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The curse of knowledge: Having access to customer information can be detrimental to monopoly’s profit

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Abstract

We show that a monopolist's profit is higher if he refrains from collecting coarse information on his customers, sticking to constant uniform pricing rather than recognizing customers' segments through their purchase history. In the Markov-perfect equilibrium with coarse information collection, after each commitment period, a new introductory price is offered to attract new customers, creating a new market segment for price discrimination. Eventually, the whole market is covered. Shortening the commitment period results in lower profits. These results sharply differ from the ones obtained when the firm can uncover the exact willingness-to-pay of each previous customer.

JEL Classification codes: L12, L15

Keywords: curse of knowledge; customers information; intertemporal price discrimination; Coase conjecture; price personalization.

1 Introduction

According to the conventional theory of price discrimination, if a monopolist can partition the customer base into segmented markets, he will be able to reap larger profits, even when. in each market segment, only linear pricing is used. Indeed, the standard model of third-degree price discrimination has shed light on some frequently observed phenomena, such as discounted prices for senior citizens on train travels, students' discounts for admissions to movies, and so on. Market segmentation in these examples is however rather crude, because ages or schooling status are only rough proxies for more relevant characteristics such as income and preferences.

With the advance in digital technology, firms are increasingly able to collect and process huge amounts of consumer-specific data, allowing them to classify consumers on the basis of their purchase histories and browsing histories, their location, what they like or dislike on social networks, their preferred sites and so on (EOP, 2015). This enables firms to implement strategies that exploit the "*consumer addressability*" features described by Blattberg and Deighton (1991).

Thanks to the *Invisible Digital Hand* (Mazlkiel, 2016), price personalization, the monopolist's old dream, has now become the norm in many sectors (Petrison et al., 1997; Mohammed, 2017). Depending on the firms' technology options and on the regulatory restrictions (such as the General Data Protection Regulation (GDPR) in the European Union), the information firms are able to collect on consumers may be more or less fine.

In the literature on third-degree price discrimination, there is a presumption that with a more refined market segmentation, the firm can tailor different prices to different consumer groups, thus increasing its profit. However, this literature has overlooked an important issue: when customers are heterogeneous with respect to their willingness to pay (WTP), their purchase histories are *endogenously* determined by the firm's dynamic pricing policy. Indeed, the number of distinct market segments based on the monopolist's grouping of customer types may well depend on his current, past, and future pricing policies. Anticipating the firm's future prices and grouping strategy, lower-type customers may have an incentive to defer their purchases until later periods in order to receive a better deal. The firm may have to counter this incentive by offering higher informational rents to the new customers it wishes to serve in each period. Under these circumstances, a firm's ability to acquire customer information might well be detrimental to its profit.

In other words, the ability to use information on customers' WTP may be a curse to the monopolist.

Our article demonstrates that the "Curse of Knowledge" may arise within a coarse information setting.[1] We propose a model where a monopolist is able to recognize former customers only on the basis of the moment of their first purchase and uses this coarse information to engage in third-degree price discrimination. We show that his use of customer information for intertemporal price discrimination reduces his aggregate profit below the level he would get if such information were not available.

The equilibrium dynamics arising in this model with coarse information on customers' preferences are later compared to the ones arising in the polar case of full information acquisition (FIA, for short), in which the monopolist is able to use consumers' purchase history to uncover their exact WTP, which enables him to engage in price personalization. These two cases lead to diametrically opposed conclusions concerning the equilibrium dynamics and its Coasian or non-Coasian features. Under FIA, the monopolist gains from his ability to acquire full information, and his profit is even greater than that obtained by a full commitment monopolist under the coarse information scenario. Interestingly, comparing profits across three scenarios (the FIA case, the coarse information collection case, and the case of complete absence of customer records) shows that firm's profit can be non-monotonic in the degree of precision of information.

To derive the above mentioned results, we set up a model of a monopolist producing a homogenous good (or service) that must be consumed instantaneously. Consumers may purchase and consume at each instant of time, whereas the monopolist is committed to making pricing decisions at discrete points in time. The length of the time interval between two consecutive price offers is called the *commitment period*. There is a continuum of consumers with heterogeneous WTP for the good. The (type-dependent) consumers' WTP is initially private information. In the base-line model with coarse information, we posit that, as time goes by, the monopolist can collect some imperfect information on the consumers' WTP and use it to implement third-degree price discrimination. If a consumer makes her first-time purchase in a given period n, the monopolist will label her as a vintage-n consumer, clustering her with other consumers who have chosen to buy the

[1] As argued later, our model sheds light on optimal dynamic pricing in several real-world set-ups, including the subscription-based business models, the telecommunications industry, the streaming music industry, the online video industry, the online betting sector and the pricing of football/baseball tickets.

good for the first time in the same period.[2] In all its future dealings with the consumer, the firm will be able to recognize her as a vintage-n consumer and price discriminate accordingly.

After having been in the market for n periods, the firm faces n groups of former customers and one group of new customers. Facing these $n + 1$ market segments, the monopolist announces $n + 1$ contractually fixed prices for the period (one price for each segment). Note that whereas the size of each of the n groups of former customers is known, the size of the new market segment is endogenously determined by a number of factors. The first factor is the introductory price that the monopolist offers to this group. The second one is the decision of each member of the new targeted group concerning whether to make her first purchase in that period, or delay it until the next period. This decision depends not only on the consumers' WTP and the current introductory price but also on her expectations regarding next period's introductory price and the price she will have to face in the future as an old customer who bought the good for the first time in period $n + 1$.

We suppose that consumers have rational expectations about future prices, and that their decisions concerning when to make their first purchase are rationally made. The firm is aware that consumers are rational. We are thus dealing with a dynamic game between the firm and the potential new customers. We characterize the Markov Perfect Equilibrium (MPE) of this game, considering first the coarsest information acquisition scenario.

In the MPE with coarse information, the dynamic pricing strategy combines elements of both introductory pricing and price skimming (see Kotler and Armstrong, 2012, for more details on these concepts). More precisely, when adding a new market segment, the monopolist adopts an introductory pricing strategy: new customers benefit from a price discount when they buy the good for the first time.[3] At the same time, when serving different cohorts of consumers, the monopolist charges different prices to different consumers' cohorts: he will set higher prices to early buyers, knowing that they have higher WTP in comparison to late buyers. This means that if we look at the prices paid by successive groups of new customers, the pricing dynamics are consistent with a price skimming strategy (more precisely, the equilibrium price dynamics feature a decreasing

[2] Differently from the FIA case, in our base-line model, we suppose that the monopolist can never know for sure a former consumer's exact WTP for the good.

[3] Conitzer et al. (2012) present a plethora of evidence on firms' introductory pricing strategies.

sequence of introductory prices).

The results of our base-line model capture some of the ingredients of the complex price-discrimination strategies shaping the new subscription-based business models that have become prevalent in many markets, such as software (e.g., Adobe Creative Cloud, Mathematica licenses), Apps (e.g., auto-renewable apps on iOS through iTunes), telecommunications or e-commerce (e.g., Amazon Subscribe and Save).[4] For example, the pioneering business model used by Adobe to market Creative Cloud has quite interesting features. On the one hand, consistent with our theoretical findings, Adobe provides an introductory plan for new customers.[5] On the other hand, it tries to implement price discrimination based on clustering customers according to their WTP.[6] Although the drivers of market segmentation are clearly not the same as in our model, the Adobe example is a good illustration that firms may be interested in combining introductory pricing strategies with third-degree price discrimination based upon an imperfect market segmentation.

Our characterization of the MPE under coarse information also unveils that (i) eventually, the whole market is covered (in sharp contrast with the static equilibrium outcome), and (ii) a shortening of the commitment interval will result in a fall in the firm's aggregate profit. In the limit, as the commitment period tends to zero, the profit vanishes, a Coasian feature, even though the good is non-durable. We also find that the monopolist's profit under the MPE is strictly lower than that obtained under the full commitment scenario, where he is able to commit, right from time $t = 0$, to a specified sequence of prices for new and old customers for each vintage.[7]Thus, the firm is paradoxically hurt when it can recognize its consumers' vintage and use this information to implement price discrimination.

This result illustrates the "curse-of-knowledge" and it is consistent with business practices observed in some markets, where firms avoid this curse by committing to very simple

[4]Indeed, the subscription-based business model is often associated with the repeated purchases, a feature that is well captured by our non-durable goods framework. Success stories include Amazon Subscribe and Save, Dollar Shave Club, Blue Apron, among others.

[5]Adobe provides trial versions to new customers. Moreover, when lauching the Creative Cloud subscription-based product, Adobe offered an introductory plan to encourage the adoption of Creative Cloud for current Creative Suit users.

[6]According to Chen (2015) Adobe's plans are segmented into four user segments: individuals, businesses, students and teachers, and schools and universities.

[7]In the full commitment scenario, the monopolist finds it optimal to treat old customers the same way as new customers. In fact, the firm is optimally committed to the policy that there will be no new customers after the initial period.

pricing strategies even though they could potentially gather information to engage in behavior-based price discrimination. Indeed, as pointed out by Rao (2015),"*Apple and Amazon, two of the leading online video retailers, maintain a relatively uniform pricing policy - $14.99/$3.99 to purchase/rent their new movies and $9.99/$2.99 to purchase/rent their catalog titles. This policy has the semblance of a commitment device, because prices are held fixed over a relatively long period of time.*"

However, there are also real-world markets in which firms exert great efforts to gather information on their customers.[8] To show how, under some circumstances, a monopolist may profit from customer recognition, thus overcoming the curse of knowledge, we develop, at the end of this paper, an extension of our base-line model to the case of full information acquisition (FIA). In this extension, the monopolist can uncover each customer's exact WTP after their first purchase. This allows us to contrast the result of vanishing profit for a non-committed monopolist (who collects coarsest information) with what happens in the polar FIA scenario: we show that under FIA, the monopolist benefits from intertemporal price discrimination and Coasian-like outcomes are not obtained. Indeed, we find that in the FIA model, shortening the period of commitment would lead to increased profits rather than vanishing profits.

Our theoretical investigations suggest that profit and welfare effects of dynamic pricing strategies based on customers' purchase history may be ambiguous. Concerning profits, when the accuracy of information on consumers' WTP is not too precise, there seems to be such a thing as blessed ignorance, but when the monopolist is able to uncover the exact WTP of his customers, he actually profits from engaging in price-personalization strategies, building up a hyper-segmented market. Concerning welfare, our analysis suggests that economists need to be cautions when assessing the effects of public policies that aim at limiting firms' ability to collect and keep data on their customers (such as GDPR in the European Union). Interestingly, if a monopolist is only able to collect coarse information on his customers, he may profit from legal or technological restrictions that prevent him to use such data.

