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# Vertically Differentiated Cournot Oligopoly: Effects of Market Expansion and Trade Liberalization on Relative Markup and Product Quality

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## Abstract

We model an oligopoly where firms are allowed to freely enter and exit the market and choose the quality level of their products by incurring different set-up costs. Using this framework, we study the mix of firms in the long-run Cournot-Nash equilibrium under different cost structures and the effects of market size on market outcomes. Specifically, we consider two alternative specifications of cost structure. In the first specification, quality upgrading requires a large increment in the set-up cost or R&D investment. Under this cost structure, we show that in the Nash equilibrium, each firm specializes in a single quality level, and an increase in the market size leads to (i) an increase in the fraction of firms that specialize in the high quality product, (ii) an increase in the market share of the high quality product, and (iii) a reduction in firms' markups and in markup dispersion. Under the second type of cost structure where quality upgrading only requires higher marginal cost, we find that all firms will produce both types of product, and the value share of the high-quality product increases as the market expands, but in quantity terms, the market share of the high quality product does not change. Finally, we find that trade liberalization has broadly similar effects to that of a market expansion, but the supply of the high-quality product from the smaller economy may decrease.

**JEL Classification:** L1, L2, F15.

**Keywords:** Multiproduct firms; Cournot competition; Vertical product differentiation; Cost structure; Market size; Trade liberalization.

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# 1 Introduction

Do increased competitive pressure and/or market expansion induce firms to upgrade the quality of their products, leading to an increase the average quality of products in the market place? Can market expansion lead to the demise of firms that produce the low-quality products? Does trade liberalization has similar effects on product quality to those of a pure market expansion? The answers to these questions may well depend on the nature of the industry under consideration, in particular its cost structure and possibly on its mode of competition. While there is an existing theoretical and empirical literature that has shed light on these questions (see, for example, Fajgelbaum et al. 2011; Berry and Waldfogel 2010; Vives 2008), there remain some gaps in the analysis of the role of the trade-off between fixed cost and marginal cost in the quality upgrading process and of firms' decision on whether they should offer a wide range of qualities, or, instead, specialize in one end or the other of the quality spectrum. Moreover, the existing studies on the relation between market size and products' quality are incomplete. We apply our theoretical framework to the study of this issue and expand the relevant literature by adding the analysis under the Cournot-oligopoly competition mode and allowing variable markups.<sup>1</sup>

In this paper, we develop model of Cournot oligopoly with vertically differentiated products and show how the effect of market expansion on the average quality depends crucially on the industry's cost structure. We demonstrate that the equilibrium features of the model depends on whether the additional investments required to produce the high-quality product involve sinking large costs (i.e., quality upgrading is intensive in overhead factors, such as R&D) or incurring higher variable costs per unit of quality (i.e., quality upgrading is intensive in the intermediate inputs). One of our main findings is that when the upgrading process is intensive in the overhead factors, in equilibrium each firm will choose to produce either the high-quality product, or the low-quality product, but not both. Another major finding is that, under this cost configuration, an increase in the market size will increase the number of high-quality producers, decrease the number of low-quality producers, leading to a fall in the relative price and relative markup of the high-quality product and an increase in the market share of high-quality firms. When, on the contrary, quality upgrading is intensive in intermediate inputs and fixed cost is independent of quality, each firm will produce both the high-quality and the low-quality products, and an increase in market size will increase the relative price, relative markup, and the market share of the high-quality product in value terms, while leaving the ratio of high-quality to low-quality outputs unchanged.

Concerning the effects of partial trade liberalization between two countries, we find that, in broad outlines, the results are similar to that of an increase in market size. However, when the two countries are of different size, we find that as the trade costs decrease, in the small country, the ratio of high-quality firms to low-quality firms may change in a non-monotonic fashion. This is due to the relocation decision of high-quality firms from the small country to the large country, which depends on trade costs. Indeed, firms choose their

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<sup>1</sup>For example, Fajgelbaum et al. 2011 did not study how the markups and the relative markup change when the market size increases. In contrast, this issue is fully analyzed in our paper.

location based on the balance of market demand and market competition intensity.

Our paper is an extension of the model of strategic product line choice by Johnson and Myatt (2006). Different from Johnson and Myatt (2006), our model assumes more general cost structures. Thus, while Johnson and Myatt (2006) assume that all firms have the same fixed cost,<sup>2</sup> we consider two different cost scenarios. In the first scenario, firms that want to produce the high-quality product must incur a higher quality-adjusted fixed cost, and this enables them to produce the high-quality product at a smaller quality-adjusted marginal cost than that of the lower-quality product. In the second scenario, all firms have the same fixed cost but marginal costs differ: the high-quality product requires much more expensive raw materials, such that the quality-adjusted marginal cost of the high-quality product is higher than that of the low-quality product. In this alternative scenario, we find that all firms will produce both products.<sup>3</sup>

Our results are broadly consistent with the empirical literature. Berry and Waldfogel (2010) examine descriptive data on the relationship between product quality and market size. They find that in one industry (restaurants), where quality is created largely through variable costs, such as food ingredients and skilled labor time, an increase in market size will increase the number of varieties, and while there will be more high-quality restaurants per capita there is “no evidence that average quality increases as well (p. 25)”. In contrast, in the newspaper industry, where quality is produced by fixed cost rather than marginal cost, Berry and Waldfogel (2010) find that larger market sizes are associated with higher average quality in the market. Our results on decreasing markups and increasing product quality under trade liberalization are also consistent with empirical studies. Indeed, using a model where intermediate good producers are oligopolists, Edmond et al. (2015) reported in their empirical study of Taiwanese firms that increased exposure to trade reduces the markup distortion by one-half. Along the same veins, using highly disaggregated firm-product level data from China over the period 2000-2006, Li and Miao (2017) found that market expansion in the form of trade liberalization induced China’s high productivity firms to increase the fraction of high quality goods they exported.

Our analysis has implications for firms’ strategies. Should a firm that is capable of producing both high and low quality products offer a large range of quality to cater for all types of consumers, or should it focus on the high-quality end of the product spectrum? According to our model, if quality is produced mainly by fixed cost, and if there are other firms in the industry that choose to incur the low fixed cost (which disqualify them from producing the high-quality product), then the high-quality firms should specialize in the high-quality product, because supplying both types of products will create an adverse cannibalization effect, reducing consumers’ demand for their high-quality good.<sup>4</sup> This result seems to be applicable to the newspaper industry, where, as argued by Berry and Waldfogel (2010), quality is produced mainly by fixed cost. We do not observe high-quality newspaper firms that sell low-quality versions of their high-quality products. For industries that operate under a different cost configuration, where quality is produced by variable costs while fixed cost is independent of quality, our model predicts that each firm would sell both

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<sup>2</sup>This assumption is stated clearly in Johnson and Myatt (2006).

<sup>3</sup>This is consistent with Johnson and Myatt (2006)’s proposition 9.

<sup>4</sup>Of course, there are other reasons, such as preservation of brand names, but these are outside the scope of our model.

the high-quality version and the low-quality version. This is consistent with the case of plasma televisions. Indeed, Sony, Samsung, and Mitsubishi all offer product lines with varying degrees of resolution.

The rest of this paper is organized as follows. In Section 2, we review the related literature and relate our model's predictions to other findings. In Section 3, we specify our general model and analyze it under two alternative cost configurations. In Section 4, we apply our theoretical framework to investigate the effects of trade liberalization on the average quality of products, on relative markups, and on the relative demand and supply of the high-quality product in two economies of different sizes. Section 5 offers some concluding remarks.

## 2 Related Literature

There is a large literature on vertically differentiated products. A number of papers postulate that all firms have the same fixed cost, on the assumption that quality is produced only with variable costs (Johnson and Myatt, 2006; Mussa and Rosen, 1978; Gal-Or, 1983; Champsaur and Rochet, 1989). This happens when quality improvement is achieved by using more expensive material inputs or more skilled labor. Other authors adopt the opposite assumption: high-quality can only be produced by incurring higher fixed cost, such as R&D and advertising (Bonanno, 1986; Ireland, 1987).<sup>5</sup> Shaked and Sutton (1987) assume that quality improvement involves mainly an increase in fixed cost: variable costs may increase only at a “modest rate” (p. 136).<sup>6</sup>

The effect of increased competitive pressure and/or market expansion on innovation was studied in Vives (2008) and Fajgelbaum et al. (2011). Unlike our model, Vives (2008) did not consider investment in quality upgrading, nor vertical product differentiation. His focus was on R&D expenditure by oligopolists to reduce production cost (i.e., process innovation).<sup>7</sup> Vives (2008) found that “increasing market size increases cost reduction expenditure per firm and has ambiguous effects on the number of varieties offered”. He also noted that Bertrand equilibria tend to be more competitive than Cournot equilibria, and that this conclusion must be qualified when strategic commitments are allowed. Indeed, in Cournot models, since outputs are strategic substitutes, it pays a firm to over-invest in order to gain an advantage. Thus, Cournot competition may result in more cost reduction efforts. However, Vives (2008) pointed out that increased competition should not be modelled as a move between Bertrand and Cournot equilibria, because the mode of competition is likely to vary with the institutional features of the market. Generally, our setting differs from that of Vives (2008) in a number of respects. First, in our model, consumers are heterogeneous while Vives (2008) assumed identical consumers. Second, we allow firms to choose their product lines strategies, which may lead to an

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<sup>5</sup>Motta (1993) considered two separate models, one in which quality improvement comes from incurring a higher fixed cost, and one in which it relies only on variable costs.

<sup>6</sup>They find that under certain additional conditions, under price competition, at least one firm continues to retain some particular level of market share, no matter how large the economy becomes Shaked and Sutton (1987) (p. 132). This result is known as the “finiteness property”. See also Shaked and Sutton (1983).

<sup>7</sup>However, Vives (2008) argued that “the cost reduction model can be interpreted as an investment in quality (in terms of product enhancement) in the context of the Cournot model.”

outcome where different firms specialize in different quality levels and achieve different levels of markup and short run profitability, while Vives (2008) assumed that firms are symmetric and behave in a symmetric way. Third, we address the impact of market expansion on the long-run equilibrium outcome while Vives' focus was on the short run. Fourth, while in Vives' model, market expansion is through population or economic growth, we consider not only that form of market expansion, but also market expansion through trade liberalization. Our approach allows us to analyze firms' quality scope strategies (balancing cost saving effect and cannibalization effect) and the effect of market expansion or increasing competitive pressure on the relative markups and market shares of different types of firms.

Fajgelbaum et al. (2011) considered a theoretical model in which products that are differentiated both vertically and horizontally. Following Gabszewicz and Thisse (1979; 1980) and Shaked and Sutton (1982; 1983), they assumed that when a person consumes more of the outside good her intensity of preference for quality of the goods provided by the oligopolists increases. This means that rich consumers appreciate the high quality product more. However, as made clear in Fajgelbaum et al. (2011), they abstracted from strategic interaction by assuming monopolistic competition: each firm thinks that its price does not affect the average price of the industry. In contrast, our model assumes Cournot oligopoly, implying that strategic interaction plays a key role. In our theoretical framework, each firm's markup is variable in different market conditions. Moreover, different from our model, Fajgelbaum et al. (2011) imposed the assumption that each firm can produce only a single-product. In the case where the industry offers two quality levels, they were able to determine the long run equilibrium number of the two types of firms. They proved that rich countries export the high-quality product and import the low-quality product. This is because of the "home market effects": due to trade costs, the larger home market bestows a competitive advantage to local firms. In their paper, comparative advantage is generated on the demand side.<sup>8</sup>

Recent empirical works by Baldwin and Harrigan (2011) and Johnson (2012) allowed for vertically differentiated products by modifying the model of Melitz (2003). They found that high-productivity firms export higher-quality products. They explained this by pointing out that these firms have greater incentive to undertake quality-improving investments. Both papers assumed monopolistic competition. Specifically, Johnson (2012) considered an extension of Melitz (2003) monopolistic competition model by allowing firms to be heterogeneous in two dimensions: quality and cost.<sup>9</sup>

Different from the above literature, our analysis of market expansion via trade liberalization (in Section 4) tackles the issue of how trade liberalization affects the distribution of product quality in a vertically differentiated international Cournot oligopoly. This is a significant departure from the bulk of recent trade models as these focus on monopolistic competition.<sup>10</sup> An exception is Eckel and Neary (2010), who study oligopolistic

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<sup>8</sup>Other models in which richer countries export the luxury goods focus on the supply side. For example, in Markusen (1986) and Bergstrand (1990), the richer country exports the luxury good because that good is capital intensive.

