,t}$ is constructed by $C_{can,t} + TB_t$ and the United States output $Y_{us,t}$ is constructed by $C_{us,t} - TB_t/S_t$, where S_t is the bilateral exchange rate between Canada and the United States.

Table 1: Prior Distributions of Structural Parameters

	Parameters	Distribution	Mean	S.D.	95 % Coverage
β	Household Subjective Discount Factor	Uniform(0,1)	—	—	[0.025 0.975]
γ_M	Deterministic (Gross) Money Growth	Gamma	1.015	0.005	[1.005 1.024]
γ_S	Deterministic EX Trend	Normal	0.000	1.500	[-2.939 2.939]
ψ	Debt Elasticity of Risk Premium	Gamma	0.010	0.001	[0.008 0.012]
λ	Technology Diffusion Speed	Beta	0.010	0.001	[0.008 0.012]
ϕ	Mean Money Demand Shock	Gamma	1.000	0.010	[0.981 1.019]
ρ_M	Permanent Money Growth AR(1) Coef.	Beta	0.100	0.010	[0.081 0.120]
ρ_q	Real EX AR(1) Coef.	Beta	0.850	0.100	[0.607 0.983]
ρ_ϕ	Money Demand AR(1) Coef.	Beta	0.850	0.100	[0.607 0.983]

Note 1. The AR(1) coefficients of the transitory money and output shocks, ρ_m and ρ_y respectively, have mass points of zero for identification.
Note 2. The standard deviations of all the structural shocks, σ_M, σ_A, σ_m, σ_y, σ_q, and σ_ϕ, have the identical inverse Gamma prior distribution, with a mean of 0.01 and standard deviation of 0.01 for the benchmark information set.
Note 3. The prior distribution of β is given by the Beta distribution, with a mean of 0.999 and standard deviation of 0.001 for the High Discount Factor model.

Table 2: Posterior Distributions of Structural Parameters

	Benchmark			HDF		
Parameters	Mean	S.D.	95 % Interval	Mean	S.D.	95 % Interval
κ	0.537	0.041	[0.457 0.618]	0.950	0.001	[0.947 0.952]
β	0.547	0.042	[0.464 0.630]	0.998	0.000	[0.998 0.999]
γ_M	1.019	0.005	[1.009 1.027]	1.051	0.001	[1.050 1.054]
γ_S	-0.000	0.002	[-0.005 0.002]	0.002	0.000	[0.002 0.003]
ψ	0.010	0.001	[0.008 0.012]	0.011	0.001	[0.009 0.012]
λ	0.009	0.001	[0.008 0.011]	0.007	0.001	[0.006 0.009]
ϕ	0.999	0.010	[0.980 1.019]	0.999	0.009	[0.980 1.018]
ρ_M	0.088	0.005	[0.083 0.097]	0.105	0.003	[0.099 0.108]
ρ_q	0.867	0.044	[0.778 0.948]	0.982	0.004	[0.974 0.989]
ρ_ϕ	0.997	0.001	[0.995 0.999]	0.928	0.002	[0.924 0.931]
σ_M	0.017	0.001	[0.016 0.019]	0.026	0.001	[0.024 0.028]
σ_A	0.006	0.000	[0.005 0.007]	0.006	0.000	[0.005 0.007]
σ_m	0.007	0.001	[0.005 0.009]	0.105	0.004	[0.098 0.111]
σ_y	0.003	0.000	[0.002 0.003]	0.003	0.000	[0.002 0.003]
σ_q	0.006	0.001	[0.004 0.009]	0.006	0.000	[0.005 0.006]
σ_ϕ	0.027	0.002	[0.024 0.031]	0.066	0.003	[0.061 0.070]
Marginal Likelihood	2148.572			1871.309		

Note 1: The "Benchmark" represents the Benchmark specification of the two-country model and the "HDF" represents the High Discount Factor specification.
Note 2: The marginal likelihoods are estimated based on Geweke's (1999) harmonic mean estimator.

Table 3: Forecast Error Variance Decompositions (%): Benchmark Model

horizon	ϵ_M	ϵ_A	ϵ_m	ϵ_y	ϵ_q	ϵ_ϕ
			(a) Exchange Rate: $\ln S_t$			
Impact	27	3	1	0	2	68
	[22 31]	[2 4]	[0 1]	[0 0]	[1 3]	[63 72]
1 *yr*	28	3	0	0	2	67
	[23 33]	[2 4]	[0 0]	[0 0]	[1 3]	[62 71]
3 *yr*	29	3	0	0	2	66
	[24 34]	[2 4]	[0 0]	[0 0]	[1 3]	[62 71]
5 *yr*	29	3	0	0	2	66
	[24 34]	[2 4]	[0 0]	[0 0]	[1 3]	[61 71]
10 *yr*	30	3	0	0	2	66
	[24 34]	[2 4]	[0 0]	[0 0]	[1 3]	[60 70]
			(b) Consumption Differential: $\ln C_t$			
Impact	0	93	0	5	2	0
	[0 0]	[91 96]	[0 0]	[3 8]	[0 3]	[0 0]
1 *yr*	0	93	0	5	2	0
	[0 0]	[89 96]	[0 0]	[2 7]	[0 4]	[0 0]
3 *yr*	0	81	0	4	14	0
	[0 0]	[73 90]	[0 0]	[2 6]	[6 23]	[0 0]
5 *yr*	0	74	0	4	22	0
	[0 0]	[63 85]	[0 0]	[2 6]	[11 33]	[0 0]
10 *yr*	0	68	0	4	29	0
	[0 0]	[53 80]	[0 0]	[2 5]	[15 43]	[0 0]
			(c) Interest Rate Differential: $(1 + r_t)$			
Impact	6	0	94	0	0	0
	[3 10]	[0 0]	[90 97]	[0 0]	[0 0]	[0 0]
1 *yr*	6	0	94	0	0	0
	[3 10]	[0 0]	[90 97]	[0 0]	[0 0]	[0 0]
3 *yr*	6	0	94	0	0	0
	[3 10]	[0 0]	[90 97]	[0 0]	[0 0]	[0 1]
5 *yr*	6	0	94	0	0	0
	[3 10]	[0 0]	[89 96]	[0 0]	[0 0]	[0 1]
10 *yr*	6	0	94	0	0	0
	[3 10]	[0 0]	[89 96]	[0 0]	[0 0]	[0 2]

Note: Part (a) reports the posterior means and the 95% Bayesian highest probability density (HPD) intervals (in square brackets) of the forecast error variance decompositions (FEVDs) of the nominal exchange rate into the six structural shocks over the impact, the one-year, three-year, five-year, and ten-year forecast horizons; Part (b) reports those of the consumption differential; and Part (c) reports those of the interest rate differential.

Table 4: Forecast Error Variance Decompositions (%): HDF Model

horizon	ϵ_M	ϵ_A	ϵ_m	ϵ_y	ϵ_q	ϵ_ϕ
			(a) Exchange Rate: $\ln S_t$			
Impact	52	0	2	0	0	46
	[50 54]	[0 0]	[2 2]	[0 0]	[0 0]	[44 48]
1 *yr*	58	0	0	0	0	41
	[56 60]	[0 0]	[0 1]	[0 0]	[0 0]	[39 43]
3 *yr*	70	0	0	0	0	30
	[68 72]	[0 0]	[0 0]	[0 0]	[0 0]	[28 32]
5 *yr*	77	0	0	0	0	22
	[86 87]	[0 1]	[0 0]	[0 0]	[0 0]	[21 24]
10 *yr*	87	0	0	0	0	13
	[76 79]	[0 1]	[0 0]	[0 0]	[0 0]	[12 14]
			(b) Consumption Differential: $\ln C_t$			
Impact	0	15	0	0	85	0
	[0 0]	[10 20]	[0 0]	[0 0]	[80 90]	[0 0]
1 *yr*	0	16	0	0	84	0
	[0 0]	[10 21]	[0 0]	[0 0]	[79 90]	[0 0]
3 *yr*	0	18	0	0	82	0
	[0 0]	[12 24]	[0 0]	[0 0]	[76 88]	[0 0]
5 *yr*	0	20	0	0	80	0
	[0 0]	[14 26]	[0 0]	[0 0]	[74 86]	[0 0]
10 *yr*	0	27	0	0	73	0
	[0 0]	[19 34]	[0 0]	[2 5]	[66 81]	[0 0]
			(c) Interest Rate Differential: $(1 + r_t)$			
Impact	0	0	87	0	0	13
	[0 0]	[0 0]	[86 87]	[0 0]	[0 0]	[12 14]
1 *yr*	0	0	68	0	0	32
	[0 0]	[0 0]	[66 69]	[0 0]	[0 0]	[31 34]
3 *yr*	0	0	53	0	0	46
	[0 0]	[0 0]	[52 55]	[0 0]	[0 0]	[45 48]
5 *yr*	0	0	50	0	0	49
	[0 0]	[0 0]	[49 52]	[0 0]	[0 0]	[48 51]
10 *yr*	0	0	49	0	0	51
	[0 0]	[0 0]	[48 50]	[0 0]	[0 0]	[49 52]

Note: Part (a) reports the posterior means and the 95% Bayesian highest probability density (HPD) intervals (in square brackets) of the forecast error variance decompositions (FEVDs) of the nominal exchange rate into the six structural shocks over the impact, the one-year, three-year, five-year, and ten-year forecast horizons; Part (b) reports those of the consumption differential; and Part (c) reports those of the interest rate differential.

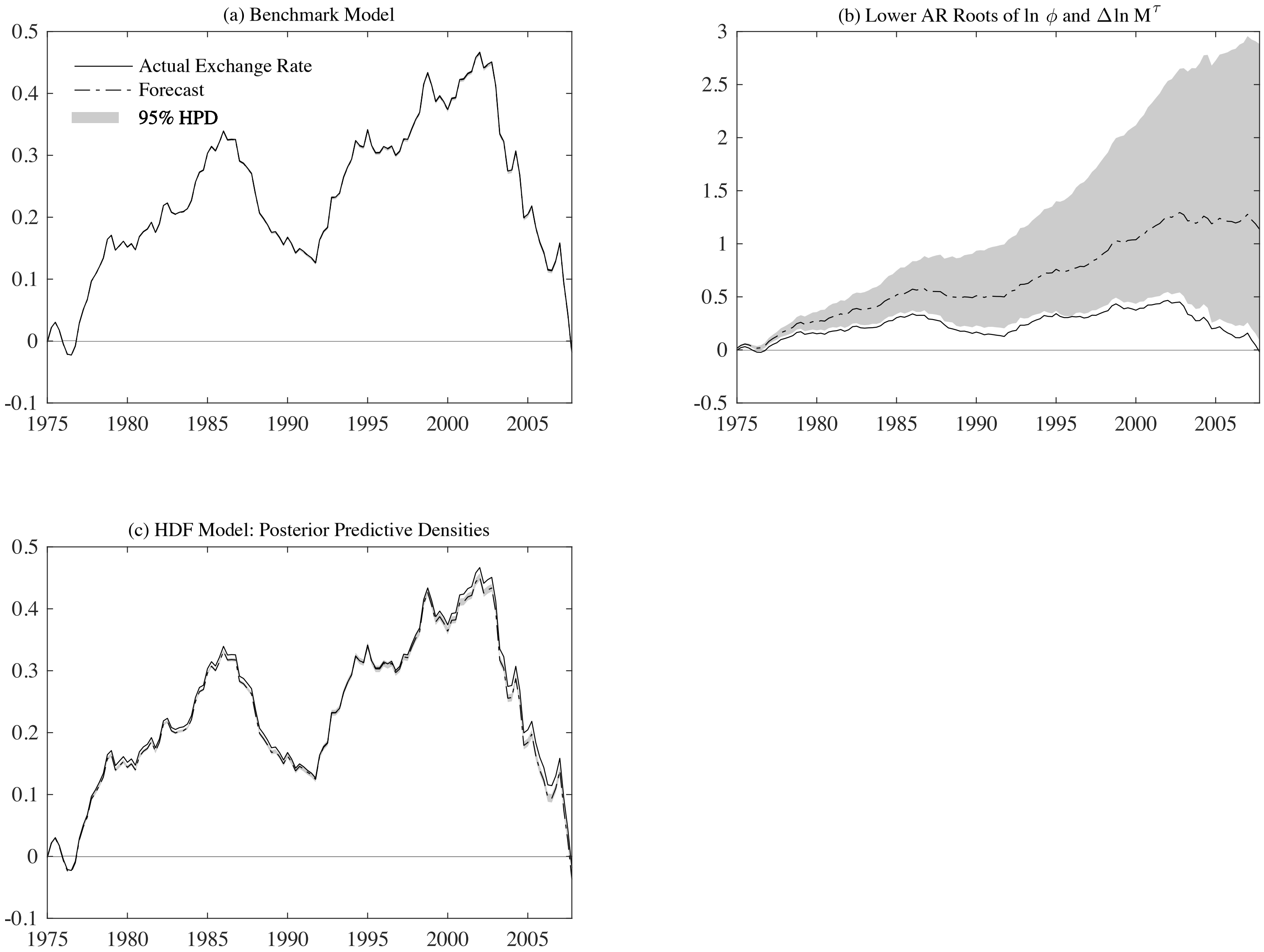

Figure 1: Exchange rates and in-sample predictions. Note: Window (a) corresponds to the Benchmark model, window (b) the Benchmark model with lower AR roots of the money demand shock differential and the permanent money supply growth rate, and window (c) the posterior high discount factor (HDF) model. In each window, the solid black line represents the actual Canadian dollar against the US dollar; the dashed black line the means of in-sample predictions; and the gray area the 95% Bayesian highest probability density (HPD) intervals of the in-sample predictions.