The rest of the paper is organized as follows: Section 2 presents an overview of the related literature. Section 3 introduces the model. Section 4 characterizes the MPE. Section 5 investigates equilibrium outcomes arising in the full commitment scenario, comparing them to the MPE. Section 6 presents the welfare analysis. Section 7 analyzes the case of

[8] We are grateful to a reviewer for encouraging us to analyze this issue.

full information acquisition. Finally, Section 8 concludes.

2 Related literature

Our model is related to three strands of literature. The first strand deals with third-degree price discrimination in a static framework, where a monopolist partitions his customer base into several market segments and charges different unit prices for different segments. For an exposition, see Tirole (1988).

The second strand of literature deals with price discrimination when firms are able to learn about consumers' purchase history and/or the personal tastes of individual consumers. More precisely, our base-line model enriches the literature on "behavior-based price discrimination" (e.g., Fudenberg and Tirole, 2000; Chen, 1997; Choe *et al.*, 2018),[9] whereas our full information acquisition model adds to the literature on price personalization.

The behavior-based price discrimination literature shares with our model the feature that firms are able to learn about consumers' preferences by accessing data on their histories of purchase and use this information to practice third degree price discrimination (see Fudenberg and Villas-Boas, 2006, for a survey). Most papers in this strand of literature assume a two-period model and focus on duopoly competition.[10] A general conclusion is that history-based pricing intensifies oligopolistic competition and erodes firms' profits. Indeed, following the seminal work by Stokey (1979), many scholars have stressed that intertemporal price discrimination could hurt firms.[11] Our article arrives at a similar conclusion on profit erosion. In our base-line model, a consumer faces a trade-off between buying today and buying tomorrow given that there is a price advantage associated with delay. Competition exists this time between the different selves of the monopolist instead of between oligopolists. Although in both settings it is the pressure of competition that

[9] Chen (1997) focuses on duopolistic behavior-based price discrimination (BBPD) with consumers' switching costs. Fudenberg and Tirole (2000) analyze BBPD in a Hotelling duopoly model with two period. In Choe *et al.* (2018), there are multiple equilibria when the duopolists have different information.

[10] Following the seminal works by Fudenberg and Tirole (2000) and Chen (1996), a number of authors have investigated the competitive effects of behavior-based price discrimination. See, for example, Chen and Pearcy (2010), Esteves (2009, 2010), Gherig et al. (2011, 2012), and Villas Boas (1999).

[11] In contrast, Akan et al. (2015) found that, if consumers learn about their own WTP under a time-dependent process, then intertemporal price discrimination may result in higher profits. Ata and Dana (2015) examined the robustness of this result.

reduces the firms' profits, the two market structures yield remarkable differences in terms of market coverage. In the monopoly case, to exert some competitive pressure on the current monopolist, some consumers need to stay out of the market (until the period when they make their first purchase). Differently, in standard duopoly models of behavior-based price discrimination (such as Fudenberg and Tirole, 2000), the market is fully covered in each period, and the competitive pressures follow from the possibility of switching to the rival firm.

The recent literature on price personalization concentrates on the competitive and profit effects associated with first-degree price discrimination in oligopoly set-ups. Following the pioneering paper by Thisse and Vives (1988), several authors (e.g., Choe et al., 2017, Anderson et al., 2015), have shown that firms may be harmed by the possibility of setting personalized prices, as this leads to asymmetric Bertrand competition for each consumer in the market.[12] Their results on price personalization are different from what we obtain in section 7 for our polar case, the FIA model: we show that the opportunity to practice first degree price discrimination among old customers actually benefits the information-seeking monopolist, even though he must offer introductory prices in each period to attract new clients.

The third strand of literature studies the market dynamics of a durable-good monopoly, and shows that a monopolist who is unable to fully commit to future prices would lose all his monopoly power if the time interval between two consecutive price offers goes to zero (Coase, 1972; Stokey, 1981; Bulow, 1982; Gul et al., 1986; Bond and Samuelson, 1987; Kahn, 1987; Hart and Tirole, 1988; Karp, 1996; Laussel et al., 2015; Correia-da-Silva, 2019; see Long, 2015, for a survey.) The logic behind this result is that early customers would not accept monopoly prices because they anticipate that future prices will be lower. As in our base-line model, it is the competition between the current and the future incarnations of the monopolist that reduces his market power.

While our model has the Coasian flavor, it is quite novel, and is not mathematically isomorphic to Coase's durable good monopoly model. To see this, we would like to draw the reader's attention to the following points.[13]

[12] There are some exceptions. Anderson et al. (2019) find that price personalisation may benefit the duopolists if the demand curve is strongly convex. Anderson and Dana (2009) find that intertemporal price discrimination is profitable when the relative change in surplus related to product upgrading increases with consumers' WTP.

[13] We are very grateful to Editor Kathryn E. Spier for suggesting these points.

First, our model brings some novel insights to the durable good monopoly problem itself. It is well known that short-term leasing can solve this problem (Tirole, 1988). With leasing, the monopolist can charge the consumers just for using the product for a short period of time. In the leasing mechanism discussed in the literature, the firm cannot distinguish between customers who leased in earlier periods and those who did not lease. Consequently, it charges the monopoly price for the lease, the same for all customers in each period, and profits are maximized. Our model is similar to a durable goods monopoly problem with leasing, except we allow the monopolist to discriminate directly based on previous lease adoption. In other words, the classical leasing solution is predicated on the assumption of customer anonymity, while in our model, information collection allows the monopolist to identify previous customers, leading to substantially different pricing strategies (as each cohort of consumers is charged a different price).

Second, in the durable goods monopoly model without commitment, the monopolist grabs successive slivers of consumers from the top of the demand curve. In our model, the firm does grab the first sliver from the top, but then in successive rounds it refines its information, raising the price charged to former consumers. Thus, for example, the marginal customer in period $n = 1$, whose type is θ_2, enjoys, if she chooses to be a first-time customer in period 1, the introductory price $p(1, 1)$ that is strictly lower than θ_2, but in all later periods $j \geq 2$, being a former customer, she must pay a higher price $p(1, j) = \theta_2$, leaving her with zero surplus.

Third, our model and the durable goods monopoly model have different empirical implications. In our model, at any point of time, the monopolist posts different prices to different segments of consumers. In the classic durable goods monopoly model, the monopolist offers a single price at each point of time.

Our article is also related to the literature on Markov-Perfect Equilibrium in games involving interactions between firms and infinitely-lived consumers with rational expectations (see, e.g., Driskill and McCafferty, 2001; Laussel, Montmarin and Long, 2004; Laussel, Long and Resende, 2015; Long, 2015).

Finally, there is some connection between some of this article's conclusions and the real options literature (Dixit and Pindyck, 1994; Abel et al., 1996). In our model, consumers have an incentive to delay purchase: with delayed entry, they could face a lower price as they would be classified into a market segment of consumers with a lower valuation of the good. By waiting, a consumer is transmitting the information that she has low valuation, thereby anticipating greater surplus in future periods. Thus, the value of waiting is associated with information transmission. In this sense, there is a connection between our model and the real options literature, which has identified the option value associated with delaying investment decisions when facing uncertainty. This value should be taken into account for the characterization of optimal investment timing: delayed investment facilitates information acquisition. Although in both set-ups, the value of waiting is associated with information transmission, there are important differences between the two problems. In particular, in our case, by waiting, the consumer will certainly get higher future surplus in the future (at the cost of forgoing current surplus), whereas in the real options theory, the expected benefit of waiting arises from reduced uncertainty.

3 The model

A monopolist produces a non-durable good (or a service) at a constant marginal cost, normalized to zero. Although the good is homogeneous, consumers have heterogeneous WTP for the good.

Time is a continuous variable. The market is populated with infinitely-lived consumers. Each consumer buys and instantaneously consumes at most one unit of the good at each instant of time. There is a continuum of consumer types. A consumer of type θ derives θ units of utility for consuming one unit of the good per unit of time. If she pays the unit price p for the good, her instantaneous net utility is $\theta - p$. The support of the distribution of θ is a closed set $\left[\underline{\theta}, \overline{\theta}\right]$.

The firm enters the market at time $t = 0$. It partitions the (non-negative) time line $[0, \infty)$ into a sequence of "commitment periods" of equal length Δ. Thus, period 0 corresponds to the interval $[0, \Delta)$ of the time line and period n corresponds to the interval $[n\Delta, (n+1)\Delta)$, where $n = 0, 1, 2, 3, \ldots$.

In period $n = 0$, there are no former customers, and the monopolist offers a price $p(0,0)$ to the top sliver of the customer base, i.e., customers with $\theta \in \left[\theta_1, \overline{\theta}\right]$, where θ_1 and $p(0,0)$ are to be optimally determined. A customer who buys the good for the first time in period i is called a vintage-i consumer. The collection of all customers of the same vintage constitutes a *market segment*. At the beginning of any period n, the monopolist offers to all former consumers of vintage i (where $i < n$) a vintage-specific price $p(i,n)$ at which they can purchase a unit of the good at each instant of time in the time interval $[n\Delta, (n+1)\Delta)$. We assume that, thanks to the firm's "big-data" capability, the monopolist can identify the vintage of all former customers and use that information to engage in third degree price discrimination: a consumer of vintage $i < n$ can acquire the good in period n only at the price $p(i,n)$ and not at any other prices. Customers who buy the good for the first time in period n are offered an introductory price $p(n,n)$.

For all $i = 0, 1, 2, .., n$, a consumer of vintage i who purchases and consumes a unit of the good at each point of time in period $n \geq i$ enjoys the instantaneous net utility $\theta - p(i,n)$. Her net utility over the whole period n (discounted back to the beginning of period n) is

$$v(\theta, i, n) \equiv (\theta - p(i,n)) \int_0^{\Delta} e^{-r\tau} d\tau \equiv (\theta - p(i,n)) \frac{1-\beta}{r}, \text{ where } \beta \equiv e^{-r\Delta}.$$

Here $r > 0$ denotes the *instantaneous* discount rate, and β is the discount factor between periods. Note that whereas r is exogenous and independent of Δ, β depends on Δ. Clearly $\beta \to 1$ when $\Delta \to 0$.

Let us consider the case where the monopolist cannot commit to future prices. Specifically, when selling to first-time consumers in period n, the firm cannot commit to offer them, in future periods, the same price as the introductory price. In addition, in any period n, the monopolist cannot commit to offer pre-determined introductory prices to future new customers. In the absence of such a commitment capability, we assume the monopolist uses third-degree price discrimination with respect to old market segments: they are discriminated on the basis of the monopolist's endogenous market segmentation (which in our set-up relates to the consumers' vintage clustering).

The cumulative distribution of θ is denoted by $F(\theta)$ and the density function is denoted by $f(\theta)$. We make the following assumptions:

Assumption A1. $f(\theta) - \theta f'(\theta) \leq 0$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

Assumption A2. $1 - \underline{\theta} f(\underline{\theta}) > 0$.

