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Quality and Price Personalization under Customer Recognition: a Dynamic Monopoly Model with Contrasting Equilibria

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Abstract

We present a model of market hyper-segmentation, where a monopolist acquires within a short time all information about the preferences of consumers who purchase its vertically differentiated products. The firm offers a new price/quality schedule after each commitment period. Lower consumer types may have an incentive to delay their purchases until next period to obtain a better introductory offer. The monopolist counters this incentive by offering higher informational rents. Considering the dynamic game played by the monopolist and its customers, we find that there is always a Markov perfect equilibrium (MPE) in which the firm immediately sells the good to all customers, offering the Mussa-Rosen static equilibrium schedule to first time customers (and getting full commitment profits). However, if the commitment period between two offers is long enough, there is another MPE with gradual market expansion. Contrary to the Coasian result for a durable-good monopoly, we find that in both equilibria the profit of the monopolist increases (and the aggregate consumers surplus decreases) as the interval of commitment shrinks.

The model yields policy implications for regulations on collection and storage of customers information.

JEL codes: L12; L15.

Key words: monopoly; product quality; customer information; intertemporal price discrimination.

1   Introduction

In recent years, the rapid growth of new-generation digital technologies has been reshaping firms' business models in many economic sectors, such as software, retail, energy, automobiles, fashion and apparels, media and entertainment, and so on. On the one hand, these technologies allow firms to collect and treat very large volumes of (big) data about their customers, which enables them to get very accurate information on consumers' true willingness to pay for their products. On the other hand, new-generation digital technologies (including machine learning, robotics, digital platforms, 3D printing, etc) also enable firms to engage in innovative personalization strategies, through which they tailor their product/price offers to suit consumers' individual preferences and needs, which the firms uncover within a short time.

The ability to process unprecedented amounts of real time data about their customers indeed permits firms achieve what is now known as hyper-segmentation of markets, increasing the scope of price and quality discrimination.[1] As a result, in recent years, many firms are committing themselves to deliver personalized product-price offers to their customers at scale.[2] Among a plethora of examples of this new trend, one can cite offers of personalized clothing items (e.g. Nike, Adidas, Longchamp, Gucci, Louis Vuitton and other luxury brands), the customizable Tesla dashboard, personalized TV scheduling (through on-demand video systems), personalized tourism experiences, customizable executive education programmes, and so on...

In this paper, we investigate the market dynamics that arise when a monopoly firm is able to: (i) collect Big Data about its customers (after their first purchase) and (ii) engage in hyper-segmentation strategies, making personalized quality-price offers to its returning customers. More precisely, we propose a dynamic extension of the static model of Mussa-Rosen (1978) on monopoly and product quality, extending the seminal Mussa-

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[1]As Zinsmeister (2016) puts it, *"both prioritization and personalization are key when it comes to your hyper-segmentation strategy. Rather than creating a few rigid personas or a large list based on broad firmographic characteristics, hyper-segmentation allows you to use all of your customer data to pinpoint specific marketing problems you can solve for smaller customer groups, aka a 'segment of one.'"* https://martechtoday.com/achieving-hyper-segmentation-reach-personalization-scale-165097 [Access date: 10/02/2019]

[2]As reported in the New York Times (2018), *"For some brands, a future of made-to-order everything is on its way to becoming a reality. Nike, for instance, is experimenting with new customization and personalization concepts in its stores, where consumers can help to create products, said John R. Hoke III, Nike's chief design officer."* https://www.nytimes.com/2018/11/19/fashion/luxury-retail-personalization.html [Access date: 10/02/2019]

Rosen model in order to account for the consumers' repeated purchases of a non-durable good for instantaneous consumption and the monopolist's gradual information collection on the preferences of its set of heterogeneous customers. We assume that the monopolist cannot modify the price-quality schedules offered to its customers during an exogenous period of finite length, called the commitment period (which corresponds to the usual contractual commitment period in dynamic models investigating the Coasian conjecture). While in our formal model, for expositional reasons, the length of the commitment period is treated as a parameter, as we shall show, the monopolist has an incentive to make this length as close as possible to an irreducible minimum consistent with the state of information technology and the legal requirements that limit firms ability to collect and store consumers information. This result allows us to discuss the policy implications of our model.[3]

At the beginning of each commitment period, the monopolist proposes to new customers a period-specific price-quality schedule intended to induce them to reveal their true type. Once a consumer has made her first purchase, the firm is able to collect "Big Data" to uncover her exact type, so that any possible misreporting (of consumers' willingness to pay for the good) has no consequence beyond the period of the first purchase.[4]

In other words, quality-price menus and data collection constitute complementary tools to implement personalization strategies within our model: the firm relies on quality-price menus to screen new customers, and as soon as they have made their first purchase, it exploits Big Data to uncover customers' true willingness to pay. This allows the monopolist to offer to each returning customer the personalized quality-price deal that maximizes her potential surplus (which is fully extracted by the firm).

Thanks to accurate data collection (after the first purchase), former customers will be unable to misreport their type in subsequent periods.[5] This means that consumers can cheat and misreport their types only in the period in which they purchase the good for the

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[3]See subsection 4.3.2 for details.

[4]For example, in the context of digital markets, such information could be collected through cookies and other tracking devices that allow firms to recover the consumers' digital footprint.

[5]By complementing price-quality menus with Big Data, the firm does not need to account for the old customers' incentive compatibility constraints in subsequent periods since their true preferences have become known after their first purchase. (If this were not the case, our dynamic game would be much more complex since we would have to account for the incentive compatibility constraints of former customers). Of course, in other contexts, data collection and the provision of personalized quality-price menus can be regarded as substitutes, given that both of them can be useful intruments that help the firm identify consumer preferences.

first time. As a result, only new consumers (who are buying the good for the first time) may earn information rents. Hence, the model combines second-degree discrimination with respect to new customers (whose preferences are unknown to the monopolist) with first-degree discrimination on former customers (whose preferences have become fully known to the monopolist after their first purchase). Consequently, the monopolist only needs to distort downward the quality offered to the successive rungs of the customers who buy the good for the first time.[6]

Our main focus is on the case where the monopolist is unable to commit to future sales decisions. Under this scenario, we characterize the Markov Perfect Equilibria of the dynamic game played by the firm and its customer base (which consists of a continuum of consumer types). The firm makes decisions on (a) the quality-price schedules to be offered to new customers, and (b) personalized quality-price offers to old customers. Knowing that the firm will be able to track their true willingness to pay immediately after their first purchase, the targeted new consumers must decide on whether to make their first purchase now, or to delay it until the next period.

In a Markov Perfect Equilibrium, the monopolist quality/price strategy maximizes its discounted lifetime profit, given consumers' expectations; and, given the monopolist's sales strategy, consumers' expectations are rational. Our first result for this model is that there exists a Markov-Perfect Equilibrium (which we will refer to as Equilibrium A from now on) in which the monopolist simply offers the static Mussa-Rosen equilibrium schedule in the initial period and all customers purchase the good for the first time in that period[7]. Afterwards, each consumer is offered a take-it-or-leave-it personalized price/quality deal which fully extracts her surplus (due to the firm's data collection ability). At this Equilibrium A, the monopolist obtains the Mussa-Rosen profits in the initial period, whereas it gets the maximum profit of a first-degree (perfectly discriminating) monopolist in all subsequent periods. This explains why the monopolist chooses to serve all the consumers in one go, offering the lowest quality at zero price to a subset of (bunched) consumers: by selling them the good immediately in the first period, the monopolist is able to collect data on their tastes, subsequently engaging in first-degree price discrimination (and

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[6]If we were to assume the absence of tracking devices and big data collection, misreporting would impact the options offered to old consumers as well. In that case, they would also be offered distorted quality levels in subsequent periods.

[7]In the Mussa-Rosen static model, a subset of customers are pooled together and offered a good at zero quality at the price $p = 0$. These customers are deemed to purchase the lowest quality good (zero quality).

collecting the corresponding profits).[8]

Our second result is that there is generically another Markov-Perfect Equilibrium (which, from now on, we will refer to as Equilibrium B) with completely different properties. In Equilibrium B, successive rungs of customers purchase the good for the first time in consecutive periods, meaning that the monopolist is expanding the market gradually (in small steps) from high-type to low-type consumers. While it remains true that once a consumer has bought the good for the first time, she will be offered the personalized first-degree price discriminatory deal in subsequent periods, in Equilibrium B, some customer types end up delaying their first-time purchase, which means that the monopolist must wait to discover their exact preferences (which can only be uncovered after consumers' first-time purchase period, which is up to their rational decision).

The intuition behind the multiplicity of MPEs in this model is similar to the one underlying self-fulfilling expectations equilibria. When all customers expect that all of them or almost everyone (except possibly a set of measure zero) will buy in the initial period, no low-type customer can hope to gain anything from unilaterally deviating by delaying her first purchase: she anticipates that there will be no new offers to first-time buyers in any period in the future. Consequently, low-type customers do not require any informational rent to induce them to buy in the first period. Facing such expectations, the monopolist covers immediately the whole market. Thus, customers' expectations are fulfilled.

In contrast, when consumers expect that the firm will not be able to sell the good in the initial period to a subset of consumers with a positive measure, they contemplate the possibility of gaining by delaying their first purchase in order to take advantage of the firm's future attractive offers to recruit new customers. Therefore, in any given period, some set of consumers could be persuaded to become first-time customers only if they are offered strictly positive informational rents, and this applies even to the marginal customer of that subset. In turn, the consumers' aspiration to get more informational rents makes the firm unwilling to sell immediately to all new customers. Thus, in Equilibrium B, consumers expectations are also fulfilled.[9]

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[8]To some extent the characteristics of this equilibrium mimic the so-called "freemium business model" in which low-quality variants of the good are offered, free of charges, and at the same time positively-priced, high-quality versions of the good are available.

[9]Notice that this argument does not work when the commitment period between two offers is too short since, in that case, the monopolist's cost of offering marginal customers' rents over such a short

Interestingly, the model exhibits non-Coasian dynamics. We find that as the length of the commitment period shrinks, for each of the two equilibria, the firm's aggregate profit increases, contrary to the well-known "vanishing profit" result obtained in the standard Coasian model of durable good monopoly. The intuition behind our result is as follows. The monopolist's discounted stream of profits is higher, the sooner it has sufficient information and ability to practice first-degree discrimination over consumers. An almost immediate consequence of this result is that the discounted stream of aggregate consumers' surplus falls when the monopolist's length of commitment period decreases. Moreover we show that, when the length of the commitment period falls below a threshold value, Equilibrium B (which is less favorable to the firm and more favorable to customers relative to Equilibrium A) disappears, and only Equilibrium A remains. Accordingly, our analysis suggests that firms and customers' interests are diametrically opposed regarding public policies aiming at reducing firms' ability to collect information on consumers (e.g., the EU's General Data Protection Regulation (GDPR) policy or the California Consumer Privacy Act of 2018[10]). Such policies tend to increase the length of time needed to completely identify consumers' tastes, thus resulting in greater consumer surplus and lower monopoly profit. We shall argue that our model may provide some hints as to why, relative to the USA, the European Union is more inclined to institute and enforce restrictive data protection laws which tend to hurt firms and benefit consumers.

2 Related literature

In recent years, the analysis of firms' (price) personalization based on consumer recognition has attracted a great deal of attention among scholars and professionals in the fields of economics and management. In particular, the recent literature on behavior- based price discrimination has investigated, using mostly two-period models, the implications

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period is not sufficiently important to outweight the monopolist's prospective gain from obtaining perfect information on all customers for immediate first-degree price discrimination.

[10]Several states (in cluding Alabama, Arizona, California, Colorado, Iowa, Louisana, Nebraska, Nevada, Oregon, South Carolina, South Dakota, Vermont, Virginia) have recently passed privacy laws. See Serrato, Cwalina, Rudawski, Coughlin and Fardelmann (2018) for details. https://www.dataprotectionreport.com/2018/07/u-s-states-pass-data-protection-laws-on-the-heels-of-the-gdpr/.

of endogenous acquisition of information on consumers using their purchase history.[11] (For a survey see Fudenberg and Villas-Boas (2006)). For example, the pioneering work by Fudenberg and Tirole (2000) has focused on the simple case of a two-period duopoly model with horizontal product differentiation, where in the second period, firms are able to engage in (third degree) price discrimination between old and new customers (depending on whether these consumers belong to the firm's own turf or to the rival's one). More recent papers (see for instance Choe et al., 2017, or Laussel and Resende, 2019) have gone further by assuming that firms' data collection capabilities (e.g. cookies and other tracking devices) allow them to identify the consumers' exact preferences (i.e., firms know the each returning customer's exact location on the Hotelling line).

In these models, the possibility of price personalization ends up hurting firms' profit because it intensifies duopolistic competition. In contrast, when targetability is imperfect (which amounts to say that first degree price discrimination is not possible), the literature has pointed out that firms may actually benefit from price discrimination based on customer recognition. In such cases, Chen et al. (2001, p. 25) show that individual marketing with imperfect targetability can lead to a win-win outcome if firms exchange with each other information about their customers.[12]

The present paper also focuses on how firms' price (and product specification) strategies may be reshaped by the possibility to implement price and product personalization strategies. We depart from previous literature by looking at price discrimination through customer recognition in a fully dynamic set-up in which a monopoly firm makes hyper-segmented quality specification offers to its customers.