Online Appendix to

Exchange Rates and Fundamentals: A General Equilibrium Exploration

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Appendix A. First order necessary conditions and stochastically de-trended system

The first-order necessary conditions (FONCs) for the household's problem in the home country are given by the budget constraint

$$B^h_{h,t} + S_t B^f_{h,t} + P_{h,t}C_{h,t} + M_{h,t} = (1 + r^h_{h,t-1})B^h_{h,t-1} + S_t(1 + r^f_{h,t-1})B^f_{h,t-1} + M_{h,t-1} + P_{h,t}Y_{h,t} + T_{h,t};$$

the Euler equation

$$\frac{1}{P_{h,t}C_{h,t}} = \beta(1 + r^h_{h,t})E_t\left(\frac{1}{P_{h,t+1}C_{h,t+1}}\right);$$

the utility-based uncovered parity condition (UIP)

$$(1 + r^h_{h,t})E_t\left(\frac{1}{P_{h,t+1}C_{h,t+1}}\right) = \frac{(1 + r^f_{h,t})}{S_t}E_t\left(\frac{S_{t+1}}{P_{h,t+1}C_{h,t+1}}\right);$$

and the money demand function

$$\frac{M_{h,t}}{P_{h,t}} = \phi_{h,t}\left(\frac{1 + r^h_{h,t}}{r^h_{h,t}}\right)C_{h,t}.$$

The foreign country's counterparts of the FONCs are the budget constraint

$$B^h_{h,t} + S_t B^f_{h,t} + P_{h,t}C_{h,t} + M_{h,t} = (1 + r^h_{h,t-1})B^h_{h,t-1} + S_t(1 + r^f_{h,t-1})B^f_{h,t-1} + M_{h,t-1} + P_{h,t}Y_{h,t} + T_{h,t};$$

the Euler equation

$$\frac{1}{P_{f,t}C_{f,t}} = \beta(1 + r^f_{f,t})E_t\left(\frac{1}{P_{f,t+1}C_{f,t+1}}\right);$$

the utility-based uncovered parity condition (UIP)

$$(1 + r^h_{f,t})E_t\left(\frac{1}{S_{t+1}P_{f,t+1}C_{f,t+1}}\right) = \frac{(1 + r^f_{f,t})}{S_t}E_t\left(\frac{1}{P_{f,t+1}C_{f,t+1}}\right);$$

and the money demand function

$$\frac{M_{f,t}}{P_{f,t}} = \phi_{f,t}\left(\frac{1+r^f_{f,t}}{r^f_{f,t}}\right)C_{f,t};$$

The stochastically de-trended versions of the FONCs of the home country consist of the budget constraint

$$p_{h,t}c_{h,t} + b^h_{h,t} + s_t b^f_{h,t} = \frac{(1+r^h_{h,t-1})b^h_{h,t-1}}{\gamma^h_{M,t}} + \frac{(1+r^f_{h,t-1})s_t b^f_{h,t-1}}{\gamma^f_{M,t}} + p_{h,t}y_{h,t};$$

the Euler equation

$$\frac{1}{p_{h,t}c_{h,t}} = \beta(1+r^h_{h,t})E_t\left(\frac{1}{\gamma^h_{M,t+1}p_{h,t+1}c_{h,t+1}}\right);$$

the UIP condition

$$s_t(1+r^h_{h,t})E_t\left(\frac{1}{p_{h,t+1}c_{h,t+1}\gamma^h_{M,t+1}}\right) = (1+r^f_{h,t})E_t\left(\frac{s_{t+1}}{p_{h,t+1}c_{h,t+1}\gamma^f_{M,t+1}}\right);$$

the money demand function

$$\frac{m_{h,t}}{p_{h,t}} = \phi_{h,t}c_{h,t}\left(\frac{1+r^h_{h,t}}{r^h_{h,t}}\right);$$

the risk premiums

$$r^h_{h,t} = r^h_{w,t}[1+\psi\{\exp(-b^h_{h,t}+\bar{d})-1\}],$$

and

$$r^f_{h,t} = r^f_{w,t}[1+\psi\{\exp(-b^f_{h,t}+\bar{d})-1\}].$$

Similarly, the stochastically de-trended versions of the FONCs of the foreign country consist of the budget costraint

$$\frac{q_t p_{h,t}c_{f,t}}{a_t} - s_t b^f_{h,t} - b^h_{h,t} = -\frac{(1+r^f_{w,t-1})s_t b^f_{h,t-1}}{\gamma^f_{M,t}} - \frac{(1+r^h_{w,t-1})b^h_{h,t-1}}{\gamma^h_{M,t}} + \frac{q_t p_{h,t}y_{f,t}}{a_t};$$

the Euler equation

$$\frac{a_t s_t}{q_t p_{h,t} c_{f,t}} = \beta(1 + r^f_{w,t}) E_t \frac{a_{t+1} s_{t+1}}{\gamma^f_{M,t+1} q_{t+1} p_{h,t+1} c_{f,t+1}};$$

the UIP condition

$$s_t(1 + r^h_{w,t}) E_t \left(\frac{a_{t+1}}{q_{t+1} p_{h,t+1} c_{f,t+1} \gamma^h_{M,t+1}} \right) = (1 + r^f_{w,t}) E_t \left(\frac{a_{t+1} s_{t+1}}{q_{t+1} p_{h,t+1} c_{f,t+1} \gamma^f_{M,t+1}} \right);$$

and the money demand function

$$\frac{a_t s_t m_{f,t}}{q_t p_{h,t}} = \phi_{f,t} c_{f,t} \left(\frac{1 + r^f_{w,t}}{r^f_{w,t}} \right).$$

Finally The stochastically de-trended PPP condition is $s_t = p_{h,t} q_t / (a_t p_{f,t})$.

If the TFP differential a_t is I(1) as assumed in NR, the above system of stochastic difference equations becomes non-stationary through the home and foreign budget constraints and there is no deterministic steady state to converge. Notice that the cross-country permanent money supply differential $\ln M^\tau_{h,t}/M^\tau_{f,t}$ does not appear in the stochastically de-trended system of the FONCs. In contrast to the TFP differential a_t, the I(1) property of $\ln M^\tau_{h,t}/M^\tau_{f,t}$ in Assumption 2 does not matter for the closing of the model. This might be an obvious result of the model's property that the super-neutrality of money holds in the money-in-utility model: Money growth does not matter for the deterministic steady state.

Notice that at the deterministic steady state, the TFP differential a^* is one. Because of the stationarity of the above system of equations, the deterministic steady state is characterized by constants c_h^*, c_f^*, p_h^*, s^*, b_h^{h*}, b_h^{f*}, r_h^{h*}, r_h^{f*}, r_w^{h*}, and r_w^{f*} that satisfy

$$\begin{aligned}
b_h^{h*} &= b_h^{f*} = \bar{d}, \\
r^* \equiv r_h^{h*} &= r_f^{f*} = r_w^{h*} = r_w^{f*} = \gamma_M/\beta - 1, \\
s^* &= \frac{y_f(\phi\gamma_M)^{-1}r^* + (y_h + y_f)(1-\beta^{-1})\bar{d}}{y_h(\phi\gamma_M)^{-1}r^* - (y_h + y_f)(1-\beta^{-1})\bar{d}}, \\
p_h^* y_h &= (1-\beta^{-1})(1+s^*)\bar{d} + (\phi\gamma_M)^{-1}r^*, \\
p_h^* c_h^* &= (\phi\gamma_M)^{-1}r^*, \\
c_f^* &= s^* c_h^*.
\end{aligned}$$

The log-linear approximation of the stochastically de-trended home budget constraint is

$$\begin{aligned}
&p_h^*(c_h^* - y_h)\hat{p}_{h,t} + p_h^* c_h^* \hat{c}_{h,t} - p_h^* y_h \hat{y}_{h,t} + \tilde{b}_{h,t}^h + \bar{d}(1-\beta^{-1})s^*\hat{s}_t + s^*\tilde{b}_{h,t}^f \\
&\quad = \beta^{-1}\bar{d}[(1+\hat{r}_{h,t-1}^h) - \hat{\gamma}_{M,t}^h] + s^*\beta^{-1}\bar{d}[(1+\hat{r}_{h,t-1}^f) - \hat{\gamma}_{M,t}^f] + \beta^{-1}\tilde{b}_{h,t-1}^h + s^*\beta^{-1}\tilde{b}_{h,t-1}^f; \quad \text{(A.1)}
\end{aligned}$$

that of the home Euler equation is

$$\hat{p}_{h,t} + \hat{c}_{h,t} + (1+\hat{r}_{h,t}^h) = E_t(\hat{p}_{h,t+1} + \hat{c}_{h,t+1} + \hat{\gamma}_{M,t+1}^h); \quad \text{(A.2)}$$

that of the home UIP condition is

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}_{h,t}^h) - (1+\hat{r}_{h,t}^f) - E_t(\hat{\gamma}_{M,t+1}^h - \hat{\gamma}_{M,t+1}^f); \quad \text{(A.3)}$$

and that of the home money demand function is

$$\hat{p}_{h,t} + \hat{c}_{h,t} - \hat{m}_{h,t} = \frac{1}{r^*}(1+\hat{r}_{h,t}^h) - \hat{\phi}_{h,t}. \quad \text{(A.4)}$$

The foreign country's counterparts are the log-linear approximation of the stochastically de-trended foreign budget constraint

$$
\begin{aligned}
&p_h^*(c_f^* - y_f)(\hat{p}_{h,t} + \hat{q}_t - \hat{a}_t) + p_h^* c_f^* \hat{c}_{f,t} - p_h^* y_f \hat{y}_{f,t} - \tilde{b}_{h,t}^h - \bar{d}(1-\beta^{-1}) s^* \hat{s}_t - s^* \tilde{b}_{h,t}^f \\
&\quad = -\beta^{-1}\bar{d}[(1+\hat{r}_{w,t-1}^h) - \hat{\gamma}_{M,t}^h] - s^*\beta^{-1}\bar{d}[(1+\hat{r}_{w,t-1}^f) - \hat{\gamma}_{M,t}^f] - \beta^{-1}\tilde{b}_{h,t-1}^h - s^*\beta^{-1}\tilde{b}_{h,t-1}^f; \quad \text{(A.5)}
\end{aligned}
$$

that of the foreign Euler equation

$$
\hat{s}_t - \hat{p}_{h,t} - \hat{c}_{f,t} - \hat{q}_t + \hat{a}_t - (1+\hat{r}_{w,t}^f) = E_t(\hat{s}_{t+1} - \hat{p}_{h,t+1} - \hat{c}_{f,t+1} - \hat{q}_{t+1} + \hat{a}_t - \hat{\gamma}_{M,t+1}^f); \qquad \text{(A.6)}
$$

that of the foreign UIP condition

$$
E_t \hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}_{w,t}^h) - (1+\hat{r}_{w,t}^f) - E_t(\hat{\gamma}_{M,t+1}^h - \hat{\gamma}_{M,t+1}^f); \qquad \text{(A.6)}
$$

and that of the home money demand function

$$
\hat{s}_t + \hat{m}_{f,t} - \hat{p}_{h,t} - \hat{c}_{f,t} - \hat{q}_t + \hat{a}_t = -\frac{1}{r^*}(1+\hat{r}_{w,t}^f) + \hat{\phi}_{f,t}. \qquad \text{(A.7)}
$$

The log-linear approximations of the home country's interest rates are

$$
(1+\hat{r}_{h,t}^h) = (1+\hat{r}_{w,t}^h) - \psi(1-\kappa)\tilde{b}_{h,t}^h, \quad \text{and} \quad (1+\hat{r}_{h,t}^f) = (1+\hat{r}_{w,t}^f) - \psi(1-\kappa)\tilde{b}_{h,t}^f. \qquad \text{(A.8)}
$$

Notice that the home interest rates (A.8) redefine the home UIP condition (A.3) as

$$
E_t \hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}_{w,t}^h) - (1+\hat{r}_{w,t}^f) - \psi(1-\kappa)(\tilde{b}_{h,t}^h - \tilde{b}_{h,t}^f) - E_t(\hat{\gamma}_{M,t+1}^h - \hat{\gamma}_{M,t+1}^f). \qquad \text{(A.9)}
$$

Comparing the above home UIP condition with the foreign UIP condition (A.6) implies that the home and foreign bonds are perfectly substitutable along the equilibrium path. Hence, the equilibrium condition $\tilde{b}_t \equiv \tilde{b}_{h,t}^h = \tilde{b}_{h,t}^f$ holds.

