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Extreme Lobbyists and Policy Convergence

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Abstract

We consider a two-candidate election model with campaign contributions. In the first stage of the game, each of two candidates chooses a policy position. In the second stage, each of n lobbyists chooses the amount of contribution to each candidate. The winning probability of each candidate depends on the total amount of contributions that she raised from the lobbyists. In any equilibrium of our model, only extreme lobbyists contribute at any subgame, and the policies converge on the unique equilibrium path. Our results suggest that extreme lobbyists and their contributions do not necessarily cause policies to diverge.

Keywords: Interest groups, campaign contributions, Hotelling model

JEL codes: C72, D72, D78

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1 Introduction

Electoral candidates run campaigns to influence the outcomes of the elections. One major determinant of the campaigns' success is contributions from interest groups. Contributions made to US presidential campaigns are quite large, thus attracting the attention of many media outlets. In the 2012 Presidential election, the winner Obama raised more than 720 million US dollars, and even the loser Romney raised about 450 million dollars. Furthermore, the candidates' policy positions seem to affect the flow of political contributions. For example, unsurprisingly, the vast majority of political contributions from gay and lesbian interest groups have gone to Democrats. In the 2012 election cycle, only 7 percent of this money went to a Republican national candidate or political committee.[1] Thus, given the impact of contributions, it is important for candidates to deliberate the way their policy positions affect the contributions, which in turn affects the outcome of the election. This paper studies how the outcomes of elections are affected by policy choices *through contributions by interest groups*.

To motivate our analysis, let us start by the observation that donors' ideological positions are often said to be extreme and polarized. For example, Barber (2016), McElwee (2016), and McElwee et al. (2016) report that such polarization occurs at various levels: presidential, senate, and mayoral elections. The effect of such a polarization on the manifest choices, however, is ambiguous. Some suggest that donors' polarization causes manifests' polarization (e.g., Verba et al. (1995) and Miller and Schofield (2003)), while others argue there is no such causal relationship (e.g., La Raja and Wiltse (2011)). This paper examines the theoretical foundation for the causal relationship. We find that only extreme lobbyists would donate, and uncover a novel force that political contributions have. Specifically, we argue that the possibilities of contributions by extreme lobbyists make the policies *converge*, not diverge. Thus, our analysis is consistent with the observation that donors are polarized, and provides a theoretical rationale for the argument that there be no causal relationship

[1] The numbers stated in this paragraph are cited from The Center for Responsible Politics (2018).

between donors' and manifests' polarizations.

Specifically, we characterize subgame perfect equilibria of a two-stage game in which two office-seeking candidates set policies in the first stage and then contributions are made by *n* interest groups in the second stage, taking into account the policy positions chosen in the first stage.[2] In order to focus on the effect of contributions on the election outcome, we postulate that the amounts of contributions are the sufficient statistic of the election outcome. Taking into account the effect that the policy choices have on the amount of contributions, candidates strategically choose their positions to maximize electability.

In our model, we show the *extremist dominance* result and the *policy convergence* result. The extremist dominance result states that in each subgame (i.e., for each realized policy profile on and off the equilibrium path), no interest groups except the two extreme groups (i.e., the leftmost and the rightmost groups) contribute to any candidates. This result is consistent with the observation that donors are polarized. However, polarization of active donors does not necessary imply that of policies: The policy convergence result states that the two candidates set the same policy in any subgame perfect equilibrium of our model.

To see the intuition for extremist dominance, consider two left-wing lobbyists where one is more extreme than the other, as in Figure 1. First of all, neither lobbyist has an incentive to contribute to the right-wing candidate. To see who will contribute to the left-wing candidate, notice that the difference of payoffs that the extreme lobbyist perceives from the two candidates is larger than that of the moderate lobbyist.[3] Thus the extreme lobbyist's marginal benefit of contribution is always larger than that of the moderate lobbyist. Given that we measure the utilities in common monetary units, this entails that the marginal cost of contribution cannot be equal to the marginal benefits for both lobbyists, implying that the moderate lobbyist's contribution must be zero.

[2]The assumption that the candidates' policy positions affect the contributors' (utilities and hence) decisions is consistent with the empirical literature (see, e.g., Bonica (2014) and Barber et al. (2017)). We however note that this is not a view for which there is a consensus: Hill and Huber (2016) suggest that ideology plays no role when individual donors distinguish among same-party candidates.

[3]Formally, this follows from the single-crossing condition that we will assume.

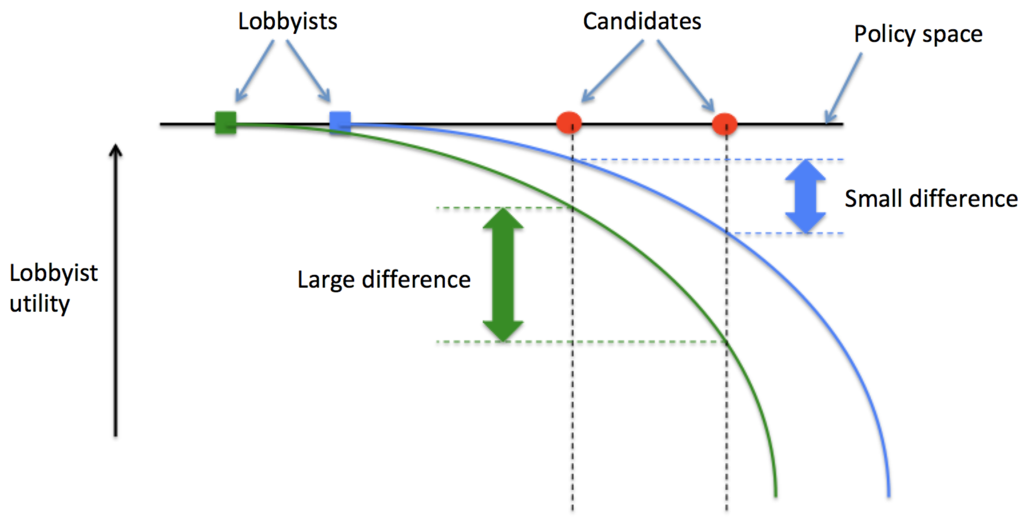

Figure 1: Graphical intuition for extremist dominance: Green and blue curves correspond to the two lobbyists' utility functions. The blue lobbyist perceives a smaller utility difference than the green lobbyist, and hence does not contribute in equilibrium.

Given extremist dominance, the heart of the intuition for policy convergence can be explained as follows: When two policies differ, a candidate's approach to the other policy has asymmetric effects on the marginal benefits from contributions by the two extreme interest groups. Specifically, consider the situation where the left-wing candidate moves to the right by a small amount. This movement decreases the amounts of contributions from both lobbyists, but our assumption that the utility functions are concave implies that, as Figure 2 illustrates, the left-wing interest group perceives a small change in payoffs from such a movement, compared to the change that the right-wing interest group perceives. Thus the amount that the right-wing lobbyist would decrease the contribution by is more significant than that of the left-wing lobbyist. This implies, with some assumptions on the winning probability function, an increase of the winning probability for the left candidate.[4]

Examining this from another angle may help: What prevents a candidate from moving towards

[4]The actual proof has more subtlety, and we formalize it in the analysis that follows.

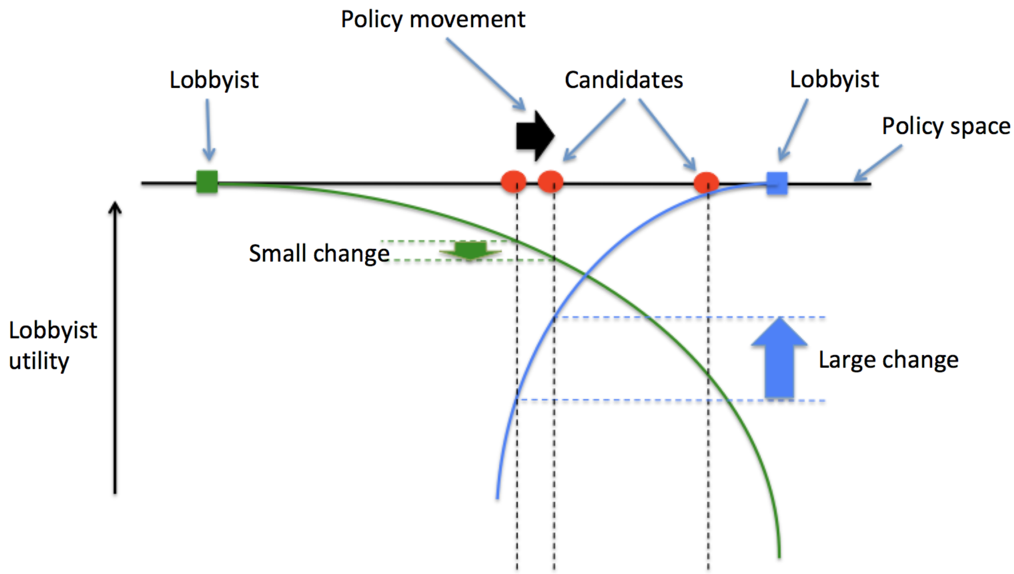

Figure 2: Graphical intuition for policy convergence: Green and blue curves correspond to the two lobbyists' utility functions (the utility functions are allowed to be asymmetric). When the left candidate moves to the right, the green lobbyist perceives a small change of the utility while the blue lobbyist perceives a large difference. This, combined with our assumption on the winning probability function, implies policy convergence.

her own donor? An usual argument would postulate that moving towards the own donor would increase the contribution from that donor. This is true in our model. At the same time, however, it also increases the contribution from the other donor to the opposition candidate. An example of this is Meg Whitman's contribution to Hilary Clinton, calling Donald Trump an "authoritarian character" and a threat to democracy (Becker, 2016). Another example would be the £400,000 donation to Best for Britain for an anti-Brexit campaign by George Soros, who is accused of "forcing his views on societies and even destroying them" (Henley, 2018). In our model, such a countervailing effect prevents a candidate from catering to the extreme donor.

To understand the contribution of this study, it would be useful to compare it with the "collective policies" model of Baron (1994).[5] He considers a two-candidate two-lobbyist model with the

[5] Baron (1994) studies the cases of "collective policies" and of "particularistic policies," and his analysis of the latter predicts policy divergence. The reason for the divergence is that in his model, given that interest group k contributes to candidate i, the amount of such a contribution is independent of the other candidate $-i$'s position. In our model,

same timeline as ours and also obtains a policy-convergence result, but for a different reason. The key assumptions behind his result are that (i) two types of voters cast votes, where “uninformed voters" only respond to campaign funds, and “informed voters" only respond to the policy positions, and that (ii) the utility functions of interest groups are linear.[6] The linearity implies that the ratio of utility differences that interest groups perceive between two candidates is independent of the policies chosen by the candidates. With his modeling assumptions, this further implies that the ratio of the campaign funds raised by the candidates is constant, and consequently, the voting behavior of the uninformed voters is also independent of policy profile. Hence, candidates only compete for votes from the informed voters. Therefore, the standard median voter theorem applies and leads to policy convergence.