Assumption A1 is sufficient to ensure that the monopolist's profit function for period n for each market segment $i < n$ is concave in the price $p(i,n)$. This assumption is satisfied by the uniform distribution and, more generally, by all distributions such that F is not too convex. Assumption A2 ensures that a static monopolist would not serve the whole market. In the uniform distribution case, it is equivalent to $\overline{\theta} > 2\underline{\theta}$.

4 Equilibrium under non-commitment

We start our analysis of the model by scrutinizing the monopolist's third degree price discrimination among former customers. This will be followed by an examination of the optimal pricing for new customers.

□ **Third-degree price discrimination among former customers.** The following observation is useful in what follows. As consumers are rational, we can deduce that, in equilibrium, if a consumer of type θ' finds it optimal to be a first-time customer in period $j \geq 0$, then any consumer of type $\theta'' > \theta'$ must find it optimal to be a first-time customer in some period i, with $i \leq j$. At the beginning of any period j, there are already j known cut-off types $\theta_1 \geq \theta_2 \geq \theta_3 \geq ... \geq \theta_j$, and the firm, through its price decisions, determines θ_{j+1}. New customers in period 0 are of type $\theta \in \left(\theta_1, \overline{\theta}\right]$, new customers in period 1 are of type $\theta \in (\theta_2, \theta_1]$, and so on. In any period $n > 1$, with $j + 1 \leq n$, customers of type $\theta \in (\theta_{j+1}, \theta_j]$ are called vintage-j customers. A customer of type θ_{j+1} is called a vintage-j *marginal customer*. In equilibrium, the vintage-j marginal customer θ_{j+1} is indifferent between making her first purchase in period j or in period $j + 1$. Thanks to his ability to keep its data records registering the moment in which each consumer enters the market, the firm is able to identify the vintage of a former customer when she comes back in subsequent periods. In our base-line model with coarsest information, the firm cannot tell the difference among former customers that belong to the same vintage. Later on, we will relax this assumption, by looking at the polar case of full information acquisition (the FIA case, studied in Section 7).

At the beginning of period j, all consumers whose types belong to $\left(\theta_j, \overline{\theta}\right]$ have already purchased the good at least once in previous periods. The monopolist would not offer a former customer the introductory price that it offers to new customers. In any period j, the firm offers former customers a vintage-dependent price, $p(i, j)$, for each vintage $i < j$ ($i = 0, 1, 2, .., j - 1$) so as to maximize the profits it makes from them. The population share of customers that belong to vintage i is $F(\theta_i) - F(\theta_{i+1})$. Clearly, offering these customers any $p(i, j) > \theta_i$ would be a dominated strategy because it would result in zero demand from that market segment. Thus, in period j, for former customers that belong to vintage-i (where $i < j$), the monopolist will offer a price $p(i,j) \leq \theta_i$. The quantity sold in period j to this market segment is

$$Q_{i,j} = \begin{cases} F(\theta_i) - F(p(i,j)) & \text{if } \theta_{i+1} \leq p(i,j) \leq \theta_i \\ F(\theta_i) - F(\theta_{i+1}) & \text{if } 0 \leq p(i,j) \leq \theta_{i+1} \end{cases}$$

Thus, the profit obtained in period j from this market segment is

$$\pi(i,j) = \frac{1-\beta}{r} \left[F(\theta_i) - \max\{F(\theta_{i+1}), F(p(i,j))\}\right] p(i,j).$$

Define $p(\theta_i)$ as the solution of the unconstrained problem $\max_p [F(\theta_i) - F(p)]\, p$. Then $p(\theta_i)$ satisfies the first order condition that $F(\theta_i) - F(p(\theta_i)) - f(p(\theta_i))p(\theta_i) = 0$.[14]

Clearly there is no point in offering former customers of vintage i a price that is below the lowest valuation among them, θ_{j+1}. Thus, the monopolist's optimal choice must satisfy the condition $p^*(i,j) \geq \theta_{i+1}$. Accordingly, the optimal price is

$$p^*(i,j) = \max\{p(\theta_i), \theta_{i+1}\} \text{ for all } i \leq j. \tag{1}$$

It follows that if $p(\theta_i) \leq \theta_{i+1}$, then the monopolist's optimal price for former vintage-i customers is exactly equal to θ_{i+1}, i.e., the price is equal to the maximum WTP of the lowest valuation customers in vintage i. We will show later that along the optimal path, the property $p(\theta_i) \leq \theta_{i+1}$ is indeed satisfied. For future reference, we record the following result as Lemma 1:

Lemma 1. Under the assumption that $p(\theta_i) \leq \theta_{i+1}$ (which will be showed to be satisfied in equilibrium), in all periods $j > i$, the monopolist's optimal price for market segment i is equal the maximum willingness to pay of the lowest type of that segment:

$$p^*(i,j) = \theta_{i+1}. \tag{2}$$

In view of equation (2), the monopolist's optimal aggregate profit in period n over all

[14] The second order condition is always satisfied thanks to Assumption A1.

the former customers is

$$\Pi_n^F = \frac{1-\beta}{r} \sum_{i=1}^{i=n} \left[F(\theta_{i-1}) - F(\theta_i)\right] \theta_i. \tag{3}$$

where the superscript F stands for "former" customers.

☐ **Monopoly pricing for first-time customers.** Let us now turn to new customers in period n. Let $U_n(\theta_{n+1})$ denote the intertemporal net utility of period-n marginal new customer. By marginal, we mean that she is indifferent between (i) being a first-time customer in period n (paying the price $p(n,n)$ for the good), and (ii) being a first-time customer in period $n+1$, paying a lower price $p(n+1,n+1) < p(n,n)$, but at the cost of having to forgo her net utility $\theta_{n+1} - p(n,n)$ at each instant of time over the time interval of length Δ.

Recall that Lemma 1 stated that marginal customers in period n will be charged an instantaneous price $p^*(n,j) = \theta_{n+1}$ in all later periods $j > n$. Thus, they get zero utility in all later periods. It follows that their intertemporal utility from period n onwards is simply equal to their utility in period n, that is $U_n(\theta_{n+1}) = \frac{1-\beta}{r}\left(\theta_{n+1} - p(n,n)\right)$. It is useful to re-write this equality as

$$p(n,n) = \theta_{n+1} - \frac{r}{1-\beta} U_n(\theta_{n+1}). \tag{4}$$

We will refer to $U_n(\theta_{n+1})$ as the *informational rent* of period-n marginal new customer.

The period-n profit over all new customers may then be written as

$$\Pi_n^N = \left(\frac{1-\beta}{r}\right)\left[F(\theta_n) - F(\theta_{n+1})\right] p(n,n) = \left[F(\theta_n) - F(\theta_{n+1})\right]\left(\frac{(1-\beta)\theta_{n+1}}{r} - U_n(\theta_{n+1})\right). \tag{5}$$

Now, any vintage-n customer of type $\theta > \theta_{n+1}$ will face the same present and future prices as the ones that the marginal customer θ_{n+1} faces, but values the good more. Therefore, the difference in their intertemporal net utility is

$$U_n(\theta) - U_n(\theta_{n+1}) = \frac{1}{r}(\theta - \theta_{n+1}). \tag{6}$$

If the firm wants to induce a new customer with $\theta \in [\theta_{n+1}, \theta_n]$ to buy in period n, it must ensure that she would not be better off waiting until period $n+1$. This participation constraint may be written as

$$\frac{1}{r}(\theta-\theta_{n+1})+U_n(\theta_{n+1})\geq\beta\left[\frac{1}{r}(\theta-\theta_{n+2})+U_{n+1}(\theta_{n+2})\right]\text{ for all }\theta\in[\theta_{n+1},\theta_n]\,. \tag{7}$$

From (6), the LHS of (7) is simply $U_n(\theta)$, the intertemporal utility obtained by a type-θ customer who chooses to be a period-n new customer. The RHS is the alternative intertemporal utility, measured from period n, that she would obtain if she chose to be a new customer in period $n+1$. Notice that if the participation constraint (7) is satisfied for type $\theta=\theta_{n+1}$ (the marginal customer), it will also be satisfied for all customers of type $\theta\in(\theta_{n+1},\theta]$. Thus, the constraint (7) is redundant if the following simpler constraint is satisfied: $U_n(\theta_{n+1})\geq\beta\left[\frac{1}{r}(\theta_{n+1}-\theta_{n+2})+U_{n+1}(\theta_{n+2})\right]$. In fact, the latter constraint is satisfied with equality,

$$U_n(\theta_{n+1})=\beta\left(\frac{1}{r}(\theta_{n+1}-\theta_{n+2})+U_{n+1}(\theta_{n+2})\right), \tag{8}$$

because, by definition, a marginal customer is indifferent between being a first-time customer in period n and being a first-time customer in period $n+1$. By repeated substitution, the difference equation (8) yields the solution

$$U_n(\theta_{n+1})=\frac{1}{r}\sum_{j=1}^{\infty}\beta^j\left[\theta_{n+j}-\theta_{n+j+1}\right], \tag{9}$$

where we have used the fact that $\lim_{j\to\infty}\beta^jU_{n+j}(\theta_{n+j+1})=0$ (as U is bounded).

□ **The Markov perfect equilibrium.** In any period n, let $X(n)\in[0,1]$ denote the fraction of the total population that has purchased the good prior to that period. Given the nature of our problem, $X(n)$ would be a natural state variable for our dynamic problem. However, it turns out to be more convenient to use as state variable the following transformation of $X(n)$: $\Theta(n)\equiv F^{-1}(1-X(n))$, where $\Theta(n)$ is our state variable and $F(.)$ is the cumulative distribution of θ. Then $\Theta(0)=\overline{\theta}\equiv\theta_0$, and $\Theta(n)\in\left[\underline{\theta},\overline{\theta}\right]$. Clearly, the real interval $\left(\Theta(n),\overline{\theta}\right]$ corresponds to the set of customers who have not purchased the good prior to period n.

As we have solved for the firm's optimal pricing for former customers, its dynamic optimization problem reduces to determining, at the beginning of each period n, given the observed value of the state variable $\Theta(n)$, the optimal size of the new market segment. It follows that the firm's Markovian strategy consists of a cut-off rule ψ, such that $\Theta(n+1)$ is determined by $\Theta(n+1) = \psi(\Theta(n)) \leq \Theta(n)$.