Appendix B. Solving the equilibrium with two symmetric countries

To understand the equilibrium transitory dynamics of the exchange rate in this model, it is informative to scrutinize a simpler version of the model that includes two symmetric countries. For this purpose, I set the parameter $\bar{d}$ to zero and assume that the transitory output components of the two countries, y_h and y_f, are equal to y. Notice that the deterministic steady state in this case is characterized by $s^* = 1$, $c_h^* = c_f^* = y$, and $p_h^* = (\phi\gamma_M)^{-1}r^*$, where $r^* = \gamma_M/\beta - 1$.

The home and foreign money demand functions, (A.4) and (A.7), and the home interest rates (A.8) yield the following interest rate differential:

$$(1 + \hat{r}^h_{w,t}) - (1 + \hat{r}^f_{w,t}) = r^*(\hat{s}_t + \hat{c}_t - \hat{m}_t + \hat{\phi}_t - \hat{q}_t + \hat{a}_t) + \psi(1 - \kappa)\tilde{b}_t.$$

Substituting the interest rate differential into the foreign UIP condition (A.6) leads to the expectational difference equation of the de-trended exchange rate $\hat{s}_t$:

$$\hat{s}_t = \kappa E_t\hat{s}_{t+1} - (1 - \kappa)\hat{c}_t + (1 - \kappa)(\hat{m}_t - \hat{\phi}_t + \hat{q}_t - \hat{a}_t) + \kappa E_t(\hat{\gamma}^h_{M,t+1} - \hat{\gamma}^f_{M,t+1}) - \psi\kappa(1 - \kappa)\tilde{b}_t.$$

I combine the log-linearized Euler equations of the home and foreign countries, (A.2) and (A.6), with those of the home country's interest rates (A.8) to yield the first-order expectational difference equation of $\hat{s}_t + \hat{c}_t - \hat{q}_t + \hat{a}_t$:

$$\hat{s}_t + \hat{c}_t - \hat{q}_t + \hat{a}_t = \kappa E_t(\hat{s}_{t+1} + \hat{c}_{t+1} - \hat{q}_{t+1} + \hat{a}_{t+1}) + \kappa E_t\hat{\gamma}_{M,t+1} + (1 - \kappa)(\hat{m}_t - \hat{\phi}_t).$$

Since κ takes a value between zero and one, the above expectational difference equation has a forward solution of $\hat{s}_t + \hat{c}_t - \hat{q}_t + \hat{a}_t = \kappa\rho_M(1 - \kappa\rho_M)^{-1}\hat{\gamma}_{M,t} + (1 - \kappa)(1 - \kappa\rho_m)^{-1}\hat{m}_t - (1 - \kappa)(1 - \kappa\rho_\phi)^{-1}\hat{\phi}_t$ under a suitable transversality condition. By exploiting this forward solution and the stochastic processes of both countries' TFPs (1), I rewrite the foreign UIP condition (A.6) as

$$E_t\hat{s}_{t+1} - \hat{s}_t = \psi(1 - \kappa)\tilde{b}_t - \frac{\kappa\rho_M(1 - \rho_M)}{1 - \kappa\rho_M}\hat{\gamma}_{M,t} - \frac{(1 - \kappa)(1 - \rho_m)}{1 - \kappa\rho_m}\hat{m}_t + \frac{(1 - \kappa)(1 - \rho_\phi)}{1 - \kappa\rho_\phi}\hat{\phi}_t, \quad \text{(B.1)}$$

Furthermore, taking a difference between the log-linearized budget constraints of the home and foreign countries, (A.1) and (A.5), I find the law of motion of the international bond holdings

$$\tilde{b}_t = \beta^{-1}\tilde{b}_{t-1} + p_h^* y^* \hat{s}_t - p_h^* y^* (\hat{q}_t - \hat{a}_t) - \frac{p_h^* y^* \kappa\rho_M}{1-\kappa\rho_M}\hat{\gamma}_{M,t} - \frac{p_h^* y^*(1-\kappa)}{1-\kappa\rho_m}\hat{m}_t + \frac{p_h^* y^*(1-\kappa)}{1-\kappa\rho_\phi}\hat{\phi}_t + p_h^* y^* \hat{y}_t, \quad \text{(B.2)}$$

where $y^* = y/4$ and $\hat{y}_t \equiv \hat{y}_{h,t} - \hat{y}_{f,t}$.

Combining equation (B.1) with equation (B.2) then yields the following second-order expectational difference equation with respect to international bond holdings:

$$E_t\tilde{b}_{t+1} - [1 + \beta^{-1} + p_h^* y^* \psi(1-\kappa)]\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1} = -\lambda p_h^* y^* \hat{a}_t + p_h^* y^* (1-\rho_q)\hat{q}_t - p_h^* y^*(1-\rho_y)\hat{y}_t \quad \text{(B.3)}$$

It is straightforward to show that equation (B.3) has two roots, one of which is greater than one and the other of which is less than one.[1] Without losing generality, let η denote the root that is less than one. Solving equation (B.3) by forward iterations then shows that the equilibrium international bond holdings level is determined by the following cross-equation restriction (CER):

$$\begin{aligned}\tilde{b}_t &= \eta\tilde{b}_{t-1} + \beta\eta\lambda p_h^* y^* \sum_{j=0}^{\infty}(\beta\eta)^j E_t\hat{a}_{t+j} + \beta\eta p_h^* y^*(1-\rho_y)\sum_{j=0}^{\infty}(\beta\eta)^j E_t\hat{y}_{t+j} - \beta\eta p_h^* y^*(1-\rho_q)\sum_{j=0}^{\infty}(\beta\eta)^j E_t\hat{q}_{t+j}, \\ &= \eta\tilde{b}_{t-1} + \frac{\beta\eta\lambda p_h^* y^*}{1-\beta\eta(1-\lambda)}\hat{a}_t + \frac{\beta\eta p_h^* y^*(1-\rho_y)}{1-\beta\eta\rho_y}\hat{y}_t - \frac{\beta\eta p_h^* y^*(1-\rho_q)}{1-\beta\eta\rho_q}\hat{q}_t. \qquad \text{(B.4)}\end{aligned}$$

Substituting equation (B.4) back into equation (B.2) provides the CER for the exchange rate (7):

$$\begin{aligned}\hat{s}_t = \frac{\beta\eta - 1}{\beta p_h^* y^*}\tilde{b}_{t-1} - \frac{1-\beta\eta}{1-\beta\eta(1-\lambda)}\hat{a}_t + \frac{1-\kappa}{1-\kappa\rho_m}\hat{m}_t - \frac{1-\kappa}{1-\kappa\rho_\phi}\hat{\phi}_t \\ - \frac{1-\beta\eta}{1-\beta\eta\rho_y}\hat{y}_t + \frac{1-\beta\eta}{1-\beta\eta\rho_q}\hat{q}_t + \frac{\kappa\rho_M}{1-\kappa\rho_M}\hat{\gamma}_{M,t}.\end{aligned}$$

Therefore, in this symmetric case, the competitive equilibrium along the balanced growth path is characterized by a lower dimensional dynamic system of $(\hat{s}_t, \tilde{b}_t, \hat{a}_t, \hat{\gamma}_{M,t}, \hat{m}_t, \hat{\phi}_t, \hat{y}_t, \hat{q}_t)$.

[1] To characterize the roots of the second-order expectational difference equation, see, for example, Sargent (1987).

Adding the log-linearized home and foreign budget constraints together implies the resource constraint $\hat{c}_{h,t} + \hat{c}_{f,t} = \hat{y}_{h,t} + \hat{y}_{f,t}$. Since the equilibrium dynamics of the consumption differential follow $\hat{c}_{h,t} - \hat{c}_{f,t} = -\hat{s}_t + \hat{q}_t - \hat{a}_t + \kappa\rho_M(1-\kappa\rho_M)^{-1}\hat{\gamma}_{M,t} + (1-\kappa)(1-\kappa\rho_m)^{-1}\hat{m}_t - (1-\kappa)(1-\kappa\rho_\phi)^{-1}\hat{\phi}_t$, the home country's consumption obeys $2\hat{c}_{h,t} = (\hat{y}_{h,t} + \hat{y}_{f,t}) - \hat{s}_t + \hat{q}_t - \hat{a}_t + \kappa\rho_M(1-\kappa\rho_M)^{-1}\hat{\gamma}_{M,t} + (1-\kappa)(1-\kappa\rho_m)^{-1}\hat{m}_t - (1-\kappa)(1-\kappa\rho_\phi)^{-1}\hat{\phi}_t$, while the foreign country's is $2\hat{c}_{f,t} = (\hat{y}_{h,t} + \hat{y}_{f,t}) + \hat{s}_t - \hat{q}_t + \hat{a}_t - \kappa\rho_M(1-\kappa\rho_M)^{-1}\hat{\gamma}_{M,t} - (1-\kappa)(1-\kappa\rho_m)^{-1}\hat{m}_t + (1-\kappa)(1-\kappa\rho_\phi)^{-1}\hat{\phi}_t$. The home country's price $\hat{p}_{h,t}$ then is determined as follows. The Euler equation and the money demand function of the foreign country, (A.6) and (A.7), imply the expectational difference equation of $\hat{s}_t - \hat{p}_{h,t} - \hat{c}_{f,t}$

$$\hat{s}_t - \hat{p}_{h,t} - \hat{c}_{f,t} - \hat{q}_t + \hat{a}_t = \kappa E_t(\hat{s}_{t+1} - \hat{p}_{h,t+1} - \hat{c}_{f,t+1} - \hat{q}_{t+1} + \hat{a}_{t+1} - \hat{\gamma}^f_{M,t+1}) - (1-\kappa)(\hat{m}_{f,t} - \hat{\phi}_{f,t}).$$

Solving the above equation by forward iterations and imposing a suitable transversality condition yields the CER $\hat{s}_t - \hat{p}_{h,t} - \hat{c}_{f,t} - \hat{q}_t + \hat{a}_t = -\kappa\rho_M(1-\kappa\rho_M)^{-1}\hat{\gamma}^f_{M,t} - (1-\kappa)(1-\kappa\rho_m)^{-1}\hat{m}_{f,t} + (1-\kappa)(1-\kappa\rho_\phi)^{-1}\hat{\phi}_{f,t}$. This CER characterizes the equilibrium home price

$$2\hat{p}_{h,t} = \hat{s}_t - (\hat{y}_{h,t} + \hat{y}_{f,t}) - \hat{q}_t + \hat{a}_t + \frac{\kappa\rho_M}{1-\kappa\rho_M}(\hat{\gamma}^h_{M,t} + \hat{\gamma}^f_{M,t}) + \frac{1-\kappa}{1-\kappa\rho_m}(\hat{m}_{h,t} + \hat{m}_{f,t}) - \frac{1-\kappa}{1-\kappa\rho_\phi}(\hat{\phi}_{h,t} + \hat{\phi}_{f,t}).$$

The money demand functions of both countries, eqs.(A.4) and (A.7), imply that the interest rates in the two countries are

$$\begin{aligned}(1+\hat{r}^h_{h,t}) &= (1-\kappa)\left(\frac{\rho_M}{1-\kappa\rho_M}\hat{\gamma}^h_{M,t} - \frac{1-\rho_m}{1-\kappa\rho_m}\hat{m}_{h,t} + \frac{1-\rho_\phi}{1-\kappa\rho_\phi}\hat{\phi}_{h,t}\right)\\ (1+\hat{r}^f_{w,t}) &= (1-\kappa)\left(\frac{\rho_M}{1-\kappa\rho_M}\hat{\gamma}^f_{M,t} - \frac{1-\rho_m}{1-\kappa\rho_m}\hat{m}_{f,t} + \frac{1-\rho_\phi}{1-\kappa\rho_\phi}\hat{\phi}_{f,t}\right).\end{aligned}$$

Finally, as the last endogenous variable, the risk-free nominal interest rate of the home bonds then fluctuates in response to the risk premium, following $(1+\hat{r}^h_{w,t}) = (1+\hat{r}^h_{h,t}) + \psi(1-\kappa)\tilde{b}_t$.