In building our model, we draw from two distinct lines of literature.[13] The first stream addresses the issue of the losses that arise from an agent's inability to commit (which may not be related to the issue of private information). The second stream deals with private

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[11]For the conditioning of prices on purchase history see for instance Acquisti and Varian (2005), Chen (2005) or Chen and Iyer (2002).

[12]The reason why imperfect targetability may lead to a win-win outcome in Chen et al. (2001) is that their model allows for misperception. They wrote that "*due to imperfect targetability, each firm mistakenly perceives some price-sensitive switchers as price-insensitive loyal customers and charge them all a high price. These misperceptions thus allow its competitors to acquire those mistargeted customers without lowering their prices and hence, reduces the rival firm's incentive to cut prices. This effect softens price competition*" (p. 23).

[13]From a methological point of view, our paper is also related to the literature on Markov-perfect equilibrium in games involving firms with market power that interact with infinitely-lived consumers who have Markovian rational expectations (see, e.g., Driskill and McCafferty, 2001; Laussel et al. 2004). For a recent survey of this literature, see Long (2015).

information issues and the resulting informational rents: consumers are heterogeneous in terms of their individual values represented by a preference parameter, which are private information until the moment they have completed their first purchase.

The first stream of literature mentioned above has its origin from Coase (1972). Assuming that consumers have rational expectations, Coase (1972) argued that, in continuous time, if a monopolist selling a durable good is not able to make a commitment about its future prices and outputs, it will lose all its monopoly power: in equilibrium, the price must be equal to the constant marginal cost, and the monopolist must serve all its customers immediately in one go. This conjecture has been subsequently proved rigorously. In particular, using a model with heterogeneous valuations where each consumer buys at most one unit, Bulow (1982) confirms this result for the "No Gap" case, defined as the situation in which the constant marginal cost is higher than the willingness to pay for the good of the (unserved) consumer with the lowest valuation. There are a number of exceptions (see, e.g. Kahn, 1987, for non-constant marginal cost, Karp, 1996, for durable goods subject to depreciation, and Mason, 2000 or Laussel, Long and Resende, 2015 for the case where the durable good generates consumption network effects). In the context of non-renewable resource markets, various authors have shown that the lack of ability to commit may reduce monopoly profit, or make monopoly power disadvantageous (Kemp and Long, 1980; Maskin and Newbery, 1990). Similarly, the macroeconomic literature is abound with the disadvantages of governmental "discretion" as compared to "rule" (Kydland and Prescott, 1977). The literature addressing the conditions leading to Coasian dynamics, has highlighted the role of the length of the period of commitment between any two successive contract offers. We assume the firm can make commitment over a short time interval, and then we take the limit as the time interval goes to zero. This modelling feature is related to the earlier literature on the length of commitment period (Reinganum and Stokey, 1985; Bond and Samuelson, 1987; Mason, 2000).

The second stream of literature, which deals with informational rents when there is a continuum of types, arguably received the greatest impetus from Mirrlees (1971), who analyzed optimal income tax under asymmetric information. This was followed by a formal analysis of the revelation principle (e.g., Holmstrom, 1977, Myerson, 1979). This principle has been applied to models of regulation (Laffont and Tirole,1986) and models of incentive contracts (see, for example, Laffont and Tirole, 1988, Laffont and Martimort, 2002). The theory has been extended to the multi-dimensional case (e.g.,

Martimort, 2006), as well as the multi-period case (e.g., the extension of Mirrlees' model to a multi-period setting, as in Kocherlakota, 2005; Golosov *et al.*, 2003, 2006, 2007, 2016a,b; Stantcheva, 2017).

By combining these two streams of literature, we are able to shed light on firms' incentives to offer personalized quality-price schedules to successively new sets of customers, deriving novel results concerning the market dynamics and the magnitude of consumers' informational rents in the context of markets for non-durable goods where firms are able to gather accurate information on their customers' preferences. Our results enrich the scarce literature that investigates dynamic monopoly models with multiple varieties. To the best of our knowledge, the contributions that account for the interplay between the Coasian conjecture and the private information issues due to asymmetric information on consumers' exact willingness to pay for the good (à la Mussa-Rosen, 1978) are relatively scarce and they are mostly focused in the case of durable goods.[14] For example, the interesting contribution by Nava and Schiraldi (2019) looks at a dynamic game where the monopolist offers multiple varieties. Considering the case of durable goods, the authors find that in any *"perfect Bayesian equilibrium of the dynamic game: (1) there is skimming, as the measure of buyers in the market at any point in time is a truncation of the original measure; and (2) the market clears instantaneously whenever the seller sets static market-clearing prices."* Our set-up differs from the Nava and Schiraldi (2019) framework since we focus on non-durable goods pricing and consumption, under the possibility of accurate data collection.

Board and Pycia (2014) also look at a dynamic monopoly with durable-goods but they consider that buyers have an outside option they may exercise each period. Such possibility leaves low-type consumers out of the market as the authors find that the model displays a unique equilibrium in which the firm charges the monopolist price every period. Other recent papers looking at dynamic monopoly set-ups with multiple varieties include the works of Inderst (2008), Takeyama (2002) and Kumar (2006). In particular, the latter also looks at a monopoly set-up with durable goods, where the firm may vary product quality over time (also engaging in inter-temporal price discrimination). In this framework, there are equilibria where prices and qualities are non-monotone with respect

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[14]In a static set-up, the issue of optimal product design has been widely studied, following the seminal contribution by Mussa and Rosen (1978). Recent static contributions on this include Deneckere and McAfee (1996) and Johnson and Myatt (2002). In competitive set-ups, a recent contribution related to the problem of optimal product design is Johnson and Myatt (2018).

to time and randomization may take place.

As mentioned earlier, all these papers focus on the case of durable goods. The recent contribution by Maestri (2017) constitutes an exception. In line with the present paper, the author also looks at the case of non-durable goods. However, differently from us, Maestri (2017) assumes that the monopolist does not have access to tracking technologies that allow the firm to uncover the consumers' true willingness to pay. The author concludes that there is a bound on the quality distortion of low types (and also on the high types rent). When the commitment period vanishes, the monopolist offers menus to quickly separate types, which allows the author to show that *"a version of the Coase conjecture holds for the nondurable goods monopoly model"*. Our paper complements this analysis by looking how results are affected by the possibility of collecting accurate data on consumers' preferences after the first purchase in a set-up without commitment.

An important result we obtain is that the game has multiple equilibria. In one of those equilibria (the Markov Perfect Equilibrium B), we find that price-quality offers vary not only across individuals (due to differences in their type-specific taste parameter) but, for a given individual, they also differ from the first-period to the subsequent periods. In particular, after their first purchase, individuals benefit from a quality upgrade. However, consumers end up worse-off as the additional surplus generated by such quality upgrade is extracted by the firm, given its ability to implement first-degree price discrimination.

3   The Model

A monopolist produces a continuum of varieties of a non-durable good. The varieties differ in terms of their quality level, which we denote by q. The set of feasible quality levels is the real interval $[0, q_{\max}]$ where 0 is the lowest quality, and $q_{\max}$ is the highest feasible quality. The unit cost of each variety depends on its quality level. The unit cost is denoted by $c(q)$.

Assumption A1: $c(0) = 0, c'(q) \geq 0, c'(0) = 0$, and $c''(q) > 0$ for all $q > 0$.

There is a continuum of consumer types. Let θ be a variable denoting the type of a consumer. Assume that θ is distributed over a closed interval $\left[\underline{\theta}, \overline{\theta}\right]$, where $\overline{\theta} > \underline{\theta} \geq 0$. The density function and the cumulative distribution function are denoted by $f(\theta)$ and

$F(\theta)$ respectively, where $f(\theta) > 0$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Let $h(\theta)$ denote the inverse hazard rate, $h(\theta) = \frac{1-F(\theta)}{f(\theta)}$.

Assumption A2: $h'(\theta) < 0$.

Assumption A2 is a standard assumption to ensure that there is no bunching in a (static) second-degree discrimination model. This assumption is satisfied by many familiar distributions, such as the uniform, exponential, normal, binomial, and Poisson distributions.[15] However, as we show below, bunching will occur in our dynamic model, because the participation constraint for new customers in any period must take account of the fact that they can delay their first purchase until next period.

Assumption A3: $h(\underline{\theta}) > \underline{\theta}$.

For the uniform distribution case, A3 is equivalent to $2\underline{\theta} < \overline{\theta}$. Taken together, Assumptions A2 and A3 ensure that, in a static second-degree discrimination framework, the market is not covered. More precisely, there must exist a *cut-off type* $\theta^c > \underline{\theta}$, defined by $\theta^c - h(\theta^c) = 0$ such that all types θ in the interval $[\underline{\theta}, \theta^c]$ are not served by the firm.

Herein, we investigate the monopolist's optimal quality-price decisions within an infinite horizon framework with customer recognition. Time is a continuous variable, $t \in [0, \infty)$ and a consumer at each time t buys either one or zero units of the good. Consumers live forever and the good is non-durable: it is consumed instantaneously, yielding the individual a net instantaneous utility equal to:

$$u(q_t(\theta), p_t(\theta), \theta) = \theta q_t(\theta) - p_t(\theta).$$

We refer to θ as the individual's marginal valuation of quality. Each individual's type is constant over time.[16] The consumer's type is initially unknown to the firm. However, as soon as a consumer makes her first purchase, the firm can use tracking devices (e.g. cookies in the case of on-line shoppers, or fidelity cards in the case of more conventional brick-and-mortar stores) to collect data about the consumers. Such data can then be

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[15]For the uniform distribution, $F(\theta) = (\theta - \underline{\theta}) / \left(\overline{\theta} - \underline{\theta}\right)$ and $h(\theta) = \overline{\theta} - \theta$.

[16]In our model, goods sold to customers who have the same θ have the same characteristics. In reality, customers who have the same valuation of quality may have different personal tastes (e.g., concerning design, as in clothes). Extending the model to account for such differences and for the cost of quality personalization would be an interesting topic for future research.

treated by powerful algorithms to uncover the relevant information for the monopolist to accurately identify the consumers' type.[17]

Assumption A4: $c(q_{\max}) > \overline{\theta} q_{\max}$.

Assumption A4 implies the firm would never produce the good at quality level $q_{\max}$, because the unit cost $c(q_{\max})$ is higher than the highest-type consumer's maximum willingness to pay for a unit of the good at that quality level.

To the assumption that consumption takes place instantaneously, we add the assumption that the monopolist cannot modify the price-quality schedules offered to its customers during a period of finite length Δ. For expositional simplicity, we treat Δ as exogenous in the model. This allows us to evaluate the welfare consequences of varying Δ. We will show later that the firm has an incentive to bring Δ to the minimum length which allows it to collect the data necessary to discover the customers' types. This minimum length, denoted by $\underline{\Delta}$, is dependent on both (i) the state of information collection technology and (ii) on the strength of legislations on consumers data protection. For the moment, we simply treat Δ as contractual, which may a priori exceed $\underline{\Delta}$.

There is an infinite sequence of periods, indexed by n, where $n = 0, 1, 2, 3, \ldots$. The first period (denoted by $n = 0$) corresponds to the subset $[0, \Delta)$ of the time line. The second period ($n = 1$) corresponds to the subset $[\Delta, 2\Delta)$, and so on. At the beginning of each period n, the monopolist offers to all potential new customers (those who have not bought the good in some earlier period) a price-quality schedule, which applies for all points of time in the interval $[n\Delta, (n+1)\Delta)$,

$$ p_n = \phi_n(q). $$

where ϕ_n is a mapping from the quality domain $[0, q_{\max}]$ to the space of non-negative prices, $\mathbb{R}_+^2 = [0, \infty)$. During each period (whose length Δ is the time that elapses between two different contract offers) the price and the quality selected by a consumer at the

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[17]The possibility of using tracking technologies to accurately uncover consumers' preferences is a distinctive feature of our model *vis-à-vis* the previous literature based on the revelation principle. Herein the information on consumers' true type is always uncovered after the consumers' first-purchase (regardless of the action taken by the buyer). Differently, in previous works (e.g., Laffont and Tirole, 1986, 88) the seller learns the valuation of a buyer with whom he interacted in the past because her actions reveals her private information. We are grateful to an anonymous reviewer for drawing our attention to this important difference between the two settings.

beginning of the period are contractually fixed. Moreover, Δ cannot be smaller than the minimum length of time it takes to the firm, given the current regulation on consumers' data protection, to collect and process the information about the consumers' tastes in order to get accurate information on the new consumers' type (so that the firm can then start implementing personalized quality-price offers in the subsequent periods). As consumers tastes are assumed to be unchanged during their life, the price-quality offers proposed by the firm to each customer will change only once in her lifetime (after the end of the period in which the customers made their first purchase). After each commitment period, the firm possesses information on the exact type of its old customers and each old consumer always receives the same offer (which corresponds to the quality-price offer that expropriates her entire potential surplus). Accordingly, the length of Δ does not affect old customers, since they always receive the same contract offer. This is not the case for new customers (who may get some informational rents during the first commitment period after their first purchase). Taking this into consideration (and also the fact that the length of Δ is not relevant for the case of old customers), one may argue that Δ must be equal to or greater than the length of time the monopolist takes to collect information on customers after their first purchase.