<sup>9</sup>In his data set, quality and quality-adjusted prices are unobserved and can only be inferred.

<sup>10</sup>While the monopolistic competition framework is convenient, the CES utility function assumed in this literature (e.g., Melitz (2003)) produces the counterfactual result that markup is a constant, independent of the market size. This is contrary to the empirical evidence, see, e.g. Edmond et al. (2015). In our model, the markups differ between low and high-quality products and vary with market size.

competition among multiproduct firms. They consider both supply and demand linkage within the firm. The supply side linkage is characterized by flexible manufacturing, while the demand linkage is via the “cannibalization effect.” According to Eckel and Neary (2010), multiproduct firms (MPFs) internalize demand linkage between the varieties they produce. This feature is called the cannibalization effect and is a defining feature of MPFs.” Unlike our model, in Eckel and Neary (2010) products are not vertically differentiated. Their emphasis is on the concept of core competence. They show that increasing trade openness tends to make each firm reduce their product varieties in order to better exploit their core competence. This leads to a trade gain, in the form of raising each firm’s average productivity (since they are better in producing goods that are in their in their core competence), but there is also an associated welfare loss, because consumers will be exposed to fewer varieties.<sup>11</sup> In a follow-up paper, Eckel et al. (2015), concentrating on short-run equilibrium, allow firms to invest to improve the consumers’ perception of the firm’s product qualities. These investments are possibly outlays on advertising. Applying their model to Mexican data, they find that the data is consistent with the model’s key prediction, namely, firms in the differentiated product sectors exhibit quality-based competence (meaning that prices fall with distance from core competence), while firms in the non-differentiated product sectors exhibit the opposite patterns. Performing comparative statics, given the fixed number of firms, they showed that an increase in the market size raises the output of all firms if firms are homogeneous, but if firms are heterogeneous, the outcome displays the so-called “super-star” property: Firms with above-average total output and outputs per variety tend to grow faster than other firms when the market size expands.<sup>12</sup>

Our model is built on Johnson and Myatt (2006), where firms can choose the quality levels from a discrete set  $\{S_1, S_2, \dots, S_m\}$ , and they compete in quantities, taking the inverse demand function for each quality type as given. However, in one of our cost configurations, we replace their assumption that all firms incur the same set-up cost (regardless of the quality of the product that firms offer) with a more plausible one: firms that wish to specialize in the lower quality product incur a lower set-up cost than that of firms that produce the high quality product. In addition, while Johnson and Myatt (2006) are mainly concerned with the short run equilibrium, where the number of firms are fixed, the focus of our model is the long run equilibrium, where free entry and exit ensures that profit is zero.

### 3 The model

We consider an oligopoly with vertically differentiated products. Specifically, we assume there are two quality levels, denoted by  $q_L$  and  $q_H$ , where  $0 < q_L < q_H$ .<sup>13</sup> A firm can produce the low-quality product, or the high-quality product, or both, incurring different variable costs and fixed costs. We will consider two

<sup>11</sup>For an empirical test of the core competence model, see Eckel et al. (2016).

<sup>12</sup>Long et al. (2011) examine the long run equilibrium in a model where firms are oligopolists with ex-ante cost heterogeneity and examine the effects of trade liberalization on R&D expenditure and industry productivity. Their paper however assumes that all the firms in the industry produce the same homogeneous product.

<sup>13</sup>In this paper, we follow the approach of Johnson and Myatt (2006) in that we do not address the issue of how the quality levels  $q_H$  and  $q_L$  are determined. The set of quality levels is discrete and given.

alternative cost configurations, as specified in subsection 3.2 below.

### 3.1 Consumers

There is a continuum of heterogeneous consumers. They differ from each other in terms of their intensity of preference for quality, which is represented by a parameter  $\theta$ , where  $0 \leq \theta \leq \bar{\theta}$ . Let  $G(\theta)$  denote the fraction of consumers whose intensity of preference is smaller than or equal to  $\theta$ . We assume that  $G(0) = 0$ ,  $G(\bar{\theta}) = 1$  and  $G'(\theta) > 0$  for all  $\theta \in (0, \bar{\theta})$ .

Each consumer buys at most one unit of the good. She must decide whether to buy one unit of the high-quality product, or one unit of the low-quality product, or neither product. A consumer of type  $\theta$  places a value  $\theta q_H$  on the consumption of a unit of the high-quality product, and a value  $\theta q_L$  on the low-quality product. Let  $P_H$  (respectively,  $P_L$ ) denote the market price of the high-quality product (respectively, low-quality product). Her net utility is  $\theta q_H - P_H$  or  $\theta q_L - P_L$ , depending on which product she buys.<sup>14</sup> We assume that  $P_H > P_L$ . Let us define the following ratios:

$$\theta_L \equiv \frac{P_L}{q_L}, \theta_H \equiv \frac{P_H}{q_H}, \theta_I \equiv \frac{P_H - P_L}{q_H - q_L} \quad (1)$$

In what follows, we assume that equilibrium prices are such that  $0 < \theta_L < \theta_H < \theta_I < \bar{\theta}$ . It is easy to show that

$$\frac{P_H - P_L}{q_H - q_L} > \frac{P_H}{q_H} \iff \frac{P_L}{q_L} < \frac{P_H}{q_H} \quad (2)$$

A consumer whose  $\theta$  equals  $\theta_L$  is indifferent between not buying the good and buying one unit of the low-quality product at the price  $P_L$ . Similarly, a consumer whose  $\theta$  equals  $\theta_H$  is indifferent between not buying the good and buying one unit of the high-quality product at the price  $P_H$ . And a consumer with  $\theta = \theta_I$  will be indifferent between the two alternative purchases. The fraction of the population who purchases the high quality product is  $G(\bar{\theta}) - G(\theta_I)$ , the fraction who purchases the low quality product is  $G(\theta_I) - G(\theta_L)$ , and the fraction who does not buy the good is  $G(\theta_L)$ .

### 3.2 Producers

We consider a two-stage game. We assume that in stage 1, firms that want to produce the high-quality product must incur an upfront cost (or set-up cost)  $F_H$ , which represents the cost of *R&D* and purchases of equipment. This fixed cost, once incurred, allow them to produce, in stage 2, both the high-quality product and the low-quality product (though in stage 2, it is possible that in equilibrium they choose to specialize in the high-quality product). Any firm that wants to produce the low quality product must incur in stage 1 a fixed cost  $F_L$ , where  $0 < F_L \leq F_H$ . In stage 2, firms choose their output levels and compete as Cournot rivals. The marginal production costs for high and low-quality products are  $C_H$  and  $C_L$  respectively, where  $C_L < C_H$ . We assume that  $C_L < C_H < \bar{\theta}q_H$ .

<sup>14</sup>In Subsection 3.4.2 we consider a more general utility function, as in Johnson and Myatt (2006), namely  $U = u(\theta, q_i) - P_i$ .



Let  $X_H$  and  $X_L$  denote the industry outputs of the high-quality and low-quality product. Letting  $P_H(X_H, X_L)$  and  $P_L(X_H, X_L)$  denote the inverse demand functions for the high-quality and the low-quality product, the profit function for the two types of firm are

$$\begin{cases} \pi_H^i &= [P_H(X_H, X_L) - C_H]x_H^i + [P_L(X_H, X_L) - C_L]x_L^i - F_H \\ \pi_L^j &= [P_L(X_H, X_L) - C_L]x_L^j - F_L \end{cases} \quad (3)$$

### 3.3 Equilibrium

The equilibrium depends on the specifications of the trade-off between fixed cost and marginal cost. Since this trade-off depends on the quality level, it turns out to be useful to define, for each of the two quality levels  $q_L$  and  $q_H$ , the fixed cost per unit of quality and the marginal cost per unit of quality for each of the two product types:

$$f_H \equiv \frac{F_H}{q_H}, \quad f_L \equiv \frac{F_L}{q_L}, \quad c_H \equiv \frac{C_H}{q_H}, \quad c_L \equiv \frac{C_L}{q_L}$$

While there are several possible configurations of  $(f_H, f_L, c_H, c_L)$ , we will restrict attention to two cost configurations, which we refer to as Cost Configuration 1 and Cost Configuration 2. Under Cost Configuration 1, higher quality requires higher fixed cost per unit of quality ( $f_H \geq f_L$ ), but involves lower marginal cost per unit of quality ( $c_H < c_L$ ), even though  $C_H \geq C_L$ . Under Cost Configuration 2, we assume that  $c_H \geq c_L$ , and  $F_H = F_L = F$ , which implies that  $f_H < f_L$ .

#### 3.3.1 Cost Configuration 1: higher quality requires higher fixed cost per unit of quality and involves lower marginal cost per unit of quality

We now consider equilibrium under Cost Configuration 1, in which the following inequalities hold:

$$f_H \geq f_L \text{ and } c_H < c_L < \bar{\theta} \quad (4)$$

We assume that any firm  $j$  that has invested only  $F_L$  is not able to produce the high-quality product. Its output is denoted by  $x_L^j$ . In contrast, any firm  $i$  that has invested  $F_H$  can produce both quality levels. Its output levels of high and low-quality products are denoted by  $x_H^i$  and  $x_L^i$ . With the cost structure (4), and any general cumulative distribution of preference  $G(\theta)$  with  $G(0) = 0$ ,  $G(\bar{\theta}) = 1$  and  $G'(\theta) \geq 0$  for all  $\theta \in (0, \bar{\theta})$ , we can prove the following Lemma about the equilibrium markups for the two types of product.

**Lemma 1:** *Assume  $f_H \geq f_L$ ,  $C_H > C_L$ , and  $c_H < c_L < \bar{\theta}$ . For any general cumulative distribution of preference  $G(\theta)$ , with  $G'(\theta) > 0$ , in equilibrium, the markup on the high-quality product is greater than the markup on the low-quality product,  $P_H/C_H > P_L/C_L$ , and the price per unit of quality is higher for the high quality product, i.e.,  $P_H/q_H > P_L/q_L$ .*

**Proof:** See the Appendix.

**Remark 1:** Using U.S. trade data, Baldwin and Harrigan (2011) found that, adjusted for quality, higher quality products are more expensive (i.e.,  $P_H/q_H > P_L/q_L$ ) and more profitable (i.e.,  $P_H/C_H > P_L/C_L$ ),

this is consistent with our Lemma 1.

Our next Lemma states that if the cumulative distribution function  $G(\theta)$  is convex (or linear, as in the case of a uniform density function), then in any Cournot equilibrium where both types of firms exist in the market, any firm that has invested  $F_H$  will specialize in the high-quality product even though it has the ability to produce both products.

**Lemma 2:** Assume  $f_H \geq f_L$  and  $c_H < c_L < \bar{\theta}$  and that  $G(\theta)$  has the following properties:

- (a)  $G(0) = 0$ ,  $G(\bar{\theta}) = 1$  and  $G'(\theta) \geq 0$  for all  $\theta \in (0, \bar{\theta})$ ,
- (b)  $G''(\theta) \leq 0$ .

Then in a Cournot equilibrium where both types of firms exist in the market, firms that have invested  $F_H$  will specialize in the high-quality product.

**Proof:** See the Appendix.

**Remark 2:** Lemma 2 is based on the standard linearity assumption that  $u(\theta, q_i) = \theta q_i$  (as in Mussa and Rosen (1978)). We will show in Section 3.4 that when we allow for a non-linear  $u(\theta, q_i)$ , the specialization result of Lemma 2 will hold if some additional assumptions are made on the cumulative distribution function  $G(\theta)$ .