Put differently, in Baron’s (1994) model of collective policies, political contributions have no effect at all to determine equilibrium policies due to the linearity of lobbyist utilities. In contrast, our model assumes concave lobbyist utilities, and we show that in such a setting, different policy profiles induce different winning probabilities *through those contributions* and hence, policy convergence arises from the competition for uninformed voters.

Persson and Tabellini (2000, Section 7.5.1) also offer a model of political contribution that has the same timing structure as ours and predicts policy convergence. Their model has two key assumptions; namely, (i) each lobbyist’s maximization problem is independent of other lobbyists’ contribution amount, and (ii) each candidate’s maximization problem is independent of other candidates’ policy position. Property (i) implies that extremist dominance fails (all lobbyists contribute whenever two policies are different), while property (ii) implies that policy convergence holds. Although policy convergence is common between the two models, their logic is different from ours because in our setting each candidate’s best response is to position as close as possible

contrastingly, k’s contribution to i increases when $-i$’s position moves away from k’s ideal policy, and this diminishes $-i$’s incentive to diverge.

[6] Baron (1994) assumes that each candidate is associated with one interest group, and each interest group contributes the amount that changes linearly with the policy position. One way to interpret this specification is to consider interest groups situated at the extreme points of the policy space and to assume that they have a linear utility function.

to the opponent candidate's policy, i.e., property (ii) fails in our model.

In contrast, Glazer and Gradstein (2005), again with the same timing structure, argue that the candidates cater to extreme lobbyists. The key assumption behind their analysis is that the candidates maximize the contribution they collect rather than the winning probabilities. This in particular implies that the effect of a candidate's policy change on the amount of the contribution the opposition collects is not taken into account. Consequently, the candidates cater to the extreme lobbyists because those lobbyists are willing to contribute more than the moderates. In our model, collecting more contribution is not necessarily good for a candidate to maximize his winning probability, as a policy choice that increases the contribution to him also increases the contribution to his opponent.

Austen-Smith (1987) and Baron (1989) also consider models with the same timing as ours (i.e., policy choice first and then contributions). Austen-Smith (1987) does not predict the exact policy positions, but only the relative change of the positions compared with the ones in the case with no contribution. In Baron's (1989) model, candidates in his model do not choose policies but "service levels" to lobbyists.

Some other papers have considered different timing structures than ours. Grossman and Helpman (1996) analyze a model of political contributions with the opposite timing of moves from ours. That is, the lobbyists first make offers that specify the amount of contribution for each policy positions, and then the politicians choose policies. In their model, two motives for lobbyists making contributions coexist: what they call the electoral motive (contributions affect winning probabilities) and the influence motive (contributions affect the policy positions). Since lobbyists in our model make contributions only after policy positions are set, our model isolates the electoral motive. In our model, although there is no explicit contract that specifies the amount of contributions contingent on policy positions, the existence of contributions indirectly affects the choice of policy positions because candidates are forward-looking in choosing their positions. In a citizen-candidate model, Felli and Merlo (2006) also consider contracts between lobbyists and politicians that specify, as in Grossman and Helpman (1996) a policy-contingent transfer. Although the model is quite different, their results share some similarity to ours in that they show lobbying induces some policy compromise. Zudenkova (2017) study the same timing structure with endogenous formation of lobbyist groups and also shows that the competition among extreme lobbies moderates the policy outcome.

A large strand of the literature specifies how candidates use campaign funds and thereby affect voters' behavior under various model specifications. For example, Coate (2004b) considers a model in which political campaigns provide information about the policy positions of policy-motivated candidates and shows that, even in equilibrium, voters are uncertain about the candidates' positions. Bailey (2002) assumes that one candidate chooses the policy position prior to the other, and that contributions can be used to target the campaign at selected people.[7] Potters et al. (1997) and Prat (2002a,b) consider the situations in which campaign contributions are used to signal the candidates' valence. Morton and Myerson (2012) consider a model in which candidates use the raised funds to provide a promised service to the voters. In contrast to the models considered in the papers mentioned here, our model considers how interest groups adjust the amount of their campaign contributions and how politicians react to such incentives while abstracting away from the consideration regarding how funds are used.

Implications similar to our extreme dominance are obtained in different models. First, Hilman and Riley (1989) model the lobbying process as an all-pay auction and show that only two highest-value lobbyists actively compete (see also Baye et al., 1993). The difference is that the values from winning the auction are exogenously fixed in their model, whereas the lobbyists' benefits from contribution are endogenously determined by the candidates' policy choice in our model. Second, Osborne et al. (2000) and Bulkey et al. (2001) consider a model where players decide whether to pay a cost and attend a meeting whose outcome is a compromise (such as median or mean) of the participants' bliss points (so there are no players who correspond to our candidates). In their

[7] Schultz (2007) also discusses such targeting.

model, only players with extreme preferences participate in equilibrium. Bergstrom et al. (1986) also make similar points in the context of public good provision.

The paper proceeds as follows: Section 2 presents the basic model. Sections 3 is devoted to the analysis of the model. Section 3.1 analyzes the case with concave lobbyist utilities and shows that extremist dominance and policy convergence hold in any equilibrium. Section 3.2 proves equilibrium existence. Section 4 concludes and provides various discussions. Appendix A extends the analysis to the case with mixed strategies, and Appendices B, C, and D analyze, respectively, the models with informed voters, with policy-motivated candidates and with convex lobbyist utilities.

2 Model

Consider the following election game played by two candidates and n lobbyists with $n \geq 2$. Each candidate, $i \in \{A, B\}$, is purely office-motivated. That is, the candidates want to maximize their own winning probability, which is determined by their campaign expenditure. Electoral campaigns need to be funded by the political contributions from the lobbyists, and each candidate commits to her policy so as to attract campaign funds. Each lobbyist, $k \in N := \{1, \ldots, n\}$, is purely policy-motivated in the sense that he cares only about the realized policy but not the identity of the winning candidate.

More precise rules of the game are as follows. The game consists of two stages. In stage 1, each candidate i simultaneously chooses her policy commitment $x_i \in \mathbb{R}$. In stage 2, after observing (x_A, x_B), each lobbyist k simultaneously determines $c_k = (c_{kA}, c_{kB})$, where $c_{ki} \geq 0$ is k's contribution to candidate i. The election campaign and the election take place after all those choices are made. During the campaign, each candidate i spends $c_i = \sum_{k \in N} c_{ki}$. Candidate i's probability of winning in the election is given by $P_i(c_A, c_B)$, where $P_i : \mathbb{R}^2_+ \to [0, 1]$ is a partially differentiable function that is nondecreasing in c_i and satisfies $P_i(c_A, c_B) \geq 0$ and $P_A(c_A, c_B) + P_B(c_A, c_B) = 1$

for any pair $(c_A, c_B) \in \mathbb{R}^2_+$. Note that we assume the winning probabilities depend only on the sum of contributions for each candidate. In particular, it does not depend on the identities of the contributors.

Under the outcome $((x_A, x_B), (c_1, \ldots, c_n))$, the payoffs of the players are specified as follows. Candidate i's payoff is 1 if elected and 0 otherwise. Lobbyist k's expected payoff is

$$U_k((x_A, x_B), (c_1, \ldots, c_n)) := (P_A(c_A, c_B)u_k(x_A) + P_B(c_A, c_B)u_k(x_B)) - (c_{kA} + c_{kB}),$$

where $u_k(x)$ is k's utility from policy x in monetary terms. We assume that the function $u_k : \mathbb{R} \to \mathbb{R}$ is strictly concave and differentiable, satisfying $u'_k(a_k) = 0$, where $a_k \in \mathbb{R}$ is interpreted to be k's ideal policy (or, bliss point).[8] Notice that $\arg\max_{x \in \mathbb{R}} u_k(x) = \{a_k\}$. The lobbyists are labeled so that $a_1 < \cdots < a_n$ and we refer to lobbyists 1 and n as extreme.[9] We further assume that u_k's satisfy the standard "single-crossing" property: $\frac{\partial u_k}{\partial x} < \frac{\partial u_{k'}}{\partial x}$ if $k < k'$. A leading example is the case in which u_k is identical across k except for the bliss points, i.e., $u_k(x) = u(x - a_k)$ for each k.

In this game, a pure strategy profile is given by $((x_A, x_B), (\kappa_1(\cdot), \ldots, \kappa_n(\cdot)))$ where (i) $x_i \in \mathbb{R}$ denotes candidate i's policy choice, (ii) $\kappa_k(\tilde{x}_A, \tilde{x}_B)$ for each $(\tilde{x}_A, \tilde{x}_B) \in \mathbb{R}^2$ denotes lobbyist k's contribution amount contingent on policy profile $(\tilde{x}_A, \tilde{x}_B)$. In the next section, we consider *subgame perfect equilibria (SPEs) in pure strategies* of the above specified game. In Appendix A, we formalize mixed-strategies and show that every mixed-strategy subgame perfect equilibrium is in pure strategies under certain regularity conditions.

[8]The concavity of $u_k(\cdot)$ is critical for the results. It is technically challenging to analyze the case of convex $u_k(\cdot)$ in general, because an equilibrium may not exist in pure strategies. We, however, provide some results for a certain special case in Appendix D. Differentiability of u_k is, however, not crucial under the assumption of concavity. Without differentiability, we would only need to redefine a^M (defined before Proposition 2) and slightly modify related analyses accordingly. We also note that the linearity of U_k with respect to the spending, although standard in the literature, is important for extremist dominance. However, one can show that the most extreme lobbyist contributes the most whenever U_k is a weakly concave (and decreasing) function of the spending.

[9]Here we assume that no two lobbyists have the same bliss point. This assumption is made only to simplify the argument, and none of our results hinges on this assumption.

3 Analysis

3.1 Properties of Pure-Strategy Equilibrium

In this section, we investigate the properties of SPEs. We are going to establish the existence in the next subsection with certain additional assumptions.

The first proposition shows that, in any subgame of any SPE, only the extreme lobbyists make positive amounts of contribution. The intuition for this result is as follows (refer back to Figure 1). Fixing a subgame, the only lobbyist who can possibly make a contribution to candidate i in any SPE is the one who perceives the highest utility difference for i over $-i$. This is because the marginal benefit from contribution is the highest for such a lobbyist. The single-crossing condition implies that it is either lobbyist 1 or n who perceives the highest utility difference.

Proposition 1 (*Extremist Dominance*)**.** *For any subgame, if a lobbyist donates a strictly positive amount, then that lobbyist must be extreme. Formally, fix a subgame after (x_A, x_B) with $x_A \leq x_B$, and suppose that $(c_k^*)_{k \in N}$ is a (pure-strategy) Nash equilibrium in this subgame. Then, $c_{ki}^* = 0$ for all $(k,i) \notin \{(1,A),(n,B)\}$. Symmetrically, $c_{ki}^* = 0$ for all $(k,i) \notin \{(1,B),(n,A)\}$ in any (pure-strategy) Nash equilibrium of any subgame after (x_A, x_B) with $x_A \geq x_B$.*

Proof. To begin, notice that the claim trivially holds if $x_A = x_B$, because if so, there is a unique pure-strategy Nash equilibrium, in which all lobbyists make zero contribution to each candidate. We provide the proof only for the case of $x_A < x_B$, as the other case is perfectly symmetric.