We assume positive discounting, such that, within each period, a dollar paid at the end of the period is worth only as much as a fraction $e^{-r\Delta}$ of a dollar paid at the beginning of the period, where $r > 0$ denotes the instantaneous discount rate (which is constant and independent of Δ). The same discounting applies to utility. Then a consumer of type θ who selects the quality $q_n(\theta)$ in period n and pays the price $p_n(\theta) \equiv \phi_n(q_n(\theta))$ at each $t \in [n\Delta, (n+1)\Delta)$ achieves the following net utility (properly discounted) for the period:[18]

$$v_n(\theta) \equiv \int_0^\Delta u(p_n(\theta), q_n(\theta), \theta)e^{-r\tau}d\tau = [\theta q_n(\theta) - p_n(\theta)] \int_0^\Delta e^{-r\tau}d\tau$$

$$= \frac{[\theta q_n(\theta) - p_n(\theta)]}{r}\left(1 - e^{-r\Delta}\right) \equiv [\theta q_n(\theta) - p_n(\theta)]\frac{1 - \beta}{r}.$$

Note that r is independent of Δ, whereas β is the discount factor between periods, i.e., $\beta \equiv e^{-r\Delta}$. Clearly $\beta \to 1$ when $\Delta \to 0$.

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[18]Defining $\theta q - p$ as the utility over period n would have the undesirable consequence that the utility per unit of time would increase when the length Δ of the period would decrease, tending to ∞ as Δ tends to 0.

At the beginning of any period n ($n = 0, 1, 2, ...$), the firm faces two disjoint subsets of customers: those whose type belongs to $\left(\theta_n, \overline{\theta}\right]$ are customers who have already bought the good in some previous periods (and therefore the monopolist is fully informed about their preferences) and those whose types belong to $[\underline{\theta}, \theta_n]$ have never purchased the good (and therefore they are potentially new consumers, whose preferences remain unknown to the firm). Among the latter, the consumers whose types θ belong to $(\theta_{n+1}, \theta_n]$, with $\theta_{n+1} < \theta_n$ will be induced to buy the good for the first time in period n. Accordingly, type θ_{n+1} consumers are called the marginal consumers in period n. In equilibrium they are indifferent between (a) being a first-time consumer in period n, at the bottom of the rung, and (b) being a first-time consumer in period $n + 1$, at the top of the rung.[19] It is not difficult to show that in equilibrium (i) if a type-θ consumer has already bought the good, so have all type-θ' customers such that $\theta' > \theta$, and that (ii) if a type-θ consumer has not purchased the good, all type-θ' customers such that $\theta' < \theta$ also have not done so. We assume that initially nobody has purchased the good, i.e., $\theta_0 = \overline{\theta}$.

In the following section, we will look at the monopolist's optimal price-quality decisions in a dynamic set-up in which the monopolist is able to fully track consumers' tastes after their first purchase (and uses this information to implement price/quality discrimination based on consumer recognition). First, we consider the case where the monopolist cannot commit to future contract offers. Afterwards, we will compare equilibrium outcomes described in the next section to the ones arising in a model where the monopolist fully commits to a sequence of price/quality offers.

4   Equilibrium under non-commitment

The monopolist's inability to commit to future prices and qualities has two aspects. First, when selling to first-time consumers in period n, the firm cannot commit to offer them in periods $n + j$ (where $j = 1, 2, 3...$) the same contract as the initial one when they come back to purchase the good. Moreover, in any period n, the monopolist cannot commit to offer pre-determined contracts to new customers in periods $n + j$, $j = 1, 2, 3..$, even though such offers could profitably induce potential first-time consumers in period n to

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[19]This implies that they obtain a higher surplus per unit consumed in period $n + 1$ (if they choose to be first-time consumers in period $n + 1$) than the surplus per unit consumed in period n (if they choose to be first-time consumers in period n), but this difference is fully offset by the fact that the surplus is delayed by Δ units of time. See eq. (12) below.

purchase in that period, in preference to delaying their purchase until the next period or later periods.

In addition, due to data collection by the firm, customers' type becomes known after their first purchase so that the monopolist always uses first-degree discrimination with respect to old customers (this amounts to say that even if the monopolist could commit to future contracts, it does not have any incentive to distort the quality offered to former customers).

Let us study the implications of these considerations.

4.1   Old customers: quality and price personalization

Contrary to the one-shot problem studied by Mussa and Rosen (1978), a consumer does not purchase the good only once. At the beginning of period n, all consumers whose types belong to $(\theta_n, \overline{\theta}]$ have already purchased the good at least once in previous periods. Because the firm maintains a dataset of former customers, when they return in subsequent periods, the firm will not offer them contracts designed for new customers. Instead, it will make them personalized price-quality offers, extracting all their consumer's surplus $\theta q - c(q)$. More precisely, the firm exclusively offers to a type-θ returning customer a good with quality level $q^*(\theta)$ which equates the consumer's marginal valuation of quality, θ, with the marginal cost of quality upgrading,

$$\theta - c'(q^*(\theta)) = 0. \tag{1}$$

and charges the corresponding surplus-eliminating price $p^*(\theta) = \theta q^*(\theta)$.

Under Assumption A1, $q^*(\theta)$ is uniquely defined by (1). Since $c'(0) \leq \underline{\theta}$, it is clear that in any period n the monopolist earns a positive profit from each former customer of type θ by making the take-it-or-leave-it offer $(q^*(\theta), \theta q^*(\theta))$.

Let us denote by $\pi^F(\theta)$ the profit that the monopolist obtains with a sale to a former customer of type θ

$$\pi^F(\theta) \equiv \theta q^*(\theta) - c(q^*(\theta)), \tag{2}$$

where the superscript F stands for "former" customers. The aggregate profit earned in period n from selling to all former customers (those whose types are in the interval $(\theta_n, \overline{\theta}]$) is then given by

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$$\Pi_n^F = \frac{1-\beta}{r} \int_{\theta_n}^{\overline{\theta}} \pi^F(\theta) f(\theta) d\theta. \tag{3}$$

4.2 New customers: screening mechanism with possible bunching

The type of new customers is unknown to the monopolist. Thus, in order to discriminate among new customers, the monopolist uses a screening mechanism. At the beginning of period n, the targeted new customers are those whose type θ belongs to the interval $(\theta_{n+1}, \theta_n]$. We will refer to members of this group as vintage-n customers. By offering a price-quality schedule $p_n(q)$ for these first-time consumers[20], it is indeed possible to induce them to select a couple $(q_n(\theta), p_n(\theta))$, provided that an incentive compatibility condition (ICC) is satisfied. The nature of the ICC is explained below.

Being rational, these first-time consumers fully expect that the monopolist, in later periods, will offer them a take-it-or-leave-it offer that extracts all their surplus. The consumer knows that the firm will gather data about her tastes, identifying her true type after her first purchase. Hence, any consumer who has reported a type $\widehat{\theta}$ (regardless of whether it is equal to her true θ or not) knows that, in all subsequent periods, her true θ will be known, and thus she will be proposed a quality $q^*(\theta)$ as defined by (1) and a price $\theta q^*(\theta)$ such that the surplus of the consumer is fully extracted by the monopolist.

Truthful reporting is optimal for a type θ-customer (when she purchases the good for the first time, say in period n) if and only if pretending to be a different type (say $\widehat{\theta} \neq \theta$) would make her worse off than telling the truth:

$$\frac{1-\beta}{r}(\theta q_n(\theta) - p_n(\theta)) \geq \frac{1-\beta}{r}\left(\theta q_n(\widehat{\theta}) - p_n(\widehat{\theta})\right),$$

where the right-hand side (RHS) term is the net utility earned in period n by misreporting one's type (pretending to be of type $\widehat{\theta}$ instead of type θ). The left-hand side (LHS) is the net utility earned in period n if the consumer is truthful. The above ICC condition may be written as

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[20]Or, equivalently, a direct mechanism $\left(q(\widehat{\theta}), p(\widehat{\theta})\right)$ depending on the type $\widehat{\theta}$ which the consumer chooses to report.

$$\theta = \arg\max_{\widehat{\theta}} \frac{1-\beta}{r} \left(\theta q_n(\widehat{\theta}) - p_n(\widehat{\theta}) \right). \tag{4}$$

Let us denote the net utility of a first-time buyer of type $\theta \in [\theta_{n+1}, \theta_n]$ in period n by

$$U_n(\theta) \equiv \max_{\widehat{\theta}} \frac{1-\beta}{r} \left(\theta q_n(\widehat{\theta}) - p_n(\widehat{\theta}) \right). \tag{5}$$

The envelope theorem implies that the pair $(q_n(\theta), p_n(\theta))$ is incentive compatible iff

$$U_n'(\theta) = \frac{1-\beta}{r} q_n(\theta). \tag{6}$$

Accordingly, by integrating (6), we find that for all $\theta \in [\theta_{n+1}, \theta_n]$, it holds that

$$U_n(\theta) = U_n(\theta_{n+1}) + \int_{\theta_{n+1}}^{\theta} \left(\frac{1-\beta}{r} q_n(s) \right) ds, \tag{7}$$

where the integral on the RHS is the difference between the informational rent of a type-θ consumer, where $\theta_{n+1} \leq \theta \leq \theta_n$, over the informational rent of a marginal type θ_{n+1}, who is indifferent between *(a)* being a first-time customer in period n, at the bottom of the rung, and *(b)* being a first-time customer in period $n + 1$, at the top of the rung. For later use, let us denote by $R(\theta_n, \theta_{n+1})$ the difference between the informational rent of first-time customer at the top rung in period n and that of the first-time customer at the bottom rung in period n:

$$R(\theta_n, \theta_{n+1}) \equiv \int_{\theta_{n+1}}^{\theta_n} \frac{1-\beta}{r} q_n(s) ds. \tag{8}$$

Equation (5) gives:

$$U_n(\theta) = \frac{1-\beta}{r} \left(\theta q_n(\theta) - p_n(\theta) \right). \tag{9}$$

Moreover, a familiar revealed preference argument yields the following inequality (second incentive-compatibility constraint):

$$\frac{1-\beta}{r} q_n'(\theta) \geq 0. \tag{10}$$

This implies, via (6), that $U_n(\theta)$ is convex in θ.

A pair $(q_n(\theta), p_n(\theta))$ is *incentive-feasible* if, in addition to the incentive compatibility

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condition (6) (or, equivalently, (7)), it also satisfies the following participation constraint (PC) for first-time buyers in period n:

$$U_n(\theta) - \beta \left[\frac{1-\beta}{r} \left(\theta q_{n+1}^b(\theta) - p_{n+1}^b(\theta) \right) \right] \geq 0, \forall \theta \in [\theta_{n+1}, \theta_n], \tag{11}$$

where $\left(q_{n+1}^b(\theta), p_{n+1}^b(\theta) \right)$ denote the best price-quality pair that a type-θ consumer would choose in period $n+1$ if she delays her first purchase to period $n+1$.

To understand the participation constraint (11), note that, for $\theta \in [\theta_{n+1}, \theta_n]$, a type θ-consumer that is targeted by the monopolist as a potential first-time customer in period n can deviate by delaying her first purchase to the next period. The firm, wishing to prevent such a deviation, must ensure that her net utility from buying as a first-time customer in period n is greater than or equal to her net utility from buying as a first-time customer in period $n+1$. Since the monopolist is able to identify the true type of all those customers who have made their first purchase and will extract their whole surplus in subsequent periods, the life-time net utility of a first-time customer in period n is simply equal to the net utility obtained in period n itself (the net utility from her purchases in periods $i > n$ being zero). The (discounted) life-time net utility obtained by a consumer of type $\theta \in [\theta_{n+1}, \theta_n]$ who delays her first purchase to period $n+1$ is then:

$$\beta \left[\frac{1-\beta}{r} \left(\theta q_{n+1}^b(\theta) - p_{n+1}^b(\theta) \right) \right].$$

Hence, such type-θ consumer will choose to buy in period n iff $U_n(\theta) \geq \beta \left[\frac{1-\beta}{r} \left(\theta q_{n+1}^b(\theta) - p_{n+1}^b(\theta) \right) \right]$. The RHS of this inequality is the reservation-utility level of the targeted type-θ consumer. It turns out that this reservation level can be easily characterized. This is the essence of Claim 1 below.

Claim 1. *A type-θ customer, where $\theta \in [\theta_{n+1}, \theta_n]$, when evaluating the merit of delaying her first-time purchase to period $n+1$ instead of buying in period n, will find it optimal (conditional on deviating) to report the highest type among consumers who buy in period $n+1$, i.e., $\widehat{\theta} = \theta_{n+1}$.*

Proof. See the Appendix. ∎

Given Claim 1, the participation constraint (11) for any type $\theta \in [\theta_{n+1}, \theta_n]$ may be written as

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$$\Delta_n(\theta) \equiv U_n(\theta) - \beta \left(\frac{1-\beta}{r} \left(\theta q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1}) \right) \right) \geq 0, \tag{12}$$

i.e., the price-quality schedule offered to new customers in period n and the schedule offered to new customers in period $n + 1$ must be such that, for all $\theta \in [\theta_{n+1}, \theta_n]$, buying for the first time in period n is at least as advantageous as doing so in period $n + 1$.