In what follows, we assume that the distribution of preferences is uniform. Then, using Lemma 2 and given the firm entry in the first stage, i.e.  $n_L$  and  $n_H$ , the equilibrium outputs are solved from the equations:

$$\begin{cases} \bar{\theta}q_H \left[ 1 - \frac{(n_H x_H)}{N} \right] - \frac{(\bar{\theta}q_L n_L x_L)}{N} - C_H &= \frac{(\bar{\theta}q_H x_H)}{N} \\ \bar{\theta}q_L \left[ 1 - \frac{(n_H x_H)}{N} - \frac{(n_L x_L)}{N} \right] - C_L &= \frac{(\bar{\theta}q_L x_L)}{N} \end{cases} \quad (5)$$

The zero profit conditions can then be written as

$$\begin{cases} \pi_H^* = \frac{\bar{\theta}q_H}{N} (x_H^*)^2 - F_H = 0 \\ \pi_L^* = \frac{\bar{\theta}q_L}{N} (x_L^*)^2 - F_L = 0 \end{cases} \quad (6)$$

Using the equations (5) and (6) we can solve for the long-run equilibrium numbers of each type of firms:<sup>15</sup>

$$\begin{cases} n_H^* = \frac{k\sqrt{1/\beta} + [1 - k + (kc_L - c_H)/\bar{\theta}] \sqrt{(\bar{\theta}N)/f_H - 1}}{1 - k} \\ n_L^* = \frac{\sqrt{\beta - (c_L - c_H)} \sqrt{N/(\bar{\theta}f_L) - 1}}{1 - k} \end{cases} \quad (7)$$

where  $f_H \equiv \frac{F_H}{q_H}$ ,  $f_L \equiv \frac{F_L}{q_L}$ ,  $c_H \equiv \frac{C_H}{q_H}$ ,  $c_L \equiv \frac{C_L}{q_L}$ ,  $\beta \equiv \frac{f_H}{f_L}$ , and  $k \equiv \frac{q_L}{q_H} < 1$ . Recalling our specification that  $c_L > c_H > 0$  and  $f_H > f_L > 0$ , we can see from equation (7) that  $n_L^* > 0$  and  $n_H^* > 0$  iff

$$\sqrt{N^{**}/\bar{\theta}} \equiv \frac{(\sqrt{f_H} - \sqrt{f_L}) + (1 - k)\sqrt{f_L}}{(c_L - c_H) + (1 - k)(\bar{\theta} - c_L)} < \sqrt{N/\bar{\theta}} < \frac{\sqrt{f_H} - \sqrt{f_L}}{c_L - c_H} \equiv \sqrt{N^*/\bar{\theta}} \quad (8)$$

<sup>15</sup>For details, please see the Appendix.

The intuition behind the restriction (8) is as follows. For both types of firm to co-exist, the market size should not be too large, nor too small. Since the high-quality firms have a higher fixed cost, as long as the market size is below  $N^*$  (i.e., as long as  $N$  is not very large), there are not enough high-quality firms around to push the low-quality firms out of the market. If the market size is too small (i.e., it is below  $N^{**}$ ), there are not enough customers to support the high fixed cost.

**Proposition 1:** *Assume  $c_H < c_L$  and  $f_H > f_L$ , and that condition (8) is satisfied. Then there exists a unique equilibrium point  $(n_L^*, n_H^*) > (0, 0)$ , and a marginal increase in  $N$  will increase  $n_H^*$  and decrease  $n_L^*$ , and will lead to an increase in the ratio of output of the high-quality product to that of the low-quality product,  $X_H/X_L$ , implying that the average quality in the market rises.*

**Proof:** see the Appendix.

Proposition 1 constructs an important property under the Cournot competition model, which is reconciled with a rich number of empirical evidences, and has been shown in Fajgelbaum et al. (2011) with a price competition model. With some restrictions on the parameters to guarantee the existence of unique and positive solution, the proportions of high quality products and the firms that produce the high quality products increase in the market size.

How do prices and markups change when the market size expands? When both types of firms co-exist in the long-run equilibrium, we find that the long-run equilibrium prices are given by

$$\begin{cases} P_H &= C_H + q_H \sqrt{\frac{\theta f_H}{N}} \\ P_L &= C_L + q_L \sqrt{\frac{\theta f_L}{N}} \end{cases} \quad (9)$$

From equation (9), we see that as the market size expands, the price of each product decreases, and so does the ratio  $P_H/P_L$ . Concerning mark-ups, let us denote them by  $\rho_H \equiv P_H/C_H$  and  $\rho_L \equiv P_L/C_L$ . Then

$$\begin{cases} \rho_H &= 1 + \frac{1}{c_H} \sqrt{\frac{\theta f_H}{N}} \\ \rho_L &= 1 + \frac{1}{c_L} \sqrt{\frac{\theta f_L}{N}} \end{cases} \quad (10)$$

Then we have the result that  $\rho_L < \rho_H$  and both markups decrease as the market size expands.

We summarize these results in Proposition 2.

**Proposition 2:** *Assume all conditions listed in Proposition 1 hold, so that there exists a unique interior equilibrium point  $(n_L^*, n_H^*)$ . Then an increase in the market size will (i) decrease the price and markups of both the high and the low-quality products, (ii) decrease the relative price and the relative markup of the high-quality product.*

**Proof:** Omitted.

Our results provide a possible theoretical explanation of the empirical finding by Edmond et al. (2015) concerning the effect of economic integration on firms' markups and distribution of markups. They reported that trade liberalization, which brings about an increase in market competition, reduces both the average

markup and the dispersion of markups. Our model shows that, under Cost Configuration 1, an increase in market size leads to a decrease in all firms' markup, and narrows the gap between high-quality markup and low-quality markup. The intuition is clear. As the market size expands, more firms enter the market, and competition becomes more fierce. On the other hand, the increase in the number of firms is proportionally less than the expansion of the market size. This makes firms find it more attractive to invest in the R&D fixed cost to upgrade the quality they offer. Thus, high-quality firms face more intensive competition and their markup declines. In contrast, since the number of low-quality firms decreases, the competitive pressure from rivals of the same type is alleviated. This explains why the relative price  $P_H/P_L$  and the relative markup  $\rho_H/\rho_L$  decrease.

### 3.3.2 Cost Configuration 2: higher quality product requires higher marginal cost per unit of quality but does not require a larger fixed cost

We now turn to the opposite case. In this case,  $F_H = F_L = F$  (an assumption made by Johnson and Myatt (2006)), and  $c_H > c_L$ , that is, the upgrading process is input intensive. We assume, as they did, that any firm that incurs a single fixed cost  $F$  can produce both quality levels; it does not have to incur  $F$  twice. As shown in Proposition 9 of Johnson and Myatt (2006), if there are two possible quality levels, all firms will produce both high-quality and low-quality products. Applying this result to our model, we can solve for the equilibrium output of each product and determine the effect of an increase in market size. We obtain the following results.

**Proposition 3:** *Assume  $F_L = F_H = F$ ,  $c_H > c_L$ ,  $(\bar{\theta} - c_L)/(\bar{\theta} - c_H) > q_H/q_L$ , and  $F$  is small relative to the market size. Then in equilibrium, each firm produces both low and high quality products, and an increase in market size will lead to*

- (i) *a less than proportionate increase in the number of firms,*
- (ii) *a fall in both  $P_L$  and  $P_H$ , but an increase in the relative price  $P_H/P_L$  and in the relative markup  $\rho_H/\rho_L$ ,*
- (iii) *an increase in the value share of the high quality product,  $P_H X_H/(P_H X_H + P_L X_L)$ , but no change in the ratio of outputs,  $X_H/X_L$ .*

**Proof:** See the Appendix.

The results for Cost Configuration 2 is quite different from the ones we obtained for Cost Configuration 1 in the preceding subsection. Recall that under Cost Configuration 1, an increase in the market size leads to an increase in the ratio of high-quality output to low-quality output,  $X_H/X_L$ . In contrast, under Cost Configuration 2, Proposition 3 shows that this ratio is unchanged. The reasons are as follows. First, under Cost Configuration 2, the low and high-quality outputs share the same fixed cost, therefore the high-quality product does not gain a cost-saving advantage over the low-quality product when the market size increases. Second, under Cost Configuration 2, each firm produces both types of products, so the head to head competition between the two types of products is substantially internalized within the firm's boundary. Thus it is not surprising that the ratio  $X_H/X_L$  does not change when the market size expands.

Even though the output ratio  $X_H/X_L$  does not change, there is an increase in the relative price  $P_H/P_L$  when the market expands, leading to an increase in the number of firms. This generates a fierce competition in attracting customers with low preference intensity who previously did not buy the good. Therefore the percentage fall in  $P_L$  is bigger.

### 3.4 Discussion

#### 3.4.1 Characteristics of the equilibrium and firms' strategies

Figure 1 presents a schematic distinction between Cost Configurations 1 and 2. When the cost structure is characterized by a common fixed cost  $F_H = F_L = F$  (which is shared by both types of product, such that any firm that has invested  $F$  can produce both types of product without paying any additional fixed cost), and the marginal cost, adjusted for quality, is higher for the high-quality product,  $c_H > c_L$  (i.e., the quality upgrade process is input intensive), each firm will find it optimal to produce both types of good. An increase in market size will attract more firms, but firms do not adjust their output ratio,  $x_H^i/x_L^i$ . Under this cost structure, the average quality in the market, defined as

$$\hat{q} = \left( \frac{X_H}{X_H + X_L} \right) q_H + \left( \frac{X_L}{X_H + X_L} \right) q_L \quad (11)$$

does not change when the market expands.

In contrast, when the quality upgrade is fixed-cost intensive, any firm currently producing the low-quality product that wishes to upgrade must incur a higher fixed cost per unit of quality,  $F_H/q_H > F_L/q_L$ . Under the variable cost specification that  $C_H > C_L$ , but  $C_H/q_H < C_L/q_L$ , we find that for a firm that has incurred  $F_H$ , its optimal product line strategy is to specialize in the high-quality product, even though it is capable of producing both vertically differentiated varieties. Thus, under this cost structure, each firm produces a single type of product. The reason is that it is not worthwhile for a firm that has invested  $F_H$  to manufacture the low-quality product, at a higher marginal cost (adjusted for quality),  $c_L > c_H$ , as it would compete with its own high-quality product. In other words, the firm wishes to avoid the phenomenon called ‘‘cannibalization’’: producing the costly low-quality good will add to the market supply of that good, reducing its price, which switches consumers away from its high-quality product. In view of this product-line specialization result, it is clear that an expansion of the market will lead to an increase in the average quality in the market, defined by (11) above. Indeed, the market expansion will lead to an increase in the number of high quality firms and a fall in the number of low quality firms. As discussed in the previous section, this result is due to the cost saving effect, i.e., an increasing market demand brings greater benefits to the product that requires a high fixed cost. From the consumers' point of view, there are two main gains from market expansion. First, the prices fall, and thus a higher fraction of consumers are served. Second, the market share of the high-quality product increases, which implies that on average, consumers have a greater access to the higher quality products.

Our theoretical results are consistent with the stylized facts in the real world, especially for the type of industry in which quality upgrading is capital intensive, in the sense that  $f_H > f_L$ . Examples of this type of industry, where the quality upgrade is largely dependent on a massive fixed investment, include the media, cosmetics, fashion goods, and pharmaceutical industries. In these industries, each firm is more likely to specialize in either the high end or the low end of the quality spectrum. For example, one does not observe a magazine or a newspaper that is sold in different version with different contents and prices. Similarly, in the pharmaceutical industry, the firms with famous brand names would not introduce a generic version of its branded drugs, unless the patent has expired. During the patent protection period, they only sell the branded drugs with the highest quality. As discussed previously, these market strategies aim at avoiding the cannibalization effect. Berry and Waldfogel (2010) found empirical support for the prediction that in an industry with several types of firms with each firm specializing in a single quality level, an expansion of the market size leads to an increase in the average quality in the market. Our prediction that an increase in market size leads to lower markups and reduces the dispersion of markups are in line with the calibration results of Edmond et al. (2015).