Suppose that $\psi = 0$: There is no debt elastic risk premium in the home country's interest rate. It is easy to show that in this case, the second-order expectational difference equation (B.3) has a unit root, i.e., $\eta = 1$, and the resulting forward solution turns out to be

$$\tilde{b}_t = \tilde{b}_{t-1} + \frac{\beta \lambda p_h^* y^*}{1 - \beta(1-\lambda)} \hat{a}_t + \frac{\beta p_h^* y^* (1-\rho_y)}{1 - \beta \rho_y} \hat{y}_t - \frac{\beta p_h^* y^* (1-\rho_q)}{1 - \beta \rho_q} \hat{q}_t.$$

Hence, the stochastic process of the de-trended international bond holding $\tilde{b}_t$ contains a permanent unit root component and never converges to the steady state. This lack of stationarity of the equilibrium balance growth path motivates this paper to allow for a positive elasticity of the risk premium with respect to the debt level.

Importantly, a permanent stochastic process of the de-trended international bond holding also emerges even when $\kappa = 1$. Because the log-linearized home country's interest rates (A.8) imply that under $\kappa = 1$, the debt elastic risk premia in play no role in determining the interest rates faced by the home country. As a result, the de-trended international bond holding $\tilde{b}_t$ contains a permanent unit root component, as in the case where $\psi = 0$. Hence, the closing of the two-country DSGE model in this paper requires the market discount factor to be strictly less than one.

The consumption logarithms of the home and foreign countries in terms of the home currency can be solved as

$$2 \ln C_{h,t} = \ln Y_{h,t} + \ln q_t Y_{f,t} + \frac{1-\beta\eta}{1-\beta\eta(1-\lambda)} \ln a_t + \frac{1-\beta\eta}{1-\beta\eta\rho_y} \ln y_t - \frac{1-\beta\eta}{1-\beta\eta\rho_q} \ln q_t + \frac{1-\beta\eta}{\beta p_h^* y^*} \tilde{b}_{t-1},$$

$$2 \ln q_t C_{f,t} = \ln Y_{h,t} + \ln q_t Y_{f,t} - \frac{1-\beta\eta}{1-\beta\eta(1-\lambda)} \ln a_t - \frac{1-\beta\eta}{1-\beta\eta\rho_y} \ln y_t + \frac{1-\beta\eta}{1-\beta\eta\rho_q} \ln q_t - \frac{1-\beta\eta}{\beta p_h^* y^*} \tilde{b}_{t-1}.$$

Each country's consumption depends on the log-linearized global aggregate endowment $\ln Y_{h,t} + \ln q_t Y_{f,t}$, the log-linearized country-specific portion of the aggregate endowment $\frac{1-\beta\eta}{[1-\beta\eta(1-\lambda)]} \ln a_t + \frac{1-\beta\eta}{(1-\beta\eta\rho_y)} \ln y_t - \frac{1-\beta\eta}{(1-\beta\eta\rho_q)} \ln q_t$, and the wealth effect of the net foreign asset position $\frac{1-\beta\eta}{\beta p_h^* y^*} \tilde{b}_{t-1}$. If the discount factor approaches one, both the log-linearized country-specific portion and the wealth effect of the net foreign asset position disappear and the log consumption levels become

$$\ln C_{h,t} = \frac{1}{2}(\ln Y_{h,t} + \ln Y_{f,t}) + \frac{1}{2} \ln q_t, \quad \ln C_{f,t} = \frac{1}{2}(\ln Y_{h,t} + \ln Y_{f,t}) - \frac{1}{2} \ln q_t.$$

Relative consumption then turns out to be correlated perfectly with the RER because

$$\ln C_{h,t} - \ln C_{f,t} = \ln q_t.$$

Appendix C. Derivation of the error correction representation (4)

Let n_t denote the fundamental of PVM (3): $n_t \equiv \ln M_t - \ln C_t - \psi\kappa\tilde{b}_t - \ln\phi_t + \ln q_t$. Consider the currency return $\Delta\ln S_t$ adjusted by the fundamental $(1-\kappa)n_{t-1}$: $\Delta\ln S_t + (1-\kappa)n_{t-1}$. PVM (3) then implies:

$$\begin{aligned}
\Delta\ln S_t + (1-\kappa)n_{t-1} &= (1-\kappa)\sum_{j=0}^{\infty}\kappa^j(E_t - E_{t-1})n_{t+i} + (1-\kappa)\sum_{j=0}^{\infty}\kappa^j E_{t-1}n_{t+i} \\
&\quad - (1-\kappa)\sum_{j=0}^{\infty}\kappa^j E_{t-1}n_{t+i-1} + (1-\kappa)n_{t-1}, \\
&= (1-\kappa)\sum_{j=0}^{\infty}\kappa^j(E_t - E_{t-1})n_{t+i} + \frac{(1-\kappa)^2}{\kappa}\sum_{i=0}^{\infty}\kappa^i E_{t-1}n_{t+i-1} - \frac{(1-\kappa)^2}{\kappa}n_{t-1}, \\
&= (1-\kappa)\sum_{j=0}^{\infty}\kappa^j(E_t - E_{t-1})n_{t+i} + \frac{1-\kappa}{\kappa}\ln S_{t-1} - \frac{(1-\kappa)^2}{\kappa}n_{t-1}.
\end{aligned}$$

This result means that the currency return has the following error correction representation, given by equation (4):

$$\begin{aligned}
\Delta\ln S_t = \frac{1-\kappa}{\kappa}&(\ln S_{t-1} - \ln M_{t-1} + \ln C_{t-1} + \psi\kappa\tilde{b}_{t-1} + \ln\phi_{t-1} - \ln q_{t-1}) \\
&+ (1-\kappa)\sum_{j=0}^{\infty}\kappa^j(E_t - E_{t-1})n_{t+i}.
\end{aligned}$$

Appendix D. The restricted UC model and posterior simulation strategy

Let $\mathbf{X}_t$ denote an unobserved state vector defined as

$$\mathbf{X}_t = [\hat{s}_t\ \hat{c}_t\ E_t\hat{s}_{t+1}\ E_t\hat{c}_{t+1}\ \tilde{b}_t\ \hat{\gamma}_{M,t}\ \hat{a}_t\ \hat{m}_t\ \hat{y}_t\ \hat{q}_t\ \hat{\phi}_t]'.$$

Furthermore, let ϵ_t and ω_t denote random vectors consisting of structural shocks and rational expectations errors: $\epsilon_t \equiv [\epsilon_{M,t}\ \epsilon_{A,t}\ \epsilon_{m,t}\ \epsilon_{y,t}\ \epsilon_{q,t}\ \epsilon_{\phi,t}]'$ and $\omega_t \equiv [\hat{s}_t - E_{t-1}\hat{s}_t\ \hat{c}_t - E_{t-1}\hat{c}_t]'$, respectively. In particular, for empirical investigation purposes, I presume that the structural shock vector ϵ_t is normally distributed, with a mean of zero and a diagonal variance-covariance matrix Σ: $\epsilon_t \sim i.i.d.N(\mathbf{0}, \Sigma)$ with $diag(\Sigma) = [\sigma_M^2\ \sigma_A^2\ \sigma_m^2\ \sigma_y^2\ \sigma_q^2\ \sigma_\phi^2]'$.

Accompanied by the stochastic processes of the exogenous forcing variables, LRE model (2) then implies that

$$\Gamma_0\mathbf{X}_t = \Gamma_1\mathbf{X}_{t-1} + \Phi_0\omega_t + \Phi_1\epsilon_t,$$

where Γ_0, Γ_1, Φ_0, and Φ_1 are the corresponding coefficient matrices. Applying Sims's (2001) QZ algorithm to the linear rational expectations model above yields a unique solution as the following stationary transition equation of the unobservable state vector:

$$\mathbf{X}_t = \mathbf{F}\mathbf{X}_{t-1} + \Phi\epsilon_t, \tag{D.1}$$

where $\mathbf{F}$ and Φ are confirmable coefficient matrices.

To construct this paper's UC model, I further expand the unobservable state vector $\mathbf{X}_t$ by the permanent money supply differential $\ln M_t^\tau$ to obtain the augmented state vector $\mathbf{Z}_t$: $\mathbf{Z}_t \equiv [\mathbf{X}_t'\ \ln M_t^\tau]'$. The stochastic process of $\ln M_t^\tau$ and the state transition (D.1) then imply the following non-stationary transition of the expanded state vector $\mathbf{Z}_t$:

$$\mathbf{Z}_t = \mathbf{G}\mathbf{Z}_{t-1} + \Psi\epsilon_t, \quad \epsilon_t \sim i.i.d.N(\mathbf{0}, \Sigma), \tag{D.2}$$

where $\mathbf{G}$ and Ψ are confirmable coefficient matrices.

In this paper, I explore time-series data on the log of the consumption differential $\ln C_t$, the log of the output differential $\ln Y_t$, the log of the money supply differential $\ln M_t$, the interest rate differential $r_t \equiv r^h_{h,t} - r^f_{f,t}$, and the log of the bilateral exchange rate $\ln S_t$. Let $\mathbf{Y}_t$ denote the information set that consists of these five time series: $\mathbf{Y}_t \equiv [\ln C_t \;\; \ln Y_t \;\; \ln M_t \;\; r_t \;\; \ln S_t]'$. It is then straightforward to show that the information set $\mathbf{Y}_t$ is linearly related to the unobservable state vector $\mathbf{Z}_t$ as

$$\mathbf{Y}_t = \mathbf{H}\mathbf{Z}_t, \tag{D.3}$$

where $\mathbf{H}$ is a confirmable coefficient matrix. The transition equation, the unobserved state (D.2), and the observation equation (D.3) jointly consist of a non-stationary state-space representation of the two-country model, which is the restricted UC model estimated in this paper.[2]

Given the data set $\mathbf{Y}^T \equiv \{\mathbf{Y}_t\}_{t=0}^{T}$, applying the Kalman filter to the UC model provides model likelihood $\mathbf{L}(\mathbf{Y}^T|\theta)$, where θ is the structural parameter vector of the two-country model. Multiplying the likelihood by a prior probability of the structural parameters, $\mathbf{p}(\theta)$, is proportional to the corresponding posterior distribution $\mathbf{p}(\theta|\mathbf{Y^T}) \propto \mathbf{p}(\theta)\mathbf{L}(\mathbf{Y}^T|\theta)$ through the Bayes law. The posterior distribution $\mathbf{p}(\theta|\mathbf{Y}^T)$ is simulated by the random-walk Metropolis-Hastings algorithm, as implemented by Schorfheide (2000), Bouakez and Kano (2006), and Kano (2009).

Appendix E. The high discount factor (HDF) model

In Figure A.1, the five windows plot the 95% HPD intervals of the forecast errors for the five time-series data sets, which are implied by the Benchmark model as the dark gray area and the HDF model as the light gray area. Windows (a) and (b) of Figure A.1, especially, reveal that the HDF model yields much larger forecast errors for the consumption differential and the money supply differential than does the Benchmark model.