Since θ_{n+1} is defined as the marginal type who is indifferent between buying for the first time at n and buying at $n + 1$, it holds that, condition (12), when evaluated at $\theta = \theta_{n+1}$, is satisfied with equality, i.e., we have $\Delta_n(\theta_{n+1}) = 0$. How can we ensure that the participation constraint (12) is satisfied also for all the inframarginal types, $\theta > \theta_{n+1}$? Lemma 1 below provides a necessary and sufficient condition.

Lemma 1. *The participation constraint (12) is satisfied for all $\theta \in [\theta_{n+1}, \theta_n]$ iff, in addition to the condition that the marginal first-time customer is indifferent, i.e., $\Delta_n(\theta_{n+1}) = 0$, the following inequality is met*

$$q_n(\theta_{n+1}) - \beta q_{n+1}(\theta_{n+1}) \geq 0. \tag{13}$$

Proof. See the Appendix. ∎

Condition (13) requires that the *lowest* quality offered to new customers in period n is greater than the (discounted) *highest* quality offered to new customers in period $n + 1$.

Recall that $q_{n+1}(\theta_{n+1})$ is the quality level intended for the highest consumer type among those who purchase for the first time in period $n + 1$. One may expect that the familiar property that there is "*no distortion at the top*" (which holds in the static setting) also applies in our dynamic model, so that $q_{n+1}(\theta_{n+1}) = q^*(\theta_{n+1})$, which turns out to be the case in equilibrium, as it will be verified afterwards. When this is indeed the case, constraint (13) reduces to:

$$q_n(\theta_{n+1}) \geq \beta q^*(\theta_{n+1}). \tag{14}$$

that is, the quality offered to the lowest-type first-time customer in period n is greater than the (discounted) first-best quality for that type.

Finally, from (9) and (11), the condition $\Delta_n(\theta_{n+1}) = 0$ implies that

$$U_n(\theta_{n+1}) = \beta U_{n+1}(\theta_{n+1}). \tag{15}$$

Equation (15) indicates that marginal consumers in period n are indifferent between being the lowest type among all new consumers in period n and being the highest type among all new consumers in period $n + 1$ (the higher surplus being exactly offset by the discount factor). With the help of equation (7) evaluated at $n + 1$, this indifference condition implies the following dynamic relationship between the net utilities obtained by the marginal first-time customers in period n and period $n + 1$:

$$U_n(\theta_{n+1}) = \beta \left(U_{n+1}(\theta_{n+2}) + \int_{\theta_{n+2}}^{\theta_{n+1}} \left[\frac{1 - \beta}{r} q_{n+1}(\theta) \right] d\theta \right). \tag{16}$$

In period n, the monopolist's profit from sales to new customers, Π_n^N, is equal to

$$\Pi_n^N = \frac{1 - \beta}{r} \int_{\theta_{n+1}}^{\theta_n} [p_n(\theta) - c(q_n(\theta))] f(\theta) d\theta \tag{17}$$

where the superscript N in Π_n^N stands for "new customers."

Re-arranging terms, we get

$$\Pi_n^N = \frac{1 - \beta}{r} \int_{\theta_{n+1}}^{\theta_n} \{ [\theta q_n(\theta) - c(q_n(\theta))] - [\theta q_n(\theta) - p_n(\theta)] \} f(\theta) d\theta. \tag{18}$$

Using the definition of $U_n(\theta)$, we can rewrite (18) as:

$$\Pi_n^N = \int_{\theta_{n+1}}^{\theta_n} \left\{ [(\theta q_n(\theta) - c(q_n(\theta))] \left(\frac{1 - \beta}{r} \right) - U_n(\theta) \right\} f(\theta) d\theta. \tag{19}$$

Equation (19) shows that the profits obtained from sales to new customers in period n is equal to the "*vintage n total virtual surplus*" (i.e., gross utility minus production cost minus the life-time rent of all first-time consumers of vintage n).

It is shown in the Appendix that the RHS of equation (19) is equivalent to

$$\int_{\theta_{n+1}}^{\theta_n} \left\{ \begin{array}{c} \left[\left(\theta - \frac{F(\theta_n) - F(\theta)}{f(\theta)} \right) q_n(\theta) - c(q_n(\theta)) \right] \left(\frac{1 - \beta}{r} \right) \\ - U_n(\theta_{n+1}) \end{array} \right\} f(\theta) d\theta. \tag{20}$$

For each $\theta \in [\theta_{n+1}, \theta_n]$, let $q_n^N(\theta)$ denote the quality level that maximizes the term

inside the square brackets [...] in eq. (20), subject to the participation constraint that

$$q_n(\theta) \geq \beta q^*(\theta_{n+1}) \text{ for all } \theta \in [\theta_{n+1}, \theta_n]$$

It it is easy to see that $q_n^N(\theta)$ is given by

$$q_n^N(\theta) = \max \left\{ \beta q^*(\theta_{n+1}), q_n^\#(\theta) \right\}$$

where $q_n^\#(\theta)$ is defined by

$$\theta - \left(\frac{F(\theta_n) - F(\theta)}{f(\theta)} \right) - c'(q_n^\#(\theta)) = 0 \tag{21}$$

i.e.

$$q_n^\#(\theta) = (c')^{-1} \left[\theta - \left(\frac{F(\theta_n) - F(\theta)}{f(\theta)} \right) \right]$$

Using the specification that the distribution $f(\theta)$ is uniform, and the cost function is quadratic, i.e.,

$$c(q) = \frac{1}{2} q^2$$

and

$$\theta - \left(\frac{F(\theta_n) - F(\theta)}{f(\theta)} \right) = 2\theta - \theta_n$$

we find that

$$q_n^\#(\theta) = 2\theta - \theta_n.$$

Then, for a uniform distribution $f(\theta)$ and a quadratic cost function, it follows that:

$$q_n^N(\theta) = \begin{cases} 2\theta - \theta_n & \text{if } 2\theta - \theta_n \geq \beta\theta_{n+1} \text{ i.e., if } \theta \in \left[\frac{\theta_n + \beta\theta_{n+1}}{2}, \theta_n \right] \\ \beta\theta_{n+1} & \text{if } \theta \in \left[\theta_{n+1}, \frac{\theta_n + \beta\theta_{n+1}}{2} \right] \end{cases} \tag{22}$$

We state this result as Lemma 2.

Lemma 2: *A screening mechanism is offered to customers with $\theta \in \left[\frac{\theta_n + \beta\theta_{n+1}}{2}, \theta_n \right]$ at the top of the rung. Bunching occurs at the bottom of the rung as a consequence of the monopolist's optimization under the participation constraint $q_n(\theta_{n+1}) \geq \beta q^*(\theta_{n+1})$: it is optimal for the monopolist to offer these consumers the same low quality level, so as to reduce the informational rents granted to the higher types.*

Remark 1: *Note that the set*

$$S_T \equiv \left\{ \theta \mid \frac{\theta_n + \beta\theta_{n+1}}{2} < \theta \le \theta_n \right\}$$

is non-empty, because $\theta_{n+1} < \theta_n$ and $0 < \beta < 1$. The set

$$S_B \equiv \left\{ \theta \mid \theta_{n+1} \le \theta \le \frac{\theta_n + \beta\theta_{n+1}}{2} \right\}$$

is a non-empty if and only if θ_{n+1} is much further to the left of θ_n, such that

$$\theta_{n+1} < \frac{\theta_n}{(2 - \beta)}. \tag{23}$$

If inequality (23) holds, we say that the monopolist expands the market by large steps. If $\theta_{n+1} > \theta_n/(2 - \beta)$, we say he expands the market in small steps. We will show in Section 4.3.1 that under the uniform distribution and quadratic cost, in equilibrium it is never optimal for the firm to expand the market in small steps given that consumers' expectations of future rents to marginal customers is quadratic.

Using Remark 1 and eq. 22, we derive Lemma 3, where we assume a quadratic quality cost specification and a uniform distribution of consumers' willingness to pay:

Lemma 3: *The profit obtained from selling to the set of screened new customers - they are of types $\theta \in \left(\frac{\theta_n + \beta\theta_{n+1}}{2}, \theta_n \right]$ - over the period in which they are first-time customers is*

$$\int_{\frac{\theta_n + \beta\theta_{n+1}}{2}}^{\theta_n} \left\{ \begin{array}{c} \left[\frac{(2\theta - \theta_n)^2}{2} \right] \left(\frac{1-\beta}{r} \right) \\ -U_n(\theta_{n+1}) \end{array} \right\} \frac{1}{\overline{\theta} - \underline{\theta}} d\theta$$

and the profit from from selling to the set of pooled (i.e. bunched) new customers of types $\theta \in \left[\theta_{n+1}, \frac{\theta_n + \beta\theta_{n+1}}{2} \right]$, if this set is non-empty, is

$$\int_{\theta_{n+1}}^{\frac{\theta_n + \beta\theta_{n+1}}{2}} \left\{ \begin{array}{c} \left[(2\theta - \theta_n)(\beta\theta_{n+1}) - \frac{1}{2}(\beta\theta_{n+1})^2 \right] \left(\frac{1-\beta}{r} \right) \\ -U_n(\theta_{n+1}) \end{array} \right\} \frac{1}{\overline{\theta} - \underline{\theta}} d\theta. \tag{24}$$

Remark 2: *Since $F(\theta_n) - F(\theta) > 0$ for all $\theta > \theta_n$, condition (21) implies that there*

is quality distortion for all first-time customers except for type θ_n. This confirms that the familiar property called "no-distortion at the top" in static models also applies to each group of new customers in our dynamic model.

Remark 3: *Given Assumptions A1 and A2, even when $f(\theta)$ is not uniform, we still can prove that $q_n^{\#}(\theta)$ is a strictly increasing function of θ provided that*

$$f'(\theta)/f(\theta) \leq f(\theta)/\left[1 - F(\theta_n)\right].$$

The latter condition ensures that higher-type consumers are offered strictly higher quality levels. To see this, rewrite (21) as

$$\theta - h(\theta) + \frac{[1 - F(\theta_n)]}{f(\theta)} = c'(q_n^{\#}(\theta)).$$

Then

$$1 - h'(\theta) - \frac{[1 - F(\theta_n)]\, f'(\theta)}{(f(\theta))^2} = c''(q_n^{\#}(\theta))\frac{dq_n^{\#}(\theta)}{d\theta},$$

where $-h'(\theta) \geq 0$, given Assumption A2. It follows that $\frac{dq_n^{\#}(\theta)}{d\theta} > 0$ provided that $f'(\theta)/f(\theta) \leq f(\theta)/\left[1 - F(\theta_n)\right]$.

4.3   Markov Perfect Equilibria

In this subsection we focus on the characterization of the Markov Perfect Equilibria (MPE for short). In a MPE, the strategy of the monopolist is expressed as function of a state variable, say $\Theta(n)$ (to be defined), and consumers have a Markovian expectations function, denoted by $\Omega(.)$, which is a function of the same state variable, such that (i) given the consumers expectations function, starting from any (date, state) pair $(n, \Theta(n))$, the monopolist's strategy maximizes its profit, and (ii) given the monopolist's strategy, the consumers expectations are rational, i.e., their expectations are correct. Let us now define the state variable, the monopolist's strategy, and the consumers expectation function.

In any period n, $n = 0, 1, 2, 3...$, let $X(n) \in [0, 1]$ denote the fraction of the total population (the customer base) that has purchased the product prior to that period, with $X(0) = 0$. It may seem natural to use $X(n)$ as the state variable. However, it turns

out to be more convenient to use as state variable $\Theta(n)$, which corresponds to a monotone decreasing transformation of $X(n)$, being defined as follows:

$$\Theta(n) \equiv F^{-1}(1 - X(n)),$$

where $F(.)$ is the cumulative distribution of θ. Then $\Theta(0) = \overline{\theta} \equiv \theta_0$, and $\Theta(n) \in \left[\underline{\theta}, \overline{\theta}\right]$.

The firm's Markovian strategy is a pair (ψ, η), that consists of two components: (a) a *Markovian cut-off rule* $\psi(.)$, specifying, at the beginning of period n, given θ_n, what is the cut-off (on the distribution of consumers' types) that defines the targeted group of first-time customers for period n; and (b) a *Markovian quality-schedule rule* $\eta(.)$, specifying the monopolist's type-dependent quality offers to first-time consumers in period n.

To be more precise, in each period n, the firm's Markovian cut-off rule is a function $\psi(.)$ that determines, for any currently-observed value $\Theta(n)$, a value $\Theta(n + 1) = \psi(\Theta(n)) \leq \Theta(n)$, where $\psi(.)$ is non-increasing and bounded below by $\underline{\theta}$. We interpret $\Theta(n + 1) = \psi(\Theta(n))$ as the lowest-type of first-time consumers in period n that the firm intends to serve.

A quality-schedule rule is a mapping $\eta(.)$ that determines for any given $\Theta(n)$ an associated quality schedule $q_n(.|\Theta(n))$, which is itself a function that assigns to each $\theta \in [\Theta(n + 1), \Theta(n)]$ a value in $[0, q_{\max}]$, where $\Theta(n + 1) = \psi(\Theta(n))$. We interpret $q_n(\theta|\Theta(n))$ as the quality level offered to a first-time customer of type $\theta \in [\Theta(n + 1), \Theta(n)]$ in period n, given that the value of the state variable is $\Theta(n)$.