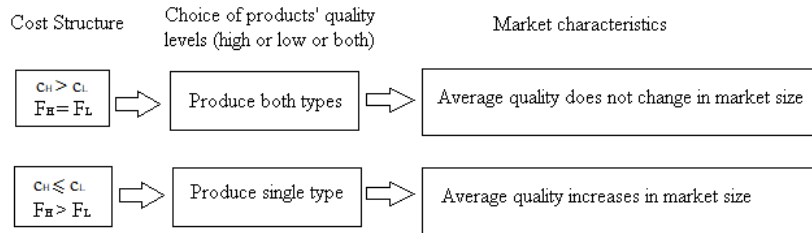


Figure 1. Cost structure, firms' strategies, and the industrial structure

### 3.4.2 The general form of the utility function

Our result that a firm that has invested  $F_H$  will specialize in the high-quality product (if  $f_H > f_L$  and  $c_L > c_H$ ) were obtained using the specification that the net utility that a consumer of type  $\theta$  derives from consuming a good at quality level  $q_j$  and price  $P_i$  is  $\theta q_j - P_j$ . A more general net utility specification would be, as proposed by Johnson and Myatt (2006),

$$U(P_j, q_j, \theta) = u(\theta, q_j) - P_j \tag{12}$$

where the gross utility  $u(\theta, q_j)$  is increasing in  $\theta$ , and, for any given  $\theta$ , it is assumed that  $u(\theta, q_H) > u(\theta, q_L)$  for  $q_H > q_L$ . In what follows, we will show that our result of specialization in the high-quality product still holds under this more general utility function, and under a more general distribution of  $\theta$ , provided that some additional assumptions are added.

Consider any cumulative distribution function  $G(\theta)$  over a compact set  $[0, \bar{\theta}]$  such that  $G'(\theta) > 0$  for all  $\theta \in (0, \bar{\theta})$  and  $G(0) = 0$ ,  $G(\bar{\theta}) = 1$ . Given  $(q_H, q_L)$  and the prices  $(P_H, P_L)$ , where  $P_H > P_L$ , let  $\theta_I$  be the consumer who is indifferent between buying a unit of the high-quality product and buying a unit of the low-quality product. Then it holds that  $u(\theta_I, q_H) > u(\theta_I, q_L)$ . Let us add the assumption that the partial derivative of  $u(\theta, q_H)$  with respect to  $\theta$ , evaluated at  $\theta_I$ , is greater than the partial derivative of  $u(\theta, q_L)$  with respect to  $\theta$ , evaluated at  $\theta_I$ :

$$u_1(\theta_I, q_H) > u_1(\theta_I, q_L) \quad (13)$$

where  $u_1$  stands for the partial derivative with respect to  $\theta$ . This property ensures that the consumer with  $\theta > \theta_I$  will buy the high quality product in preference to the low quality one. For simplicity, let us normalize the population size to 1. Then  $G^{-1}(1 - X_H) = \theta_I$  and  $G^{-1}(1 - X_H - X_L) = \theta_L$ . For a representative firm  $i$  that has invested  $F_H$  and produces the quantity  $x_H^i$ , let us denote by  $\Psi(x_H^i)$  the increase in gross utility if the marginal consumer  $\theta_I$  switches from the low-quality product to the high-quality product

$$\Psi(x_H^i) = u[G^{-1}(1 - X_H^{-i} - x_H^i), q_H] - u[G^{-1}(1 - X_H^{-i} - x_H^i), q_L] \quad (14)$$

This expression is equal to the difference between the equilibrium prices of the two products,  $P_H - P_L$ . We now make the assumption that by increasing  $x_H^i$  by one unit and reducing  $x_L^i$  by one unit, the gain in revenue exceeds the increase in cost,

$$\Phi'(x_H^i) > C_H - C_L. \quad (15)$$

where  $\Phi(x_H^i) \equiv \Psi(x_H^i)x_H^i$ .

We can now state the following proposition.

**Proposition 4:** *Assume that the general utility function satisfies condition (13) and that the difference in marginal cost is not too large, so that condition (15) is met at the optimal output level  $x_H^i$ . Then the firm that has invested in the fixed cost  $F_H$  (which enables them to produce both type of products) will find it optimal to specialize in the high-quality product.<sup>16</sup>*

**Proof:** See the Appendix.

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<sup>16</sup> Our results are different from the Proposition 11 of Johnson and Myatt (2006), where they claimed that the firms that are able to produce both types of products will not specialize in producing only one type of product. However, there is an implicit assumption behind that Proposition. In Proposition 11 of Johnson and Myatt (2006), they assume  $Z_L^* > Z_H^*$ , where  $Z_i^*$  denotes the equilibrium quantity of the products whose quality is higher than or equal to  $i$  and  $i = H, L$ . This assumption contains an implicit condition, i.e. the marginal-cost difference between the two types of products is large enough. Otherwise, the low quality products won't be offered by the high type firms due to the cannibalization effect. In contrast, Proposition 4 of our paper assumes that the difference of the marginal cost between the high and low quality products is low enough. If the high type firms provide the low quality products, they won't save a lot from lowering the marginal cost, but suffer the so called cannibalization effect.

## 4 The effects of economic integration

In this section, we use our theoretical framework to explore the effects of economic integration, modeled as fall in trade costs. Intuitively, lowering the trade barriers between two trading countries is similar to an expansion in the market size faced by the firms from each country. Therefore, if two countries enter into a free trade agreement and commit to reduce the trade barriers between them, we can expect that there will be an increase in the market share of the high-quality product in each country, and a reduction in the markups and in the dispersion of markups. We restrict attention to Cost Configuration 1, so that firms that have incurred the higher fixed cost will not want to produce the low-quality product, and firms that have incurred the low fixed cost are unable to produce the high-quality product. If the two economies are of the same size, we are able to obtain analytical expressions for long run equilibrium prices and markups of each product type in each market, and sales of each type of firm in the domestic and in the export market. When countries are of different sizes, we must resort to numerical simulations, and report our results with graphical illustrations.

### 4.1 Trade liberalization with a vertically differentiated international oligopoly

Suppose there are two countries, called the home country, denoted by  $h$ , and the foreign country, denoted by  $f$ . In each country, there are two industries. Industry 1 produces two vertically differentiated products: a high-quality product and a low-quality product. Their quality levels are denoted by  $q_H$  and  $q_L$ , where  $q_H > q_L > 0$ . Firms in industry 1 must incur fixed costs,  $F_H$  and  $F_L$ , and we assume that  $f_H > f_L$  where  $f_H = F_H/q_H$  and  $f_L = F_L/q_L$ . These firms are Cournot oligopolists. Marginal costs are  $C_H$  and  $C_L$ , where  $C_H > C_L$ , and we assume that the quality adjusted marginal costs,  $c_H = C_H/q_H$  and  $c_L = C_L/q_L$  are such that  $c_H < c_L$ . Industry 2 produces a homogeneous good under constant returns to scale, and operates under perfect competition. The homogeneous good serves as the numeraire good: its price is unity. There is only one factor of production, called labour. Each unit of labour can produce one unit of the numeraire good. Therefore the wage rate is also unity. The costs  $F_H, F_L, C_H$  and  $C_L$ , and the prices  $P_H$  and  $P_L$  are all expressed in terms of the numeraire good. There are  $N_h$  consumers in  $h$  and  $N_f$  consumers in  $f$ .<sup>17</sup>

To simplify matters, we assume, as is standard in the recent literature on trade liberalization (see, for example, Krugman (1995), Feenstra (2015), Long et al. (2011)) that trade barriers take the form of an “ice-berg transport cost” rather than tariffs, and that only the differentiated products are subject to trade barriers. When a domestic producer of the differentiated product sends  $x$  units of the good to the foreign country, only  $\delta x$  units arrive, where  $0 < \delta < 1$ . Partial trade liberalization is modeled as an increase in  $\delta$ .

Each home firm that produces the high-quality product must decide on the quantity  $x_H^{hh}$  that it supplies

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<sup>17</sup>For simplicity we assume that utility derived from consuming the numeraire good is linear in quantity consumed. Each consumer is endowed with  $M$  units of labor, which they supply inelastically to the labor market. Since we focus in the long run equilibrium (i.e., firms earn zero profit), there are no positive profits to be distributed to consumers. Consequently, each consumer’s income is  $M$ . Each consumer spends her entire income on consumption. Each buys at most one unit of the good produced by industry 1, and the remaining income is spent on the numeraire good. We assume that  $M > P_H > P_L$  so that buying one unit of the differentiated good does not exhaust the consumer’s income.



to the domestic market, and the quantity  $x_H^{hf}$  that it exports to the foreign market. Similarly, each home firm that produces the low-quality product must decide on  $x_L^{hh}$  and  $x_L^{hf}$ . The symbols  $n_H^f \delta x_H^{fh}$  and  $n_L^f \delta x_L^{fh}$  represents foreign supplies to the domestic market. Then total supplies of the two products in the domestic market are

$$\begin{cases} X_H^h = n_H^h x_H^{hh} + n_H^f \delta x_H^{fh} \\ X_L^h = n_L^h x_L^{hh} + n_L^f \delta x_L^{fh} \end{cases} \quad (16)$$

The profit functions for home firms are

$$\begin{cases} \pi_H^h = \bar{\theta} \left[ q_H \left( 1 - \frac{X_H^h}{N_h} \right) - q_L \frac{X_L^h}{N_h} \right] x_H^{hh} + \bar{\theta} \left[ q_H \left( 1 - \frac{X_H^f}{N_f} \right) - q_L \frac{X_L^f}{N_f} \right] \delta x_H^{hf} - C_H(x_H^{hh} + x_H^{hf}) - F_H \\ \pi_L^h = \bar{\theta}_{QL} \left( 1 - \frac{X_H^h}{N_h} - \frac{X_L^h}{N_h} \right) x_L^{hh} + \bar{\theta}_{QL} \left( 1 - \frac{X_H^f}{N_f} - \frac{X_L^f}{N_f} \right) \delta x_L^{hf} - C_L(x_L^{hh} + x_L^{hf}) - F_L \end{cases} \quad (17)$$

For the foreign firms, similar formulas apply. In the long run equilibrium, profits are zero, and we obtain the following expressions for long run equilibrium prices in the home country:

$$\begin{cases} P_H^h = \sqrt{\frac{\bar{\theta} q_H F_H}{(1+\delta^2) N_h}} + C_H \\ P_L^h = \sqrt{\frac{\bar{\theta}_{QL} F_L}{(1+\delta^2) N_h}} + C_L \end{cases} \quad (18)$$

Equation (18) shows that equilibrium prices fall as trade costs fall (i.e., as  $\delta$  increases). With  $N_h = N/2$ , as  $\delta$  tends to 1 (complete liberalization), this equation coincides with the long run price equation in Section 3. Concerning long run equilibrium quantities sold by each firm in the domestic market and the export markets, we obtain the following expressions:

$$\begin{cases} x_H^{hh} = \sqrt{\frac{N_h F_H}{\bar{\theta} q_H (1+\delta^2)}} \\ x_L^{hh} = \sqrt{\frac{N_h F_L}{\bar{\theta}_{QL} (1+\delta^2)}} \\ x_H^{hf} = \sqrt{\frac{N_f F_H}{\bar{\theta} q_H (1+\frac{1}{\delta^2})}} \\ x_L^{hf} = \sqrt{\frac{N_f F_L}{\bar{\theta}_{QL} (1+\frac{1}{\delta^2})}} \end{cases} \quad (19)$$

Thus, as  $\delta$  increases (trade becomes more liberalized), each firm's sales in the domestic market fall (because of competition from foreign firms) while its sales in the export market increase.