Fix (x_A, x_B) with $x_A < x_B$ and a Nash equilibrium after (x_A, x_B), denoted by $(c_k^*)_{k \in N}$. First, we establish that lobbyist k contributes a positive amount to candidate i only when k has the largest utility gain from i winning over j; i.e.,

$$c_{ki}^* > 0 \Longrightarrow k \in \arg\max_{\ell \in N} \left[u_\ell(x_i) - u_\ell(x_j)\right], \text{ for each } k \in N \text{ and } i \in \{A,B\}, \tag{1}$$

where $j \in \{A,B\} - \{i\}$. To see this, suppose that $c^*_{ki} > 0$. Then, by the first-order condition,

$$\frac{\partial U_k((x_A,x_B),(c^*_\ell)_{\ell\in N})}{\partial c_{ki}} = (u_k(x_i) - u_k(x_j)) \cdot \frac{\partial P_i(c^*_A,c^*_B)}{\partial c_i} - 1 = 0.$$

Since $\frac{\partial P_i(c^*_A,c^*_B)}{\partial c_i} \geq 0$ by assumption, $u_k(x_i) - u_k(x_j) > 0$ and $\frac{\partial P_i(c^*_A,c^*_B)}{\partial c_i} > 0$ must hold. Hence, if there exists $k' \in N$ such that $u_{k'}(x_i) - u_{k'}(x_j) > u_k(x_i) - u_k(x_j)$, then, we have

$$\frac{\partial U_{k'}((x_A,x_B),(c^*_\ell)_{\ell\in N})}{\partial c_{k'i}} = (u_{k'}(x_i) - u_{k'}(x_j)) \cdot \frac{\partial P_i(c^*_A,c^*_B)}{\partial c_i} - 1 > 0.$$

Therefore, by the definition of partial derivatives, there exists $\varepsilon > 0$ such that

$$U_{k'}((x_A,x_B),(c^*_{k'}+\varepsilon,(c^*_\ell)_{\ell\neq k'})) > U_{k'}((x_A,x_B),(c^*_\ell)_{\ell\in N}),$$

which is a contradiction to the supposition that $(c^*_\ell)_{\ell\in N}$ is a Nash equilibrium.

Now, it suffices to establish that $\arg\max_{\ell\in N}\left[u_\ell(x_i) - u_\ell(x_j)\right]$ is equal to $\{1\}$ for $i = A$ and to $\{n\}$ for $i = B$. To show this, fix two lobbyists $\widehat{k}$ and $\widetilde{k}$ such that $\widehat{k} < \widetilde{k}$. Then the single-crossing property of u_ℓ's implies

$$[u_{\widehat{k}}(x_B) - u_{\widehat{k}}(x_A)] - [u_{\widetilde{k}}(x_B) - u_{\widetilde{k}}(x_A)] < 0,$$

which leads to

$$u_{\widehat{k}}(x_A) - u_{\widehat{k}}(x_B) > u_{\widetilde{k}}(x_A) - u_{\widetilde{k}}(x_B).$$

This implies $\arg\max_{\ell\in N}[u_\ell(x_A) - u_\ell(x_B)]$ is equal to $\{1\}$. A symmetric argument shows that $\arg\max_{\ell\in N}[u_\ell(x_B) - u_\ell(x_A)]$ is equal to $\{n\}$, and thus the proof is complete. ■

The second proposition shows that policies converge on the path of play of any pure-strategy SPE. In order to rule out some uninteresting indeterminacy, we hereafter assume that if $x_i < x_j < a_1$

or $a_n < x_j < x_i$ with $i \neq j$, then the contribution amount from any lobbyist is zero for both candidates and candidate j wins with probability 1. Let us further impose the following assumptions on (P_A, P_B).

Assumption 1. For any $c \geq 0$, $P_A(c,c) = P_B(c,c)$.[10]

Assumption 2. If $c_i \geq c_j$, then $\partial P_i / \partial c_i \leq \partial P_j / \partial c_j$.

Assumption 3. $\partial P_i / \partial c_i = \infty$ at $c = (0,0)$.[11]

Assumption 1 is the symmetry between the candidates. Assumption 2 states that an additional unit of campaign spending is weakly less effective for a candidate who spends more than the other. Assumption 3 rules out corner solutions. Notice that the commonly-used Tullock function, $P_i(c) = c_i^r/(c_A^r + c_B^r)$ if $c \neq (0,0)$ and $P_i(0,0) = 1/2$, satisfies all these assumptions. The assumptions are also satisfied by Hirshleifer's (1989) "difference form" of contest success functions that depends only on the difference of contributions, so there exists a function $Q : \mathbb{R} \to [0,1]$ such that $P_i(c_i, c_j) = Q(c_i - c_j)$, provided that $Q'(0) = \infty$.

Let $a^M \in (a_1, a_n)$ be the unique solution of the equation $u_1'(a^M) = -u_n'(a^M)$. That is, a^M is the policy position such that the two extreme lobbyists perceive the same magnitude of utility difference for a marginal change in policy. Note that a^M exists because differentiability and concavity of u_1 and u_n implies that these functions are also continuously differentiable.

Proposition 2 (Policy Convergence)**.** *Suppose that Assumptions 1–3 hold. Then, policies converge in any SPE. Formally, if $((x_A^*, x_B^*), (\kappa_1^*(\cdot), \ldots, \kappa_n^*(\cdot)))$ is a pure-strategy SPE, then $x_A^* = x_B^* = a^M$ holds.*

Proof. Fix (x_A, x_B) such that $x_i = a^M \neq x_j$ and a Nash equilibrium $(c_k^*)_{k \in N}$ in the subgame after (x_A, x_B). It suffices to show that i's winning probability is strictly higher than 1/2. Without loss of

[10] In fact, it suffices to assume $P_i(c,c) = P_i(c',c')$ for any $c, c' \geq 0$ for each $i = A, B$ (not necessarily requiring symmetry across candidates) to prove the results in this paper. For simplicity, however, we stick to the current Assumption 1.

[11] The infinite partial derivative is defined in the standard manner: $\partial P_i / \partial c_i = \infty$ at (c_i, c_{-i}) if $\lim_{\varepsilon \to 0} \frac{P_i(c_i + \varepsilon, c_{-i})}{\varepsilon} = \infty$.

generality, suppose $x_A = a^M < x_B$. Towards a contradiction, suppose that $c_A^* \le c_B^*$. By Proposition 1 we have $c_A^* = c_{1A}^*$ and $c_B^* = c_{nB}^*$. Further, $c_B^* > 0$ must hold since $c_A^* + c_B^* > 0$ by Assumption 3. Therefore, the first-order condition for the optimality of c_{nB}^* can be written as

$$\frac{\partial U_n((a^M, x_B), (c_k^*)_{k \in N})}{\partial c_{nB}} = [u_n(x_B) - u_n(a^M)] \cdot \frac{\partial P_B(c_A^*, c_B^*)}{\partial c_B} - 1 = 0.$$

Since $\frac{\partial P_B(c_A^*, c_B^*)}{\partial c_B} \ge 0$ by assumption, $u_n(x_B) - u_n(a^M) > 0$ and $\frac{\partial P_B(c_A^*, c_B^*)}{\partial c_B} > 0$ must hold. This implies, however,

$$\frac{\partial U_1((a^M, x_B), (c_k^*)_{k \in N})}{\partial c_{1A}} = [u_1(a^M) - u_1(x_B)] \cdot \frac{\partial P_A(c_A^*, c_B^*)}{\partial c_A} - 1 > 0, \tag{2}$$

because (i) the strict concavity of $u_k(\cdot)$'s and the definition of a^M imply

$$\begin{aligned} u_1(a^M) - u_1(x_B) &> u_1'(a^M)(a^M - x_B) \\ &= u_n'(a^M)(x_B - a^M) > u_n(x_B) - u(a^M) > 0, \end{aligned}$$

and (ii) Assumption 2 and the supposition of $c_A^* \le c_B^*$ entail $\frac{\partial P_A(c_A^*, c_B^*)}{\partial c_A} \ge \frac{\partial P_B(c_A^*, c_B^*)}{\partial c_B} > 0$. Equation (2) means that lobbyist 1 has an incentive to marginally increase his contribution to candidate A, which is a contradiction to the assumption that $(c_k^*)_{k \in N}$ is a Nash equilibrium. Therefore, we must have $c_A^* > c_B^*$ and hence $P_A(c^*) > P_B(c^*)$ because of Assumption 1 and $\frac{\partial P_B(c_A^*, c_B^*)}{\partial c_B} > 0$. This implies that $P_A(c^*) > \frac{1}{2}$ and the proof is complete. ■

Note that the equilibrium policy, $x_A^* = x_B^* = a^M$, depends only on the preferences of the extremists but not of the other lobbyists. This is because, given extremist dominance, candidates only care about the potential contributions from lobbyists 1 and n. Given that only lobbyists 1 and n are relevant for the policy choices, let us now explain why it is at a^M that the equilibrium policies converge. If $a^M = x_A < x^B$, the definition of a^M and the strict concavity of lobbyist utility functions imply that lobbyist 1 experiences a larger utility difference between the two candidates than lobbyist n does. With our assumptions, this leads candidate A to receive more contributions and to have a greater chance to win. In other words, a^M is a Condorcet winner and thus, in equilibrium both candidates take that policy.

Taking the two propositions together, we obtain the following result. We will comment on this property of the SPE in Section 4.[12]

Corollary 1. *On the equilibrium path of play of any pure-strategy SPE, no lobbyist contributes to any candidate.*

3.2 Equilibrium Existence

In this section, we establish the existence of a pure-strategy SPE under mild technical conditions. First, given the proof of Proposition 2, it suffices to establish the existence in the second-stage subgames.

Lemma 1. *If Assumptions 1–3 hold and a pure-strategy Nash equilibrium exists for each second-stage subgame, then a pure-strategy SPE exists.*

Proof. By supposition, there exists $(\kappa_1(\cdot),\ldots,\kappa_n(\cdot))$ that induces a pure-strategy Nash equilibrium in each second stage subgame. Given any such $(\kappa_1(\cdot),\ldots,\kappa_n(\cdot))$, the proof of Proposition 2 shows that i's winning probability is strictly lower than 1/2 whenever $x_i \neq a^M = x_j$. Therefore, taking the lobbyists' strategies $(\kappa_1(\cdot),\ldots,\kappa_n(\cdot))$ and the other candidate j's position $x_j = a^M$ as given, choosing $x_i = a^M$ is a best response for candidate i. That is, $\big((a^M,a^M),(\kappa_1(\cdot),\ldots,\kappa_n(\cdot))\big)$ is a pure-strategy SPE and hence, a pure-strategy SPE exists. ■

Second, the following regularity assumptions are sufficient for the existence of a Nash equilibrium in each subgame.