Also, at the beginning of period n, given the observation $\Theta(n)$, consumers who have not purchased the good need to decide on the optimal period to enter the market. To perform this calculation, they must form their expectations on the informational rents of the marginal customers for all successive future periods. When characterizing the MPE, we assume that consumers have a Markovian expectations rule $\Omega(.)$ that predicts the life-time rent of the marginal first-time customer in period n, i.e., $U_n(\theta_{n+1})$. Rational expectations require that, given (9), we have

$$\Omega(\Theta(n)) = \frac{1}{r}\sum_{j=1}^{\infty} \beta^j \left(\Theta^*(n+j) - \Theta^*(n+j+1)\right) = U_n(\theta_{n+1}), \tag{10}$$

where $\{\Theta^*(.)\}_n^\infty$ is the path of the state variable Θ induced by the strategy of the monopolist.

A Markovian strategy $\psi(.)$ chosen by the monopolist is called a best reply to the consumer expectations function $\Omega(.)$ if (a) it yields a sequence of cut-off values θ_{n+1} that maximizes profits, starting from any pair $(n, \Theta(n))$, and (b) the rational expectations condition (10) is satisfied by such a sequence. In order to obtain a sharp characterization of MPE, in this section we replace Assumptions A1 and A2 with a stronger assumption:

Assumption B. The distribution of θ is uniform, with $\overline{\theta} = 1$ and $\underline{\theta} = 0$.

In what follows, some Propositions rely on Assumption B, while others rely only on the weaker Assumptions A1 and A2. We will state explicitly which proposition relies on Assumption B and which one relies only on the weaker Assumptions A1-A2.

Using Assumption B, with the help of equation (3), the profit obtained in period n from old customers is

$$\Pi_n^F = \frac{1-\beta}{r}\sum_{i=1}^{i=n} (\theta_{i-1} - \theta_i)\,\theta_i. \tag{11}$$

Corresponding to eq. (5), the profit obtained in period n from first-time customers in that period is

$$\Pi_n^N = (\theta_n - \theta_{n+1}) \left(\theta_{n+1} \frac{1-\beta}{r} - \Omega(\Theta(n)) \right),$$

where we have substituted $\Omega(\Theta(n))$ for $U_n(\theta_{n+1})$ because of the rational expectations requirement.

It follows that, under Assumption B, the Bellman equation for the monopolist is[15]

$$\begin{aligned} V(\Theta(n)) &= \max_{\Theta(n+1)} \left\{ \frac{1-\beta}{r} \left(\sum_{i=1}^{i=n} \left(\Theta(i-1) - \Theta(i)\right) \Theta(i) \right) + \right. \\ & \left. (\Theta(n) - \Theta(n+1)) \left(\Theta(n+1) \frac{1-\beta}{r} - \Omega(\Theta(n)) \right) + \beta V(\Theta(n+1)) \right\}. \end{aligned}$$

Given Assumption B and the general structure of the problem, we conjecture the that the MPE consists of a *linear cut-off rule*

$$\Theta(n+1) = \psi(\Theta(n)) = \gamma\Theta(n) \tag{12}$$

and a *linear expectations function*[16]

$$\Omega(\Theta(n)) = \lambda\Theta(n) \tag{13}$$

where both γ and λ are to be determined. We must show that these rules are best replies to each other if λ and γ are appropriately chosen.

Using our conjectured expectations rule (13), the rational expectations requirement, with the help of equation (8), can be rewritten as

$$\lambda\theta_n = \beta \left(\lambda\theta_{n+1} + \frac{1}{r}(\theta_{n+1} - \theta_{n+2}) \right). \tag{14}$$

Using the conjectured equilibrium cut-off rule, we have $\Theta(n+1) = \gamma\Theta(n)$ and $\Theta(n+2) = \gamma^2\Theta(n)$, and equation (14) becomes $\lambda\theta_n = \beta\left[\lambda\gamma\theta_n + (1/r)(\gamma\theta_n - \gamma^2\theta_n)\right]$,from which we obtain

[15] $V(\Theta(n))$ depends also on the values of $\Theta(n-1)$, $\Theta(n-2), .., \Theta(0)$. This is omitted for the sake of notational simplicity. Note that in period n these given values have no relevance for the decision on the optimal $\Theta(n+1)$.

[16] Starting from a more general conjectured linear-quadratic expectations functions would lead to a simple linear expectations function, so we take this shortcut for the sake of simplicity.

$$r\lambda = \frac{\beta(\gamma - \gamma^2)}{1 - \beta\gamma}. \tag{15}$$

This equation says that, given the monopolist's cut-off rule, represented by γ, the consumers expectations are rational (best reply) if and only if

$$\lambda = \lambda(\gamma; \beta, r) = \frac{\beta(\gamma - \gamma^2)}{r(1 - \beta\gamma)}. \tag{16}$$

Maximizing the Bellman equation with respect to $\Theta(n+1)$ yields

$$(1 - \beta)\frac{\Theta(n) - 2\Theta(n+1)}{r} + \lambda\Theta(n) + \beta V'(\Theta(n+1)) = 0. \tag{17}$$

Differentiating the Bellman equation with respect to $\Theta(n)$ and using the Envelope Theorem, we obtain

$$V'(\Theta(n)) = \frac{1 - \beta}{r}\left[\Theta(n-1) - 2\Theta(n) + \Theta(n+1)\right] - \lambda\Theta(n) - \lambda\left[\Theta(n) - \Theta(n+1)\right]. \tag{18}$$

From (18), we can obtain an expression for $\beta V'(\Theta(n+1))$ and substitute it into (17). We finally obtain the Euler equation,

$$(1 - \beta^2 + r\lambda)\Theta(n) - 2(1 - \beta^2 + \beta r\lambda)\Theta(n+1) + \beta(1 - \beta + r\lambda)\Theta(n+2) = 0. \tag{19}$$

If the monopolist uses a linear cut-off rule, then $\Theta(n+2) = \gamma\Theta(n+1) = \gamma^2\Theta(n)$ and equation (19) becomes

$$(1 - \beta^2 + r\lambda) - 2(1 - \beta^2 + \beta r\lambda)\gamma + \beta(1 - \beta + r\lambda)\gamma^2 = 0 \tag{20}$$

Equation (20) says that, taking the parameter γ of the consumers' expectations rule as given, the monopolist's linear cut-off rule is an optimal response (best reply) if and only if

$$\gamma = \frac{2(1-\beta^2+\beta r\lambda) \pm \sqrt{4(1-\beta^2+\beta r\lambda)^2 - 4(1-\beta^2+r\lambda)\beta(1-\beta+r\lambda)}}{2\beta(1-\beta+r\lambda)}. \tag{21}$$

The two best-response functions (16) and (21) yield a unique fixed point (γ^*, λ^*) which characterizes the Markov-perfect Nash equilibrium. Indeed, we can show that, for any given $\beta \in (0,1)$, γ^* is the unique positive real root of the equation

$$E(\gamma,\beta) = 1-\beta^2+(2\beta^2+\beta^3-\beta)\gamma+(2\beta-3\beta^2-2\beta^3)\gamma^2+(2\beta^2+\beta^3)\gamma^3-\beta^2\gamma^4 = 0. \tag{22}$$

This is a polynomial of degree 4 in γ. With $0 < \beta < 1$, we find that the polynomial has a unique positive real root γ^*. In the limiting case where $\beta = 0$, we have $\gamma^* = 1/2$, and when $\beta = 1$, we have $\gamma^* = 1$.[17] Figure 1 shows that γ^* is strictly increasing in β. In the figure, the horizontal axis represents β, whereas the vertical axis represents the equilibrium value of γ as a function of β.

Insert Figure 1

The following proposition characterizes the MPE.

Proposition 1. Under Assumption B, there exists a MPE in which the monopolist's optimal cutoff rule and consumers' expectations rule are both linear in the state variable, where the equilibrium parameters (γ^*, λ^*) are such that $\gamma^*(\beta) \in \left[\frac{1}{2}, 1\right]$ is the solution of equation (22) and $\lambda^*(\beta)$ is given by

$$\lambda^*(\beta) = \frac{\beta(\gamma^*(\beta) - \gamma^*(\beta)^2)}{r(1-\beta\gamma^*(\beta))}. \tag{23}$$

Proof See the Appendix.■

Proposition 1 identifies the equilibrium parameter λ^* of the consumers' expectations rule, $\Omega(\Theta(n)) = \lambda^*\Theta(n)$ and the parameter γ^* of the monopolist's optimal market expansion rule, $\Theta(n+1) = \gamma^*\Theta(n)$. In the MPE characterized in Proposition 1, the monopolist

[17] When $\beta = 1$, $\gamma = 0$ is also a real root. It is the limit of a negative real root (when $\beta \to 1$) and as such may be ruled out.

adopts a simple market expansion rule: in any period n, the fraction of the customer base that is not served is $(\gamma^*)^{n+1}$. The measure of customers that have been served by the end of period n is $(1-(\gamma^*)^{n+1})$. Hence the market is gradually fully covered, although full market coverage is only reached asymptotically, as n tends to infinity. The result that full market coverage is eventually reached provides a sharp contrast to the static equilibrium case and, as we shall see, also to the dynamic case with commitment, as in both of these cases only partial coverage occurs.

Proposition 1 also allows us to shed light on the prices targeted to successive cohorts of new customers and the magnitude of the rent of the marginal customer that chooses to purchase for the first-time in period n. Using eq. (9) and the linear cut-off rule, so that $\theta_{n+j}-\theta_{n+j+1}=(\gamma^*)^n(1-\gamma^*)(\gamma^*)^j$, the informational rent of the marginal customer that rationally enters the market in period n is equal to:

$$U_n(\theta_{n+1})=\frac{\gamma^n(1-\gamma)}{r}\left(\frac{\beta\gamma}{1-\beta\gamma}\right).$$

Accordingly, the monopolist's introductory price to first-time customers in period n is

$$p(n,n)=\gamma^{n+1}-\frac{\gamma^n(1-\gamma)}{1-\beta}\left(\frac{\beta\gamma}{1-\beta\gamma}\right),\qquad n=0,1,2,3,...$$

To give a concrete flavor to Proposition 1, let time be measured in days, and suppose the instantaneous interest rate is $r=0.01$, then, if $\Delta=160$ days (i.e., if $\beta\equiv e^{-r\Delta}\simeq 0.20$), using eq. (22), we obtain the monopolist's equilibrium cut-off parameter, $\gamma^*=0.55$, i.e., the monopolist initially serves only the top 45% of the customer base (during the first 160 day period). Thus, in period $n=0$, the bottom 55% of the potential market is not served. In the second round of offers ($n=1$), the monopolist expands the market, so that the fraction of unserved customers shrinks to $(\gamma^*)^2$, i.e., around 30%. In period $n=1$, the monopolist serves two market segments: all the vintage-0 customers (which consist of the top 45% of the customer base) must pay a higher price, $p(0,1)=\gamma=0.55$ (higher than the introductory price $p(0,0)$ that they paid in the previous period, which was approximately $p(0,0)\simeq 0.48$). The period 1's first-time customers are asked to pay only the introductory price $p(1,1)$, which is approximately $p(1,1)\simeq 0.264\,26<p(0,0)<p(0,1)$. It is easy to verify that period 1's marginal first-time customers (whose type is $\theta_1=\gamma^*$) are indifferent between (a) making their first purchase at the beginning period 1 and (b) delaying their

first purchase until the beginning of period 2. Indeed, condition (22) ensures that they would be indifferent, as expectations rule is given by (23). And their expectations are in fact rational.