Home firms' average markups are<sup>18</sup>

$$\begin{cases} \rho_H^h &= \frac{\sqrt{F_H(1+\delta^2)}}{C_H \left[ \sqrt{\frac{N_h}{\theta q_H}} + \delta^2 \sqrt{\frac{N_f}{\theta q_H}} \right]} + 1 \\ \rho_L^h &= \frac{\sqrt{F_L(1+\delta^2)}}{C_L \left[ \sqrt{\frac{N_h}{\theta q_L}} + \delta^2 \sqrt{\frac{N_f}{\theta q_L}} \right]} + 1 \end{cases} \quad (20)$$

Concerning the equilibrium number of firms of each type in each country, we are able to obtain an explicit expression only in the symmetric case, where  $N_h = N_f = N/2$ .

$$\begin{cases} n_H &= \frac{\sqrt{\frac{(1+\delta^2)N}{2\theta}} [\bar{\theta}(q_H - q_L) + C_L - C_H] + \sqrt{q_L F_L} - \sqrt{q_H F_H}}{(1-k)(1+\delta^2)\sqrt{q_H F_H}} \\ n_L &= \frac{k\sqrt{\frac{(1+\delta^2)N}{2\theta}} (C_H - C_L/k) + k\sqrt{q_H F_H} - \sqrt{q_L F_L}}{(1-k)(1+\delta^2)\sqrt{q_L F_L}} \end{cases} \quad (21)$$

where  $k \equiv q_L/q_H$ . Using the above results, we can obtain the following analytical results concerning the long-run effect of trade liberalization.

**Proposition 5:** *Consider a two-country, two-quality-product trade model where the market equilibrium is in long run, i.e. the firms are allowed to free entry-exit. A decrease of the trade cost in both country will lead to the following changes:*

- [1] price levels of both products in each country decrease;
- [2] total consumption on differentiated products increase.

Moreover, if the following additional conditions hold, i.e. both countries are symmetric, i.e. they share the same population number,  $N_h = N_f = N/2$ , and the relative fixed cost of high quality product is large enough, i.e.  $\beta > \left( \frac{\bar{\theta} - c_H}{\bar{\theta} - c_L} \right)^2$ , then we further get:

- [3] relative number of high quality firms increase;
- [4] relative quantity of high quality products supplied in each country increases;
- [5] average markup of each firm decreases;
- [6] the relative average markup of the high quality firm decreases.

**Proof :** See the Appendix.

Trade liberalization thus has similar effects to an expansion of market size. It increases consumers' accessibility to foreign firms' products and thus increases the intensity of competition. Proposition 5, which deals with the effects of trade liberalization, offer results that are qualitatively similar to Propositions 1 and 2, which deal with the case of a pure market expansion. In the next subsection, we illustrate the effects of various degrees of trade liberalization on a number of key variables, both for the case of two trading partners with identical population size and for the case where one country is smaller than the other.

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<sup>18</sup>Since the prices differ across markets, and the marginal cost in the foreign market, inclusive of the iceberg transport cost, is higher than the marginal cost in the domestic market, the markups reported here are computed using the firm's average price and average marginal cost.

## 4.2 Simulation results

This subsection reports simulation results. For both the case of two identical trading partners and the case of two countries that differ in population size, we use the following parameter values:  $\bar{\theta} = 1$ ,  $q_H = 10$ ,  $q_L = 8$ ,  $F_H = 1$ ,  $F_L = 0.1$ ,  $C_H = 0.1$ ,  $C_L = 0.08$ .

We consider different values  $\delta$  from the set  $\{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ . Note that we do not consider the value  $\delta = 1$ , for in that case there is no trade barriers, and thus the total number of firms of each type in each country become indeterminate.<sup>19</sup> When the two countries have identical population size, we set  $N_h = N_f = 105$ . When their population sizes differ, we set  $N_h = 100$  and  $N_f = 110$ .

Figures 2 to 6 report results for the case of two countries with the same population size. (To save space, these Figures refer only to the home country, because what happens to the foreign country is the same.)

The upper panel of Figure 2 shows that as  $\delta$  increases (i.e., as trade becomes more and more liberalized), the number of domestic firms of each type falls (but of course each domestic consumers are served by more firms thanks to the trade liberalization). The lower panel depicts the ratio of high quality firms to low quality firms,  $n_H/n_L$ . Figure 3 shows that with more liberalized trade, the price of each type of product falls, but that of the high-quality product falls by more. The upper panel of Figure 4 shows the falls in markups, and the narrowing of the gap between the two markups. The lower panel of Figure 4 shows the fall in the relative markup of the high-quality product. Figure 5 reports the total supply of each type of products by the firms that located in each country. It shows that the supplied quantity of the high-quality product increases while that of the low-quality product decreases in  $\delta$ . The relative quantity supply of high quality products increase in  $\delta$  as well. Figure 6 reports the total demand of each type of products in each country. We observe a similar result, i.e. the demand for high quality products increases while the demand for low quality products decreases in  $\delta$ .

Figures 7 to 11 report simulation results for the case of two countries with different population sizes. Overall, these simulations show similar results to the case of symmetric countries. The main difference is that in the asymmetric case, when the trade costs decrease (i.e.,  $\delta$  increases) the small country's ratio of high-quality firms to low-quality firms decreases if the initial trade barriers are small, i.e., the initial  $\delta$  is large (Figure 7). This is reflected in the ratio of the supply of high-quality product to the supply of low-quality product by the small country (Figure 10). The result is due to the relocation of high-quality firms from the small country to the large country in some range of trade cost, because firms choose their location based on the balance of market demand and market competition intensity.

The top left-hand panel of Figure 7 shows that the smaller country's firm number of each type declines when trade is liberalized. Their ratio changes in a non-monotone way, as shown in the bottom left-hand panel of Figure 7. The right-hand panels report what happens to firm numbers in the larger country. Figure 8 shows the effects of trade liberalization on the prices of each type of product in each country. While all prices fall with trade liberalization, the price of the of both products are lower in the larger country, and they remain

<sup>19</sup>When both countries are fully integrated, it's equivalent for firms to choose either country to locate in. Thus, we cannot decide the firm number in each country.

so even when trade liberalization has substantially reduced trade cost (i.e., at  $\delta = 0.9$ ). Figure 9 shows that markups and the relative markup of the high quality product decline as trade liberalization proceeds. Figure 10 reports the effects on the total quantity supplied by each type of firm in each country. Interestingly, the supply of high quality products increases in size when trade costs decrease ( $\delta$  increases), while the opposite happens to the small country. As stated before, this is because of the relocation of high-quality firms from the small country to the large country. Finally, Figure 11 reports changes in the quantity demanded of each type of product in each country. The ratio of consumption of the high-quality product to that of the low-quality product rises as trade liberalization proceeds. In the long run, as firms earn zero profits, each country's welfare depends only on the prices  $P_H$  and  $P_L$ , and since these prices fall monotonically as trade costs decrease, welfare obviously increases.

## 5 Concluding Remarks

This paper studies the Cournot equilibrium of an oligopoly producing vertically differentiated products under free entry and exit. The model highlights how prices, markups and average quality depend on the market size and on the cost structure.

Our main focus was on the cost configuration where the set-up cost depends on the quality of the product. In the first stage of the game, firms decide whether they incur the high fixed cost, which enables them to produce the high-quality product (and also the low-quality one), or only the low fixed cost, which only permits them to produce the low-quality product. In the second stage, they decide on their output levels. We showed that in equilibrium firms that have incurred the high fixed cost will specialize in the high-quality product if the fixed cost for the low-quality product, adjusted for the quality level, is lower than that for the high-quality product. Under that cost configuration, we showed that an increase in market size leads to (i) an increase in the fraction of firms that specialize in the high-quality product, (ii) an increase in the market share of the high-quality product, (iii) a decrease in the prices and markups of both types of product, (iv) a decrease in the relative markup of high-quality product.

We also considered the opposite case where the fixed cost is independent of quality level while the quality-adjusted marginal cost of the high-quality product is higher than that of the low-quality product. Under that cost configuration, we found that all firms will produce both quality levels and an increase in market size will increase the value share of the high quality product, but leaves its quantity share unchanged.

As an extension of our model, we applied our theoretical framework to analyze the effects of trade liberalization. We found that a reduction in trade costs has similar effects as a market expansion: the prices and markups of both types of product decrease, while the share of the high-quality product increases.

Our study constitutes an extension of the existing literature on markets with vertical product differentiation in two directions. First, we offer a detailed characterization of the long run equilibrium mix of firms under alternative specifications of the cost of quality upgrading. Second, we explore the effect of market size on market outcomes, especially on the market shares and markups of high-quality versus low-quality

product, when firms are Cournot rivals within groups as well as across groups. The theoretical predictions of our model are broadly consistent with empirical studies and stylized facts. For example, the firms that have made massive investment in quality upgrading are more likely to specialize in the high end of the quality spectrum in order to avoid the cannibalization effect. Trade liberalization induces firms to upgrade the product quality and reduces firms' markup. Consumers gain from trade liberalization, not only from lower prices, but also from greater access to high quality products.

For simplicity, we have assumed that all firms of each type have the same level of productivity. A natural extension would be to allow for heterogeneity in productivity.

## Appendix A

### Proof of Lemma 1

Since  $G'(\theta) > 0$  over  $[0, \bar{\theta}]$ , we can define the inverse function  $\Omega(y)$ , where  $y = G(\theta)$ . Then the inverse demand functions are

$$\begin{cases} P_H &= \Omega\left(1 - \frac{X_H}{N}\right)(q_H - q_L) + \Omega\left(1 - \frac{X_H + X_L}{N}\right)q_L \\ P_L &= \Omega\left(1 - \frac{X_H + X_L}{N}\right)q_L \end{cases} \quad (\text{A.1})$$

From the first order conditions of profit maximization, we obtain

$$\frac{P_H}{C_H} - \frac{P_L}{C_L} = \frac{q_H}{C_H} \left[ \Omega\left(1 - \frac{X_H}{N}\right)(1 - k) + \Omega\left(1 - \frac{X_H + X_L}{N}\right)\left(k - \frac{c_H}{c_L}\right) \right] \quad (\text{A.2})$$

Now, since  $k = q_L/q_H < 1$  and  $c_H \leq c_L$ , and since  $\Omega(\cdot)$  is monotone increasing, the RHS of equation (A.2) must be positive. It follows that the LHS must be positive, too. Finally,

$$\frac{P_H}{C_H} - \frac{P_L}{C_L} \geq \frac{q_H}{C_H} \Omega\left(1 - \frac{X_H + X_L}{N}\right) \left(1 - \frac{c_H}{c_L}\right) > 0. \quad (\text{A.3})$$

**Q.E.D.**

### Proof of Lemma 2

Recall that the inverse demand functions are given by equation (A.1) above. Firm  $i$ 's profit function is

$$\pi_i = (P_H - C_H)x_H^i + (P_L - C_L)x_L^i - F_H \quad (\text{A.4})$$

The F.O.C.s are

$$(P_H - C_H) + x_H^i \frac{\partial P_H}{\partial X_H} + x_L^i \frac{\partial P_L}{\partial X_H} \leq 0, \quad (= 0 \text{ if } x_H^i > 0) \quad (\text{A.5})$$

$$(P_L - C_L) + x_L^i \frac{\partial P_L}{\partial X_L} + x_H^i \frac{\partial P_H}{\partial X_H} \leq 0, \quad (= 0 \text{ if } x_L^i > 0) \quad (\text{A.6})$$

We now show that if both  $x_H^i$  and  $x_L^i$  were strictly positive, a contradiction would emerge. For then conditions (A.5) and (A.6) would give

$$\frac{P_H}{q_H} - \frac{C_H}{q_H} - \frac{1}{N}\Omega' \left(1 - \frac{X_H}{N}\right) (q_H - q_L) x_H^i - \frac{1}{N}\Omega' \left(1 - \frac{X_H + X_L}{N}\right) k(x_H^i + x_L^i) = 0 \quad (\text{A.7})$$

$$\frac{P_L}{q_L} - \frac{C_L}{q_L} - \frac{1}{N}\Omega' \left(1 - \frac{X_H + X_L}{N}\right) (x_H^i + x_L^i) = 0 \quad (\text{A.8})$$

Subtracting (A.7) from (A.8), we obtain

$$\begin{aligned} \frac{C_H}{q_H} - \frac{C_L}{q_L} = & \left\{ \frac{P_H}{q_H} - \frac{P_L}{q_L} \right\} + \left\{ \frac{1-k}{N} x_L^i \Omega' \left(1 - \frac{X_H + X_L}{N}\right) \right\} \\ & + \left\{ \frac{1-k}{N} x_H^i \left[ \Omega' \left(1 - \frac{X_H}{N}\right) - \Omega' \left(1 - \frac{X_H + X_L}{N}\right) \right] \right\} \end{aligned} \quad (\text{A.9})$$