[2] The state-space form of the model, (D.2) and (D.3), decomposes the I(1) difference-stationary information set $\mathbf{Y}_t$ into permanent and transitory components exploiting the theoretical restrictions provided by the two-country model. Recursion of the Kalman filter for a non-stationary state-space model is explained in detail by Hamilton (1994).

The large forecast error of the HDF model in the consumption differential results from the model's implying tight positive correlation between the consumption differential and the PPP deviation shock. Figure A.1 plots as the light gray area the 95% HPD interval of the forecast error in the consumption differential that the HDF model draws. The same figure also displays as the dark gray area the 95% HPD interval of the forecast error in the consumption differential generated *only by the PPP deviation shock.* Observe that these two HPD intervals overlap each other to quite a large degree. This simply means that the PPP deviation shock is the main contributor to the large variations in the forecast error in the consumption differential that the HDF model implies. Therefore, it is the high dependence of the consumption difference on the PPP deviation shock that leads to the significant failure in the HDF model to explain the actual data of the consumption differential. The data, as a result, forces the subjective discount factor to be much lower to increase the empirical fit as in the Benchmark model.

Understanding the difficulty of the HDF model in explaining the money supply differential requires several steps. First, recall that the AR(1) coefficient of the transitory money supply differential, ρ_m, is assumed to be zero for identification. Additionally, as reported in Table 2, the posterior mean of the AR(1) coefficient of the money growth rate differential, ρ_M, is estimated to be small around 0.088, while that of the money demand shock differential, ρ_ϕ, is estimated to be close to one. Equation (8), then, shows that when the discount factor κ approaches one, the interest rate differential becomes dependent only on the transitory money supply differential and the money demand shock differential with very small sensitivities.

Second, applying the Kalman filter to interest rate differential equation (8) yields the one-period-ahead forecast of the interest rate differential. Importantly, the forecast of the interest rate differential should be dominated by the persistent money demand shock differential $\ln \phi_t$ accompanied by the white noise forecast error $\ln m_t$. Because of the flat money demand function, the forecast of the money demand shock differential should be very volatile to explain the actual data of the interest rate differential.

Third, transitory exchange rate equation (6), then, predicts that under the HDF model the forecast of the transitory exchange rate should be dominated by the volatile forecast of the money demand shock differential. Indeed, the negative correlation between the forecasts of $\ln s_t$ and $\ln \phi_t$ is almost perfect. Figure A.3(a) plots the 95% HPD intervals of the forecasts of $\ln s_t$ as the dark gray area and of $\ln \phi_t$ as the light gray area, as predicted by the HDF model. The figure confirms that the forecast of $\ln \phi_t$ is very volatile and is correlated negatively with that of $\ln s_t$. The HDF model, therefore, predicts the volatile forecast of the transitory exchange rate.

Fourth, the forecast of the permanent money supply differential $\ln M_t^\tau$ should be volatile and correlated negatively with that of the transitory exchange rate $\ln s_t$. This is due to the stochastic de-trending of the exchange rate $\ln S_t \equiv \ln s_t + \ln M_t^\tau$. To keep the fit of the model to the actual $\ln S_t$ good, the volatile forecast of $\ln s_t$ should be offset by a volatile forecast of $\ln M_t^\tau$. Figure A.3(b) plots the 95% HPD intervals of the forecasts of $\ln s_t$ and $\ln M_t^\tau$ as the dark and light gray areas, respectively. The figure also displays the actual $\ln S_t$ as the black solid line. Observe in the figure that the forecast of $\ln M_t^\tau$ is very volatile and associated negatively with that of $\ln s_t$. Moreover, the volatile forecasts of $\ln s_t$ and $\ln M_t^\tau$ offset each other to fit to the actual exchange rate data $\ln S_t$. As a result, the forecast of the permanent money supply differential turns out to be as volatile as that of the money demand shock differential.

Finally, remember the decomposition of the money supply differential into the permanent and transitory components $\ln M_t \equiv \ln M_t^\tau + \ln m_t$. With this decomposition the Kalman filter interprets that the forecast of the money supply differential $\ln M_t$ should be dominated only by the volatile forecast of the permanent money supply differential $\ln M_t^\tau$.[3] This is the direct reason why the HDF model fails to explain the money supply differential.[4]

In sum, the high discount factor, as in the HDF model, imposes counterfactual restrictions on the consumption differential and the money supply differential. In the Benchmark model, the

[3]The corresponding forecast error matches the transitory money supply differential $\ln m_t$. Indeed, the Kalman smoother of the transitory money supply shock equals the forecast errors of both the money supply differential and the interest rate differential.

[4]This result does not depend crucially on the identification assumption of the white noise transitory money supply differential $\ln m_t$. Even when I allow for a high serial correlation of this shock, the flat money demand function requires large volatilities of the monetary demand shock differential and results in a worse fit of the HDF toward the money supply differential than the Benchmark model.

two aspects of the data require β to be much lower than the prior value of the HDF model. These restrictions, indeed, are so strictly binding that they jointly pin down the low value of β with a high degree of precision, as shown by the corresponding small posterior standard deviation reported in Table 2.

Appendix F. A two-country model with non-tradable goods

F.1. The model

In this augmented model, each country is endowed with a representative household whose objective is the lifetime money-in-utility

$$\sum_{j=0}^{\infty} \beta^j E_t \left\{ \ln C_{i,t+j} + \phi_{i,t+j} \ln \left(\frac{M_{i,t+j}}{P_{i,t+j}} \right) \right\}, \quad 0 < \beta < 1, \quad \text{for } i = h, f,$$

where $C_{i,t}$, $M_{i,t}$, and $P_{i,t}$ represent the ith country's consumption, money stock, and price index, respectively. The money-in-utility function is subject to a persistent money demand shock $\phi_{i,t}$. To extend the basic model of this paper with non-tradable goods, I assume that in each country, the representative household consumes both tradable and non-tradable goods, $C_{T,i,t}$ and $C_{N,i,t}$ for $i = h, f$, and raises utility from the Cobb-Douglas type consumption aggregator

$$C_{i,t} = G(C_{T,i,t})^{\alpha}(C_{N,i,t})^{1-\alpha}, \quad 0 < \alpha < 1, \quad \text{for } i = h, f,$$

where G is a constant.

Notice that the expenditure minimization problem across the tradable and non-tradable goods derives the following demand functions for the two goods

$$C_{T,i,t} = G^{-1} \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \left(\frac{P_{T,i,t}}{P_{N,i,t}} \right)^{\alpha-1} C_{i,t}, \quad \text{and} \quad C_{N,i,t} = G^{-1} \left(\frac{\alpha}{1-\alpha} \right)^{-\alpha} \left(\frac{P_{T,i,t}}{P_{N,i,t}} \right)^{\alpha} C_{i,t}, \quad \text{(F.1)}$$

where $P_{T,i,t}$ and $P_{N,i,t}$ are the prices of the tradable and non-tradable goods, respectively. When constant G is set to $\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{\alpha}$, the unit price of the consumption basket, $P_{i,t}$, is $(P_{T,i,t})^{\alpha} (P_{N,i,t})^{1-\alpha}$.

Moreover, due to the property of the constant elastic function, the expenditure share of the tradable good is proportional to the total consumption expenditure $P_{T,i,t}C_{T,i,t} = \omega P_{i,t}C_{i,t}$ where $\omega \in (0,1)$.

Below I follow the conventional assumption that the law of one price holds in the tradable goods across the two countries: $S_t P_{T,f,t} = P_{T,h,t}$. It is the straightforward to show that the real exchange rate q_t is given as

$$q_t \equiv \frac{S_t P_{f,t}}{P_{h,t}} = \frac{S_t P_{T,f,t}^{\alpha} P_{N,f,t}^{1-\alpha}}{P_{T,f,t}^{\alpha} P_{N,f,t}^{1-\alpha}} = \frac{\left(\frac{P_{N,f,t}}{P_{T,f,t}}\right)^{1-\alpha}}{\left(\frac{P_{N,h,t}}{P_{T,h,t}}\right)^{1-\alpha}} = \left(\frac{C_{N,h,t}}{C_{N,f,t}}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{C_{f,t}}{C_{h,t}}\right)^{\frac{1-\alpha}{\alpha}}, \tag{F.2}$$

where the last equality is obtained by using the demand functions of the non-tradable goods, equation (F.1).

The representative households in the home and foreign countries maximize their lifetime utility functions subject to the home budget constraint

$$B_{h,t}^{h} + S_t B_{h,t}^{f} + P_{h,t}C_{h,t} + M_{h,t} = (1 + r_{h,t-1}^{h})B_{h,t-1}^{h} + S_t(1 + r_{h,t-1}^{f})B_{h,t-1}^{f} + M_{h,t-1} + P_{h,t}Y_{h,t} + T_{h,t},$$

and its foreign counterpart

$$\frac{B_{f,t}^{h}}{S_t} + B_{f,t}^{f} + P_{f,t}C_{f,t} + M_{f,t} = (1 + r_{f,t-1}^{h})\frac{B_{f,t-1}^{h}}{S_t} + (1 + r_{f,t-1}^{f})B_{f,t-1}^{f} + M_{f,t-1} + P_{f,t}Y_{f,t} + T_{f,t},$$

respectively, where $B_{i,t}^{l}$, $r_{i,t}^{l}$, $Y_{i,t}$, $T_{i,t}$, and S_t denote the ith country's holdings of the lth country's nominal bonds at the end of time t, the ith county's returns on the lth country's bonds, the ith country's output level, the ith country's government transfers, and the level of the bilateral nominal exchange rate, respectively.

Each country's output $Y_{i,t}$ is given as an exogenous endowment that consists of the tradable and non-tradable endowments with a Cobb-Douglas type aggregator: $Y_{i,t} = (Y_{T,i,t})^{\alpha} (Y_{N,i,t})^{1-\alpha}$. Notice that $P_{i,t}Y_{i,t} = P_{T,i,t}Y_{T,i,t} + P_{N,i,t}Y_{N,i,t}$ holds. The stochastic processes of the tradable and non-tradable endowments are $Y_{T,i,t} = y_{T,i,t}A_{T,i,t}$ and $Y_{N,i,t} = y_{N,i,t}A_{N,i,t}$, where $y_{T,i,t}$ and $y_{N,i,t}$ are the transitory components and $A_{T,i,t}$ and $A_{N,i,t}$ are the permanent components of the tradable and non-tradable goods. Below, I interpret the permanent components as the TFPs in the underlying production technologies of the two goods.

The first-order necessary conditions (FONCs) of the home country's household are given by the budget constraint, the Euler equation

$$\frac{1}{P_{h,t}C_{h,t}} = \beta(1 + r^h_{h,t})E_t\left(\frac{1}{P_{h,t+1}C_{h,t+1}}\right),$$

the utility-based uncovered parity condition (UIP)

$$(1 + r^h_{h,t})E_t\left(\frac{1}{P_{h,t+1}C_{h,t+1}}\right) = \frac{(1 + r^f_{h,t})}{S_t}E_t\left(\frac{S_{t+1}}{P_{h,t+1}C_{h,t+1}}\right),$$

and the money demand function

$$\frac{M_{h,t}}{P_{h,t}} = \phi_{h,t}\left(\frac{1 + r^h_{h,t}}{r^h_{h,t}}\right)C_{h,t}.$$

The foreign country's FONC counterparts are the budget constraint, the Euler equation

$$\frac{1}{P_{f,t}C_{f,t}} = \beta(1 + r^f_{f,t})E_t\left(\frac{1}{P_{f,t+1}C_{f,t+1}}\right),$$

the utility-based uncovered parity condition (UIP)

$$(1 + r^h_{f,t})E_t\left(\frac{1}{S_{t+1}P_{f,t+1}C_{f,t+1}}\right) = \frac{(1 + r^f_{f,t})}{S_t}E_t\left(\frac{1}{P_{f,t+1}C_{f,t+1}}\right),$$

and the money demand function

$$\frac{M_{f,t}}{P_{f,t}} = \phi_{f,t}\left(\frac{1 + r^f_{f,t}}{r^f_{f,t}}\right)C_{f,t}.$$

Each country's government transfers the seigniorage to the household as a lump-sum. Hence, the government's budget constraint is

$$M_{i,t} - M_{i,t-1} = T_{i,t}, \quad \text{for } i = h, f.$$

The money supply $M_{i,t}$ is specified to consist of permanent and transitory components, $M_{i,t}^{\tau}$ and $m_{i,t}$: $M_{i,t} \equiv m_{i,t} M_{i,t}^{\tau}$ for $i = h, f$.