[12] This result parallels that of Ledyard's (1984) although he studies a quite different context. In his model, candidates choose their policy positions, and then voters make abstention decisions. In the "strong rational election equilibrium" that he defines, policies converge and, as a consequence, no one votes.

Assumption 4. Each $P_i(\cdot,\cdot)$ is (jointly) continuous.

Assumption 5. Each P_i is weakly concave in c_i.

Proposition 3. *All second-stage subgames have a Nash equilibrium in pure strategies if Assumptions 4– 5 hold. If Assumptions 1–5 hold , hence, an SPE exists in pure strategies.*

Proof. Given Lemma 1, it suffices to show the first part of the claim. To do so, fix an arbitrary (x_A, x_B). First notice that for lobbyist k, if $u_k(x_i) \geq u_k(x_j)$ with $i \neq j$, then any contribution amount $c_{kj} > 0$ is strictly dominated by the contribution amount $c_{kj} = 0$. Hence, each lobbyist k contributes a positive amount to at most one candidate in any Nash equilibrium of the subgame after (x_A, x_B), and the identity of the candidate to whom lobbyist k contributes a positive amount does not depend on other lobbyists' strategies. For each lobbyist k, let $\imath(k)$ be the candidate i satisfying $u_k(x_i) > u_k(x_j)$ if such i exists and let $\imath(k) = A$ if $u_k(x_A) = u_k(x_B)$. The preceding argument implies that it is without loss of generality to regard each lobbyist's action to be a nonnegative scalar, i.e., $c_{k\imath(k)} \in \mathbb{R}_+$. Now, define $\bar{U} := \max_{k \in N} |u_k(x_A) - u_k(x_B)| < \infty$. Since no lobbyist has an incentive to make a contribution greater than $\bar{U}$, it is without loss of generality to restrict the action space for each lobbyist to $[0, \bar{U}]$ in this subgame. With these transformations, each U_k is continuous in $c_{k\imath(k)}$ by Assumption 4 and is quasi-concave in $c_{k\imath(k)}$ by Assumption 5. Hence, the theorems by Debreu (1952), Glicksberg (1952), and Fan (1952) guarantee the existence of an equilibrium.[13] ∎

Notice that the Tullock function violates Assumption 4, for it is discontinuous at the origin. However, as in the standard Tullock models, a pure-strategy equilibrium can be shown to exist in each subgame with the Tullock P_i function as long as $r \leq 2$ holds. Thus, even for such P_i functions, the results in Propositions 1 and 2 are relevant.

[13]Debreu (1952), Glicksberg (1952), and Fan (1952) show that any normal-form game whose action spaces are nonempty compact convex subsets of a Euclidian space with payoff functions that are quasi-concave on each player's action space and are jointly continuous on the space of action profiles has a pure-strategy Nash equilibrium.

4 Discussions

We considered a two-candidate election model in which candidates set their policies first and then lobbyists contribute, which in turn determines the winning probabilities. Under the assumption of concave lobbyist utility functions, only extreme lobbyists can contribute in any Nash equilibrium of the subgame after any policy profile on and off the equilibrium path of play, and the policies converge. As a consequence, no lobbyists contribute to any candidate on the equilibrium path.

Let us conclude by making a comment on the interpretation of our contribution. The SPE outcome of our model that no lobbyists contribute apparently contradicts the reality in which campaign contributions do exist. We would not necessarily interpret such discrepancy between the outcome of the model and the reality as a weakness of the model, but as a benchmark for understanding the drivers of campaign contributions.[14] Specifically, we interpret this discrepancy as suggesting that perhaps some other components that we do not model are the key drivers of campaign contributions. Let us consider four possibilities of such missing drivers and discuss whether and how they change the insights from our baseline model.

Informed voters: First, one may wonder whether our conclusions depend on our assumption that the winning probabilities depend only on campaign contributions, and in particular how they would change if we introduce informed voters who respond to the implemented policy as in Baron (1994). In Appendix B, we discuss a model in which the winning probability depends not only on the contribution profile but also on the policy profile. We show that policy convergence holds under certain natural assumptions on how the policy profile affects the winning probability, although the convergent point may be different from the one in our baseline model.

[14]This is in the same spirit as how one would interpret Hotelling's convergence result: it is a useful benchmark even though it does not necessarily fit the reality.

Restricted policy space: Second, there may be various constraints on part of candidates that prevent them from freely choosing their policy positions. For instance, a Republican candidate may be constrained not to announce a radically liberal policy. In such a situation, policy divergence may occur in the policy-setting stage, and if that happens then the campaign contributions would take place in our model. More specifically, we can show that even if the candidates' choice sets are exogenously restricted while being convex and disjoint, each candidate would try to move as close as possible to the other candidate, which is analogous to our policy convergence result.[15] Furthermore, our extremist dominance result would be helpful in identifying who would be making contributions when policies diverge.[16] The proof of extremist dominance also suggests that, with additional technical assumptions on the P_i function, we would obtain positive correlations between the contribution amounts and the degree of policy divergence.

Policy motivated candidates: Third, candidates may receive utility from the policy implemented by the winning candidate. In Appendix C, we present a model where each candidate gains some utility from the implemented policy, in addition to the utility from being elected. There, we show that a unique policy profile emerging from any pure-strategy SPE exhibits divergence if and only if the weight on the policy preferences (relative to the weight on the utility from winning the election) is above some cutoff level. Moreover, when the weight is above the cutoff, the divergence and the total amount of contributions are increasing in that weight, as well as in the degree of divergence of the extreme lobbyists' bliss points and that of the candidates' bliss points.

Furthermore, we extend the model to allow for caps on contribution amounts, and find an interesting role that contribution caps can play. Specifically, we show that once a sufficiently tight

[15] The proof is analogous to the one for Proposition 2. To gain some intuition, refer back to Figure 2.

[16] One may criticize extremist dominance on the basis of multiple interest groups contributing to a single party in reality. This is again a discrepancy that future research could address. One possibility for avoiding such an outcome would be to assume that each lobbyist is budget-constrained. In such a model, multiple extreme lobbyists from each side would make a positive amount of contributions, and policy would still converge. Another possibility would be that the policy space is multi-dimensional, and each policy issue (corresponding to each dimension of the policy space) is associated with multiple lobbyists. In such a case, only two lobbyists from each dimension would make a contribution in any subgame, but the overall number of lobbyists making positive contributions would be greater.

cap is imposed, there is no pure-strategy SPE in which the contribution amount is zero no matter how small the weight on the policy preferences is. This is in a stark contrast with our result that, when there is no cap, the contribution amount is zero in the unique pure-strategy SPE if the weight on the policy preferences is small enough. Moreover, when sufficiently tight caps are imposed, we show that there is a mixed-strategy SPE in which the contribution amount is strictly positive with strictly positive probability.[17] These results shed new light on the discussion on the role of contribution caps, which have been long argued in the literature (see, e.g., Levitt (1994) and Coate (2004a)).

Convex lobbyist utility: Fourth, our analysis depends on the shape of the utility function of the interest groups, and it is not our purpose to argue that concave utility functions are more plausible than others. Rather, by examining various preferences, we would like to offer an understanding of the role of contributions in determining electoral outcomes. In Appendix D, we supplement our analysis by examining how our result changes when we consider other sorts of utility functions. Specifically, we consider the case where the utilities are single-peaked and are convex with respect to the distance between the bliss point and the implemented policy.[18] In such a context, we still obtain policy convergence (albeit to a different position) and no contributions unless a certain strong symmetry condition on the lobbyist utility function holds. This suggests that changing the shape of utility functions would not be very helpful in avoiding the non-contribution result.

As we have discussed above, there are many ways to modify the baseline model to examine various forces in elections and their implications. Although the no-contribution result is not obtained in some of those variants, the economic force that generates the extreme dominance and policy convergence in the baseline model continues to play a key role: Each lobbyist solves their

[17] There may not exist any pure-strategy SPE when the weight on the policy preferences is small.

[18] Such preferences may seem unusual at first glance, but in certain contexts they may even be more natural than other specifications. Again, we do not intend to justify any particular types of preferences. We refer interested readers to Osborne (1995) and Kamada and Kojima (2014) for extensive discussion on the contexts in which convexity naturally arise.

maximization problem taking into account the fact that other lobbyists are facing similar problems, and a candidate's move to an extreme lobbyist not only increases contributions from that lobbyist but also increases contributions from the opposing lobbyist to the opposing candidate. This force prevents the candidates diverging to the opposing extremes.

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APPENDICES

A Mixed Strategies

In this section, we introduce mixed strategies and show that every SPE is in pure strategies once we slightly strengthen Assumption 5 to the following:

Assumption 6. Each P_i is strictly concave in c_i.

That is, it is almost without any loss of generality that we restrict our attention to pure-strategy SPEs in the main body of the paper.

Let ξ_i denote candidate i's mixed strategy, specifying a Borel measure over the policy positions in $\mathbb{R}$. Also, let $\gamma_k(x_A, x_B)$ denote lobbyist k's mixed strategy contingent on policy profile (x_A, x_B), specifying a Borel measure over the contribution amounts in $\mathbb{R}^2_+$. A mixed strategy profile is given by $((\xi_A, \xi_B), (\gamma_1(\cdot), \ldots, \gamma_n(\cdot)))$. With the specifications so far, the expected payoff to each player at each subgame of the entire game is well-defined, and thus a *mixed-strategy subgame perfect equilibrium (SPE)* is defined in the standard manner.

The next proposition shows that, under Assumption 6 along with Assumptions 1–4, any SPE in mixed strategies should be indeed in pure strategies. In other words, the policy convergence result in Proposition 2 extends to mixed strategies. The proof first shows that in any SPE, the second-stage strategies have to be pure. To show this, we observe that every lobbyist contributes to at most one candidate, and the strict concavity of P_i ensures that the contribution amount is deterministic. To further show that the first-stage strategies are pure, we show that a^M is the Condorcet winner.