On the consumers' side, their Markovian expectations rule $\Omega(.)$ predicts, given $\Theta(n)$, the life-time rent of *the marginal first-time customer in period n*, i.e., $U_n(\theta_{n+1})$. When consumers have the ability to perfectly anticipate future market outcomes, the expectations function $\Omega(.)$ must reflect rational expectations, i.e.,

$$\Omega(\Theta(n)) = \sum_{j=1}^{\infty} \beta^j \left(\int_{\Theta^*(n+j+1)}^{\Theta^*(n+j)} \left[\frac{1 - \beta}{r} q_{n+j}(s) \right] ds \right) = U_n(\theta_{n+1}), \tag{25}$$

where $\{\Theta^*(.)\}_n^{\infty}$ is the path of the state variable Θ induced by the strategic behavior of the monopolist from period n, when the state variable takes the value $\Theta(n)$, and where $q_{n+j}(s) = q_{n+j}(s|\Theta^*(n + j))$, i.e., the quality schedule that the consumers expect to be offered in period $n + j$ is the same as the schedule that the monopolist's equilibrium

strategy would select. Note that, due to (16), $\Omega(\Theta(n))$ satisfies the following equation:

$$\Omega(\Theta(n)) = \beta \left(\Omega(\Theta(n+1)) + \int_{\theta_{n+2}}^{\theta_{n+1}} \left[\frac{1-\beta}{r} q_{n+1}(s) \right] ds \right). \tag{26}$$

A Markovian strategy $(\eta(.), \psi(.))$ chosen by the monopolist is called a best reply to the consumer expectations function $\Omega(.)$ if (a) it yields a sequence of schedules $q_n(.)$ and cut-off values θ_{n+1} that maximize the monopolist's expected profits from any starting (date, state) pair $(n, \Theta(n))$, and (b) the rational expectations condition (25) is satisfied by such a sequence.

In what follows, we analyze in more detail the firm's optimal Markovian sales and quality strategies and we provide an analytical characterization of the consumers' optimal Markovian expectation rules, under the assumptions that the distribution of θ is uniform and the production cost is quadratic.

4.3.1 Equilibrium strategies under quadratic cost and uniform distribution

In order to obtain a closed form solution for the firm's optimal strategy, let us now suppose that the distribution of types is uniform on $[0,1]$ [21] and that $c(q)$ is quadratic: $c(q) = \frac{1}{2}q^2$.

Because of the quadratic cost, the first best quality offered to old customers of type θ is $q^*(\theta) = \theta$. Then, using (2) and (3), the profit obtained in period n from old customers is:

$$\Pi_n^F \equiv \frac{1-\beta}{r} \int_{\theta_n}^{\overline{\theta}} \pi^F(\theta) f(\theta) d\theta = \left(\frac{1-\beta}{6r} (\overline{\theta}^3 - \Theta(n)^3) \right). \tag{27}$$

It follows that the Bellman equation for the monopolist (where we have set $\overline{\theta} = 1$ and $\underline{\theta} = 0$) is

$$V(\Theta(n)) = \max_{q_n(.), \Theta(n+1)} \left\{ \begin{array}{c} \frac{(1-\beta)(1-\Theta(n)^3)}{6r} + I\left[\Theta(n), \Theta(n+1), q_n(.|\Theta(n))\right] \\ + \beta V(\Theta(n+1)) \end{array} \right\}, \tag{28}$$

where $I\left[\Theta(n), \Theta(n+1), q_n(.|\Theta(n))\right]$ denotes the profit obtained from first-time customers in period n (this is specified in appendix, before the proof of Lemma 4). In the Bellman

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[21] Assuming that the distribution is uniform on $[0, \overline{\theta}]$ would not make any important difference.

equation (28), the choice of $q_n(.)$ is subject to the participation constraint (13), which is

$$q_n(\theta_{n+1}|\Theta(n)) - \beta q_{n+1}(\theta_{n+1}|\Theta(n+1)) \geq 0. \tag{29}$$

and the choice of $\Theta(n+1)$ is subject to the non-negativity constraint, $\Theta(n+1) \geq 0$.

Turning to the consumers' expectations of the life-time rents of marginal first-time customers in period n, they depend on whether consumers expect the market to expand in large steps or in small steps. More precisely, we obtain from (26) and (22) the following:

$$\Omega(\Theta(n)) = \beta \left(\Omega(\Theta(n+1)) + \frac{1-\beta}{r} \left(\int_{\Theta(n+2)}^{\Theta(n+1)} (2\theta - \Theta(n+1)) \, d\theta \right) \right), \tag{30}$$

if $\Theta(n+2) \geq \frac{\Theta(n+1)}{2-\beta}$, i.e., if they expect the market to expand in small steps; and

$$\Omega(\Theta(n)) = \beta \left(\begin{array}{c} \Omega(\Theta(n+1)) \\ +\frac{1-\beta}{r} \left(\begin{array}{c} \int_{\frac{\Theta(n+1)+\beta\Theta(n+2)}{2}}^{\Theta(n+1)} (2\theta - \Theta(n+1)) \, d\theta + \\ \int_{\Theta(n+2)}^{\frac{\Theta(n+1)+\beta\Theta(n+2)}{2}} \beta\Theta(n+2) d\theta \end{array} \right) \end{array} \right), \tag{31}$$

if $\Theta(n+2) \leq \frac{\Theta(n+1)}{2-\beta}$, i.e., if they expect the market to expand in large steps.

Given the linear-quadratic structure of the model, let us focus on Markov-perfect equilibria with a linear cut-off rule by the monopolist

$$\Theta(n+1) = \psi(\Theta(n)) = \gamma\Theta(n), \text{ with } \gamma \in [0,1] \tag{32}$$

and, for consumers, a quadratic expectation function,

$$\Omega(\Theta(n)) = \lambda\Theta(n)^2, \lambda > 0 \tag{33}$$

where the parameters γ and λ are to be determined. Note that $\gamma = 0$ means that the market is fully covered in one period, and $\gamma \in \left(0, \frac{1}{2-\beta} \right]$ means that the monopolist chooses to expand market in large steps (such that some bunching occurs in every periods among the new customers with low θ), $\gamma \in \left(\frac{1}{2-\beta}, 1 \right)$ means market expansion in small steps (no bunching in any period), and finally $\gamma = 1$ means that $\Theta(n+1) = \Theta(n) = ... = \Theta(1) = \Theta(0) = 1$, i.e., the monopolist does not offer the good to anyone in any period.

Substituting the quadratic expectations function and the linear cut-off rule into eq. (30) and (31), we deduce that for the quadratic expectations function to be a best reply to the linear cutoff rule γ, the parameter λ must be such that:

$$r\lambda = \frac{\beta(1-\beta)(1-\gamma)\gamma^3}{1-\beta\gamma^2}, \tag{34}$$

for $\gamma \geq \frac{1}{2-\beta}$ (i.e., for slower rates of market expansion), and

$$r\lambda = \frac{(1-\beta)\beta\gamma^2(1+\beta\gamma\left(2+(\beta-4)\gamma\right))}{4(1-\beta\gamma^2)}, \tag{35}$$

for $\gamma \leq \frac{1}{2-\beta}$ (i.e., for faster rates of market expansion). Notice that $r\lambda$ is continuous with respect to γ at $\gamma = \frac{1}{2-\beta}$ where it equals $\frac{\beta(1-\beta)}{(4-\beta)(2-\beta)^2}$.

Let us now investigate what is the monopolist's best reply if the consumers' expectations function is quadratic, such that $\Omega(\Theta(n)) = \lambda\Theta(n)^2$. The following Lemma shows that, facing consumers expectations rule $\Omega(\Theta(n)) = \lambda\Theta(n)^2$, the monopolist's optimal response to any strictly positive λ consists of choosing a small cut-off parameter γ, which implies that the market is expanded by large steps (and thus in each period there is bunching of the bottom rung of that period's first-time customers).

Lemma 4. *The monopolist's best reply to the consumers' expectations must satisfy the first-order condition that*

$$\frac{\Theta(n)^2}{4r} \left[\begin{array}{c} 4r\lambda(1+\beta\gamma^2(-3+2\gamma)) - (1-\beta)\beta\times \\ (1+(2\beta-8)\gamma+(13-2\beta+\beta^2)\gamma^2 \\ +2\beta\gamma^3+\beta(\beta-4)\gamma^4) \end{array} \right] + \kappa = 0, \tag{36}$$

where $\kappa \geq 0$ is the Kuhn-Tucker multiplier associated with the constraint that $\Theta(n+1) \geq 0$. This equation implies that, facing customers' quadratic expectations rule with strictly positive λ, the monopolist's optimal response, γ^*, must lie in the interval $\left[0, \frac{1}{2-\beta}\right)$, which means either the monopolist serves all customers in the initial period ($\gamma = 0$) or he expands the market by large steps, $\gamma \in \left(0, \frac{1}{2-\beta}\right)$.
Proof. See the Appendix. ∎

It follows from Lemma 4 that a Markov-Perfect Equilibrium must be fully characterized by (i) an expectations parameter λ that satisfies equation (35), which describes the

consumers' rational expectations for given $\gamma \in \left[0, \frac{1}{2-\beta}\right)$, and (ii) a cut-off parameter γ that satisfies equation (36), which describes the monopolist's best reply to the customers' expectations function. We characterize below two contrasting Markov-perfect Equilibria, which we call Equilibrium A and Equilibrium B, respectively. As we shall see later, the two equilibria have substantially different properties regarding the way the market is covered. In particular, in equilibrium A, the market is immediately covered in a **"twinkle of an eye"** (with some customers being bunched to enter the market immediately by getting the lowest-quality version at zero price). In contrast, in equilibrium B, the market is covered gradually. Also, as reported in Propositions 1 and 2 below, an equilibrium type A always exists for any $\beta \in [0, 1]$, whereas equilibrium B exists if and only if β is small enough in the sense that $\beta \leq 0.525$ (approximately). If $\beta \in (0.525, 1]$, the two reaction functions (35) and (36) fail to intersect in the relevant region where $\lambda > 0$ and $\gamma \in \left(0, \frac{1}{2-\beta}\right)$. To see intuitively why an equilibrium of type B does not exist if $\beta \in (0.525, 1]$, recall that $\beta \to 1$ means that the period of commitment tends to infinity. With a long period of commitment between any two consecutive offers, consumers do not have a strong incentive to delay their purchases, so they expect immediate market coverage, and the future rent of any potential future marginal customer must be zero. And the monopolist, guessing that consumers expectation parameter is $\lambda^* = 0$, will choose the optimal reply, $\gamma = 0$.

Let us now turn our attention to Equilibrium A. As described in Proposition 1, the monopolist chooses to serve all the customers from the very beginning. In this case, the monopolist's optimal strategy partitions the first-period market into two segments. For the higher types of consumers, i.e., $\theta \in \left[\frac{1}{2}, 1\right]$, the firm offers the Mussa-Rosen price-quality schedule. In contrast, for the lower types, $\theta \in \left[0, \frac{1}{2}\right)$ it induces them to choose a low quality ($q = 0$) at price 0 (leading to bunching at the bottom rung of first-time buyers).

While the profit earned from this lower segment is zero in the first period, it is worth noting that the possibility of full customer recognition after the first purchase (using the monopolist's Big Data) means that the immediate full market coverage allows the firm to immediately get information on all the customers' exact willingness to pay for the good, including those consumers who would not be willing to get the good at a positive price. In all subsequent periods, such information will allow the monopolist to make the quality-price offer that extracts from each customer the maximum possible surplus.

Accordingly, the equilibrium type A provides a theoretical foundation for the adoption

of the so-called "*freemium business model*" when a new good or service is brought to the market. Indeed, under this business strategy, the firm offers some low-quality variant of a good or service for free and, simultaneously, offers higher-quality variant(s) of the good or service at a strictly positive price. In line with our model, the latter offers intend to screen consumers with high willingness to pay for this good or service.

Proposition 1 (*Equilibrium A*): *There always exists a Markov-Perfect Equilibrium (called Equilibrium A) such that* $\gamma^*(\beta) = 0$ *and* $\lambda^*(\beta) = 0$, $\forall \beta \in [0,1]$. *In this MPE, all consumers buy the good for the first time in the initial period, in which the monopolist offers them the static Mussa-Rosen price-quality schedule, i.e., a quality* $2\theta - 1$ *at a price* $\theta^2 - \frac{1}{4}$ *(screening), for all types* $\theta \in \left[\frac{1}{2}, 1\right]$ *and a quality* 0 *at a price* 0 *(bunching) for all types* $\theta \in \left[0, \frac{1}{2}\right]$.