To close the model within an incomplete international financial market, I allow for a debt-elastic risk premium in the interest rates faced only by the home country:

$$r_{h,t}^{l} = r_{w,t}^{l}[1 + \psi\{\exp(-B_{h,t}^{l}/M_{l,t}^{\tau} + \bar{d}) - 1\}], \quad \bar{d} \leq 0, \quad \psi > 0, \quad \text{for} \quad l = h, f$$

where $r_{w,t}^{l}$ is the equilibrium world interest rate of the lth country's bond. The risk premium is given as an externality: The household does not take into account the effect of the debt position on the risk premium when maximizing the lifetime utility function. On the other hand, I do not attach a risk premium to the foreign country's interest rates: $r_{f,t}^{l} = r_{w,t}^{l}$ for $l = h, f$.

The market-clearing conditions of the two countries' bond markets are

$$B_{h,t}^{h} + B_{f,t}^{h} = 0 \quad \text{and} \quad B_{h,t}^{f} + B_{f,t}^{f} = 0,$$

i.e., along an equilibrium path, the world net supply of nominal bonds is zero on a period-by-period basis. Also the non-tradable good market is cleared in each country:

$$C_{N,i,t} = Y_{N,i,t}, \quad \text{for} i = h, f.$$

The logarithm of the permanent component of the money supply, $\ln M_{i,t}^{\tau}$, is I(1) for $i = h, f$, and the cross-country differential in the permanent component of money supply, $\ln M_{h,t}^{\tau} - \ln M_{f,t}^{\tau}$, is also I(1). I specify each country's monetary growth rate $\Delta \ln M_{i,t}^{\tau}$ to be an independent AR(1) process:

$$\Delta \ln M_{i,t}^{\tau} = (1-\rho_M)\ln \gamma_M + \rho_M \Delta \ln M_{i,t-1}^{\tau} + \epsilon_{M,t}^{i}, \quad \text{for } i = h, f.$$

where $\ln \gamma_M$ and ρ_M are the mean and AR root, respectively, of the money supply growth rate common to the two countries.

I assume that on the one hand each country's TFPs are I(1). On the other hand, the cross-country TFP differentials, $\ln a_{T,t} \equiv \ln A_{T,h,t} - \ln A_{T,f,t}$ and $\ln a_{N,t} \equiv \ln A_{N,h,t} - \ln A_{N,f,t}$, are assumed to be I(0) to guarantee the balanced growth path. These two requirements jointly imply that the TFPs of the home country must be cointegrated with that of the foreign country: for $j = T, N$

$$\Delta \ln A_{j,h,t} = \ln \gamma_j - \frac{\lambda_j}{2}(\ln A_{j,h,t-1} - \ln A_{j,f,t-1}) + \epsilon_{j,A,t}^{h},$$
$$\Delta \ln A_{j,f,t} = \ln \gamma_j + \frac{\lambda_j}{2}(\ln A_{j,h,t-1} - \ln A_{j,f,t-1}) + \epsilon_{j,A,t}^{f},$$

where $\gamma_{j,A} > 1$ is the common drift term and $\lambda_j \in [0,1)$ is the adjustment speed of the error correction mechanism (ECM). The ECMs imply that the cross-country TFP differentials are I(0) because

$$\ln a_{j,t} = (1-\lambda_j)\ln a_{j,t-1} + \epsilon_{j,A,t}^{h} - \epsilon_{j,A,t}^{f}.$$

Importantly, if the adjustment speed λ_j is sufficiently close to zero, the cross-country TFP differential can be realized near I(1), as maintained by NR.

The stochastic process of the logarithm of the transitory output component for each country, $\ln y_{j,i,t}$, is specified as the following AR(1) process:

$$\ln y_{j,i,t} = (1-\rho_j)\ln y_i + \rho_j \ln y_{j,i,t-1} + \epsilon_{j,y,t}^{i},$$

for $i = h, f$. Similarly, the stochastic process of the logarithm of the transitory money supply component for each country, $\ln m_{i,t}$, is specified as the following AR(1) process:

$$\ln m_{i,t} = (1-\rho_m)\ln m_i + \rho_m \ln m_{i,t-1} + \epsilon_{m,t}^{i},$$

for $i = h, f$. The three other structural shocks, the home and foreign money demand shocks $\phi_{h,t}$ and $\phi_{f,t}$ follow persistent stationary processes. Specifically, they are characterized by AR(1) processes in terms of the following logarithm:

$$\ln \phi_{i,t} = (1 - \rho_\phi) \ln \phi + \rho_\phi \ln \phi_{i,t-1} + \epsilon^i_{\phi,t},$$

Throughout this paper, I assume that all structural shocks are distributed independently.

E.2. The log-linear approximation of the stochastically de-trended system

Define stochastically de-trended variables as $c_{i,t} \equiv C_{i,t}/A_{i,t}$ $c_{j,t,t} \equiv C_{j,i,t}/A_{j,i,t}$, $p_{i,t} \equiv P_{i,t}A_{i,t}/M^\tau_{i,t}$, $p_{j,i,t} \equiv P_{j,i,t}A_{j,i,t}/M^\tau_{i,t}$, $b^l_{i,t} \equiv B^l_{i,t}/M^\tau_{l,t}$, $\gamma^i_{M,t} \equiv M^\tau_{i,t}/M^\tau_{i,t-1}$, and $s_t \equiv S_t M^\tau_{f,t}/M^\tau_{h,t}$. Taking the stochastic de-trending of the FONCs, I construct the stochastically de-trended system of the FONCs as follows.

The stochastically de-trended versions of the FONCs of the home country consist of the budget constraint

$$\omega p_{h,t} c_{h,t} + b^h_{h,t} + s_t b^f_{h,t} = \frac{(1 + r^h_{h,t-1}) b^h_{h,t-1}}{\gamma^h_{M,t}} + \frac{(1 + r^f_{h,t-1}) s_t b^f_{h,t-1}}{\gamma^f_{M,t}} + p_{T,h,t} y_{T,h,t};$$

the Euler equation

$$\frac{1}{p_{h,t} c_{h,t}} = \beta (1 + r^h_{h,t}) E_t \left(\frac{1}{\gamma^h_{M,t+1} p_{h,t+1} c_{h,t+1}} \right);$$

the UIP condition

$$s_t (1 + r^h_{h,t}) E_t \left(\frac{1}{p_{h,t+1} c_{h,t+1} \gamma^h_{M,t+1}} \right) = (1 + r^f_{h,t}) E_t \left(\frac{s_{t+1}}{p_{h,t+1} c_{h,t+1} \gamma^f_{M,t+1}} \right);$$

the money demand function

$$\frac{m_{h,t}}{p_{h,t}} = \phi_{h,t} c_{h,t} \left(\frac{1 + r^h_{h,t}}{r^h_{h,t}} \right);$$

the risk premiums

$$r^h_{h,t} = r^h_{w,t} [1 + \psi \{ \exp(-b^h_{h,t} + \bar{d}) - 1 \}],$$

and

$$r^f_{h,t} = r^f_{w,t}[1 + \psi\{\exp(-b^f_{h,t} + \bar{d}) - 1\}].$$

Similarly, the stochastically de-trended versions of the FONCs of the foreign country consist of the budget constraint

$$\omega s_t p_{f,t} c_{f,t} - s_t b^f_{h,t} - b^h_{h,t} = -\frac{(1 + r^f_{w,t-1}) s_t b^f_{h,t-1}}{\gamma^f_{M,t}} - \frac{(1 + r^h_{w,t-1}) b^h_{h,t-1}}{\gamma^h_{M,t}} + s_t p_{T,f,t} y_{T,f,t};$$

the Euler equation

$$\frac{1}{p_{f,t} c_{f,t}} = \beta(1 + r^f_{w,t}) E_t \frac{1}{\gamma^f_{M,t+1} p_{f,t+1} c_{f,t+1}};$$

the UIP condition

$$s_t(1 + r^h_{w,t}) E_t \left(\frac{1}{s_{t+1} p_{f,t+1} c_{f,t+1} \gamma^h_{M,t+1}} \right) = (1 + r^f_{w,t}) E_t \left(\frac{1}{p_{f,t+1} c_{f,t+1} \gamma^f_{M,t+1}} \right);$$

and the money demand function

$$\frac{m_{f,t}}{p_{f,t}} = \phi_{f,t} c_{f,t} \left(\frac{1 + r^f_{w,t}}{r^f_{w,t}} \right).$$

The real exchange rate is determined by

$$q_t = \left(\frac{c_{f,t}}{c_{h,t}} \right)^{\frac{1-\alpha}{\alpha}} \left(\frac{y_{N,h,t}}{y_{N,f,t}} \right)^{\frac{1-\alpha}{\alpha}} a_t^{\frac{\alpha-1}{\alpha}} a_{N,t}^{\frac{1-\alpha}{\alpha}}.$$

The home price of the tradable good is

$$p_{T,h,t} = p_{h,t} \left[(1 - \omega) \frac{c_{h,t}}{y_{N,h,t}} \right]^{\frac{\alpha-1}{\alpha}}$$

The stochastically detrended PPP deviation is $q_t = s_t p_{f,t} a_t / p_{h,t}$. Finally, the stochastically de-trended LOP of the tradable goods is $a_{T,t} s_t p_{T,f,t} = p_{T,h,t}$.

The resulting fourteen equations determine the fourteen endogenous variables $c_{h,t}$, $c_{f,t}$, $p_{h,t}$,

$p_{f,t}$, $p_{T,h,t}$, $p_{T,f,t}$, s_t, q_t, $b^h_{h,t}$, $b^f_{h,t}$, $r^h_{h,t}$, $r^f_{h,t}$, $r^h_{w,t}$, and $r^f_{w,t}$, given eight exogenous variables $\gamma^h_{M,t}$, $\gamma^f_{M,t}$, $a_t \equiv (a_{T,t})^\alpha (a_{N,t})^{1-\alpha}$, $m_{h,t}$, $m_{f,t}$, $y_{h,t}$, and $y_{f,t}$.

Let $\hat{x}$ denote a percentage deviation of any variable x_t from its deterministic steady state value x^*, $\hat{x} \equiv \ln x_t - \ln x^*$. Also, let $\tilde{x}$ denote a deviation of x from its deterministic steady state, $\tilde{x} = x - x^*$.[5] The log-linear approximation of the stochastically de-trended home budget constraint is

$$\begin{aligned}
&\omega p^*_h y_h(\hat{p}_{h,t} + \hat{c}_{h,t}) - p^*_{T,h} y_{T,h}(\hat{p}_{T,h,t} + \hat{y}_{h,t}) + \tilde{b}^h_{h,t} + \bar{d}(1-\beta^{-1}) s^* \hat{s}_t + s^* \tilde{b}^f_{h,t} \\
&\quad = \beta^{-1}\bar{d}[(1+\hat{r}^h_{h,t-1}) - \hat{\gamma}^h_{M,t}] + s^*\beta^{-1}\bar{d}[(1+\hat{r}^f_{h,t-1}) - \hat{\gamma}^f_{M,t}] + \beta^{-1}\tilde{b}^h_{h,t-1} + s^*\beta^{-1}\tilde{b}^f_{h,t-1}; \quad \text{(F.3)}
\end{aligned}$$

that of the home Euler equation is

$$\hat{p}_{h,t} + \hat{c}_{h,t} + (1+\hat{r}^h_{h,t}) = E_t(\hat{p}_{h,t+1} + \hat{c}_{h,t+1} + \hat{\gamma}^h_{M,t+1}); \quad \text{(F.4)}$$

that of the home UIP condition is

$$E_t \hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}^h_{h,t}) - (1+\hat{r}^f_{h,t}) - E_t(\hat{\gamma}^h_{M,t+1} - \hat{\gamma}^f_{M,t+1}); \quad \text{(F.5)}$$

and that of the home money demand function is

$$\hat{p}_{h,t} + \hat{c}_{h,t} - \hat{m}_{h,t} = \frac{1}{r^*}(1+\hat{r}^h_{h,t}) - \hat{\phi}_{h,t}. \quad \text{(F.6)}$$

The foreign country's counterparts are the log-linear approximation of the stochastically de-trended