Proposition 4. *Suppose that Assumptions 1–4 and Assumption 6 hold. Then any SPE must be in pure strategies. Hence, if $((\xi_A, \xi_B), (\gamma_1(\cdot), \ldots, \gamma_n(\cdot)))$ is a mixed-strategy SPE, then $\xi_A(a^M) = \xi_B(a^M) = 1$ and, for each $k \in N$ and $(x_A, x_B) \in \mathbb{R}^2$, there exists $c_k \in \mathbb{R}^2_+$ such that $\gamma_k(x_A, x_B)(c_k) = 1$.*

Proof. Fix any lobbyist k. We check that under Assumption 6, k's best reply must be a pure strategy for any (x_A, x_B) and any profile of mixed strategies played by lobbyists $\ell \neq k$. First, if $u_k(x_A) = u_k(x_B)$, $(c_{kA}, c_{kB}) = 0$ is a strictly dominant strategy for k and hence, k's best reply must be a pure strategy. Second, suppose that $u_k(x_A) > u_k(x_B)$. Then it is apparent that k has no incentive to make a positive contribution to B and hence, k's best reply must choose $c_{kB} = 0$ with probability one. In addition, given $c_{kB} = 0$ and the mixed strategies played by the other lobbyists, k's optimization problem is given by

$$\max_{c_{kA} \in [0,\bar{U}]} \left\{ \mathbb{E}\left[P_A \left(c_{kA} + \sum_{\ell \neq k} c_{\ell A}, \sum_{\ell \neq k} c_{\ell B} \right) \right] u_k(x_A) + \mathbb{E}\left[P_B \left(c_{kA} + \sum_{\ell \neq k} c_{\ell A}, \sum_{\ell \neq k} c_{\ell B} \right) \right] u_k(x_B) - c_{kA} \right\},$$

where $\bar{U} := \max_{k \in N} |u_k(x_A) - u_k(x_B)| < \infty$ and the expectations are taken with respect to the mixed strategies of $\ell \neq k$. By Assumption 6, the objective function is strictly concave with respect to $c_{kA} \in [0, \bar{U}]$ and hence, the problem has a unique maximizer c^*_{kA} and k's best reply must be $(c_{kA}, c_{kB}) = (c^*_{kA}, 0)$. The case of $u_k(x_A) < u_k(x_B)$ is perfectly symmetric. In sum, any best reply of any lobbyist must be a pure strategy and hence, any mixed-strategy equilibrium of any second stage subgame must be in pure strategies.[19]

Now we show that first-stage actions in any SPE are also pure. Towards a contradiction, fixing a (pure-strategy) Nash equilibrium of each of the second-stage subgames, suppose that candidate i chooses $x_i = a^M$ with a probability strictly smaller than one. Then, by the arguments in the proof of Proposition 2, the other candidate j can guarantee a winning probability strictly greater than $1/2$ by taking $x_j = a^M$ with probability one. Therefore, if i's strategy is a part of an equilibrium, then i's winning probability should be strictly smaller than $1/2$. However, i can also guarantee a winning probability weakly greater than $1/2$ by picking $x_i = a^M$ with probability one, which is a

[19]This follows because there does not exist a mixed-strategy equilibrium that assigns strictly positive probability to contribution amounts in $(\bar{U}, \infty)$. To see this, suppose that in a Nash equilibrium in the subgame after some (x_A, x_B), lobbyist k uses a mixed strategy μ such that $\mu([0,\bar{U}]^2) < 1$. Then, given any strategies of other lobbyists, it is straightforward to see that deviating to another mixed strategy μ' strictly improves k's payoff in this subgame, where $\mu'(Q) = \mu(Q)$ for any measurable $Q \subseteq [0,\bar{U}]^2 \setminus \{(0,0)\}$ and $\mu'(\{0,0\}) = 1 - \mu([0,\bar{U}]^2 \setminus \{(0,0)\})$.

contradiction. ■

B Model with Informed Voters

In this section, we consider a model in which the winning probability is determined not only by the contributions by lobbyists but also by how voters respond to the policy positions. Specifically, suppose that when the contribution profile is (c_A, c_B) and the policy profile is (x_A, x_B), each candidate i's winning probability is given by

$$R_i(c_A, c_B, x_A, x_B) = (1-\theta)P_i(c_A, c_B) + \theta Q_i(x_A, x_B),$$

where $\theta \in [0,1)$, the function P_i is defined as in our main model, and Q_i is a continuous function satisfying the following properties.

1. There exists a unique $a^V \in (a_1, a_n)$ such that the following hold: If $x_A \leq a^V \leq x_B$ (resp. if $x_A \geq a^V \geq x_B$), then there exists $x'_A \in [a^V, x_B]$ and $x'_B \in [x_A, a^V]$ (resp. $x'_A \in [x_B, a^V]$ and $x'_B \in [a^V, x_A]$) such that $Q_A(x'_A, x_B) \geq Q_A(x_A, x_B)$ and $Q_B(x_A, x'_B) \geq Q_B(x_A, x_B)$.

2. For each $i, j \in \{A, B\}$ with $i \neq j$, $Q_i(x_A, x_B)$ is nonincreasing in x_i over (x_j, ∞) if $a^V \leq x_j$, and $Q_i(x_A, x_B)$ is nondecreasing in x_i over $(-\infty, x_j)$ if $x_j \leq a^V$.

3. $Q_A(x_A, x_B) + Q_B(x_A, x_B) = 1$ for any $(x_A, x_B) \in \mathbb{R}^2$.

A possible interpretation of the above R_i functions is as follows: With probability $1-\theta$ the voters are not well informed about the manifests, and respond only to the campaign expenditure (e.g., the amount of advertisements), which is financed by the contributions from the lobbyists. Thus the winning probability only depends on the contribution profile (c_A, c_B) in this case. With probability θ, the voters are well informed, respond to the policy profile, and the winning probability only depends on the policy profile (x_A, x_B).

To introduce examples of Q_i satisfying the above conditions, let f be a function such that $f(x) > 0$ over some nonempty interval $\mathscr{V}$ in $\mathbb{R}$ and $\int_{x\in\mathscr{V}} f(x)dx = 1$. In any of these examples, a^V is given by the unique solution to $\int_{-\infty}^{a^V} f(x)dx = 1/2$.

1. A mass of voters are distributed over $\mathscr{V}$ according to density f and each voter votes for the closer candidate, while an indifferent voter votes for each candidate with probability $1/2$. For any given policy profile (x_A, x_B), let $Q_i(x_A, x_B) = 1$ if i attracts strictly more than $1/2$ of the voters, $Q_i(x_A, x_B) = 1/2$ if i attracts exactly $1/2$ of the voters, and $Q_i(x_A, x_B) = 0$ if i attracts strictly less than $1/2$ of the voters.

2. The median voter is distributed over $\mathscr{V}$ according to the probability density $f(x)$ and the candidate closer to the median voter wins while if the two candidates have the same distance then each wins with probability $1/2$. The function Q_i denotes the wining probability conditional on voters being informed. Specifically, if $x_A < x_B$, $Q_A(x_A, x_B) = \int_{-\infty}^{\frac{x_A+x_B}{2}} f(x)dx = 1 - Q_B(x_A, x_B)$. If $x_A = x_B$, then $Q_A(x_A, x_B) = Q_B(x_A, x_B) = 1/2$. The case with $x_B < x_A$ is symmetric.

3. The above two examples involve discontinuity of Q at $x_A = x_B$. To define an example of continuous Q, let $\bar{Q}$ be the function defined to be Q in the second example. Then, define $Q_A(x_A, x_B) = (1 - e^{-\frac{1}{|x_A - x_B|}})\frac{1}{2} + e^{-\frac{1}{|x_A - x_B|}}\bar{Q}_A(x_A, x_B) = 1 - Q_B(x_A, x_B)$ if $x_A \neq x_B$ and $Q_A(x_A, x_B) = Q_B(x_A, x_B) = 1/2$ if $x_A = x_B$.

For the sake of simplicity, we further assume that $P_i(c_A, c_B) = c_i/(c_A + c_B)$ for each $i \in \{A, B\}$, with a convention of $\frac{0}{0+0} = \frac{1}{2}$. Note that with this Tullock from, Assumptions 1–3 are met and hence Proposition 1 holds. When $a_1 < x_A < x_B < a_n$ (i.e., when lobbyists 1 and n donate to candidates A and B, respectively), the first-order conditions for the lobbyists imply that

$$\frac{c_A^*}{c_A^* + c_B^*} = \frac{u_1(x_A) - u_1(x_B)}{[u_1(x_A) - u_1(x_B)] + [u_n(x_B) - u_n(x_A)]}, \tag{3}$$

where $(c_A^*, c_B^*) = (c_{1A}^*, c_{nB}^*)$ is the (unique) profile of equilibrium funds raised by the candidates. This explicit solution makes it easy to examine the changes in P_i when we analyze the SPE below.

Call the model we have specified above the model with informed voters. The following result shows that the introduction of informed voters (as is done in this section) does not lead to policy divergence.

Proposition 5. *In the model with informed voters, if $((x_A^*, x_B^*), (\kappa_1^*(\cdot), \ldots, \kappa_n^*(\cdot)))$ is a pure-strategy SPE, then $x_A^* = x_B^*$ holds.*[20]

Proof. Fix a pure-strategy SPE and let $\widetilde{P}_i(x_A, x_B)$ denote candidate i's winning probability in the subgame after (x_A, x_B). Let (x_A^*, x_B^*) be an SPE outcome and assume without loss that $a^V < a^M$. Suppose for contradiction that $x_A^* \neq x_B^*$. Without loss, we further assume $x_A^* < x_B^*$.

First, $u_1(x_A^*) > u_1(x_B^*)$ and $u_n(x_A^*) < u_n(x_B^*)$ must hold for the following reason. Suppose towards a contradiction that $u_1(x_A^*) \leq u_1(x_B^*)$. Then, no lobbyist donates to A and hence, $\widetilde{P}_A(x_A^*, x_B^*) = 0$. If $a^V \leq x_B^*$, there exists by assumption $x_A \in [a^V, x_B^*]$ such that $Q_A(x_A, x_B^*) \geq Q_A(x_A^*, x_B^*)$. Further, we must also have $\widetilde{P}_A(x_A, x_B^*) > 0$, as $u_1(x_A) > u_1(x_B^*)$ if $x_A \in [a^V, x_B^*)$ and $\widetilde{P}_A(x_A, x_B^*) = 1/2$ if $x_A = x_B^*$. That is, A has an incentive to deviate to such x_A if $a^V \leq x_B^*$. If $x_B^* < a^V$, by the second assumption on Q_i and continuity, $Q_A(x_B^*, x_B^*) \geq Q_A(x_A^*, x_B^*)$. Since we also have $\widetilde{P}_A(x_B^*, x_B^*) = 1/2 > \widetilde{P}_A(x_A^*, x_B^*)$, A has an incentive to deviate to $x_A = x_B^*$ if $x_B^* < a^V$. Therefore, we must have $u_1(x_A^*) > u_1(x_B^*)$. A symmetric argument implies that $u_n(x_A^*) < u_n(x_B^*)$ holds as well. Note that these inequalities imply that $\widetilde{P}_A$ is given by equation (3). Further, this entails that $\widetilde{P}_A(x_A, x_B^*)$ is increasing in x_A over $[x_A^*, x_B^*)$.