Proof. See Appendix. ∎

As it is stated, Proposition 1 depends on the (implicit) assumption, embodied in the way participation constraints are written, that the low types (bunched) customers buy the good even though this does not dominate staying out. Suppose, on the contrary, that a strictly positive measure of these low-type customers would choose to stay out of the market and the monopolist were aware of that. It would then be optimal for the monopolist to offer them a contract in the next period, granting the top ones among them strictly positive rents. This would mean that $\Omega(\Theta(1)) = \lambda^* \times (\Theta(0))^2 = 0$ would not meet the rational expectations requirement, and thus the Proposed Equilibrium A would not be an MPE. However, this difficulty is resolved if one replaces the assumption of a *continuum* of consumer types with a *fine grid* with, say, a difference ε between any two adjacent types, ε being a very small positive number. Then a cut-off rule $\psi(\Theta(n)) = \underline{\theta} + \varepsilon$, $\forall \Theta(n)$ and an expectations rule $\Omega(\Theta(n)) = 0$, $\forall \Theta(n)$ would form an MPE. Indeed the monopolist would optimally offer to type $\underline{\theta} + \varepsilon$ customers a first best quality $q^*(\underline{\theta} + \varepsilon) > 0$ and a price that leaves them with no surplus. Given constraint (14), a strictly positive quality level $\beta q^*(\underline{\theta} + \varepsilon)$ would then be offered to bunched customers. This would provide them with a strictly positive surplus, breaking indifference. By appropriately choosing ε, this MPE can be made as close as desired to the full commitment equilibrium.

Notice that the monopolist's profits at Equilibrium A equals

$$\Pi^A(\beta) = \frac{1 - \beta}{12r} + \frac{\beta}{6r} = \frac{1 + \beta}{12r} \tag{37}$$

Over the first period of length Δ, aggregate consumer surplus, in present value terms, is

$$CS^A(\beta) = \frac{1}{24} \int_0^\Delta e^{-rt}dt = \frac{1-\beta}{24r} \qquad (38)$$

It follows that social welfare at Equilibrium A is

$$W^A(\beta) = \frac{1+\beta}{12r} + \frac{1-\beta}{24r} = \frac{3+\beta}{24r}. \qquad (39)$$

We now show in Proposition 2 that there may also exist another MPE at which market coverage occurs only gradually.

Proposition 2 (Equilibrium B): *Iff $\beta \leq \beta^* \simeq 0.525$, there exists a Markov-Perfect Equilibrium (called Equilibrium B) such that $\gamma^*(\beta)$ is the greatest real root in the interval $\left[0, \frac{1}{2-\beta}\right]$ of the seventh-order polynomial*

$$\begin{aligned}
0 = \ & -1 + (8 - 2\beta)\gamma + (-12 + 3\beta - \beta^2)\gamma^2 \qquad (40) \\
& + (-8\beta + 2\beta^2)\gamma^3 + (10\beta - 2\beta^2 + \beta^3)\gamma^4 \\
& + (2\beta - 4\beta^2)\gamma^5 + (12\beta^2 - 2\beta^3)\gamma^6 + (-8\beta^2 + 2\beta^3)\gamma^7
\end{aligned}$$

while $\lambda^*(\beta)$ satisfies (35) and is strictly positive for all $\beta \in (0,1)$. At this equilibrium the market is only gradually covered. In each period, the higher-type first-time buyers are screened but the lower types of customers are bunched.

Proof. See Appendix. ∎

Remark 4: *Let us define Δ^* by*

$$e^{-r\Delta^*} \equiv \beta^*.$$

Then the condition $\beta \leq \beta^* \simeq 0.525$ is equivalent to the condition

$$\Delta \geq \Delta^* \simeq \frac{0.644\,36}{r}.$$

In light of Remark 4, the above Proposition asserts that for any given $\Delta \in [\Delta^*, \infty)$, there exists a corresponding Equilibrium B such that the monopolist chooses to expand

market in large steps (implying that some bunching occurs in all periods among the new customers with low θ). In such equilibrium, we obtain that $\gamma^*(\beta)$ is the the greatest real root in the interval $\left[0, \frac{1}{2-\beta}\right]$ of the above seventh-order polynomial in γ. Correspondingly, consumers expectations $\lambda^*(\beta)$ satisfies condition (35).

From Propositions 1 and 2, it follows that there exist two contrasting Markov-Perfect equilibria when the length of time between two different contractual offers is not too small. The intuition behind this multiplicity of equilibria is as follows. Why is it not always optimal for the monopolist to cover instantaneously the market by offering the Mussa-Rosen static equilibrium schedule in the initial period? It is indeed optimal when the consumers expect the firm to do so because, in that case, $\lambda = 0$ (and thus marginal customers do not require any rents to buy immediately). In that case, the firm's profit is equal to its full commitment profit. But when the customers, on the contrary, expect market expansion to be gradual, they always ask for positive rents to buy the good for the first time ($\lambda > 0$). This may dissuade the firm to expand the market too quickly. This is indeed the case when β is not too large (i.e., Δ is not too small). Notice that, at Equilibrium B, $\lambda^*(\beta)$ is a hump-shaped function of β: when β is too high, the corresponding low value of λ is not sufficient to dissuade the monopolist from selling immediately to all customers (because the commitment period is very short in this case, making it profitable to get full information on the customers' exact willingness to pay from the very beginning).

4.3.2 Profits, Consumer Surplus and Welfare: Equilibrium Comparisons and Policy Implications

Since, for sufficiently low values of β, there exist two MPE, it may be interesting to compare the profits, the welfare and the consumers surplus at these equilibria. Investigating how these values are affected by a decrease in the length of the commitment period Δ will enable us to derive some interesting policy implications regarding the recent discussions on legislations that regulate the collection, storage, and trading of data on consumers information.

The first step, however, is to compare the equilibrium values of λ, since it may help to understand the multiplicity of equilibria. At Equilibrium A, we have that $\lambda^*(\beta)$ is identically equal to zero, $\forall \beta \in [0, 1]$. In contrast, at Equilibrium B, which exists for all β in the interval $[0, \beta^*]$, we have that $\lambda^*(\beta)$ varies with β, as pictured below in Figure 1.

As mentioned earlier, in the case of Equilibrium B, $\lambda^*(\beta)$ is a hump-shaped function of β where $\beta \in [0, \beta^*]$.

Insert Figure 1 here with the caption below

Figure 1. $\lambda^*(\beta)$ at Equilibrium B

From straightforward computations, we can express profits as a function of β, with $\Pi(\beta)$ being equal to

$$\frac{\begin{array}{c} 1 + \beta + \beta\gamma[-3 + 3\beta + 8\gamma + \beta(-13 + 3\beta)\gamma+ \\ (-11 + \beta(12 + (\beta - 4)\beta))\gamma^2 + 6(1 - \beta)\beta\gamma^3- \\ \beta(-2 + (\beta - 3)(-1 + \beta)\beta)\gamma^4] \end{array}}{12r(1 - \beta\gamma^2)(1 - \beta\gamma^3)}, \tag{41}$$

where $\gamma = 0$ at Equilibrium A and γ is the solution of (40) at Equilibrium B.

Figure 2 shows that the monopolist's profits are increasing in β (i.e., decreasing in Δ) at both equilibria: profits are greater, the smaller is the length of time, Δ, between two contractual offers. Since the firm is eager to be able to quickly extract all old customers' surplus at the end of their initial period of purchase, this negative relationship between profit and Δ is not a surprising result, but it is worth pointing out that it is clearly a non-Coasian feature. We also learn from Figure 2 that the profit is greater at Equilibrium A than at Equilibrium B. This is to be expected, since the former corresponds the monopolist's full commitment profit.[22]

Insert Figure 2 here with the caption below

Figure 2. Equilibrium profits (Eq. A *vs* Eq. B)

The monotone decreasing relationship between profit and Δ shows that the monopolist has strong incentives to set the value of Δ, the length of the commitment period, to its minimum feasible value, which we will denote as $\underline{\Delta}$. The parameter $\underline{\Delta}$ can be interpreted as the time needed, given the state of the information collection technology and the

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[22]Notice however that the difference between the two is always smaller than 10%.

strength of the legislation on consumers' data protection, to uncover customers' exact WTPs after their first purchases. Arguably, legislations on the protection of consumers privacy have the effect of increasing the minimum feasible value $\underline{\Delta}$, thus hurting firms' profits. Our theoretical result on the negative relationship between Π and Δ seems to be supported by the empirical finding that many US firms[23] would be so much hurt by the E.U.'s General Data Protection Regulation (GDPR) policy that they prefer to block European users entirely from accessing their web sites rather than to comply with it.[24]

The next step is to consider aggregate consumer surplus (CS). From straightforward computations, we can express it as a function of β, with $CS(\beta)$ being equal to:

$$\frac{1 + \beta\gamma\left\{9 + \gamma\left[6\gamma - 19 + \beta\left[3 + \gamma(3 - \beta + 3\left(\beta - 4\right)\gamma + (12 + (\beta - 6)\beta)\gamma^2)\right]\right]\right\}}{24r(1 - \beta\gamma^2)(1 - \beta\gamma^3)} \times (1 - \beta)$$

where $\gamma = 0$ at Equilibrium A and γ is the solution of (40) at Equilibrium B. Alternatively, $CS(\beta)$ may be written as

$$\frac{1 + 9\beta\gamma + (3\beta - 19)\beta\gamma^2 + (\beta(3 - \beta) + 6)\beta\gamma^3 + 3(\beta - 4)\beta^2\gamma^4 + (12 + (\beta - 6)\beta)\beta^2\gamma^5}{24r(1 - \beta\gamma^2)(1 - \beta\gamma^3)}(1 - \beta)$$

$$(42)$$

For both MPEs, we find that the aggregate consumer surplus is a decreasing function of β (i.e., an increasing function of Δ). This is not surprising since the consumers are able to derive a positive surplus only as new customers. Due to the monopolist's ability to track consumers' willingness to pay for the good, the entire surplus of old customers is completely extracted by the monopolist. Moreover, as shown in Figure 3 below, the aggregate consumer surplus is always greater at Equilibrium B (depicted in green). This is because at Equilibrium A, the bottom 50% of customers never have any surplus.

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[23]See for instance Lanxon, Nate (25 May 2018). "Blocking 500 Million Users Is Easier Than Complying With Europe's New Rules" https://www.bloomberg.com/news/articles/2018-05-25/blocking-500-million-users-is-easier-than-complying-with-gdpr

[24]Additionally, recall that, according to Proposition 2, there is a unique threshold value $\Delta^* > 0$ such that if $\Delta < \Delta^*$ then Equilibrium B ceases to exist. Since firms prefer equilibrium A to equilibrium B, any relaxation of regulations which enable $\underline{\Delta}$ to fall below the threshold Δ^* would be most welcome by firms.

Insert Figure 3 here with the caption below

Figure 3. Expected consumers' surplus (Eq. A *vs* Eq. B)

Our above findings concerning profits and consumer surplus show that the monopolist's and the consumers' interests are diametrically opposed. First, the firm is better off at Equilibrium A than at Equilibrium B while the opposite ranking holds for the consumers. Second, the monopolist's profit is decreasing in Δ whereas the aggregate consumer surplus is increasing in Δ. Any regulation policy on firms' collection of data on consumers, such as the EU's General Data Protection Regulation (GDPR) policy, which ends up affecting in an important way the length of time over which firms remain unable to completely identify their consumers' tastes, in sofar as it raises Δ, is clearly beneficial to customers but detrimental to the firms.

As is well known, while the EU has passed a rather restrictive regulation policy on firms' data pertaining to consumers, in the USA, there is no equivalent federal law and experts seem to agree that, if a similar law is to be passed, it will certainly have to be much more lenient than the European one.[25] Apart from the well known philosophical differences between the two political landscapes, our model offers an economic rationale behind these diverging legislations. Namely, as online industries and tech-companies (such the GAFA), are much more often US-based than European-based, it is natural that, on matters relating to customers information, the EU cares more about consumers' interests while the US politicians care more about firms' interests. Accordingly, consistent with our analysis, the EU is more readily inclined to pass restrictive data protection legislations.

Consider finally social welfare. For the moment, let us temporarily adopt the usual definition of welfare, namely, the sum of the discounted streams of consumers' surplus and profit. We can then express welfare as a function of β, with $W(\beta)$ being equal to:

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[25]A U.S. law wouldn't simply copy the GDPR, according to Eduardo Ustaran, codirector of the privacy practice at law firm Hogan Lovells. "Privacy and data protection are fundamental rights from the EU perspective but not in the U.S.," he says. "*That is a major philosophical difference between the two jurisdictions, and that will be reflected in the law.*" (See David Meyer (2018), "In the Wake of GDPR, Will the U.S. Embrace Data Privacy?" Fortune, November 29, 2018.)

$$\frac{3 + \beta + 3\beta(1-\beta)\gamma - 3(1-\beta)\beta^2\gamma^2 + \left[24\beta + 3\beta^2(\beta-5) - 16\right]\beta\gamma^3}{24r(1-\beta\gamma^3)} \tag{43}$$

where $\gamma = 0$ at Equilibrium A and γ is the solution of (40) at Equilibrium B.