Now, suppose the period length is only 7 days ($\Delta = 7$). Then $\beta \simeq 0.93$ and $\gamma^* = 0.86$. That is, the monopolist serves only 14% of the customer base during the first 7 day period. However, after 23 periods (i.e., after 161 days), the measure of customers that have been served by the end of period $n = 22$ is $(1 - (\gamma^*)^{23}) = (1 - 0.86^{23})$, that is 97% of the customer base. This numerical example suggests that as the length of the period of commitment shrinks, the market coverage at any given date increases.

Let us scrutinize more formally the role of the length of the commitment period, Δ. A natural enquiry is whether a shortening of the commitment period will make the market coverage larger at any given point of time. Let t be a continuous variable denoting time, where $t = 0$ is the beginning of period 0, and, more generally, $t = n\Delta \geq \Delta$ at the beginning of period $n \geq 1$. Recall that γ^* is a function of $\beta \equiv e^{-r\Delta}$, so that $\gamma^*(\beta) \equiv \gamma(e^{-r\Delta})$. Therefore the fraction of the consumer base that is served by the monopolist at time t is given by $M(t;\Delta,r) \equiv \left[1 - (\gamma(e^{-r\Delta}))^{\frac{t}{\Delta}+1}\right]$.

Let us investigate how, at any given t, the market coverage $M(t;\Delta,r)$ depends on Δ. A shortening of Δ generates two opposite effects. On one hand, since $\gamma'(\beta) > 0$, γ^* increases as Δ gets shorter, i.e., the market expansion from one period to the next is smaller (meaning the monopolist is doing a finer partition of its customers). We call this the "step size effect". On the other hand, the interval between two periods becomes smaller and thus over any time interval $[0,t]$ market expansions occur more often. We call this the "frequency effect". The two effects work in opposite direction. When Δ shrinks, the frequency effect tends to increase M at any given t, while the step size effect tends to decrease M at any given t. Numerical simulations indicate that the frequency effect dominates the step size effect if Δ is not too large. As an illustration, please refer to Figure 2 below.

Insert Figure 2

Figure 2 shows that, given a future date, $t = 10$, and letting $r = 10\%$, the frequency effect outweighs the step size effect: starting from any $\Delta \leq 10$, a shortening of the commitment period Δ speeds up market expansion. Such dynamics are clearly Coasian. While this result seems robust in our simulations, analytical results of the effect of a decrease in

Δ on market coverage can only be obtained in the limiting case where Δ tends to zero, as reported in Claim 1 below.

We prove in Claim 1 below that full coverage occurs instantaneously as Δ tends to zero, meaning that all consumers will be served in a twinkle of an eye when Δ becomes infinitesimal.

Claim 1. Under Assumption B, when the length of time Δ between two different proposals to two consecutive sets of new customers becomes infinitesimal, the market is covered instantaneously. For any given time $t > 0$, as the length of the period of commitment Δ tends to 0, the fraction of the customer base that has been served up to that time tends to $(1 - e^{-\gamma'(1)rt})$, which equals 1 because $\gamma'(\beta) \to +\infty$ as $\beta \to 1$:

$$\lim_{\Delta \to 0} \left(\gamma(e^{-r\Delta})\right)^{\frac{t}{\Delta}+1} = e^{-\gamma'(1)rt} = 0. \tag{24}$$

Proof See the Appendix.■

What happens to the consumer equilibrium expectations coefficient, λ, when Δ shrinks? A numerical illustration is presented in Figure 3 below, where β is measured along the horizontal axis, and $r\lambda$ is measured along the vertical axis (the picture is drawn for $r = 1$).

Insert Figure 3

Notice that $r\lambda$ is increasing in β.[18] Interestingly, starting at any strictly positive β, a marginal increase in β implies that for any given number of previous customers, the rent which is to be left to marginal customers in any period n (the type θ_{n+1}, i.e., the lowest type among the first-time consumers in period n) increases.[19] In other words, the rent increases as the length of the period of commitment shrinks. The reason is that as Δ gets smaller, the marginal first-time customers have a stronger incentive to delay their first purchase to the following period, unless this incentive is countered by giving them more rent. In the limit as $\Delta \to 0$, $r\lambda \to 1$ (this is shown easily by using L'Hospital's rule: the ratio of the first-order derivatives of the numerator and denominator of (23) equals in the

[18] As $\beta \to 0$, we see $\gamma \to 1/2$ and $\lambda \to 0$. This means that when the commitment period is infinite, the firm serves only customers whose type θ belongs to $\left[\frac{1}{2}, 1\right]$, and the rent of the lowest type served by the monopolist is zero.

[19] Note, however, the type θ_{n+1} (when β increases) is not the same as the type θ_{n+1} at the initial β.

limit $\frac{\gamma'(1)}{(1+\gamma'(1))r} = 1$ as $\gamma'(1) = +\infty$). In other words, in the limit, the consumers capture all the benefits of the relationship and no profit is left to the monopolist.

Let us confirm that aggregate profit vanishes as the length of the commitment period tends to zero. The monopolist's equilibrium aggregate profit, or the value of the firm (i.e., the sum of all future discounted profits), as viewed at time $t = 0$, can be easily expressed as a function of β, as given below:

$$\Pi(\beta) = \frac{1}{r}\left(\frac{(1-\beta)\gamma^*(\beta)\left[1-\gamma^*(\beta)\right]}{\left[1-\beta\gamma^*(\beta)\right]\left[1-\beta\gamma^*(\beta)^2\right]}\right). \tag{25}$$

The relationship between Π and β is pictured in Figure 4 below.[20] The x-axis represents β, whereas the y-axis depicts equilibrium aggregate profit.

Insert Figure 4

The figure shows that the aggregate profit is decreasing as β increases, i.e., as the length of the period Δ of commitment shrinks. This result is intuitively plausible: one expects that aggregate profit is greatest when the monopolist can fully commit right from the beginning to a sequence of contracts. (As we shall show in the next section, this implies sticking forever to the monopolist's initial price.) Intuitively, the opportunity of reoptimizing repeatedly very soon ends up being detrimental to the firm, because rational customers would then expect that they do not have to wait very long to benefit from the future surplus, and accordingly they would buy in the current period only if they are offered greater current rents (lower prices) to purchase immediately.

When Δ tends toward infinity (so that $\beta \to 0$), $\Pi(0) = \frac{1}{4r}$.This is equivalent to an infinite repetition of the static equilibrium. For the other polar case, where $\beta \to 1$ (i.e., $\Delta \to 0$), Claim 2 below states formally that the limit of $\Pi(\beta)$ when $\Delta \to 0$ is equal to 0. This is again a Coasian result but in our set-up it applies to a monopolist producing a non-durable good (instead of the durable goods case already studied in the literature). The Coase Conjecture indeed states that in the limiting case where the time interval that elapses between two offers tends to zero, the *durable*-good monopolist's equilibrium price is equal to the constant marginal cost, i.e., the profit is zero.

[20] The picture is drawn for $r = 1$ (or, alternatively, the figure depicts $r\Pi(\beta)$ for any value of r).

Claim 2. Under Assumption B, the aggregate profit falls as Δ decreases, and tends toward zero as Δ tends toward zero.

Proof. See the Appendix.

5 Comparison with the case of commitment

Suppose now that, contrary to what has been assumed in the preceding sections, the monopolist is able to commit, right from the beginning, to a sequence of pre-determined prices. This commitment capability is supposed to include the ability to commit both with regard to the price charged to new customers and to the prices charged the different groups of former customers, who are segmented according to the date of first purchase. Thus, at the initial time $t_0 = 0$, the monopolist announces (i) the old-customer prices $p(n, n+j)$ that it will offer at any future period $n+j$ $(j = 1, 2, 3...)$ to consumers who choose to make their first purchase of the good in period n and (ii) the new-customer prices $p(n, n)$ that it will offer at each period n to consumers who have not bought the good before. When solving for the optimal sequence of pre-determined contracts, the monopolist has no knowledge of any consumer. The following analysis does not rely on Assumption B; instead, we only need the weaker Assumptions A1-A2.

Let us consider consumers who are first-time buyers in period n. They face (a) a price $p(n, n) = p_n^N$, when buying for the first time in period n and (b) a price $p(n, n+j) = p_n^O$, $\forall j \geq 1$ for purchasing in all the remaining periods $n+j$ (the superscripts N and O stand for new customers and old customers, respectively).[21] These contract offers are announced from the outset, i.e., at time $t = 0$.

For heuristic reasons, we first consider the fictitious case when not only the monopolist but also the consumers commit right from the beginning, i.e., we suppose provisionally that when a customer purchases the good for the first time (say in period n), she not only commits to purchase the good at all points of time during period n (paying during this period the new-customer price p_n^N), but she also commits to purchase the good in subsequent periods (at the old-customer price p_n^O, that was initially agreed on). Technically, this means that we only need to consider the consumers' initial participation constraints: they are not allowed to renege later on their contractual commitments. (Afterwards,

[21] Allowing contract offers made to old customers to be (possibly) different across periods would not change the results.

we will show that when we remove this supposition of commitment by customers, the monopolist's optimal path and consumers' equilibrium choices are not affected).

Under this assumption of commitment, let $U_n(\theta_{n+1})$ denote the marginal customers' intertemporal net utility in period n (that results from all purchases in period n and in all subsequent periods):

$$U_n(\theta_{n+1}) = \frac{(1-\beta)(\theta_{n+1} - p_n^N)}{r} + \frac{\beta\left(\theta_{n+1} - p_n^O\right)}{r} = \frac{\theta_{n+1}}{r} - \left[\frac{(1-\beta)p_n^N}{r} + \frac{\beta p_n^O}{r}\right]. \quad (26)$$

Then, for all $\theta \in [\theta_n, \theta_{n+1}]$, we have $U_n(\theta) = \theta/r + \left[(1-\beta)p_n^N + \beta p_n^O\right]/r$. This implies that

$$U_n'(\theta) = \frac{1}{r} \text{ for } \theta \in [\theta_n, \theta_{n+1}]. \quad (27)$$

By a familiar argument, it follows from (27) that

$$U_n(\theta_{n+1}) = \frac{1}{r}\sum_{j=1}^{\infty} \beta^j \left(\theta_{n+j} - \theta_{n+j+1}\right). \quad (28)$$

Clearly, equation (28) is the same as equation (9).