Now, we consider the following three (exhaustive) cases: Suppose first that $x_A^* < x_B^* \leq a^V$. Then A has an incentive to deviate to any $x_A \in (x_A^*, x_B^*)$ because $\widetilde{P}_A(x_A, x_B^*) > \widetilde{P}_A(x_A^*, x_B^*)$ and $Q_A(x_A, x_B^*) \geq Q_A(x_A^*, x_B^*)$. Note that the second inequality follows from the second assumption on the function Q_i. Suppose second that $x_A^* < a^V < x_B^*$. Then, the first assumption on Q_i guarantees the existence of $x'_A \in [a^V, x_B^*]$ with $Q_A(x_A, x_B^*) \geq Q_A(x_A^*, x_B^*)$. If $x'_A < x_B^*$, A has an incentive to deviate to x'_A because $\widetilde{P}_A(x'_A, x_B^*) > \widetilde{P}_A(x_A^*, x_B^*)$ as argued above. If $x'_A = x_B^*$, then there exists $\varepsilon > 0$ small enough

[20] Existence of the SPE would follow if we impose further assumptions on the Q_i functions as for the P_i functions.

such that A has an incentive to deviate to $x'_A - \varepsilon$, because Q_A is assumed to be continuous and $\widetilde{P}_A(x_A, x_B^*)$ is increasing in x_A over $[x_A^*, x_B^*)$ as argued above. Suppose third that $a^V \leq x_A^* < x_B^*$. Then B has an incentive to deviate to any $x_B \in (x_A^*, x_B^*)$ because $\widetilde{P}_B(x_A^*, x_B) > \widetilde{P}_B(x_A^*, x_B^*)$ and $Q_B(x_A^*, x_B) \geq Q_B(x_A^*, x_B^*)$. Again, the second inequality follows from the second assumption on the function Q_i. Overall, it cannot be the case that $x_A^* < x_B^*$. In a symmetric manner, $x_B^* < x_A^*$ cannot hold. Hence $x_A^* = x_B^*$. ■

C Model with Policy-Motivated Candidates

Here we consider a model with policy preferences. The model differs from the one in the baseline model in three ways: lobbyists' utility functions are assumed to be quadratic, the candidates' payoffs vary with the implemented policy, and the winning probabilities are determined by the Tullock function. Specifically, first, we assume lobbyist k's utility function is given by $u_k = -m(x - a_k)^2$ where x is the implemented policy (i.e., the policy committed by the winner of the election) and $m > 0$ is a parameter. As in the main model, one can show extremist dominance holds, and thus we will not discuss contributions from lobbyists $2, \ldots, n-1$ hereafter. For simplicity, we assume that there exists $a \in \mathbb{R}_{++}$ such that $a_1 = -a$ and $a_n = a$. Second, candidate A has a bliss point $-b$ and candidate B has a bliss point $b > 0$. We assume that candidates are less extreme than the most extreme lobbyists, i.e., $b < a$. Candidate A receives utility 1 upon winning an election, and loses $k(x - (-b))^2$ where x is the implemented policy and $k > 0$ is a parameter. Similarly, candidate B receives utility 1 upon winning an election, and loses $k(x - b)^2$. Third, if candidates A and B collect contributions c_A and c_B, respectively, then A's winning probability is $\frac{c_A}{c_A + c_B}$, with a convention of $\frac{0}{0+0} = \frac{1}{2}$. Call this model the model with policy-motivated candidates.

Proposition 6. *In the model with policy-motivated candidates, there exists a unique pure-strategy*

SPE. Moreover, in the unique pure-strategy SPE $((x_A^*, x_B^*), (\kappa_1^*(\cdot), \dots, \kappa_n^*(\cdot)))$*, the following holds:*

$$-x_A^* = x_B^* = \max\left\{0, \frac{ab - \frac{1}{4k}}{a+b}\right\}.$$

Proof. Suppose that (x_A^*, x_B^*) is the first-stage outcome of a pure-strategy equilibrium. First, we show $x_A^* \leq x_B^*$. Suppose the contrary, i.e., that $x_B^* < x_A^*$. Without loss of generality, suppose also that $x_A^* + x_B^* \leq 0$, which further implies that B's winning probability is no more than $1/2$.[21] We consider the following three (exhaustive) cases.

1. Suppose $x_A^* \leq -a$. Then B's winning probability is zero and A's policy is implemented with probability 1. Thus B can strictly increase his payoff by deviating to $x_B = x_A^*$, a contradiction.

2. Suppose $-a < x_A^* \leq b$. Then B can strictly increase his payoff by deviating to $x_B = x_A^*$ as it will weakly increase the winning probability and strictly increase the expected payoff from the policy.

3. Suppose $x_A^* > b$. In this case $x_B^* \leq -b$ must also hold because we have assumed $x_A^* + x_B^* \leq 0$. Then B can strictly increase his payoff by deviating to $x_B = b$ as it will strictly increase the winning probability and strictly increase the expected payoff from the policy.

Second, we show that $x_A^* \leq b$ and $-b \leq x_B^*$. To show $x_A^* \leq b$, suppose to the contrary that $x_A^* > b$. Then, since $x_A^* \leq x_B^*$, candidate B's winning probability is weakly less than $\frac{1}{2}$. By symmetry of the utility function for policies, if candidate B deviates to $x_B = (x_A^* + b)/2$, her winning probability strictly increases to a number above $\frac{1}{2}$ and the expected utility from the implemented policy strictly increases. Hence, we have $x_A^* \leq b$. In the symmetric manner, we can show that $-b \leq x_B^*$.

Third, we show $-b \leq x_A^*$ and $x_B^* \leq b$. To see this, suppose to the contrary that $x_A^* < -b$. Recalling we have established that $-b \leq x_B^*$, we have $x_A^* < x_B^*$. If $-b < x_B^*$, A's winning probability given

[21] By the symmetry of the model, if (x_A^*, x_B^*) is an equilibrium outcome, $(x_A, x_B) = (-x_B^*, -x_A^*)$ is also an equilibrium outcome. Therefore it is without loss to assume $x_A^* + x_B^* \leq 0$. This inequality, together with $x_B < x_A$ entails that $|u_1(x_A) - u_1(x_B)| \leq |u_n(x_A) - u_n(x_B)|$ and hence, B's winning probability is no more than $1/2$.

B's policy x_B^* is weakly increasing in x_A over $[x_A^*, x_B^*) \ni -b$. If $x_B^* = -b$, A's winning probability is less than $1/2$ if she chooses $x_A = x_A^*$, while it is $1/2$ if $x_A = -b$. In either case, by deviating to $x_A = -b$, A can weakly increase her winning probability as well as strictly increase the payoff upon winning. This is a contradiction. The symmetric argument can show $x_B^* \leq b$.

The arguments so far imply that we must have $-b \leq x_A^* \leq x_B^* \leq b$. From here on, we first restrict our attention to $(x_A, x_B) \in [-b, b]^2$ with $x_A \leq x_B$ to pin down a unique candidate for (x_A^*, x_B^*), and then confirm that it is actually an equilibrium. To do so, we now analyze the second-stage and compute the winning probability for each candidate, taking $(x_A, x_B) \in [-b, b]^2$ as given. When the candidates commit to $(x_A, x_B) \in [-b, b]^2$ such that $x_A < x_B$ (we will consider the case with $x_A = x_B$ later), lobbyist 1's utility is

$$-\frac{c_A}{c_A + c_B} m(x_A + a)^2 - \frac{c_B}{c_A + c_B} m(x_B + a)^2 - c_A.$$

The first-order condition with respect to c_A is

$$\frac{c_B}{(c_A + c_B)^2} \left(m(x_B + a)^2 - m(x_A + a)^2 \right) - 1 = 0. \tag{4}$$

Similarly, the first-order condition for lobbyist n is

$$\frac{c_A}{(c_A + c_B)^2} \left(m(x_A - a)^2 - m(x_B - a)^2 \right) - 1 = 0.$$

These two equations imply that in the equilibrium of the subgame,

$$\frac{c_A}{c_A + c_B} = \frac{(x_B + a)^2 - (x_A + a)^2}{[(x_B + a)^2 - (x_A + a)^2] + [(x_A - a)^2 - (x_B - a)^2]} = \frac{1}{2} + \frac{x_B + x_A}{4a}.$$

Given the above winning probability, we now investigate the equilibria of the first stage. Given a policy profile $(x_A, x_B) \in [-b, b]^2$ such that $x_A < x_B$, candidate B's payoff is

$$U_B(x_A, x_B) = \left(\frac{1}{2} - \frac{x_A + x_B}{4a}\right)(1 - k(b - x_B)^2) - \left(\frac{1}{2} + \frac{x_A + x_B}{4a}\right) k(b - x_A)^2. \tag{5}$$

This implies that

$$\frac{\partial U_B(x_A, x_B)}{\partial x_B} = -\frac{1}{4a}(1 - k(b - x_B)^2) + \left(\frac{1}{2} - \frac{x_A + x_B}{4a}\right) 2k(b - x_B) - \frac{1}{4a} k(b - x_A)^2. \tag{6}$$

Substituting $x_B = b$, the left-hand side of (6) becomes $-\frac{1}{4a} - \frac{1}{4a}k(b - x_A)^2 < 0$. Since $U_B(x_A, x_B)$ is cubic in x_B and the coefficient on x_B is strictly positive by (5), either (i) $U_B(x_A^*, x_B)$ is strictly decreasing in x_B over $(x_A^*, b]$, or (ii) there exists a unique $x_B^0 \in (x_A^*, b)$ such that the right hand side of (6) is 0 and $U_B(x_A^*, x_B)$ is strictly increasing in x_B over $(x_A^*, x_B^0]$ and strictly decreasing in x_B over $[x_B^0, b]$. In case (i), there is no best response to $x_A = x_A^*$ for candidate B in $(x_A^*, b]$, a contradiction. Hence the only possibility is case (ii), which implies that we must have $x_B^* = x_B^0$. The symmetric argument can be made for x_A^*. Hence, if $x_A^* < x_B^*$, both the values of (6) and

$$\frac{\partial U_A(x_A, x_B)}{\partial x_A} = \frac{1}{4a}(1 - k(b + x_A)^2) - \left(\frac{1}{2} + \frac{x_A + x_B}{4a}\right) 2k(x_A + b) + \frac{1}{4a} k(b + x_B)^2 \tag{7}$$

must be equal to 0 at $(x_A, x_B) = (x_A^*, x_B^*)$.

Adding the right-hand sides of (6) and (7) and rearranging, we have:

$$\frac{k}{a}(x_A + x_B)(x_B - x_A - a - b).$$

Substituting x_A^* and x_B^* into x_A and x_B, respectively, and equating the expression to zero, we get $x_B^* = x_A^* + a + b$ or $x_B^* = -x_A^*$. However, if $x_B^* = x_A^* + a + b$, then $-b \le x_A^*$ implies $x_B^* \ge -b + a + b = a > b$, which contradicts our earlier conclusion that $x_B^* \le b$. Hence $x_B^* = -x_A^*$ holds. Substituting this back into (6), we obtain

$$x_B^* = \frac{ab - \frac{1}{4k}}{a+b} \leq b. \tag{8}$$

For this to satisfy $x_A^* < x_B^*$, it is necessary and sufficient that this is strictly positive, and for that, $k > 1/(4ab)$ is necessary and sufficient. That is, an SPE such that $x_A^* < x_B^*$ can exist only if $k > 1/(4ab)$ holds, and the only candidate for such an equilibrium outcome is given by $x_A^* = -x_B^*$ and (8).