Using our eq. for welfare, we find that, over the range of values of β for which both equilibria exist, i.e., $0 \leq \beta \leq \beta^* \simeq 0.525$, the social welfare under Equilibrium A is greater than that under Equilibrium B only if the length of time between two contractual offers is very long (i.e., if β is close to zero). Indeed, $W^A(\beta)$ is greater than $W^B(\beta)$ for $\beta \in [0, 0.0765631]$ but it is smaller than $W^B(\beta)$ for $\beta \in [0.0765631, 0.525]$.[26] This is pictured in Figure 4 below for $r = 1$, where we can see that for $\beta \in [0, 0.0765631]$, the welfare differential is quite small. However, as is quite obvious looking at Figure 4 below, comparing the social welfare levels at the two equilibria over the range of parameter values over which both equilibria exist is not the whole story. When β is greater than 0.525 (approximately), only equilibrium A exists and for all values of $\beta \in [0.652607, 1]$, it turns out that $W^A(\beta)$ is greater than the highest possible welfare level for Equilibrium B, for $\beta \in [0, 0.525]$.[27]

Insert Figure 4 here with the caption below

Figure 4. Equilibrium Social Welfare (Eq. A *vs* Eq. B)

Clearly, the above findings about welfare are subject to several qualifications. First, the above definition of welfare assumes that consumers do not attach any value to privacy *per se.* Second, in the absence of non-distortionary transfers between "gainers" and "losers" (consumers *versus* monopolist or *vice-versa*), the procedure of "*treating a dollar as a dollar, no matter to whom it accrues*" has been criticized (see, e.g., Hendren, 2014; Saez

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[26] At $\beta = 0.0765631$, we have $\gamma = 0.488\,55$, and $W^A = W^B \simeq 0.128\,19/r$.

[27] Suppose we accept this welfare definition, then it follows that if in the absence of data protection regulation, the minimum period length (given the state of technology), say $\underline{\Delta}$, that permits the monopolist to uncover consumers tastes corresponds to some β in the interval $[0.652607, 1]$ then any proposed legislation that has the effect of making $\underline{\Delta}$ somewhat longer (i.e., making β somewhat smaller) would reduce welfare, and thus it should not be adopted. If the corresponding β is the interval $[0, 0.525]$, it also follows that $\underline{\Delta}$ should not be lengthened, as welfare would be reduced. However, if the corresponding β is in the interval $(0.525, 0.652607)$, then passing a law that makes $\underline{\Delta}$ a bit longer, so that the coresponding β falls just below 0.525 might be welfare improving if it induces a switch from Equilibrium A to Equilibrium B.

and Stantcheva, 2013). This criticism gains even more weight when the owners of the firm do not reside in the same tax jurisdiction (e.g. in our global economy, it is often the case that multinational firms with headquarters in the US serve a substantial fraction of consumers in other countries, which patronize different levels of consumer's privacy protection rules).

Indeed, one may argue that the fact that there are proportionately more US than European firms which may be hurt by legislations restricting access to consumer information is a good explanation of why potential consumer protection laws similar to the EU's General Data Protection Regulation (GDPR) policy are less likely to be passed in the US.

5 Commitment to Future Sales Plans

Suppose now that, contrary to what has been assumed in the preceding sections, the monopolist is able to commit, right from the beginning, to offer a sequence of pre-determined contracts.[28] This commitment capability is supposed to include the ability to commit both with regard to new customers and to former customers. Thus, at time $t = 0$, the monopolist announces (i) the *old-customer contracts* it will offer at any future period $n+j$ ($j = 1, 2, 3...$) to consumers who first purchase the good in period n; and (ii) *the new-customer contracts* to consumers who have not yet purchased the good (and consequently the firm cannot identify them since it has not yet had the possibility to collect data about their tastes).

The aggregate discounted profit (made by the monopolist who can commit) from period 0 is the sum of discounted profits from all periods:

$$\Pi^C \equiv \sum_{n=0}^{\infty} \beta^n \pi_n,$$

where, given Lemma 3,

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[28]The contracts proposed to a given consumer are, as in the non-commitment case, fixed (but now announced at time $t = 0$ and irrevocably committed to) for periods of fixed length Δ.

$$\begin{aligned}
\pi_n \;=\;& \frac{(1-\beta)(1-\theta_n^3)}{6r} \\
& + \int_{\frac{\theta_n+\beta\theta_{n+1}}{2}}^{\theta_n} \left\{ \left[\frac{(2\theta-\theta_n)^2}{2} \right] \left(\frac{1-\beta}{r} \right) - U_n(\theta_{n+1}) \right\} d\theta + \\
& \int_{\theta_{n+1}}^{\frac{\theta_n+\beta\theta_{n+1}}{2}} \left\{ \begin{array}{c} \left[(2\theta-\theta_n)(\beta\theta_{n+1}) - \frac{1}{2}(\beta\theta_{n+1})^2 \right] \left(\frac{1-\beta}{r} \right) \\ -U_n(\theta_{n+1}) \end{array} \right\} d\theta.
\end{aligned}$$

Proposition 3 *When the monopolist is able to commit to future contractual offers:*

(i) new customers with $\theta \in \left[\frac{1}{2}, 1 \right]$, are offered distorted qualities $q_n(\theta) = 2\theta - 1$, and prices $p_n(\theta) = \theta^2 - \frac{1}{4}$, $\forall n \geq 0$, while new customers with $\theta \in \left[0, \frac{1}{2} \right]$ are bunched and offered quality $q_n(\theta) = 0$ and price $p_n(\theta) = 0$, $\forall n \geq 0$, i.e; in all periods, new consumers face the same Mussa-Rosen static equilibrium schedule,

(ii) old consumers are offered personalized qualities $q_n(\theta) = \theta$ and prices $p_n(\theta) = \theta^2$, $\forall n \geq 0$, i.e; they face first-degree price discrimination.

Proof. See Appendix. ■

Accordingly, the full commitment equilibrium mimics Equilibrium A described above. Hence, when comparing the equilibrium with commitment to the no-commitment equilibria, either the results are the same (in the case of equilibrium A) or the monopolist benefits with the possibility of commitment (this is so, when we compare the commitment equilibrium to equilibrium B). In contrast, consumers either are indifferent or better-off when the monopolist loses the ability to commit to future prices.

6   Conclusion

We have developed a dynamic version of Mussa and Rosen (1978) product quality model, in which a monopolist sells over an infinite number of periods a vertically differentiated non-durable good to consumers. The monopolist has access to modern data collection technologies (e.g., cookies in online websites or datasets with consumers' previous purchase records), which allow him to track the exact preferences of consumers after their first purchase and subsequently use such information to make price-quality personalized offers that allow the monopolist to fully extract the consumers' surplus.

We have shown that there always exists a Markov-Perfect Equilibrium in which the monopolist offers to new customers the static Mussa-Rosen price/quality schedule (second-degree price discrimination) and all consumers purchase the good when it is launched (i.e., the market is immediately covered). Thereafter, in subsequent periods, old customers receive personalized price-quality offers, which result in a quality upgrade for all customers. Nonetheless, the personalized price-quality offers made to old customers leave them with no surplus, since the quality upgrade is accompanied by an increase in the price.

We have also proved that, if the length of time between two different contractual offers is not too small, there exists a second Markov-Perfect Equilibrium with substantially different economic properties. In particular, in this equilibrium, the market is only gradually covered. Instead of all consumers buying the good in the moment it is launched, in this second equilibrium, we get that subsequent cohorts of consumers are sequentially added each period. For each cohort of consumers entering the market, the highest types select a screening price-quality schedule, whereas the types at the bottom of the rung are bunched.

The intuition behind the multiplicity of equilibria follows from the fact that we are dealing with equilibria with self-fulfilling expectations. When consumers expect the monopolist to expand the market immediately, they conclude that they would not gain by delaying their first purchase, even though a subset of low-type consumers get no rent in the initial period. Given such consumers' expectations and the consumers' low informational rents, it is clearly in the interest of the monopolist to expand the market immediately, and thus consumers' expectations are fulfilled. An argument along the same line applies when consumers anticipate gradual expansion of the market. Though the two MPEs differ in a number of respects, they share a common feature: the monopolist's profits increase when the length of the commitment period decreases. This is a non-Coasian outcome but very intuitive: since the firm's profits are greater under first-degree than under second-degree price discrimination, the sooner it can exercise first-degree price discrimination, the better off the monopolist is. On the contrary, for equally intuitive reasons, the consumers are hurt by a reduction of the length of time between different offers.

Finally, we have argued that our model may shed some light on the regulatory divergence between the USA and the EU regarding consumers data protection. As the latter hurts firms and benefit consumers, and given that a disproportionate number of high-tech firms are US, it is natural that the EU is more inclined to defend its consumers' interests.

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APPENDIX

Proof of Claim 1

Incentive compatibility at $n + 1$ implies that if a consumer is of type θ_{n+1}, then reporting any $\theta' < \theta_{n+1}$ would not be optimal, i.e.,

$$\frac{1 - \beta}{r} \left(\theta_{n+1} q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1}) \right)$$

$$\geq \frac{1 - \beta}{r} \left(\theta_{n+1} q_{n+1}(\theta') - p_{n+1}(\theta') \right), \ \forall \theta' \leq \theta_{n+1}$$

And any type-θ consumer such that $\theta > \theta_{n+1}$ who deviates to be first-time consumers in period $n + 1$ would obtain a net utility at least as high as that of a consumer of type θ_{n+1} who is a first-time customer in period $n + 1$:

$$\frac{1 - \beta}{r} \left(\theta q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1}) \right)$$

$$\geq \frac{1 - \beta}{r} \left(\theta_{n+1} q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1}) \right), \forall \theta \geq \theta_{n+1}$$

Combining the two inequalities, we find that $\forall \theta \geq \theta_{n+1}$ and $\forall \theta' \leq \theta_{n+1}$,

$$\frac{1 - \beta}{r} \left(\theta q_{n+1}(\theta_{n+1}) - p_{n+1}(\theta_{n+1}) \right)$$

$$\geq \frac{1 - \beta}{r} \left(\theta_{n+1} q_{n+1}(\theta') - p_{n+1}(\theta') \right).$$

Consequently, $\widetilde{\theta}(\theta) = \theta_{n+1}$ for all $\theta \in [\theta_{n+1}, \theta_n]$. ∎

Proof of Lemma 1

From (6), (10) and (12), $\Delta_n(\theta)$ is a convex function. Accordingly, since $\Delta_n(\theta_{n+1}) = 0$, if follows that $\Delta_n(\theta) \geq 0 \ \forall \theta \in [\theta_{n+1}, \theta_n]$ iff $\Delta'_n(\theta_{n+1}) \geq 0$. Let us determine the sign of $\Delta'_n(\theta_{n+1})$. From (6) and (12),

$$\Delta'_n(\theta) = U'_n(\theta) - \beta \left(\frac{1 - \beta}{r} q_{n+1}(\theta_{n+1}) \right)$$

$$= \frac{1 - \beta}{r} q_n(\theta) - \beta \left(\frac{1 - \beta}{r} q_{n+1}(\theta_{n+1}) \right)$$

Evaluating $\Delta'_n(\theta)$ at $\theta = \theta_{n+1}$, we obtain

$$\Delta'_n(\theta_{n+1}) = \frac{1-\beta}{r}(q_n(\theta_{n+1}) - \beta q_{n+1}(\theta_{n+1})).$$

It follows that condition stated in Lemma 1 is necessary and sufficient for the participation constraint $\Delta_n(\theta) \geq 0$ (for all $\theta \in [\theta_{n+1}, \theta_n]$) to be satisfied.∎

Derivation of the equation (20) for Π_n^N

By integrating the last terms in eq (19) by parts, we obtain

$$- \int_{\theta_{n+1}}^{\theta_n} U_n(\theta)f(\theta)d\theta$$
$$=$$
$$- [U_n(\theta_n)F(\theta_n) - U_n(\theta_{n+1})F(\theta_{n+1})] + \int_{\theta_{n+1}}^{\theta_n} U'_n(\theta)F(\theta)d\theta$$

where the integral on the RHS is equal to

$$\int_{\theta_{n+1}}^{\theta_n} \left[\frac{1-\beta}{r}q_n(\theta)\right] \left(\frac{F(\theta)}{f(\theta)}\right) f(\theta)d\theta$$

Thus Π_n^N can be expressed as

$$\Pi_n^N = \int_{\theta_{n+1}}^{\theta_n} \left[\frac{1-\beta}{r}\left(\theta q_n(\theta) - c(q_n(\theta)) + \frac{F(\theta)}{f(\theta)}q_n(\theta)\right)\right] f(\theta)d\theta$$
$$- [U_n(\theta_n)F(\theta_n) - U_n(\theta_{n+1})F(\theta_{n+1})] \tag{44}$$

Now, the second term on the RHS of (44) can be written as

$$[U_n(\theta_{n+1})F(\theta_{n+1}) - U_n(\theta_{n+1})F(\theta_n)]$$
$$+ [U_n(\theta_{n+1})F(\theta_n) - U_n(\theta_n)F(\theta_n)]$$

which is equal to $-U_n(\theta_{n+1})\int_{\theta_{n+1}}^{\theta_n} f(\theta)d\theta - F(\theta_n)\int_{\theta_{n+1}}^{\theta_n} U'_n(\theta)d\theta$.