The aggregate profit (discounted to the beginning of period n) which the monopolist makes from *vintage-n consumers* over their whole life time is then equal to:

$$\pi_n = [F(\theta_n) - F(\theta_{n+1})]\left[\frac{1-\beta}{r}p_n^N + \frac{\beta}{r}p_n^O\right] = [F(\theta_n) - F(\theta_{n+1})]\left[\frac{\theta_{n+1}}{r} - U_n(\theta_{n+1})\right].$$

The aggregate discounted profit is the sum of discounted profits from all vintages, $\Pi^C \equiv \sum_{n=0}^{\infty} \beta^n \pi_n$. Using (28), this can be rewritten as

$$\Pi^C \equiv \frac{(1-\beta)}{r}\sum_{n=0}^{\infty} \beta^n (1 - F(\theta_{n+1}))\theta_{n+1}. \quad (29)$$

Point-wise maximization with respect to θ_{n+1} leads to $\theta_{n+1} = \theta^{**}$, $\forall n \geq 0$, where θ^{**} is the solution of

$$(1 - F(\theta^{**})) - f(\theta^{**})\theta^{**} = 0. \quad (30)$$

Thanks to Assumptions A1 and A2 such a solution exists and is interior, i.e., $\theta^{**} \in \left(\underline{\theta}, \overline{\theta}\right)$.

This implies that in the initial period ($n = 0$), the monopolist sells only to those consumers with $\theta \in [\theta^{**}, \overline{\theta}]$, and in subsequent periods, $n = 1, 2, 3...$, only these old customers are served. In other words, $\theta_1 = \theta^{**}$ and the monopolist does not expand the market coverage after the initial period. This result, together with (28), implies that $U_0(\theta_1) = 0$, i.e., the marginal customers are left with no surplus. Now, from (26), it follows that the equilibrium prices p_n^N and p_n^O must satisfy the condition

$$\frac{1-\beta}{r} p_n^N + \frac{\beta}{r} p_n^O = \frac{\theta^{**}}{r}, \ \forall n \geq 0. \tag{31}$$

Up to now, we supposed that the customers cannot renege on their initial contractual commitments: when vintage n customers become "old customers" (in periods $n+j$, where $j \geq 1$) they are forced to buy at the contractual price p_n^O even if they would be better off not buying. Notice that equation (31) does not determine uniquely p_n^N and p_n^O: only their weighted sum is determined. It is easy to see that the monopolist can set any $p_n^O \leq \theta^{**}$ for all $n \geq 0$ to guarantee that all former customers always purchase the good, even when allowed to renege on initial agreements. Therefore, there is a continuum of commitment equilibrium prices, but they all correspond to the same extent of market coverage, the same consumers' intertemporal utilities and the same aggregate profit for the firm. At these equilibria, the consumers are offered the same weighted price for purchasing the good, independently of the period of their first purchase.

Proposition 2. Assume A1 and A2 hold. If the monopolist can commit right from the start to any sequence of contract offers, then only consumers whose type θ belongs to the interval $[\theta^{**}, \overline{\theta}]$ will be served, and they all make their first purchase in the initial period, $n = 0$. Moreover,

(i) in all periods, they are offered the same time-invariant weighted price $(\frac{1-\beta}{r} p_n^N + \frac{\beta}{r} p_n^O) = \frac{1}{r}\theta^{**}$, with $p_n^O \leq \theta^{**}$;

(ii) the firm does not use the information it has acquired on first-time customers.

The equilibrium described in Proposition 2 (which does not rely on Assumption B) is the infinite repetition of the static non-discriminatory monopoly equilibrium. The firm defines its optimal sequence of contracts at the beginning of the game, at a time when it only knows the distribution of types and, therefore, it is unable to attribute a type neither to any specific individual customer nor to any group of customers. The monopolist thus optimally commits to refrain from selling to a new set of consumers in subsequent periods, denying his future selves the "opportunity" to exploit, in subsequent periods, the information that he has acquired on his customers' preferences. Such a commitment has the effect of reducing the high type customers' informational rents, thus diminishing their incentives to delay their first purchase.

The monopolist's aggregate profit under commitment is then, from (29) and (30),

$$\Pi^C = \frac{(\theta^{**})^2 f(\theta^{**})}{r}. \tag{32}$$

In order to compare numerically the profit under full commitment and that obtained under non-commitment (the MPE profit), in what follows, we use Assumption B, which implies that eq. (32) reduces to $\Pi^C = \frac{1}{4r}$. Comparing with (25), we see that $\Pi^C > \Pi(\beta)$ if $0 < \beta \leq 1$, which is of course to be expected. Our comparison of aggregate profits with and without price commitment reveals our curse of knowledge result. The monopolist would be better off in the absence of records of the period in which each consumer enters the market. Indeed, if the monopolist were unable to collect data about consumers, he would neither segment the market (according to the time of consumers' first purchase) nor price discriminate among different cohorts of consumers, sticking to a uniform pricing strategy instead.

Obviously the optimal strategy under commitment is not time-consistent. In other words, if at some future period $m > 0$ the firm is released from its commitment, it will have an incentive to deviate from its commitment. There are two reasons for this. First, if allowed to reoptimize, the monopolist would always benefit from using his data on the dates of old consumers' first purchase to carry out third-order price discrimination among them. Second, when allowed in period $m > 0$ to make new offers to potential new customers, he would benefit from selling to at least some of them in order to identify them (and make profits from them in all subsequent periods). Rational customers who expect to benefit from better offers in the future would then delay their purchase to the next period unless they are granted lower present prices (higher rents).[22]

[22] A monopolist that can commit is in effect an "open-loop Stackelberg leader," and it is well-known that open-loop Stackelberg leaders are typically beset by the time-inconsistency problem. For more detailed discussion, see Chapter 5 of Dockner et al. (2000), and Chapter 1 of Long (2010).

6 Welfare under MPE

In this section, our objective is two-fold: (i) to investigate whether the welfare in the MPE increases or decreases when the period of commitment becomes longer; and (ii) to compare the welfare outcome under the MPE with that under the commitment equilibrium. For the sake of simplicity, in this section, we rely on Assumption B.

Let $w(n)$ denote the social welfare at the MPE in period n. We define social welfare as the sum of consumer surplus and profit. Consumer surplus is gross utility minus the payment made to the firm. Profit is the sum of consumers' payments to the firm (because we assume that the cost of production is zero). Therefore welfare in any period is simply the gross utility of consumers who are served in that period. Thus the social welfare that accrues in period n is

$$w(n) = \frac{1-\beta}{2r}\left[1-\theta_{n+1}^2\right] = \frac{1-\beta}{2r}(1-(\gamma^{n+1})^2). \tag{33}$$

Notice that, as can be seen from eq. (33), for any given r, a shortening of the period of commitment Δ leads to a decrease in both the terms $(1-\beta)$ and $(1-(\gamma^{n+1})^2)$. Thus a decrease in Δ unambiguously reduces the social welfare that accrues in period n. This is because of two factors: (a) the length of the period itself decreases, and (b) the measure of customers who are not served in period n, γ^{n+1}, becomes larger.

The overall aggregate social welfare is the sum of the present values of $w(n)$, over all n:

$$W(\beta) = \sum_{n=0}^{\infty} \beta^n w(n) = \frac{1}{2r}\left(\frac{1-\gamma(\beta)^2}{1-\beta(\gamma(\beta))^2}\right), \tag{34}$$

where $\gamma = \gamma(\beta)$ is the solution of the Euler equation (22). Even though $w(n)$ unambiguously decreases as β increases, the effect of a marginal increase in β on $\beta^n w(n)$ turns out to be ambiguous.

A close look at the expression for aggregate social welfare, eq. (34), shows that for a given value of r, an increase in β (i.e., a decrease in Δ) has two opposite effects on aggregate social welfare: a direct effect (i.e., keeping γ constant) which is positive and an indirect effect, through the induced increase of γ, which is negative. In this respect, it is worth noting that the welfare per unit of time, $w(n)$, is a step-wise increasing function, the length of each step being equal to Δ. Accordingly, for a *given* market expansion rule (i.e., for a given value of $\gamma > 0$, or a given height $(1 - (\gamma^{n+1})^2)$ of the steps), a decrease of Δ (i.e., shorter steps) leads to a greater aggregate intertemporal welfare. This is because over any interval $[0, t]$, there are more steps when Δ decreases.

Working in the opposite direction, the indirect effect takes place because a greater β implies a slower market expansion from period to period (i.e., $\gamma(\beta)$ becomes greater, and thus the height $(1 - (\gamma^{n+1})^2)$ of the steps falls, when Δ shrinks). Does one effect dominates the other? Yes, in a global sense, the direct effect dominates. One can indeed show that $\lim_{\beta \to 1} W(\beta) = 1/(2r)$, which is greater than $W(0) = \frac{3}{8r}$, so that W is globally an increasing function of β (a decreasing function of Δ).[23] Despite being globally increasing in β over the interval $0 < \beta \leq 1$, it is worth noting that, as shown in Figure 5, W is not a monotonic function of β within this sub-domain: starting from $\beta = 0$, aggregate social welfare is at first (slightly) decreasing as β increases marginally, before beginning to increase with β (after β reaches approximately 0.20).

Insert Figure 5

When the length of time Δ is nearly infinite (i.e., when β is sufficiently close to 0), the negative effect of a marginal reduction of Δ on welfare (due to a slower market expansion) dominates the positive one. To show this analytically, let us compute $dW/d\beta$, and evaluate it at $\beta = 0$:

$$\frac{dW}{d\beta} = \frac{\partial W}{\partial \beta} + \frac{\partial W}{\partial \gamma}\frac{d\gamma}{d\beta} = \frac{1}{2r}\left[(1-\gamma^2)\gamma^2 - 2\gamma\,(1-\beta)\,\frac{d\gamma}{d\beta}\right]$$

where

$$\frac{d\gamma}{d\beta} = -\frac{E_\beta}{E_\gamma} = -\frac{-2\beta + (4\beta + 3\beta^2)\gamma + (2 - 6\beta - 6\beta^2)\gamma^2 + (4\beta + 3\beta^2)\gamma^3 - 2\beta\gamma^4}{(-2 + 2\beta^2 + \beta^3) + 2(2\beta - 3\beta^2 - 2\beta^3)\gamma + 3(2\beta^2 + \beta^3)\gamma^2 - 4\beta^2\gamma^3}.$$

We know that at $\beta = 0$, γ takes the value $1/2$. Hence, the sign of $\frac{dW}{d\beta}$ at $(\beta, \gamma) = (0, 1/2)$ is given by $-0.062\,5 < 0$. This shows that, starting at $\beta = 0$, a marginal increase in β indeed leads to a fall in welfare, consistent with Figure 5.