Next, consider the case where $x_A^* = x_B^*$. Suppose that $x_A^* = x_B^* > 0$. This is not an equilibrium because candidate A can deviate to $-x_A^*$ to win with the same probability of $\frac{1}{2}$ and to strictly increase her expected payoff from the policy because $x_A^* \leq b$. Since the symmetric argument can be made for $x_A^* = x_B^* < 0$, we have that $x_A^* = x_B^*$ implies $x_A^* = x_B^* = 0$. For $(x_A^*, x_B^*) = (0,0)$ to be an equilibrium the value of (6) must be nonpositive, because otherwise B has an incentive to deviate and marginally increase x_B. Substituting $(x_A^*, x_B^*) = (0,0)$ into (6), we obtain

$$\frac{\partial U_B(0,0)}{\partial x_B} = -\frac{1}{4a} + kb \leq 0,$$

which is equivalent to $k \leq 1/(4ab)$. That is, an SPE such that $x_A^* = x_B^*$ can exist only if $k \leq 1/(4ab)$ holds, and the only candidate for such an equilibrium outcome is $x_A^* = x_B^* = 0$.

Combining the conclusions in the previous two paragraphs, we have thus obtained a unique candidate for a symmetric equilibrium,

$$-x_A^* = x_B^* = \max\left\{0, \frac{ab - \frac{1}{4k}}{a+b}\right\}.$$

Now we show that this unique candidate is actually an equilibrium.

If $k > 1/(4ab)$, then we know that $\{x_B^*\} = \arg\max_{x_B \in (x_A^*, b]} U_B(x_A^*, x_B)$. Hence, to prove that x_B^* is a best response to x_A^*, it suffices to check that candidate B has no incentive to deviate to

$x_B \le x_A^*$ or to $b < x_B$. First, for any $x_B < x_A^*$, since $x_A^* < 0$, B's winning probability under (x_A^*, x_B) is strictly less than $1/2$. Since the utility from the policy conditional on winning is strictly less under (x_A^*, x_B) than under (x_A^*, x_A^*), we have that $U_B(x_A^*, x_B) < U_B(x_A^*, x_A^*)$ for any $x_B < x_A^*$. Since (x_A^*, x_A^*) and (x_A^*, x_B^*) yield the same winning probability (i.e., $1/2$) for candidate B while the latter gives a strictly higher payoff from the expected policy, we have $U_B(x_A^*, x_A^*) < U_B(x_A^*, x_B^*)$. Combining the two inequalities, we conclude that $U_B(x_A^*, x_B) < U_B(x_A^*, x_B^*)$ for any $x_B \le x_A^*$. Second, for any $x_B > b$, $U_B(x_A^*, x_B) < U_B(x_A^*, b)$ holds because the wining probability is strictly higher under (x_A^*, b) than under (x_A^*, x_B), and the utility from the policy conditional on winning is also strictly higher under (x_A^*, b) than under (x_A^*, x_B). That is, $U_B(x_A^*, x_B) < U_B(x_A^*, x_B^*)$ for any $x_B > b$. Overall, x_B^* is a best response to x_A^*. By symmetry, we also have that x_A^* is a best response to x_B^*. Hence, (x_A^*, x_B^*) is an equilibrium outcome of the first stage.

If $k \le 1/(4ab)$, then, given $(x_A^*, x_B^*) = (0,0)$, equation (6) and the argument that follows it imply that $U_B(x_A^*, x_B)$ is strictly decreasing in x_B over $(x_A^*, b]$. Moreover, since the winning probability under policy profile $(0, x_B)$ is continuous at $x_B = 0$, $U_B(0, x_B)$ is continuous in x_B at $x_B = 0$, which in turn implies that $U_B(x_A^*, x_B)$ is strictly decreasing in x_B over $[x_A^*, b]$. Hence, any deviation of candidate B to $x_B \in (0, b]$ results in the decrease of the payoff. Also, as before, any deviation to $x_B > b$ is strictly less profitable than the deviation to $x_B = b$. Finally, any deviation to $x_B < 0$ gives the same winning probability as and a strictly lower expected utility from the policy than the deviation to $x_B' = -x_B$. Hence, $(x_A^*, x_B^*) = (0,0)$ is an equilibrium outcome of the first stage. ■

The above analysis has three implications. First, a sufficiently large weight on the policy preferences (i.e., $k > 1/(4ab)$) is necessary for policy divergence in the unique symmetric equilibrium. In other words, our prediction of policy convergence in the main model is robust for a small weight on the policy preferences. Second, in equilibrium, the total amount of contribution positively correlates with the degree of policy divergence. To see this, let $c^* = c_A^* = c_B^*$ and $D^* = x_B^* - x_A^* = 2x_B^*$, where the latter represents the degree of policy divergence. Then equation (4) reduces to

$$\frac{1}{4c^*}\left(m\left(\frac{D^*}{2}+a\right)^2-m\left(-\frac{D^*}{2}+a\right)^2\right)-1=0 \Longleftrightarrow c^*=\frac{am}{2}D^*.$$

Since $am>0$, the contribution to each candidate (c^*) is strictly increasing in the degree of the policy divergence (D^*) fixing a and m.[22] Third, when k is sufficiently large, $x_B^*=\frac{ab-(1/4k)}{a+b}$ is strictly increasing in k, a, and b. Hence, the degree of policy divergence (D^*) is also increasing in the weight on the candidates' policy preferences (k), divergence of lobbyists' bliss points (a), and divergence of candidates' bliss points (b). We summarize the finding in the following corollary.

Corollary 2. *In the model with policy-motivated candidates, let* $((x_A^*,x_B^*),(\kappa_1^*(\cdot),\ldots,\kappa_n^*(\cdot)))$ *be the unique pure-strategy SPE and define* $D^*=x_B^*-x_A^*$ *and* $c^*=c_A^*=c_B^*$*, where* $\kappa_1(x_A^*,x_B^*)=(c_A^*,0)$ *and*$\kappa_n(x_A^*,x_B^*)=(0,c_B^*)$*. Then we have the following:*

1. $D^*=c^*=0$ *if* $k\leq\frac{1}{4ab}$*,*

2. $c^*=\frac{am}{2}D^*$ *for any parameter values, and*

3. *both* D^* *and* c^* *are strictly in creasing in k, a, and b if* $k>\frac{1}{4ab}$*.*

Now we consider the possibility that there is an exogenously given *contribution cap* on the amount that each lobbyist can spend. Let $W\geq 0$ denote the cap. We assume that this cap applies to both lobbyists, that is, $c_{ki}\leq W$ must hold for any lobbyist k and candidate i.[23] We show that once a sufficiently tight cap is imposed, there is no pure-strategy SPE in which the contribution amount is zero no matter how small the weight on the policy preferences is. This is in a stark contrast with our result that, when there is no cap, the contribution amount is zero in the unique pure-strategy SPE if the weight on the policy preferences is small enough. Moreover, we show the existence of a mixed-strategy SPE. Combining these results, we have that in any SPE of the model with policy

[22] Note that the relationship $c^*=\frac{am}{2}D^*$ holds in the case $k\leq 1/(4ab)$ as well because $c^*=D^*=0$.

[23] Since each lobbyist has an incentive to contribute to at most one candidate, the analysis will remain exactly the same if we instead assume a cap in the form of $c_{kA}+c_{kB}\leq W$.

motivated candidates and a sufficiently tight contribution cap, the contribution amount is strictly positive with strictly positive probability.

Proposition 7. *In the model with policy-motivated candidates and contributions caps, a mixed-strategy SPE exists. There is $W^* > 0$ such that for all $W \in (0, W^*)$, in any SPE $((\xi_A, \xi_B), (\gamma_1, \gamma_n))$ with W, the probability that both lobbyists pay a strictly positive contribution amount is strictly positive.*

Proof. First of all, it is without loss to assume that candidates assign probability zero to positions outside $[-a, a]$ because of our assumption for the cases of $x_i < x_j < -a$ or $x_i > x_j > a$.[24] Then taking the contribution cap W as given, each subgame after (x_A, x_B) is a special case of the model of Grossmann and Dietl (2012). For $(x_A, x_B) \in [-a, a]^2$ such that $x_A \leq x_B$ and $|x_A| \leq |x_B|$, the equilibrium levels of contributions are given by

$$(c^*_{1,A}, c^*_{n,B}) = \begin{cases} \left(\frac{R_1(x_A,x_B)^2 R_n(x_A,x_B)}{(R_1(x_A,x_B)+R_n(x_A,x_B))^2}, \frac{R_1(x_A,x_B) R_n(x_A,x_B)^2}{(R_1(x_A,x_B)+R_n(x_A,x_B))^2}\right) & \text{if } W \geq \frac{R_1(x_A,x_B)^2 R_n(x_A,x_B)}{(R_1(x_A,x_B)+R_n(x_A,x_B))^2}, \\ \left(W, \sqrt{W R_n(x_A,x_B)} - W\right) & \text{if } \frac{R_1(x_A,x_B)^2 R_n(x_A,x_B)}{(R_1(x_A,x_B)+R_n(x_A,x_B))^2} > W \geq \frac{R_n(x_A,x_B)}{4}, \text{ and} \\ (W, W) & \text{otherwise,} \end{cases}$$

and consequently, the winning probabilities for the candidates are

$$(P_A, P_B) = \begin{cases} \left(\frac{R_1(x_A,x_B)}{R_1(x_A,x_B)+R_n(x_A,x_B)}, \frac{R_n(x_A,x_B)}{(R_1(x_A,x_B)+R_n(x_A,x_B))}\right) & \text{if } W \geq \frac{R_1(x_A,x_B) R_n(x_A,x_B)^2}{(R_1(x_A,x_B)+R_n(x_A,x_B))^2}, \\ \left(\sqrt{W/R_n(x_A,x_B)},\ 1 - \sqrt{W/R_n(x_A,x_B)}\right) & \text{if } \frac{R_1(x_A,x_B)^2 R_n(x_A,x_B)}{(R_1(x_A,x_B)+R_n(x_A,x_B))^2} > W \geq \frac{R_n(x_A,x_B)}{4}, \text{ and} \\ (1/2, 1/2) & \text{otherwise,} \end{cases}$$

where $R_1(x_A, x_B) = u_1(x_A) - u_1(x_B)$ and $R_n(x_A, x_B) = u_n(x_B) - u_n(x_A)$.[25] Note that $R_1 \leq R_n$ under the assumptions on (x_A, x_B). All the other cases of (x_A, x_B) are symmetric.

[24] Under our assumption, for any any $x_i \notin [-a, a]$ and $x_j \in \mathbb{R}$, there is $x'_i \in [-a, a]$ that gives candidate i a strictly higher payoff than x_i against candidate j's x_j.