[5] In particular, for an interest rate r_t, $(1+\hat{r}_t) = (r_t - r^*)/(1+r^*)$.

foreign budget constraint

$$
\begin{aligned}
&\omega p_f^* c_f^* (\hat{p}_{f,t} + \hat{c}_{f,t}) - p_{T,f}^* y_{T,f} (\hat{p}_{T,f,t} + \hat{y}_{f,t}) - \tilde{b}_{h,t}^h - \bar{d}(1-\beta^{-1}) s^* \hat{s}_t - s^* \tilde{b}_{h,t}^f \\
&\quad = -\beta^{-1}\bar{d}[(1+\hat{r}_{w,t-1}^h) - \hat{\gamma}_{M,t}^h] - s^*\beta^{-1}\bar{d}[(1+\hat{r}_{w,t-1}^f) - \hat{\gamma}_{M,t}^f] - \beta^{-1}\tilde{b}_{h,t-1}^h - s^*\beta^{-1}\tilde{b}_{h,t-1}^f; \quad \text{(F.7)}
\end{aligned}
$$

that of the foreign Euler equation

$$
\hat{p}_{f,t} + \hat{c}_{f,t} + (1+\hat{r}_{w,t}^f) = E_t(\hat{s}_{t+1} + \hat{p}_{f,t+1} + \hat{c}_{f,t+1} + \hat{\gamma}_{M,t+1}^f); \tag{F.8}
$$

that of the foreign UIP condition

$$
E_t\hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}_{w,t}^h) - (1+\hat{r}_{w,t}^f) - E_t(\hat{\gamma}_{M,t+1}^h - \hat{\gamma}_{M,t+1}^f); \tag{F.9}
$$

and that of the home money demand function

$$
\hat{p}_{f,t} + \hat{c}_{f,t} - \hat{m}_{f,t} = \frac{1}{r^*}(1+\hat{r}_{w,t}^f) - \hat{\phi}_{f,t}. \tag{F.10}
$$

The log-linear approximations of the home country's interest rates are

$$
(1+\hat{r}_{h,t}^h) = (1+\hat{r}_{w,t}^h) - \psi(1-\kappa)\tilde{b}_{h,t}^h, \quad \text{and} \quad (1+\hat{r}_{h,t}^f) = (1+\hat{r}_{w,t}^f) - \psi(1-\kappa)\tilde{b}_{h,t}^f. \tag{F.11}
$$

The log-linear approximation of the real exchange rate is

$$
\hat{q}_t = \frac{1-\alpha}{\alpha}(\hat{c}_{f,t} - \hat{c}_{h,t}) + \frac{1-\alpha}{\alpha}(\hat{y}_{N,h,t} - \hat{y}_{N,f,t}) - (1-\alpha)(\hat{a}_{T,t} - \hat{a}_{N,t}), \tag{F.12}
$$

The log-linear approximation of the home price of the tradable good is

$$
\hat{p}_{T,h,t} = \hat{p}_{h,t} - \frac{1-\alpha}{\alpha}(\hat{c}_{h,t} - \hat{y}_{N,h,t}). \tag{F.13}
$$

The log-linear approximation of the PPP deviation is

$$\hat{q}_t = \hat{s}_t + \hat{p}_{f,t} - \hat{p}_{h,t} + \alpha \hat{a}_{T,t} + (1-\alpha)\hat{a}_{N,t}. \tag{F.14}$$

Finally, the log-linear approximation of the LOP is

$$\hat{a}_{T,t} + \hat{s}_t + \hat{p}_{T,f,t} = \hat{p}_{T,h,t}. \tag{F.15}$$

Notice that the home interest rates (F.11) redefine the home UIP condition (F.5) as

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1+\hat{r}^h_{w,t}) - (1+\hat{r}^f_{w,t}) - \psi(1-\kappa)(\tilde{b}^h_{h,t} - \tilde{b}^f_{h,t}) - E_t(\hat{\gamma}^h_{M,t+1} - \hat{\gamma}^f_{M,t+1}).$$

Comparing the above home UIP condition with the foreign UIP condition (F.9) implies that the home and foreign bonds are perfectly substitutable along the equilibrium path. Hence, the equilibrium condition $\tilde{b}_t \equiv \tilde{b}^h_{h,t} = \tilde{b}^f_{h,t}$ holds.

F.3. Equilibrium random-walk property of nominal exchange rates

Let c_t, $y_{T,t}$, $y_{N,t}$, m_t, and ϕ_t denote the de-trended consumption ratio, the de-trended tradable output ratio, the de-trended non-tradable output ratio, the transitory money supply ratio, the money demand shock ratio between the two countries, $c_t \equiv c_{h,t}/c_{f,t}$, $y_{T,t} \equiv y_{T,h,t}/y_{T,f,t}$, $y_{N,t} \equiv y_{N,h,t}/y_{N,f,t}$, $m_t \equiv m_{h,t}/m_{f,t}$, and $\phi_t \equiv \phi_{h,t}/\phi_{f,t}$, respectively. Furthermore, let M^τ_t denote the ratio of the permanent money supplies of the home and foreign countries $M^\tau_{h,t}/M^\tau_{f,t}$; let M_t foreign money supplies of the home to the foreign countries $M_{h,t}/M_{f,t} \equiv m_t M^\tau_t$; let $\gamma_{M,t}$ denote the ratio of the permanent money supply growth rate $\gamma_{M,t} \equiv \gamma^h_{M,t}/\gamma^f_{M,t}$; let C_t denote the ratio of the consumptions of the home and foreign countries $C_{h,t}/C_{f,t}$. Below, the steady state value of the nominal market discount factor is denoted by $\kappa \equiv 1/(1+r^*) = \beta/\gamma_M$. Under the symmetric case with $\bar{d} = 0$, FONCs (F.3)-(F.15) are degenerated to the following four expectational difference equations with respect to the four endogenous variables $\hat{s}_t$, $\hat{q}_t$, $\hat{c}_t$, and $\tilde{b}_t$, given the six exogenous variables $\hat{\gamma}_{M,t}$, $\hat{m}_t$, $\hat{a}_{T,t}$,

$\hat{a}_{N,t}$, $\hat{y}_{T,t}$, $\hat{y}_{N,t}$, and $\hat{\phi}_t$:

$$
\begin{aligned}
\hat{s}_t &= \kappa E_t \hat{s}_{t+1} - (1-\kappa)(\hat{c}_t - \hat{q}_t + \hat{a}_t) + (1-\kappa)(\hat{m}_t - \hat{\phi}_t) + \kappa E_t \hat{\gamma}_{M,t+1} - \psi\kappa(1-\kappa)\tilde{b}_t, \\
\hat{s}_t + \hat{c}_t - \hat{q}_t + \hat{a}_t &= \kappa E_t(\hat{s}_{t+1} + \hat{c}_{t+1} - \hat{q}_{t+1} + \hat{a}_{t+1}) + (1-\kappa)(\hat{m}_t - \hat{\phi}_t) + \kappa E_t \hat{\gamma}_{M,t+1}, \\
\tilde{b}_t &= \beta^{-1}\tilde{b}_{t-1} + p_T^* y_T^* (\hat{a}_{T,t} + \hat{y}_{T,t} - \hat{c}_t + \hat{q}_t - \hat{a}_t), \\
\hat{q}_t &= \frac{\alpha-1}{\alpha}\hat{c}_t + \frac{1-\alpha}{\alpha}\hat{y}_{N,t} - (1-\alpha)(\hat{a}_{T,t} - \hat{a}_{N,t}),
\end{aligned}
\tag{F.16}
$$

where $y_T^* = y_T/4$ and $y_T = y_{T,h} = y_{T,f}$. In particular, the first equation of the linear rational expectations (LRE) system (F.16) represents the stochastically de-treded UIP after solving the interest rate differential through the money demand functions of the two countries; the second equation the cross-country difference in the Euler equation; the third equation the law of motion of net foreign asset position; and the fourth equation the real exchange rate.

The first and second equations of LRE system (F.16) result in

$$
\hat{c}_t + \hat{a}_t - \hat{q}_t = E_t(\hat{c}_{t+1} + \hat{a}_{t+1} - \hat{q}_{t+1}) + \psi(1-\kappa)\tilde{b}_t,
$$

The third equation of LRE system (F.16) and the above equation implies the second order difference equation of the net foreign asset position $\tilde{b}_t$

$$
E_t\tilde{b}_{t+1} - [1 + \beta^{-1} + p_T^* y_T^* \psi(1-\kappa)]\tilde{b}_t + \beta^{-1}\tilde{b}_{t-1} = -p_T^* y_T^* \lambda_T \hat{a}_{T,t} + p_T^* y_T^* (\rho_T - 1)\hat{y}_{T,t}. \tag{F.17}
$$

It is straightforward to show that equation (F.17) has two roots, one of which is greater than one and the other of which is less than one. Without losing generality, let η denote the root that is less than one. Solving equation (F.17) by forward iterations then shows that the equilibrium international bond holdings level is determined by the following cross-equation restriction (CER):

$$\begin{aligned}\tilde{b}_t &= \eta \tilde{b}_{t-1} + \beta\eta\lambda_T p_T^* y_T^* \sum_{j=0}^{\infty} (\beta\eta)^j E_t \hat{a}_{T,t+j} + \beta\eta p_T^* y_T^* (1-\rho_T) \sum_{j=0}^{\infty} (\beta\eta)^j E_t \hat{y}_{T,t+j}, \\ &= \eta \tilde{b}_{t-1} + \frac{\beta\eta\lambda_T p_T^* y_T^*}{1-\beta\eta(1-\lambda_T)} \hat{a}_{T,t} + \frac{\beta\eta p_T^* y_T^* (1-\rho_T)}{1-\beta\eta\rho_T} \hat{y}_{T,t}. \end{aligned} \tag{F.18}$$

Substituting the fourth equation of LRE system (F.16) into the third equation yields $\tilde{b}_t = \beta^{-1}\tilde{b}_{t-1} + p_T^* y_T^* \left(\hat{y}_{T,t} + \frac{1-\alpha}{\alpha} \hat{y}_{N,t} - \frac{1}{\alpha} \hat{c}_t \right)$. CER (F.18) then implies that the equilibrium consumption ratio is

$$\hat{c}_t = \frac{\alpha(1-\eta\beta)}{p_T^* y_T^* \beta} \tilde{b}_{t-1} - \frac{\alpha\eta\beta\lambda_T}{1-\eta\beta(1-\lambda_T)} \hat{a}_{T,t} + \frac{\alpha(1-\eta\beta)}{1-\eta\beta\rho_T} \hat{y}_{T,t} + (1-\alpha)\hat{y}_{N,t}. \tag{F.19}$$

Using the fourth equation of LRE system (F.16), the equilibrium RER is determined as

$$\hat{q}_t = -\frac{(1-\alpha)(1-\eta\beta)}{p_T^* y_T^* \beta} \tilde{b}_{t-1} - \frac{(1-\alpha)(1-\eta\beta)}{1-\eta\beta(1-\lambda_T)} \hat{a}_{T,t} - \frac{(1-\alpha)(1-\eta\beta)}{1-\eta\beta\rho_T} \hat{y}_{T,t} + (1-\alpha)\ln Y_{N,t}. \tag{F.20}$$

Now a forward iteration of the second equation of LRE (F.16) toward an infinite future leads to

$$\begin{aligned}\hat{s}_t + \hat{c}_t + \hat{a}_t - \hat{q}_t &= \sum_{i=1}^{\infty} \kappa^i E_t \hat{\gamma}_{M,t+i} - (1-\kappa) \sum_{i=0}^{\infty} \kappa^i E_t (\hat{\phi}_{t+i} - \hat{m}_{t+i}), \\ &= \frac{\kappa\rho_M}{1-\kappa\rho_M} \hat{\gamma}_{M,t} - \frac{1-\kappa}{1-\kappa\rho_\phi} \hat{\phi}_t + \frac{1-\kappa}{1-\kappa\rho_m} \hat{m}_t \end{aligned}$$

Substituting equilibrium consumption ratio (F.19) and equilibrium RER (F.20) into the above equation provides the equilibrium de-trended nominal exchange rate

$$\hat{s}_t = \frac{\eta\beta - 1}{p_T^* y_T^* \beta} \tilde{b}_{t-1} + \frac{\kappa\rho_M}{1-\kappa\rho_M} \hat{\gamma}_{M,t} - \frac{1-\kappa}{1-\kappa\rho_\phi} \hat{\phi}_t + \frac{1-\kappa}{1-\kappa\rho_m} \hat{m}_t - \frac{1-\eta\beta}{1-\eta\beta(1-\lambda_T)} \hat{a}_{T,t} - \frac{1-\eta\beta}{1-\eta\beta\rho_T} \hat{y}_{T,t} \tag{F.21}$$