[23] In evaluating $\lim_{\beta \to 1} W(\beta)$, we make use of L'Hospital's rule. The limit is ratio between the first order derivatives of the numerator and the denominator of (34). The proof is completed by appealing to the fact that $\gamma'(1) = +\infty$.

These results provide useful insights for the welfare comparison between the full commitment and the non-commitment scenario. Recall that in the non-commitment case (MPE), for $\beta = 0$, we have $W(\beta) = \frac{3}{8r}$. As described above, when β is sufficiently close to zero, welfare decreases in β, and afterwards it starts to increase, with the highest welfare occurring when β tends to 1, with $\lim_{\beta \to 1} W(\beta) = \frac{1}{2r} > \frac{3}{8r}$. In the full commitment scenario, the firm adopts the uniform pricing policy and the welfare is simply equal to

$$W^{commit}(\beta) = \sum_{n=0}^{\infty} \beta^n w^{commit}(n),$$

with $w^{commit}(n) = \int_0^{\Delta} e^{-rt} \left[\int_{\frac{1}{2}}^{1} \theta d\theta\right] dt$. Therefore $W^{commit}(\beta) = \frac{3}{8r} = W(0)$. In other words, the welfare outcome in the commitment scenario coincides with that under MPE when β tends to zero, $W(0) = \frac{3}{8r}$. This was to be expected, since as β tends to 0, the commitment period Δ tends to infinity and therefore in that limiting case the MPE without commitment is isomorphic to the commitment scenario. Accordingly, since $W(1) > W(0)$, we can conclude that provided β is sufficiently close to 1 (i.e., the commitment period is sufficiently short), the equilibrium welfare under MPE will be greater than the equilibrium welfare with commitment, because the monopolist's market power is eroded.

7 Comparison with the polar case of full information acquisition

In the previous sections, although the monopolist can collect information about consumers' purchase history, he cannot perfectly identify consumers' type. The information is coarse: consumers are classified in terms of vintage, but not in terms of their exact preferences. When only such coarse information is available, the monopolist's future selves will have an incentive to segment the markets and practice third degree price discrimination, offering several prices, one for each vintage of former consumers, and an introductory price to customers who have not purchased before. Consumers anticipate this, and therefore they demand a higher informational rent to purchase earlier rather than later. Under these conditions, we are able to demonstrate the curse of knowledge, and show that Coasian dynamics obtain in the sense that, as the commitment interval shrinks to zero, the monopoly power vanishes.

However, casual observations suggest that some firms exert great efforts to gather consumer information. This apparently suggests they would profit from such information. It is then worth investigating conditions under which the collection of information benefits the monopolist. In this section, we show that if information is sufficiently fine to let the monopolist identify precisely each consumer's WTP, the curse of knowledge will no longer hold.

To this end, we distinguish two polar cases of information quality and we then compare the MPE of the resulting games. In both cases, the monopolist is able to collect some information about the customers in the first period in which they purchase the good. In the first polar case, the monopolist is only able to recognize a previous customer and to remember in which period she first purchased the good (we call this the Purchase History Information case, PHI for short, which is the scenario treated in sections 1 to 6 of this paper). The second polar case corresponds to the Full Information Case (FIA for short), in which at the end of each commitment period, the firm becomes fully informed about the exact WTP of each customer who has purchased during the period. Accordingly, in the FIA case, the monopolist is capable of charging each returning old customer a personalized price equal to her WTP, i.e., he exercises first-degree price discrimination. Differently, in the PHI case, the monopolist only recognizes the customers' vintage and therefore he can only exercise third-degree price discrimination.

What happens then in the FIA case? Using the same framework and equilibrium concepts as in the main part of this model, we can briefly characterize the resulting MPE of the FIA model in the linear-quadratic case. The main changes are the following.

First, in each period n, in the FIA case, the firm is able to extract all the surplus from old customers when they return to make their purchase, i.e., the monopolist can practice first-degree price discrimination among former customers. The profits made in period n from all former customers are

$$\Pi_n^F = \frac{1-\beta}{r} \int_{\theta_n}^{1} \theta d\theta = \frac{1-\beta}{2r}(1-\theta_n^2) \tag{35}$$

as compared with its counterpart in the PHI case, equation (3).

Second, instead of equation (8), the arbitrage equation becomes

$$U_n(\theta_{n+1}) = \beta \left(U_{n+1}(\theta_{n+2}) + \frac{1-\beta}{r}(\theta_{n+1} - \theta_{n+2}) \right). \tag{36}$$

Applying then the same arguments as in Section 4 but using these different functions yields the following results:

Proposition 3. Suppose that Assumption B is satisfied. The Markov Perfect Equilibrium under full information acquisition is such that (i) the monopolist's cut-off rule is linear, with $\gamma^*_{FIA}(\beta)$ being the unique real solution in $(0,1)$ of the following fourth-degree polynomial in γ_{FIA}

$$1 - (2+\beta)\gamma_{FIA} + (2\beta - \beta^2)\gamma^2_{FIA} + 2\beta^2\gamma^3_{FIA} - \beta^2\gamma^4_{FIA} = 0, \text{ with } 0 \leq \beta < 1, \tag{37}$$

and (ii) the consumer expectations rule is linear, with $\lambda^*(\beta)$ given by

$$\lambda^*_{FIA}(\beta) = \frac{1}{r}\frac{\beta(1-\beta)(\gamma^*_{FIA}(\beta) - \gamma^*_{FIA}(\beta)^2)}{1 - \beta\gamma^*_{FIA}(\beta)},$$

The resulting aggregate discounted profit is given by

$$\Pi_{FIA} = \frac{1}{r}\frac{(1-\gamma_{FIA})(2\gamma_{FIA} + \beta(1 - (3 - \beta(1-\gamma_{FIA}))\gamma_{FIA}))}{2(1-\beta\gamma_{FIA})(1-\beta\gamma^2_{FIA})},$$

where $\gamma_{FIA} = \gamma^*_{FIA}(\beta)$.

Proof. The arguments are along the lines used in the derivation of the MPE in the PHI case. The full proof is available upon request. ■

Π_{FIA} is an increasing function of β (i.e., a decreasing function of Δ) as shown in Figure 6.[24] The profit is lowest when $\beta = 0$, with $\Pi_{FIA} = \frac{1}{4r}$; whereas the highest profit obtains for $\beta = 1$, with $\Pi_{FIA} = \frac{1}{2r}$.

Insert Figure 6

We have that $\gamma^*_{FIA}(\beta)$ tends to $\frac{1}{2}$, when β tends to zero. Moreover $\gamma^*_{FIA}(\beta)$ is decreasing in β, which means it is always smaller than $\frac{1}{2}$, tending toward ≈ 0.445042 as β tends toward 1. The equilibrium $\lambda^*_{FIA}(\beta)$ is pictured below in Figure 7, evolving non-monotonically

[24] This picture is drawn for $r = 1$.

with β.

Insert Figure 7

Comparing the PHI case with the FIA case, we find that the two corresponding Markov Perfect Equilibria are strikingly different, with very contrasting implications for the monopolist's profits. Recalling that the equilibrium profits in the commitment scenario were $\Pi^C = \frac{1}{4r}$, Proposition 3 above shows that full information acquisition is actually beneficial to the firm: it always makes more profit compared to the case where it cannot recognize its old customers**.** In contrast, collecting and using coarsest information on customers' purchase history hurts the firm: its profits are smaller than under no information acquisition. If the monopolist cannot prevent his future selves from exploiting the information to create additional market segments and rational consumers correctly anticipate the firm's future prices, the ability to engage in third-degree price discrimination hurts the monopolist, which would not be the case if the information was fine enough to implement first-degree price discrimination, as shown in our FIA extension of the model. Our results suggest that if the degree of coarseness of information can be represented by a continuous variable, say $\mu \in [0, 1]$ then one should be able to prove that profit is non-monotonic in μ. For instance one could suppose that after a consumer has first purchased the good in period n, then, with probability $1 - \mu$, the monopolist is able to discover her exact WTP, whereas with probability μ he simply learns that $\theta \in [\theta_{n+1}, \theta_n]$. The parameter μ is then a measure of the degree of coarseness of the monopolist's information.

Finally, it is interesting to note that under PHI, the monopolist's aggregate profit decreases when the interval of time Δ needed for changing the price offered to new customers itself decreases, consistent with standard Coasian dynamics. In contrast, under FIA, the monopolist's profits are greater the smaller is Δ, departing from Coasian dynamics.

8 Conclusion

We have shown that the acquisition of purchase history information about customers to implement third degree price discrimination based on customer recognition can hurt a monopolist. Accordingly, the monopolist would be better off by committing to a policy of not keeping any information about customers, so as to prevent his future selves from engaging in opportunistic behavior. In other words, it may pay to tie one's own hands.

We have illustrated this general point by formulating and analyzing a dynamic extension of the static monopoly model, such that the monopolist may segment the set of consumers according to the date of their first purchase. We first looked at the equilibrium outcomes when the monopolist can only collect coarse information, i.e., only customers' first date of purchase. Then, we compared such outcomes with the ones arising in the polar case in which the monopolist is able to get full information on the consumers' exact WTP after they have made their first purchase.

Starting with the coarsest information case, we characterized the equilibrium under two alternative scenarios: the non-commitment scenario (MPE) and the full commitment scenario. In the former scenario, we found that the lack of commitment on the part of the monopolist makes customers demand more informational rents, and this is detrimental to the firm. In addition, we found that the Markov-Perfect Equilibrium of the game exhibits Coasian dynamics: when the length of time between two different price offers shrinks to zero, the market is covered instantaneously and the monopolist's profit vanishes. By contrast, if the monopolist could credibly commit to any sequence of contract offers, he would commit to a policy of non-discrimination between new and old customers and to stick forever to his initial contract offer.

Our analysis demonstrated that in the absence of commitment, the acquisition of knowledge can be harmful to a monopolist. This is what we call the curse of knowledge. This suggests that in some monopoly markets, firms may actually benefit from public policies designed to limit firms' ability to collect and keep data records about their customers. Moreover, comparing the theoretical results of the coarsest information case to those obtained in the polar case of full information acquisition, we found that in the latter case the curse of knowledge does not arise. Instead, in this polar case firms are able to engage in price personalization upon recognizing their returning customers, thus achieving more profit (higher than the one obtained under full commitment to refrain from coarse knowledge acquisition, which, in turn, is higher than the MPE profit under the coarse knowledge scenario). These results suggest that a firm's profit can be non-monotonic in any continuous measure of its ability to gather customers' information.