[25] Note that $\frac{R_1(x_A,x_B)^2 R_n(x_A,x_B)}{(R_1(x_A,x_B)+R_n(x_A,x_B))^2} \geq \frac{R_n(x_A,x_B)}{4}$ holds whenever $R_1(x_A, x_B) \leq R_n(x_A, x_B)$.

We first show that an SPE exists in mixed strategies, and then show that the probability that both lobbyists pay a strictly positive contribution amount is strictly positive in any SPE. To show that an SPE exists in mixed strategies, note that there exists a unique Nash equilibrium in each subgame played by the lobbyists, just as in the main text. Let $V_i(x_A, x_B)$ be candidate i's payoff function given the policy profile (x_A, x_B), provided that the lobbyists play the unique Nash equilibrium in the subgame after (x_A, x_B). First recall that we are considering a game in which each candidate chooses a policy from $[-a, a]$. Second, note that each V_i is bounded and continuous except on the subset $\{(x_A, x_B) | x_A = x_B\}$. Finally, note that for $x \geq 0$,

$$\lim_{x_A \uparrow x, x_B \downarrow x} V_A(x_A, x_B) \geq V_A(x, x) \geq \lim_{x_A \downarrow x, x_B \uparrow x} V_A(x_A, x_B) \tag{9}$$

and

$$\lim_{x_A \uparrow x, x_B \downarrow x} V_B(x_A, x_B) \leq V_B(x, x) \leq \lim_{x_A \downarrow x, x_B \uparrow x} V_B(x_A, x_B) \tag{10}$$

hold, where the left inequality in (9) is strict if and only if the left inequality in (10) is strict, and the right inequality in (9) is strict if and only if the right inequality in (10) is strict. Also, for $x < 0$,

$$\lim_{x_A \uparrow x, x_B \downarrow x} V_B(x_A, x_B) \geq V_B(x, x) \geq \lim_{x_A \downarrow x, x_B \uparrow x} V_B(x_A, x_B) \tag{11}$$

and

$$\lim_{x_A \uparrow x, x_B \downarrow x} V_A(x_A, x_B) \leq V_A(x, x) \leq \lim_{x_A \downarrow x, x_B \uparrow x} V_A(x_A, x_B) \tag{12}$$

hold, where the left inequality in (11) is strict if and only if the left inequality in (12) is strict, and the right inequality in (11) is strict if and only if the right inequality in (12) is strict. All in all, the conditions for the existence of a mixed-strategy equilibrium in Dasgupta and Maskin (1986, Theorem 5b) are satisfied. Hence, a mixed-strategy equilibrium exists in the game with payoff functions (V_A, V_B). Therefore, an SPE exists in mixed strategies.

To show that the probability that both lobbyists pay a strictly positive contribution amount is strictly positive in any SPE, it is sufficient to show that there is $W^* > 0$ such that for all $W \in (0, W^*)$, there is no pure-strategy SPE in which both candidates choose the same policy. Towards a contradiction, suppose that $W < \frac{mb^2}{4}$, that such an SPE exists, and that the candidates choose $x_A = x_B = x$ with probability one on the equilibrium path of play. Suppose $x \leq 0$ so that $R_1(x,b) \geq u_1(0) - u_1(b) > mb^2$ and $R_n(x,b) \geq u_n(b) - u_n(0) = mb^2$. Then, since $\min\left\{\frac{R_1(x,b)}{4}, \frac{R_n(x,b)}{4}\right\} > \frac{mb^2}{4}$, candidate B's winning probability is $\frac{1}{2}$ if she deviates to policy b. Hence, the deviation to b does not change the winning probability while strictly increasing the payoff upon winning. This contradicts the assumption that both candidates choosing x with probability 1 is part of a SPE strategy profile. Since the symmetric argument applies to the case when $x \geq 0$, we have proven the desired result. ■

D Convex Lobbyist Utility

As we saw in the main text, the concavity of lobbyists' utility functions is a key to the policy convergence result. Moreover, extremist dominance arises because the extremists perceive the largest utility gaps between two policies, and this is because we assume the single crossing property of the lobbyists' utility functions. Note that, in the simple setting with $u_k(x) = u(x - a_k)$ for each k, strict concavity of u implies the single-crossing property. In order to better understand the implication of concavity assumption on lobbyist preferences, here we examine convex utility functions. The point of this exercise is not to argue for a particular assumption about lobbyist preferences, but to better understand the roles of different assumptions about preferences of the lobbyists.

Specifically, consider the "convex utility model" that is exactly the same as the one with mixed strategies presented in Appendix A, except for the following three features. First, there exists $u : \mathbb{R} \to \mathbb{R}$ such that, for each $k \in N$, $u_k(x) = u(x - a_k)$. Second, u is *decreasing* and *strictly convex* in the following sense: For any $x, y \in \mathbb{R}$ such that $x \cdot y > 0$ and $x > y$,

$$(u(x) - u(y)) \cdot x < 0 \text{ and } u(\alpha x + (1-\alpha)y) < \alpha u(x) + (1-\alpha)u(y) \text{ for all } \alpha \in (0,1). \tag{13}$$

That is, u is strictly convex in the standard sense on $\mathbb{R}_+$ and $\mathbb{R}_-$ separately, although not on $\mathbb{R}$ as a whole. Third, u is assumed to be *skewed*: for any $x > 0$, $u(-x) < u(x)$.[26] Under these assumptions, we can characterize SPEs as follows.

Proposition 8. *Suppose that Assumptions 1-4 and Assumption 6 hold. In the convex utility model, an SPE exists, and any SPE* $((\xi_A, \xi_B), (\gamma_1(\cdot), \ldots, \gamma_n(\cdot)))$ *is pure and satisfies the following:*

1. *(Policy Convergence)* $\xi_A(a_n) = \xi_B(a_n) = 1$.
2. *(At Most Two Candidates Contribute) For any profile* $(x_A, x_B) \in \mathbb{R}^2$ *and any* $i = A, B$*, if there are* $k, k' \in N$ *such that* $k \neq k'$ *and* $c_{ki}, c_{k'i} \in \mathbb{R}_{++}$ *with* $\gamma_k(x_A, x_B)(c_{ki}) > 0$ *and* $\gamma_{k'}(x_A, x_B)(c_{k'i}) > 0$*, then for all* $k'' \in N \setminus \{k, k'\}$, $\gamma_{k''}(x_A, x_B)(0,0) = 1$.
3. *(No Contribution on Path)* $\gamma_k(a_n, a_n)(0,0) = 1$ *for all* $k \in N$.

Proof. First, note that the part of the proof of Proposition 4 showing that the second-stage strategies in any SPE are in pure strategies does not use the assumption that u is strictly concave, and they go through even if it is convex. Hence, an SPE exists, and any SPE $((\xi_A, \xi_B), (\gamma_1(\cdot), \ldots, \gamma_n(\cdot)))$ is such that γ_k is pure for each $k \in N$.

It is straightforward to see that part 2 holds, by noting that the proof for the claim (1) in the proof of Proposition 1 does not use any assumption about the functional form of u. It is also immediate that part 3 is implied by part 1. Therefore, we only need to show part 1. To do so, take any $x \neq a_n$, let $k \in \arg\max_{\ell \in N}[u_\ell(x) - u_\ell(a_n)]$, and fix a Nash equilibrium $(c^*_{\ell A}, c^*_{\ell B})_{\ell \in N}$, which must be in pure strategies as argued in the previous paragraph, of the subgame after policy profile $(x_A, x_B) = (x, a_n)$.

[26] We will note on different assumptions on u at the end of this section.

First, suppose that $u_k(x) - u_k(a_n) \leq 0$. Then $c^*_{\ell A} = 0$ must hold for any $\ell \in N$. Moreover, since $\min_{\ell \in N} [u_\ell(x) - u_\ell(a_n)] \leq u_n(x) - u_n(a_n) < 0$, Assumption 3 implies that $c_{1B} = \cdots = c_{nB} = 0$ cannot hold. That is, we must have $c^*_A = \sum_{\ell \in N} c^*_{\ell A} = 0 < \sum_{\ell \in N} c^*_{\ell B} = c^*_B$, and hence, $P_B(c^*_A, c^*_B) > 1/2$.

Second, suppose that $u_k(x) - u_k(a_n) > 0$. Note that this implies $x < a_n$. If $x \geq a_k$, then the strict convexity of u implies $u(x - a_k) - u(a_n - a_k) \leq u(0) - u(a_n - x)$. If $x < a_k$, then the same inequality holds because $u(0) > u(x - a_k)$ and $u(a_n - a_k) > u(a_n - x)$. In either case, thus, we have

$$
\begin{aligned}
u_k(x) - u_k(a_n) = u(x - a_k) - u(a_n - a_k) &\leq u(0) - (a_n - x) \\
&< u(a_n - a_n) - u(x - a_n) = u_n(a_n) - u_n(x),
\end{aligned}
$$

where the second inequality holds by the assumption that u is skewed. Then, as in the proof of Proposition 2, we can conclude that $P_B(c^*_A, c^*_B) > 1/2$.

In sum, we have shown that $P_B(c^*_A, c^*_B) > 1/2$ in any Nash equilibrium of the subgame after $(x_A, x_B) = (x, a_n)$ with $x \neq a_n$. Given this, we can establish that $\xi_A(a_n) = \xi_B(a_n) = 1$ must hold in any SPE, exactly in the same way as in the proof of Proposition 2. ■

There are two differences from the conclusion of our analysis with concave utility function. First, although the policies converge again, they converge to the position of one of the extreme lobbyists, not to the middle. Second, the proof shows that the contribution pattern differs in subgames where the policies diverge. More specifically, it is possible that a single candidate attracts contributions from two lobbyists. Moreover, the lobbyists who contribute are not the extreme lobbyists, but those who are the closest to the candidate. These are in contrast with the case of concave utilities, where each candidate raises a positive amount of contribution from one of the extreme lobbyists whenever the policies diverge, and polices converge to a^M in equilibrium.

Two further remarks are in order. First, although a symmetric argument can be made if we assume u is skewed in the other direction, i.e., $u(-x) > u(x)$ for all $x > 0$, a different conclusion holds in the knife-edge case of symmetric lobbyist utilities, i.e., $u(-x) = u(x)$ for all $x > 0$. In such a case,

(ξ_A, ξ_B) is the profile of candidates' strategies in an SPE if and only if $\xi_A, \xi_B \in \Delta(\{a_1, \ldots, a_n\})$. Thus, in particular, multiple equilibria exist. Second, one can extend our model to the case of multiple policy dimensions, where there are $n(d)$ lobbyists in dimension d and all lobbyists in the same dimension share the same shape of the utility function, as in Kamada and Kojima (2014). Then, one can show that in the dimension for which lobbyists' utilities are concave, policies converge to a middle of the two extreme lobbyists in that dimension, and in the dimension for which lobbyists' utilities are convex, policies converge to the position of one of the two extreme lobbyists in that dimension.