The first bracketed term on the RHS of (A.9) is positive because

$$\frac{P_H}{q_H} - \frac{P_L}{q_L} = (1-k) \left[ \Omega \left(1 - \frac{X_H}{N}\right) - \Omega \left(1 - \frac{X_H + X_L}{N}\right) \right] > 0. \quad (\text{A.10})$$

The second bracketed term on the RHS of (A.9) is also positive, because  $\Omega' > 0$ . Recalling that  $1 - \frac{X_H}{N} > 1 - \frac{X_H + X_L}{N}$ , the third bracketed term on the RHS of (A.9) is positive if  $\Omega' \left(1 - \frac{X_H}{N}\right) > \Omega' \left(1 - \frac{X_H + X_L}{N}\right)$ , i.e., if  $\Omega'(y)$  is increasing in  $y$ , i.e., if  $\Omega(y)$  is convex, which holds iff  $G(\theta)$  is concave. On the other hand, by assumption,  $c_H \leq c_L$ , so that the LHS of (A.9) is negative. Therefore, if both  $x_H^i$  and  $x_L^i$  were strictly positive, we would have a contradiction. Therefore, either  $x_H^i > 0$  or  $x_L^i > 0$ , but not both. But if  $x_H^i = 0$  and  $x_L^i > 0$ , it would not make sense for firm  $i$  to invest in  $F_H$ . It follows that  $G'' \leq 0$  implies that the firm that has invested  $F_H$  will specialize in the high quality product. **Q.E.D.**

### Proof of Proposition 1

Given Lemma 2, each type of firm specializes in one type of product. In this case, the firms' profit function in the second stage can be written as:

$$\begin{cases} \pi_H = (P_H - C_H) x_H - F_H \\ \pi_L = (P_L - C_L) x_L - F_L \end{cases} \quad (\text{A.11})$$

Take F.O.C., we get:

$$\begin{cases} (\partial P_H / \partial x_H) x_H + P_H - C_H = 0 \\ (\partial P_L / \partial x_L) x_L + P_L - C_L = 0 \end{cases} \quad (\text{A.12})$$

As the inverse demand function is:

$$\begin{cases} P_H &= (1 - \frac{X_H}{N})\bar{\theta}q_H - \frac{X_L}{N}\bar{\theta}q_L \\ P_L &= (1 - \frac{X_H}{N} - \frac{X_L}{N})\bar{\theta}q_L \end{cases} \quad (\text{A.13})$$

we can re-write equations A.12 as:

$$\begin{cases} -\frac{\bar{\theta}q_H}{N}x_H^* + P_H - C_H &= 0 \\ -\frac{\bar{\theta}q_L}{N}x_L^* + P_L - C_L &= 0 \end{cases} \quad (\text{A.14})$$

Which implies that

$$\begin{cases} \pi_H^* &= \frac{\bar{\theta}q_H}{N}x_H^{*2} - F_H \\ \pi_L^* &= \frac{\bar{\theta}q_L}{N}x_L^{*2} - F_L \end{cases} \quad (\text{A.15})$$

The equations A.14 imply that all firms of the same type will choose the same quantity strategy. In this case, we can get  $X_H = n_H x_H$  and  $X_L = n_L x_L$ . Next, we need to solve for the firm numbers in the first stage. The firm numbers are solved with the free entry-exit condition, i.e. when the profit equals to zero for each type of firms.

$$\begin{cases} \pi_H^* &= 0 \\ \pi_L^* &= 0 \end{cases} \quad (\text{A.16})$$

Combining equations A.13, A.14, A.15, A.16, and the fact that  $X_H = n_H x_H$  and  $X_L = n_L x_L$ , we can solve for the equilibrium firm numbers as:

$$\begin{cases} n_H^* &= \frac{k\sqrt{1/\beta + [1 - k + (kc_L - c_H)/\bar{\theta}]} \sqrt{(\bar{\theta}N)/f_H} - 1}{1 - k} \\ n_L^* &= \frac{\sqrt{\beta - (c_L - c_H)} \sqrt{N/(\bar{\theta}f_L)} - 1}{1 - k} \end{cases} \quad (\text{A.17})$$

where  $f_H \equiv \frac{F_H}{q_H}$ ,  $f_L \equiv \frac{F_L}{q_L}$ ,  $\beta \equiv \frac{f_H}{f_L}$ ,  $c_H \equiv \frac{C_H}{q_H}$ , and  $c_L \equiv \frac{C_L}{q_L}$ . Then, under the conditions demonstrated in Proposition 1, it is easy to obtain that  $n_H^* > 0$ ,  $n_L^* > 0$ ,  $\partial n_H^*/N > 0$ , and  $\partial n_L^*/N \leq 0$ . In this case, the number share of the high quality firms increases. Next, we will show that the quantity share of high quality products increases in market size. The quantity share of the high quality products is  $X_H/X_L = n_H x_H/n_L x_L$ . From equations A.15 and A.16, we know that  $x_H/x_L$  is constant. In this case, the quantity ratio is totally determined by the firm number ratio. Thus, the quantity share of the high quality products increases in the market size as well.

**Q.E.D.**

**Proof of Proposition 3**

Our first step is to show that if  $x_H^* > 0$  and  $x_L^*$  must be positive. Consider the first order conditions of firm  $i$  with respect to  $x_H^i$  and  $x_L^i$  respectively,

$$\left(1 - \frac{X_H}{N}\right) - \frac{kX_L}{N} - \frac{x_H^i}{N} - \frac{kx_L^i}{N} = \frac{C_H}{\theta q_H} \quad (\text{A.18})$$

$$-\frac{x_H^i}{N} + \left(1 - \frac{X_H + X_L}{N}\right) - \frac{x_L^i}{N} \leq \frac{C_L}{\theta q_L} \quad (\text{with equality holding if } x_L^i > 0) \quad (\text{A.19})$$

Subtracting, and using  $X_L = nx_L^i$ , we get

$$\frac{C_H}{\theta q_H} - \frac{C_L}{\theta q_L} \leq \frac{1}{N}(X_L + x_L^i)(1 - k) = \frac{1}{N}(n_L + 1)x_L^i \quad (\text{A.20})$$

Since  $c_H > c_L$ , it must hold that  $x_L^i > 0$ , which implies that

$$-\frac{x_H^i}{N} + \left(1 - \frac{X_H + X_L}{N}\right) - \frac{x_L^i}{N} = \frac{C_L}{\theta q_L} \quad (\text{A.21})$$

Using equations (A.18) and (A.21), we obtain

$$\frac{X_L^*}{N} = \frac{\frac{C_H}{\theta q_H} - \frac{C_L}{\theta q_L}}{\left(1 + \frac{1}{n}\right)(1 - k)} \quad (\text{A.22})$$

and

$$\frac{X_H^*}{N} = \frac{n}{1 + n} - \frac{n}{(1 + n)(1 - k)} \left(\frac{C_H}{\theta q_H} - k \frac{C_L}{\theta q_L}\right) \quad (\text{A.23})$$

Then  $X_H > 0$  iff

$$k < \frac{1 - \frac{C_H}{\theta q_H}}{1 - \frac{C_L}{\theta q_L}} < 1. \quad (\text{A.24})$$

From equations (A.22) and (A.23), the ratio  $X_L/X_H$  is independent of the market size. Solving for the equilibrium prices, we get

$$\begin{cases} P_H^* &= \frac{nC_H + \theta q_H}{n+1} \\ P_L^* &= \frac{nC_L + \theta q_L}{n+1} \end{cases} \quad (\text{A.25})$$

Since  $c_L < c_H$ , we obtain  $P_L^*/q_L < P_H^*/q_H$ .

With all firms producing both types of product, the zero profit condition for the industry is simply

$$(P_H^* - C_H) X_H^* + (P_L^* - C_L) X_L^* = n^* F \quad (\text{A.26})$$



Thus we can solve for  $n^*$

$$n^* = \sqrt{\frac{q_H N \bar{\theta} \Delta}{(1-k)F}} - 1 \quad (\text{A.27})$$

where

$$\Delta \equiv \left(1 - \frac{C_H}{\theta_{q_H}}\right) \left(1 - k + k \frac{C_L}{\theta_{q_L}} - \frac{C_H}{\theta_{q_H}}\right) + k \left(1 - \frac{C_L}{\theta_{q_L}}\right) \left(\frac{C_H}{\theta_{q_H}} - \frac{C_L}{\theta_{q_L}}\right) > 0. \quad (\text{A.28})$$

**Q.E.D.**

**Proof of Proposition 4**

To simplify our expressions, we let  $E \equiv 1 - X_H$  and  $F \equiv 1 - X_H - X_L$ . Let  $G^{-1}(\cdot)$  be the inverse of  $G(\cdot)$ . Then for a firm  $i$  that has invested in  $F_H$  has the following profit function

$$\begin{aligned} \pi_i = & \left\{ u(G^{-1}(E), q_H) - u(G^{-1}(E), q_L) + u(G^{-1}(F), q_L) - C_H \right\} x_H^i \\ & + \left[ u(G^{-1}(F), q_L) - C_L \right] x_L^i \end{aligned} \quad (\text{A.29})$$

Consider the partial derivatives with respect to  $x_H^i$  and  $x_L^i$

$$\begin{aligned} \frac{\partial \pi_i}{\partial x_H^i} = & \left\{ u(G^{-1}(E), q_H) - u(G^{-1}(E), q_L) + u(G^{-1}(F), q_L) - C_H \right\} \\ & + x_H^i \left\{ -G^{-1'}(E) u_1(G^{-1}(E), q_H) + G^{-1'}(E) u_1(G^{-1}(E), q_L) - G^{-1'}(F) u_1(G^{-1}(F), q_L) \right\} \\ & - x_L^i G^{-1'}(F) u(G^{-1}(F), q_L) \end{aligned} \quad (\text{A.30})$$

$$\frac{\partial \pi_i}{\partial x_L^i} = \left[ u(G^{-1}(F), q_L) - C_L \right] - x_L^i G^{-1'}(F) u(G^{-1}(F), q_L) - x_H^i G^{-1'}(F) u_1(G^{-1}(F), q_L) \quad (\text{A.31})$$

where  $u_1(\cdot)$  denotes the partial derivative of  $u_1(\cdot)$  with respect to the first variable. Then  $\frac{\partial \pi_i}{\partial x_H^i} - \frac{\partial \pi_i}{\partial x_L^i}$  has the same sign as

$$\left\{ u(G^{-1}(E), q_H) - u(G^{-1}(E), q_L) + x_H^i \left[ u_1(G^{-1}(E), q_L) - u_1(G^{-1}(E), q_H) \right] G^{-1'}(E) \right\} - (C_H - C_L) \quad (\text{A.32})$$

If this expression is strictly positive, then the firm will not produce both types of products, and thus  $x_H^i > 0$  and  $x_L^i = 0$ . Note that the term inside  $\{\dots\}$  is  $\Phi'(x_H^i)$ , where  $\Phi(x_H^i) \equiv \left[ u(G^{-1}(E), q_H) - u(G^{-1}(E), q_L) \right] x_H^i$ . Under the condition that  $\Phi'(x_H^i) > C_H - C_L$ , the expression above is positive.