Our theoretical results are broadly consistent with pricing strategies in many markets, such as telecommunication, software, electricity, TV and home video, etc., as reported in Section 1. Moreover, as the subscription based economy starts to boom, the sophisticated price discrimination strategies considered in our model may apply to a wider range of sectors. Our comparison of the coarse information case and the full information case allows us to explain two very distinct trends arising in these markets: in some markets (e.g. online flights/ hotel booking platforms), firms are exerting great efforts to gather consumer information and using it to implement increasingly sophisticated price strategies; while in some other markets (e.g., iTunes, online videos), firms stick to rather simple pricing strategies. Both types of strategies may indeed make sense, since we showed that the profitability of engaging in price discrimination critically depends on the accuracy of the firm's information on customers tastes.

Finally, let us note that in our simple setting, a consumer has only one choice: to buy now, or to buy later. In practice, consumers may choose other actions, such as frequency to visit a seller's website, quantity and frequency of purchases, etc. If the monopolist can collect customer-specific information on these actions, he will be able to have a more refined segmentation of the customer base. These cases lie in between the two polar cases, FIA and PHI, that we analyzed. A natural extension of the present analysis would be to allow the degree of coarseness of information to take any intermediate value between the two cases. We conjecture that the curse of knowledge result would hold if the monopolist's acquired information is sufficiently coarse. Coasian-like results would fail if acquired information is fine enough.

APPENDIX

Proof of Proposition 1

Equation (22) follows directly from (19). It defines the equilibrium value of $\gamma^*(\beta)$ as the solution of a fourth order polynomial. As shown in Figure 1, (22) has one positive solution for all $\beta > 0$. It has also a negative one but only the former can be a valid solution. Accordingly $\gamma^*(\beta)$ is strictly increasing in β, from $\gamma = \frac{1}{2}$ when $\beta = 0$ to $\gamma = 1$ when $\beta = 1$. ■

Proof of Claim 1

Since $\gamma(1) = 1$, $\lim\limits_{\Delta \to 0} \left(\gamma(e^{-r\Delta})\right)^{\frac{t}{\Delta}+1} = \lim\limits_{\Delta \to 0} \left(\gamma(e^{-r\Delta})\right)^{\frac{t}{\Delta}}$. Notice that $\ln\left(\gamma(e^{-r\Delta})\right)^{\frac{t}{\Delta}} = \frac{t}{\Delta}\ln\left(\gamma(e^{-r\Delta})\right)$. Since $\gamma(1) = 1$, $\frac{t}{\Delta}\ln\left(\gamma(e^{-r\Delta})\right)$ is undefined when $\Delta = 0$. Then, in order to determine the limit value of $\frac{\ln\left(\gamma(e^{-r\Delta})\right)}{\Delta}$ when $\Delta \to 0$, we have to use L'Hospital's rule and evaluate the ratio of the derivatives of the numerator and the denominator at $\Delta = 0$. This ratio turns out to be equal to $-r\gamma'(1)$. It follows that $\lim\limits_{\Delta \to 0} \left(\gamma(e^{-r\Delta})\right)^{\frac{t}{\Delta}} = e^{-r\gamma'(1)t}$.

Now let us determine $\gamma'(1)$. Differentiating the identity (22), we obtain $E'_\beta(\beta, \gamma(\beta)) + E'_\gamma(\beta, \gamma(\beta))\gamma'(\beta) = 0$. Remembering that $\gamma(1) = 1$, it turns out that $E'_\beta(1, \gamma(1)) = E'_\gamma(1, \gamma(1)) = 0$ so that the ratio $-\frac{E'_\beta(1,\gamma(1))}{E'_\gamma(1,\gamma(1))}$ is undetermined. Again, we need to use L'Hospital's rule to find the value of $\gamma'(1)$. Then, we have to differentiate the numerator and the denominator to finally obtain $\gamma'(\beta) \to +\infty$ as $\beta \to 1$. It follows that $\lim\limits_{\Delta \to 0} \left(\gamma(e^{-r\Delta})\right)^{\frac{t}{\Delta}} = 0$.■

Proof of Claim 2

According to equation (25), the value of $\Pi(\beta)$ is undetermined when $\beta = 1$ since both the numerator and the denominator equal zero. Let us then apply L'Hospital's rule. The ratio of the first order derivatives of the numerator and the denominator is again undetermined. So we consider the ratio of the second order derivatives. It is equal to

$$\frac{\gamma'(1)}{(1 + 2\gamma'(1))(1 + \gamma'(1))}.$$

Using the fact that $\gamma'(1) = +\infty$, we obtain $\Pi(1) = 0$.■

Derivation of equation (29)

Using the expression for π_n, we obtain

$$\Pi^C = \beta^0 \left[F\left(\overline{\theta}\right) - F(\theta_1)\right]\left[\frac{\theta_1}{r} - U_0(\theta_1)\right] + \beta^1\left[F(\theta_1) - F(\theta_2)\right]\left[\frac{\theta_2}{r} - U_1(\theta_2)\right] + ...etc$$

Substituting for $\left[\frac{\theta_1}{r} - U_0(\theta_1)\right]$ and $\left[\frac{\theta_2}{r} - U_1(\theta_2)\right]$ etc., we obtain

$$\left[\frac{\theta_1}{r} - U_0(\theta_1)\right] = \frac{\theta_1}{r} - \beta\left(\frac{\theta_1 - \theta_2}{r}\right) - \beta^2\left(\frac{\theta_2 - \theta_3}{r}\right) + ...$$

$$\left[\frac{\theta_2}{r} - U_1(\theta_2)\right] = \frac{\theta_2}{r} - \beta\left(\frac{\theta_2 - \theta_3}{r}\right) - \beta^2\left(\frac{\theta_3 - \theta_4}{r}\right) + ..$$

Cancelling out terms, we finally get

$$\begin{aligned}
\Pi^C &= \beta^0 F\left(\overline{\theta}\right)\left(\frac{1-\beta}{r}\theta_1\right) + \beta F\left(\overline{\theta}\right)\frac{(1-\beta)}{r}\theta_2 + \beta^2 F\left(\overline{\theta}\right)\frac{(1-\beta)}{r}\theta_3 \text{ etc.} \\
&\quad - F(\theta_1)\left(\frac{1-\beta}{r}\theta_1\right) - \beta F(\theta_2)\left(\frac{1-\beta}{r}\right)\theta_2 - \beta^3 F(\theta_2)\left(\frac{1-\beta}{r}\right)\theta_3 \text{ etc.}
\end{aligned}$$

This completes the proof. ■

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FIGURES

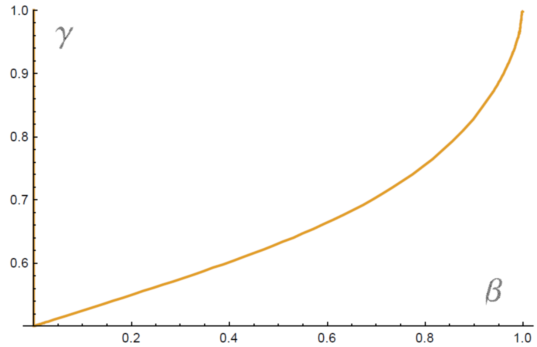

Figure 1. Equilibrium cut-off rule $\gamma^*(\beta)$

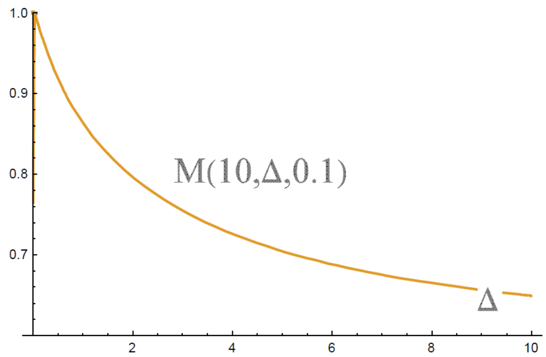

Figure 2. Market coverage as a function of Δ

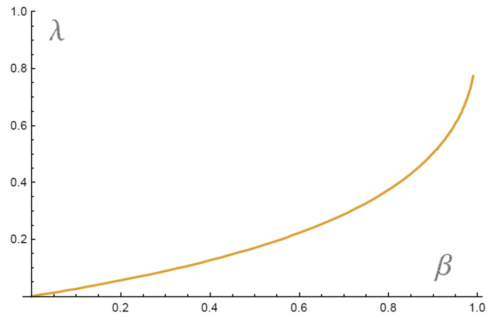

Figure 3. Equilibrium expectations rule $\lambda(\beta)$

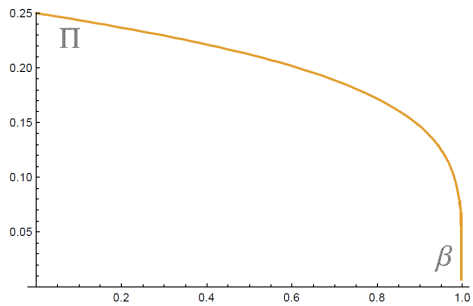

Figure 4. Equilibrium aggregate profit $\Pi(\beta)$

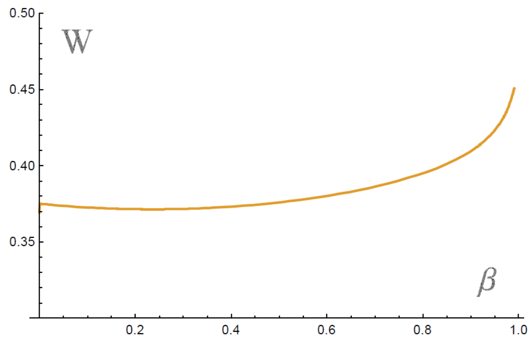

Figure 5. Equilibrium aggregate social welfare, $W(\beta)$.

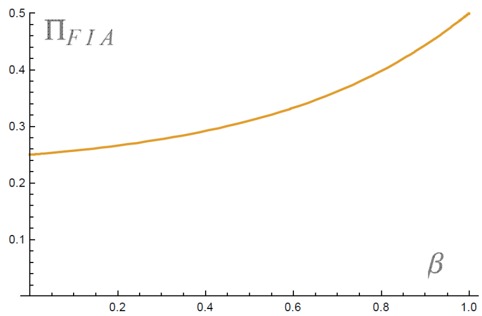

Figure 6. Π_{FIA} as a function of β $(r = 1)$

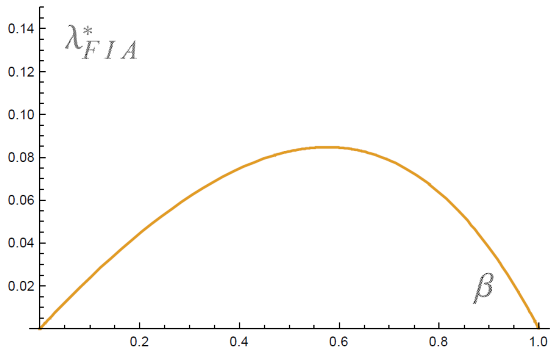

Figure 7. λ^*_{FIA} as a function of $\beta(r = 1)$