Finally, substituting the equilibrium de-trended nominal exchange rate (F.21) into the relation

$\ln S_t = \ln M_t^{\tau} + \hat{s}_t$ and taking the first difference of the result yields the equilibrium currency return

$$\Delta \ln S_t = \psi(1-\kappa)\tilde{b}_{t-1} + \frac{\rho_M(1-\kappa)}{1-\kappa\rho_M}\hat{\gamma}_{M,t-1} + \frac{(1-\kappa)(1-\rho_\phi)}{1-\kappa\rho_\phi}\hat{\phi}_{t-1} - \frac{(1-\kappa)(1-\rho_m)}{1-\kappa\rho_m}\hat{m}_{t-1}$$
$$+ \frac{1}{1-\kappa\rho_M}\epsilon_{M,t} - \frac{1-\kappa}{1-\kappa\rho_\phi}\epsilon_{\phi,t} + \frac{1-\kappa}{1-\kappa\rho_m}\epsilon_{m,t} - \frac{1-\eta\beta}{1-\eta\beta\rho_T}\epsilon_{y,t} - \frac{1-\eta\beta}{1-\eta\beta(1-\lambda_T)}\epsilon_{T,t}. \quad \text{(F.22)}$$

where $\epsilon_{M,t} \equiv \epsilon^h_{M,t} - \epsilon^f_{M,t}$, $\epsilon_{\phi,t} \equiv \epsilon^h_{\phi,t} - \epsilon^f_{\phi,t}$, $\epsilon_{m,t} \equiv \epsilon^h_{m,t} - \epsilon^f_{m,t}$, $\epsilon_{T,y,t} \equiv \epsilon^h_{T,y,t} - \epsilon^f_{T,y,t}$, and $\epsilon_{T,A,t} \equiv \epsilon^h_{T,A,t} - \epsilon^f_{T,A,t}$. Therefore, at the limit of the unit discount factor $(\kappa, \beta, \eta \to 1)$, the currency return follows a white noise process.

$$\lim_{\kappa,\beta,\eta\to 1} \Delta \ln S_t = \frac{1}{1-\kappa\rho_M}\epsilon_{M,t}.$$

F.4. Backus and Smith's puzzle at the limit

Recall that unwinding the stochastic trend into the equilibrium consumption ratio leads to $\ln C_t = \ln a_t + \hat{c}_t$. Equilibrium de-trended consumption ratio (F.19) then implies

$$\ln C_t = \frac{\alpha(1-\eta\beta)}{p_T^* y_T^* \beta}\tilde{b}_{t-1} + \frac{\alpha(1-\eta\beta)}{1-\eta\beta(1-\lambda_T)}\hat{a}_{T,t} + \frac{\alpha(1-\eta\beta)}{1-\eta\beta\rho_T}\hat{y}_{T,t} + (1-\alpha)\ln Y_{N,t}.$$

At the limit of the unit discount factor the equilibrium consumption differential turns out to be

$$\lim_{\kappa,\eta,\beta\to 1} \ln C_t = (1-\alpha)\ln Y_{N,t}$$

Similarly at the limit of the unit discount factor the RER becomes

$$\lim_{\kappa,\eta,\beta\to 1} \ln q_t = (1-\alpha)\ln Y_{N,t}$$

Therefore, the consumption differential is perfectly correlated with the RER at the limit.

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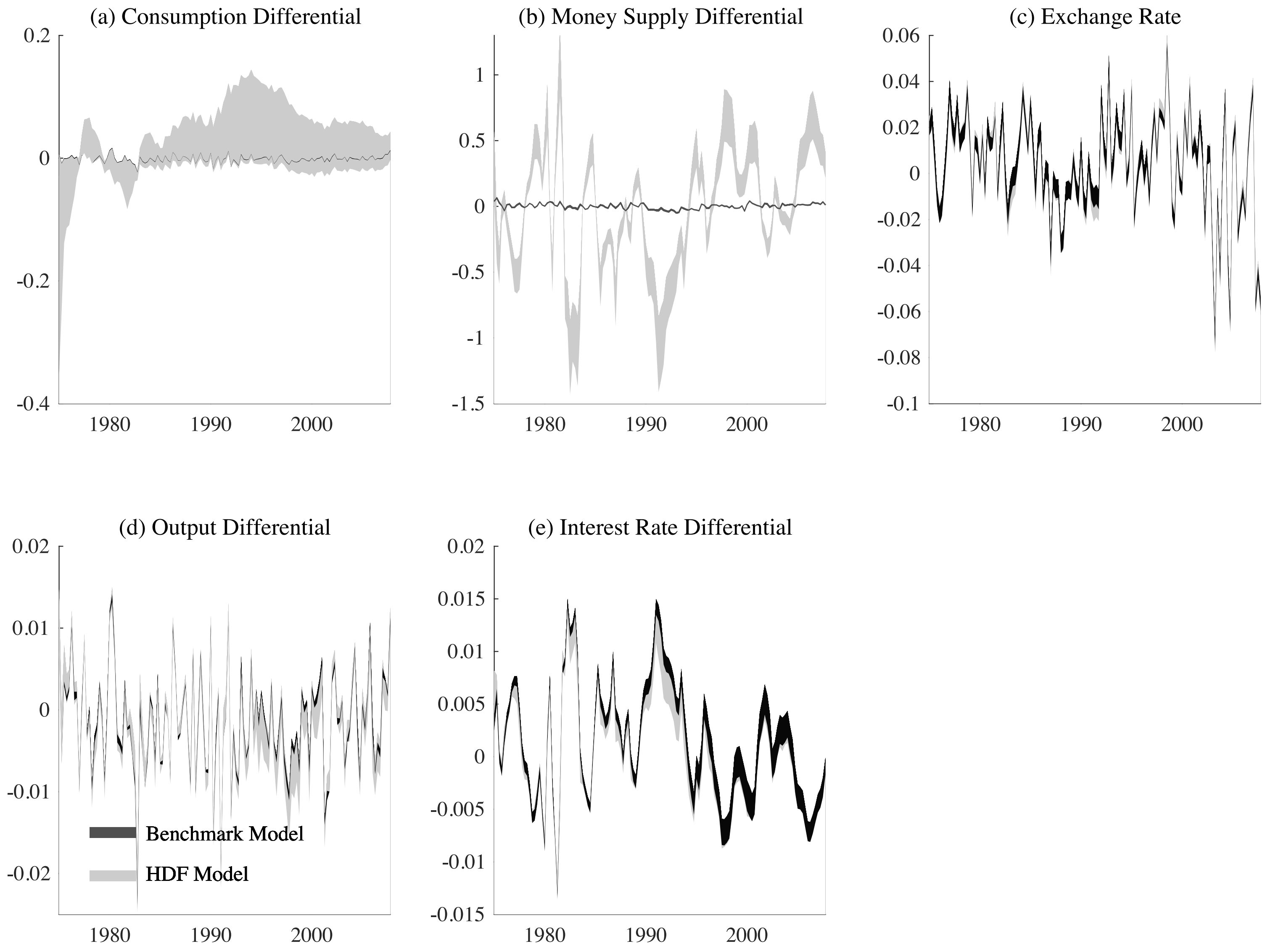

Figure A.1 : Forecast errors for observations. Note: In each window corresponding to each time series observation, the dark and light gray areas display the 95% Bayesian highest probability density (HPD) intervals of the one-period-ahead forecast error calculated through the Kalman forward recursion based on the state space representation of the Benchmark and high discount factor (HDF) models, respectively.

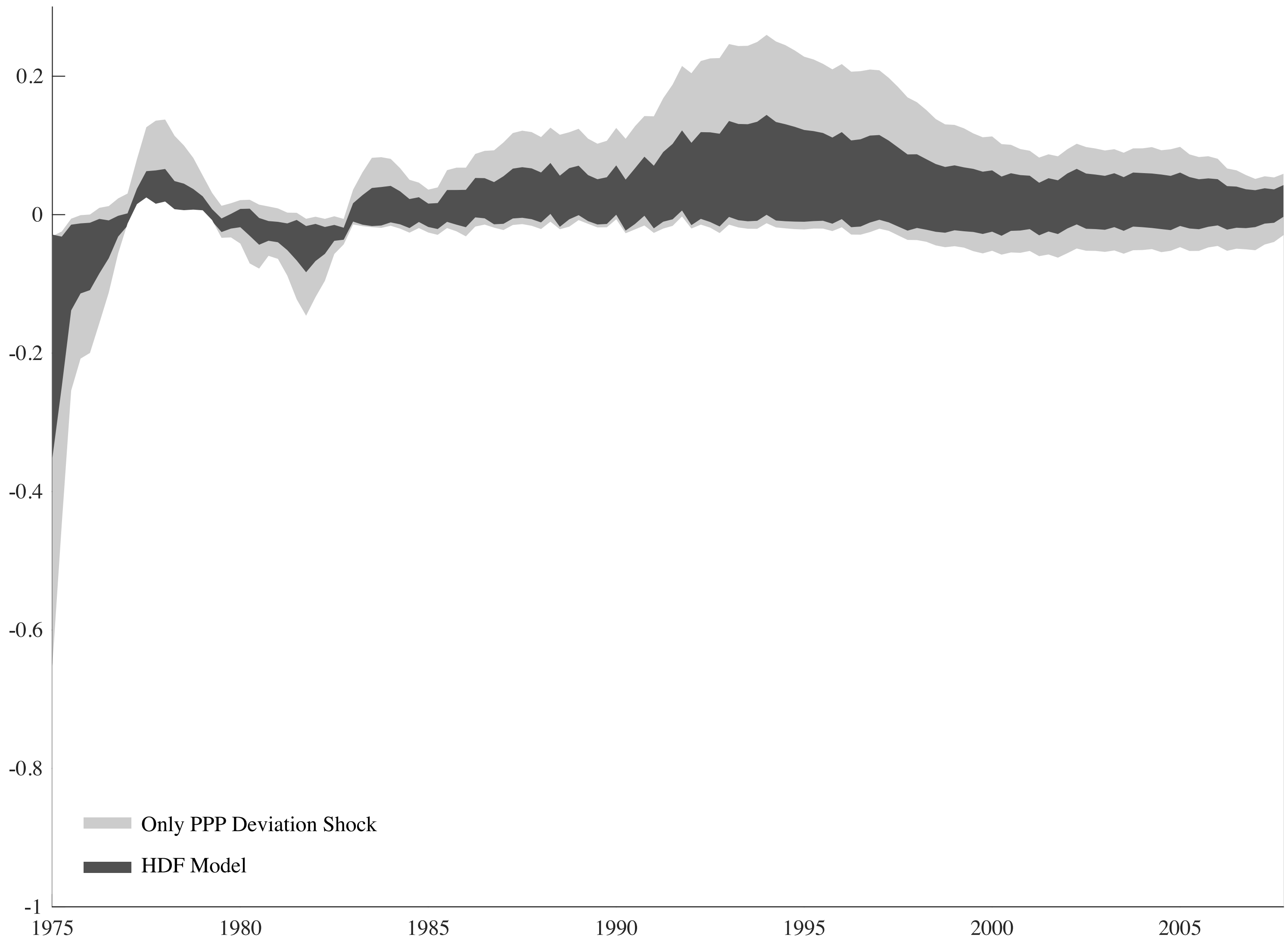

Figure A.2 : Forecast error of consumption differential Note: The dark gray area depicts the 95% Bayesian highest probability density (HPD) intervals attached to the forecast error of the consumption differential, which is implied by the high discount factor (HDF) model. The light gray area indicates the 95% HPD intervals of the forecast error of the consumption differential, which is generated only by the purchasing power parity (PPP) deviation shock.

Figure A.3 : Forecast errors for observations. Note: In each window corresponding to a particular observation, the dashed blue and dotted red lines display the 95% Bayesian highest probability density (HPD) intervals of the one-period-ahead forecast error calculated through the Kalman forward recursion based on the state space representation of the Benchmark and high discount factor (HDF) models, respectively.