**Q.E.D.**

**Proof of Proposition 5**

From equation (18), we get that  $\partial P_i^j / \partial \delta < 0$  for all  $i = H, L$ , and  $j = h, f$ . Thus, the price levels of both products in each country decrease. As the price levels of differentiate good decrease, more consumers that initially consume the numeraire good will turn to buy the differentiate good. Thus, more consumers will be access to the differentiate good. Next, we will assume that the two countries are symmetric in population size, i.e.  $N_h = N_f = N/2$ . From equation (21), we get the relative number of the high quality firms as

$$n_H/n_L = \sqrt{\frac{kF_L}{F_H} \frac{\sqrt{(1+\delta^2)N/2} [\bar{\theta}(q_H - q_L) + (C_L - C_H)] - \sqrt{\bar{\theta}}(\sqrt{q_H F_H} - \sqrt{q_L F_L})}{k [N/2(1+\delta^2)(C_H - C_L/k) + \sqrt{\bar{\theta}}(\sqrt{q_H F_H} - \sqrt{q_L F_L})]}} \quad (\text{A.33})$$

For convenience, we denote the relative number of high quality firms as  $R_n \equiv n_H/n_L$ . Under condition  $\beta > \left(\frac{\bar{\theta}-c_H}{\bar{\theta}-c_L}\right)^2$ , we have  $\partial R_H/\partial\delta > 0$ . Thus, the relative number of the firms that produce the high quality products increases. From equations (19) and (21), we can get the relative quantity of the high quality products sold in each country as:

$$R_x = \sqrt{\frac{q_L F_H}{q_H F_L}} R_n \quad (\text{A.34})$$

As  $\sqrt{\frac{q_L F_H}{q_H F_L}}$  is constant, the sign of  $\partial R_x/\partial\delta$  is the same as  $\partial R_n/\partial\delta$ , i.e.  $\partial R_n/\partial\delta > 0$ . Thus, in response to a lower trade cost, the relative quantity of high quality products supplied in each country increases. With the assumption of symmetric countries, we rewrite the markups for high and quality firms as:

$$\begin{cases} markup_H = \frac{1}{C_H} \sqrt{\frac{2\bar{\theta}q_H F_H}{N(1+\delta^2)}} + 1 \\ markup_L = \frac{1}{C_L} \sqrt{\frac{2\bar{\theta}q_L F_L}{N(1+\delta^2)}} + 1 \end{cases} \quad (\text{A.35})$$

Obviously,  $\partial markup_i/\partial\delta < 0$  for all  $i = H, L$ . Thus, average markup of each firm decreases, when trade cost reduces. Last, we will show that the relative markup of the high quality firms decreases. Based on the equation (A.35), the indicator for this relative markup is specified as:

$$R_m = \left[ \frac{1}{C_H} \sqrt{\frac{2\bar{\theta}q_H F_H}{N(1+\delta^2)}} + 1 \right] / \left[ \frac{1}{C_L} \sqrt{\frac{2\bar{\theta}q_L F_L}{N(1+\delta^2)}} + 1 \right] \quad (\text{A.36})$$

Recall that the relative fixed cost of high quality products is high enough, i.e.  $\beta > \left(\frac{\bar{\theta}-c_H}{\bar{\theta}-c_L}\right)^2 \geq 1$ . ( $c_H \leq c_L$ ). Then we have  $\beta > 1 > \left(\frac{c_H}{c_L}\right)^2$ . That is  $\frac{q_H F_H}{q_L F_L} > \left(\frac{C_H}{C_L}\right)^2$ . Thus, we have  $\partial R_m/\partial\delta < 0$ . The relative markup of the high quality firms decreases. **Q.E.D.**

## Appendix B

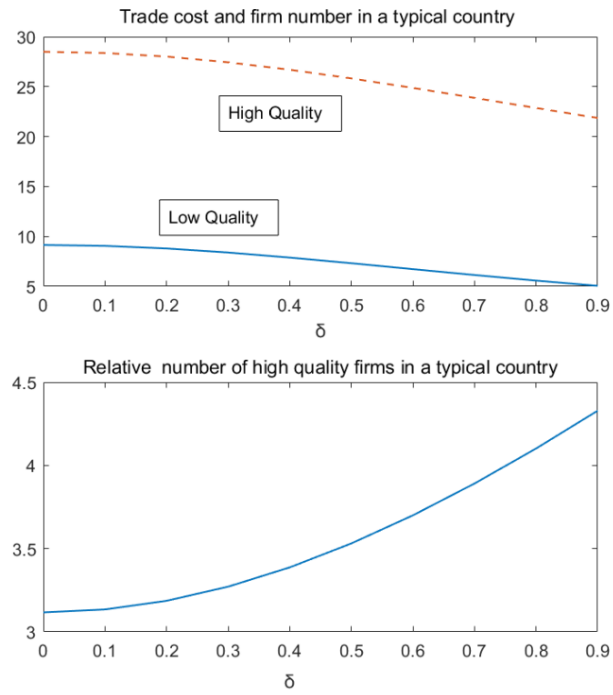


Figure 2: Effects of trade liberalization on firm numbers, the case of symmetric countries

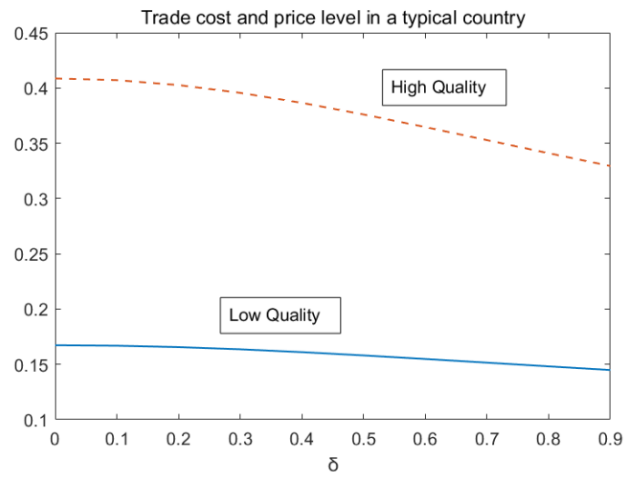


Figure 3: Effects of trade liberalization on price levels, the case of symmetric countries

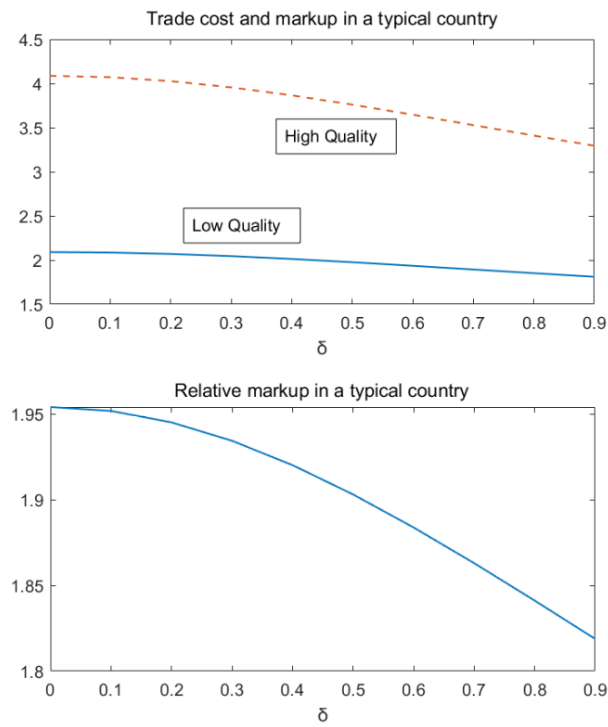


Figure 4: Effects of trade liberalization on markups, the case of symmetric countries

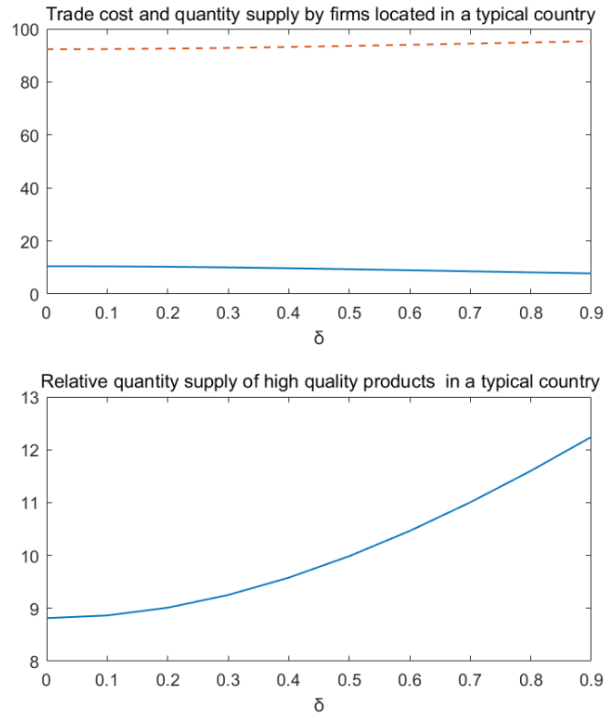


Figure 5: Effects of trade liberalization on firms' production, the case of symmetric countries

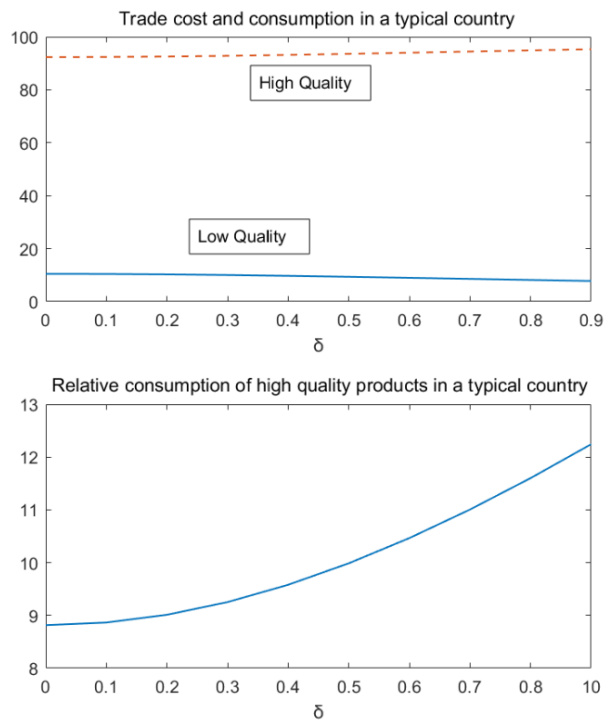


Figure 6: Effects of trade liberalization on market demand, the case of symmetric countries

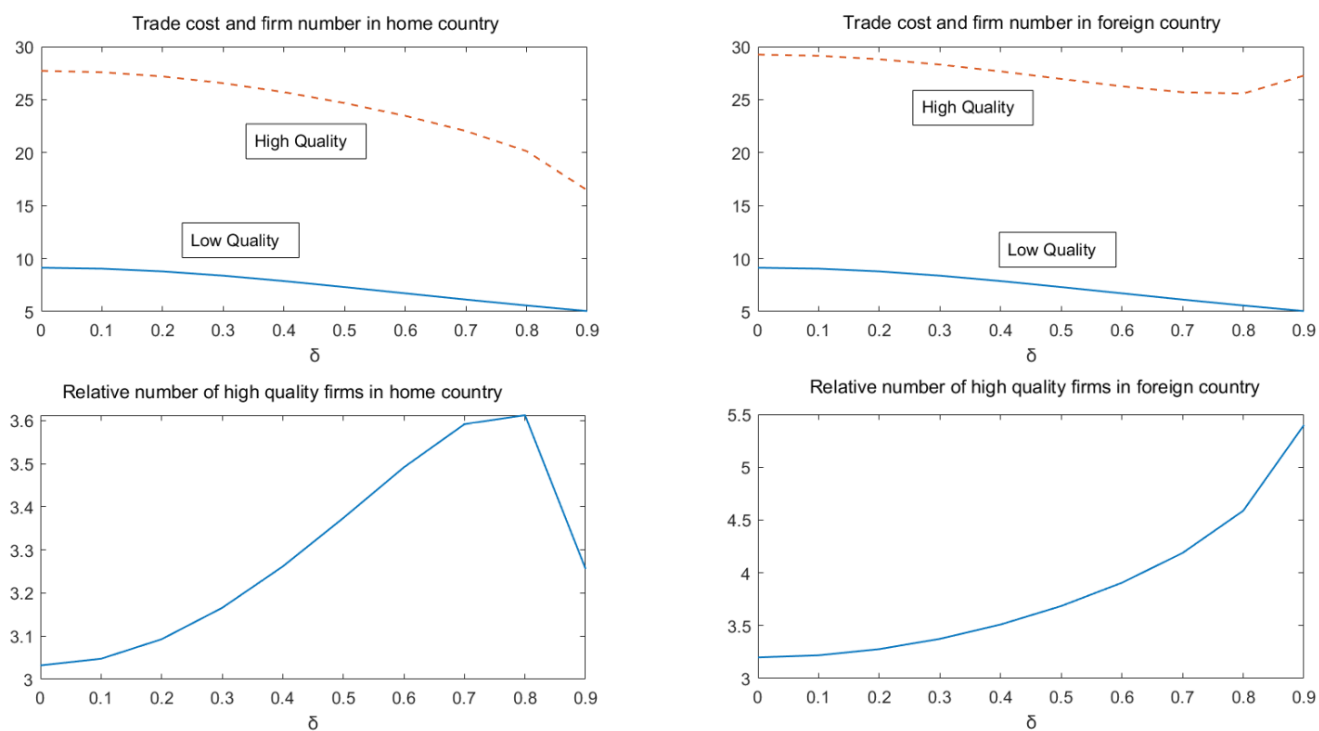


Figure 7: Effects of trade liberalization on firm numbers, the case of asymmetric countries

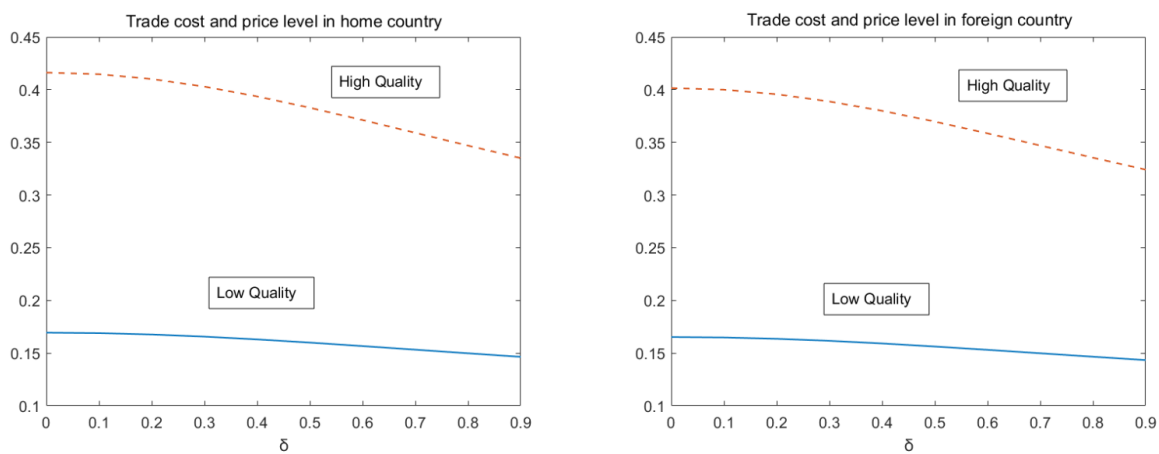


Figure 8: Effects of trade liberalization on price levels, the case of asymmetric countries

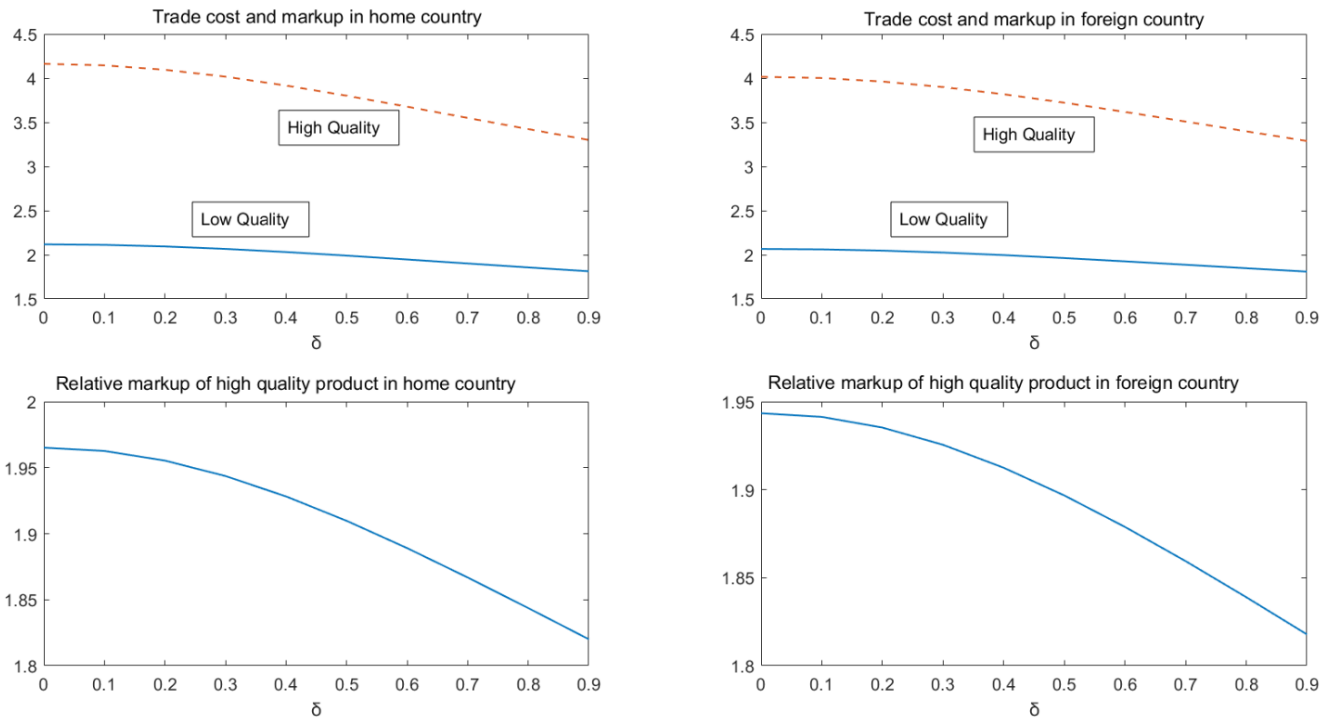


Figure 9: Effects of trade liberalization on markups, the case of symmetric countries



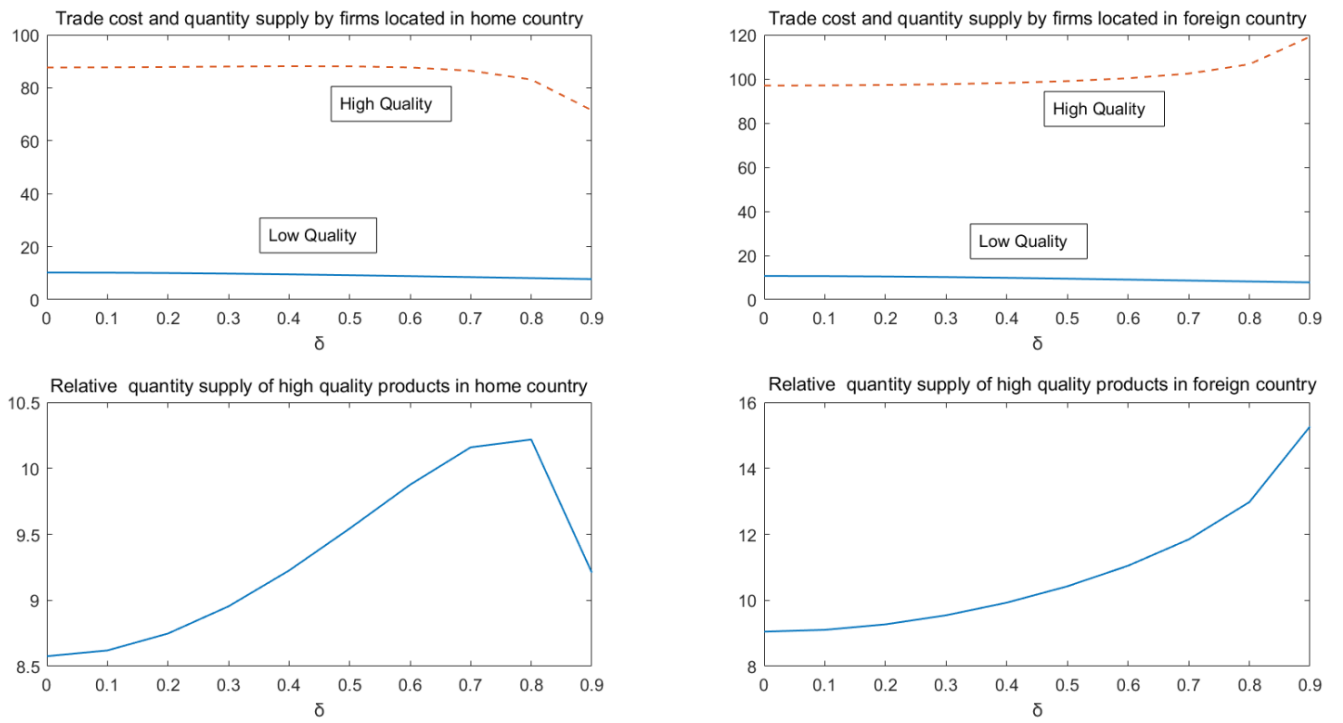


Figure 10: Effects of trade liberalization on firms' production, the case of asymmetric countries

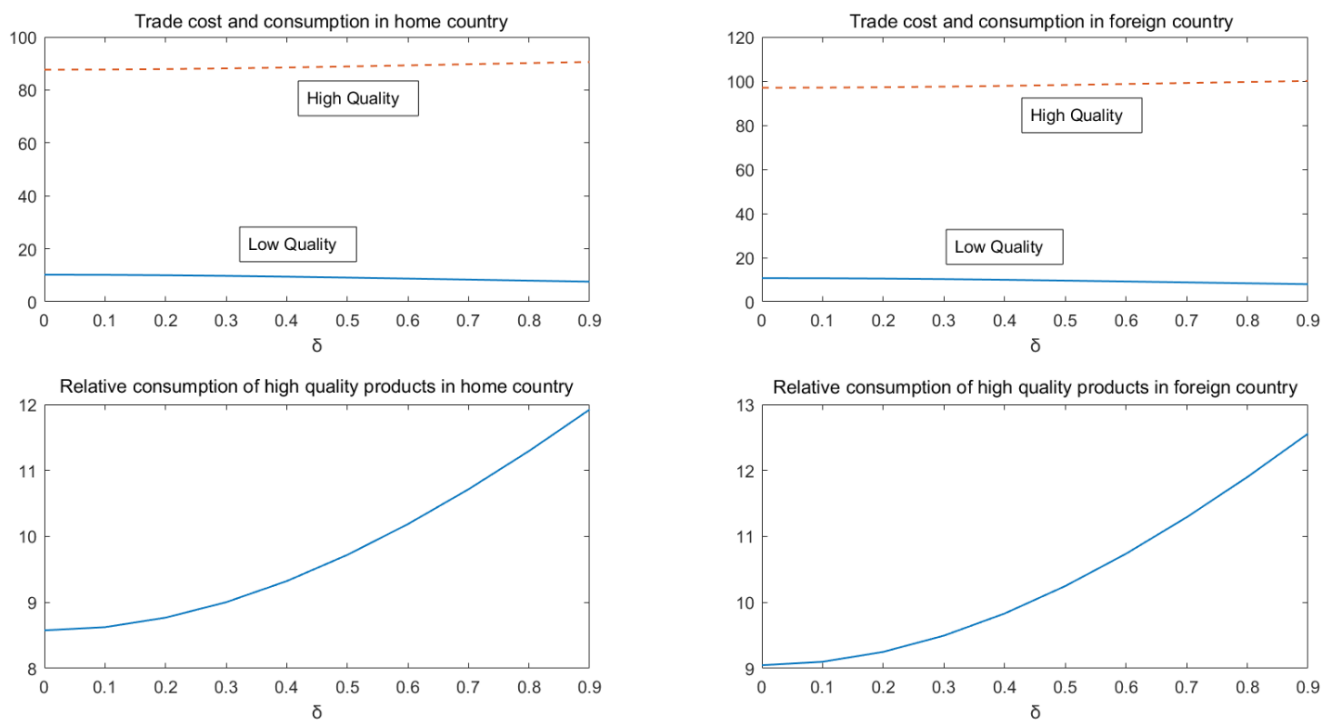


Figure 11: Effects of trade liberalization on market demand, the case of asymmetric countries

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