This is, in turn, equal to

$$- \int_{\theta_{n+1}}^{\theta_n} U_n(\theta_{n+1})f(\theta)d\theta - \int_{\theta_{n+1}}^{\theta_n} \left(\frac{F(\theta_n)}{f(\theta)}\right)\left[\frac{1-\beta}{r}q_n(\theta)\right] f(\theta)d\theta.$$

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After substitution, we obtain the equation (20) for Π_n^N.■

Specification of the profit from new customers $I\left[\Theta(n), \Theta(n+1)\right]$

From Lemma 3, this profit equals

$$I\left[\Theta(n), \Theta(n+1)\right] = \int_{\Theta(n+1)}^{\Theta(n)} \left\{ \begin{array}{c} \left[\frac{(2\theta-\Theta(n))^2}{2}\right]\left(\frac{1-\beta}{r}\right) \\ -\Omega(\Theta(n)) \end{array} \right\} d\theta, \tag{45}$$

if $\Theta(n+1) \geq \frac{\Theta(n)}{2-\beta} > \frac{\Theta(n)}{2}$, (i.e., if the market expansion proceeds by small steps), and it equals

$$I\left[\Theta(n), \Theta(n+1)\right]$$

$$= \int_{\frac{\Theta(n)+\beta\Theta(n+1)}{2}}^{\Theta(n)} \left\{ \left[\frac{(2\theta-\Theta(n))^2}{2}\right]\left(\frac{1-\beta}{r}\right) - \Omega(\Theta(n)) \right\} d\theta \tag{46}$$

$$+ \int_{\Theta(n+1)}^{\frac{\Theta(n)+\beta\Theta(n+1)}{2}} \left\{ \left[\begin{array}{c} (2\theta-\Theta(n))\beta\Theta(n+1) \\ -\frac{(\beta\Theta(n+1))^2}{2} \end{array} \right]\left(\frac{1-\beta}{r}\right) \\ -\Omega(\Theta(n)) \end{array} \right\} d\theta, \tag{47}$$

if $\Theta(n+1) \leq \frac{\Theta(n)}{2-\beta}$, (i.e., if the market expansion proceeds by large steps). Note that we have substituted $\Omega(\Theta(n))$ for $U_n(\theta_{n+1})$ because of the rational expectations requirement.■

Proof of Lemma 4.

The first-order condition for maximizing the right-hand side of (28) with respect to $\Theta(n+1)$, subject to the constraint $\Theta(n+1) \geq 0$, is obtained as

$$\frac{\partial I\left[\Theta(n), \Theta(n+1), q_n(.|\Theta(n))\right]}{\partial \Theta(n+1)} + \beta V'(\Theta(n+1)) + \kappa = 0, \tag{48}$$

where κ is the Kuhn-Tucker multiplier associated with the constraint $\Theta(n+1) \geq 0$, and where

$$\frac{\partial I\left[.\right]}{\partial \Theta(n+1)} = -\frac{1-\beta}{2r}\left(2\Theta(n+1) - \Theta(n)\right)^2 + \Omega(\Theta(n)), \tag{49}$$

if $\Theta(n+1) \geq \frac{1}{2-\beta}\Theta(n)$, and,

$$\frac{\partial I\,[.]}{\partial \Theta(n+1)} = -\frac{(1-\beta)}{r} \left\{ \begin{array}{c} (2\Theta(n+1) - \Theta(n))\,(\beta\Theta(n+1)) \\ -\frac{1}{2}(\beta\Theta(n+1))^2 \\ +\frac{\beta}{4}\,(\Theta(n) - (2-\beta)\,\Theta(n+1))^2 \end{array} \right\} \tag{50}$$

$$+\Omega(\Theta(n)), \tag{51}$$

if $\Theta(n+1) \leq \frac{1}{2-\beta}\Theta(n)$.

Notice that $\frac{\partial I\,[.]}{\partial \Theta(n+1)}$ is continuous at $\Theta(n+1) = \frac{1}{2-\beta}\Theta(n)$, where it is equal to $-\frac{(1-\beta)}{r}\left[\frac{\beta^2\Theta(n)^2}{2(2-\beta)^2}\right] + \Omega(\Theta(n))$.

Finally, differentiating the Bellman equation with respect to $\Theta(n)$ and using the envelope theorem, we obtain the value of $V'(\Theta(n))$, from which we deduce that

$$V'(\Theta(n+1)) = -\Omega(\Theta(n+1)) \tag{52}$$

$$+\int_{\Theta(n+2)}^{\Theta(n+1)} \left[\begin{array}{c} -(2\theta - \Theta(n+1))\left(\frac{1-\beta}{r}\right) \\ -\Omega'(\Theta(n+1)) \end{array} \right] d\theta, \tag{53}$$

when $\Theta(n+2) \geq \frac{1}{2-\beta}\Theta(n+1)$ (i.e., $\gamma \geq \frac{1}{2-\beta}$, the market expansion proceeds by small steps) and

$$V'(\Theta(n+1)) = -\Omega(\Theta(n+1)) \tag{54}$$

$$+\int_{\frac{\Theta(n+1)+\beta\Theta(n+2)}{2}}^{\Theta(n+1)} \left[\begin{array}{c} -(2\theta - \Theta(n+1))\left(\frac{1-\beta}{r}\right) \\ -\Omega'(\Theta(n+1)) \end{array} \right] d\theta + \tag{55}$$

$$\int_{\Theta(n+2)}^{\frac{\Theta(n+1)+\beta\Theta(n+2)}{2}} \left[\begin{array}{c} (-\beta\Theta(n+2))\left(\frac{1-\beta}{r}\right) \\ -\Omega'(\Theta(n+1)) \end{array} \right] d\theta,$$

when $\Theta(n+2) \leq \frac{1}{2-\beta}\Theta(n+1)$ (i.e., $\gamma \leq \frac{1}{2-\beta}$, the market expansion proceeds by large steps).

Notice that the two terms coincide when $\Theta(n+2) = \frac{1}{2-\beta}\Theta(n+1)$.

We now show that it is not optimal to expand the market by small steps. To show this, let us substitute for $V'(\Theta(n+1))$ in (48) using its value from (52). Then using the

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conjectured cut-off rule, we obtain the derivative of the right-hand side of the Bellman equation (28) wrt $\Theta(n + 1)$ as

$$\frac{\Theta(n)^2}{2r} \left[r\lambda\beta\gamma^2(4\gamma - 6) + (1 - \beta) \left(-1 + 4\gamma - 4\gamma^2 - 2\beta\gamma^3 + 2\beta\gamma^4 \right) \right], \qquad (56)$$

when $\gamma \geq \frac{1}{2-\beta}$. This expression is negative for all $\lambda \geq 0$ and $\gamma \in [\frac{1}{2-\beta}, 1]$, which implies that to select a cut-off rule such that $\gamma \geq \frac{1}{2-\beta}$ is never a best reply to a quadratic customers' expectations rule.[29] We conclude that the monopolist's best reply, γ, must belong to $[0, \frac{1}{2-\beta})$. Accordingly, the FOC of the Bellman equation is obtained by substituting for $V'(\Theta(n + 1))$ in (48) using its value from (54). Then, using the conjectured cut-off rule, we obtain the monopolist's best reply to the customers' expectations function:

$$\frac{\Theta(n)^2}{4r} \left[\begin{array}{c} 4r\lambda(1 + \beta\gamma^2(-3 + 2\gamma)) - (1 - \beta)\beta \\ (1 + (2\beta - 8)\gamma + (13 - 2\beta + \beta^2)\gamma^2 \\ +2\beta\gamma^3 + \beta(\beta - 4)\gamma^4) \end{array} \right] + \kappa = 0. \qquad (57)$$

■

Proof of Proposition 1

(i) Let $\gamma = 0$, $\forall\beta \in [0, 1]$. From (35), $\lambda = 0$.

(ii) Let $\lambda = 0$. (57) then becomes

$$\frac{\Theta(n)^2}{4r} \left[\begin{array}{c} -(1 - \beta)\beta(1 + (2\beta - 8)\gamma \\ +(13 - 2\beta + \beta^2)\gamma^2 + 2\beta\gamma^3 \\ +\beta(\beta - 4)\gamma^4) \end{array} \right] + \kappa = 0. \qquad (58)$$

At $\gamma = 0$, the bracketed term is negative, implying that $\gamma = 0$ corresponds at least to a local maximum. In order to show that this is also a global maximum, it is enough to notice that the monopolist's aggregate profit equals the one under full commitment which is the maximum that the firm may obtain. ■

Proof of Proposition 2

In the bracketed term of the LHS of (57), let us substitute for λ its value from (35); at any interior equilibrium[30] $\kappa = 0$, so that condition (40) must hold. This equation has two

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[29]More precisely, the negative value of (56) would imply, via (48), that $\kappa > 0$, i.e., $\Theta(n + 1) = 0$, which contradicts with $\Theta(n + 1) = \gamma\Theta(n) \geq \Theta/(2 - \beta)$.

[30]The only possible corner equilibrium is the one characterized in Proposition 1.

solutions $\gamma_1(\beta) \geq \gamma_2(\beta)$ between 0 and $\frac{1}{2-\beta}$ iff $\beta \leq 0.648628$, implying that no interior MPE can exist for values of $\beta > 0.648628$. The two solutions are pictured below in Figure 5.

Insert Figure 5 here with the caption below

Figure 5. Candidate interior equilibrium cut-off parameters

Only the greatest of the two, $\gamma_1(\beta)$ may correspond to a MPE. Let $H(\gamma, \beta, r\lambda)$ denote the bracketed term in the LHS of (57). Denote $R(\gamma, \beta)$ the RHS of (35). We obviously verify that

$$H(\gamma_i(\beta), \beta, R(\gamma_i(\beta), \beta)) = 0, i = 1, 2.$$

In order for $\gamma_i(\beta)$ to correspond to a local maximum of the monopolist's problem, it must be[31] in addition that

$$\frac{\partial H(\gamma_i(\beta), \beta, R(\gamma_i(\beta), \beta))}{\partial \gamma} \leq 0.$$

This never holds at $\gamma_2(\beta)$ where the derivative is always strictly positive, indicating that this solution corresponds to a local minimum. This always holds at $\gamma_1(\beta)$ where the derivative is negative, indicating that this solution corresponds to a local maximum. Below we picture as an illustration $H(\gamma, \beta, R(\gamma_2(\beta), \beta))$ in the case for $\beta = 0.4$ (and thus $\gamma_2(0.4) = 0.206063$) and then $H(\gamma, \beta, R(\gamma_1(\beta), \beta))$ also for the case $\beta = 0.4$. See Figures 6 and 7, respectively.

Insert Figure 6 here with the caption below

Figure 6. $H(\gamma, 0.4, 0.00286051)$

Insert Figure 7 here with the caption below

Figure 7. $H(\gamma, 0.4, 0.0123984)$

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[31]A local concavity condition.

That $\gamma_1(\beta)$ corresponds to a local maximum of the monopolist's problem does not ensure that it corresponds to a global maximum. Indeed, as shown for instance above for $\beta = 0.4$, there is another local maximum at $\gamma = 0$, where $H'_\gamma(0, \beta, , R(\gamma_1(\beta), \beta)) < 0$. We need to compare the firm's profits at these two values.

When the monopolist selects $\gamma = 0$, its profit is equal to

$$\Pi^A = \frac{1-\beta}{r}\frac{1}{12} + \frac{\beta}{r}\frac{1}{6} - R(\gamma_1(\beta), \beta). \tag{59}$$

When it chooses $\gamma = \gamma_1(\beta)$, its profit is

$$\Pi^B = \frac{\begin{array}{c} -1 + \beta + \beta\gamma(-3 + 3\beta + 8\gamma + \beta(-13 + 3\beta)\gamma+ \\ (-11 + \beta(12 + (\beta - 4)\beta))\gamma^2 + 6(1 - \beta)\beta\gamma^3- \\ \beta(-2 + (\beta - 3)(-1 + \beta)\beta)\gamma^4 \end{array}}{12r(1 - \beta\gamma^2)(1 - \beta\gamma^3)}. \tag{60}$$

Computations show that the second is greater than the first iff β is approximately greater than 0.525 and that, accordingly, candidate equilibrium B is indeed an MPE under this condition. ∎

Proof of Lemma 3

1. Point-wise maximization of Π^C with respect to the quality levels $q(\theta)$, subject to $q(\theta) \geq 0$, leads to

   $q_n^N(\theta) = q^{**}(\theta)$, $\forall n \geq 0$, where $q^{**}(\theta)$ is the solution of

   $$\theta - h(\theta) - c'(q^{**}(\theta)) \leq 0, \text{ with equality holding if } q^{**}(\theta) > 0.$$

   That is, all new customers are offered distorted quality levels (except the customers of type $\bar{\theta}$).

2. $q_n^O(\theta) = q^*(\theta)$, $\forall n \geq 0$, where $q^*(\theta)$ is the solution of (1), i.e., to each type, the monopolist offers the corresponding first-best quality level.

Proof of Proposition 3

1. It is always optimal to offer quality θ and price θ^2 to old customers' types θ in order to maximize the gross surplus out of these customers and leave them with zero net surplus (first-degree discrimination). The old customers' types are known with certainty by the firm. There is no benefit after the initial purchasing period to consumers from misreporting their type. Accordingly the monopolist has no incentive whatsoever to leave positive net rents to old customers.

2. Point-wise maximization of Π^C with respect to the quality levels $q_n(\theta)$ for new customers, subject to $q_n(\theta) \geq 0$, leads to $q_n(\theta) = 2\theta - 1$, $\forall \theta \in \left[\frac{1}{2}, 1\right]$, $\forall n \geq 0$, and $q_n(\theta) = 0$, $\forall \theta \in \left[0, \frac{1}{2}\right]$, $\forall n \geq 0$, that is, all new customers are offered distorted quality levels (except the customers of type $\overline{\theta}$).

3. Equilibrium new customers' prices follow from the fact that the monopolist does not leave any rent to the lowest types customers.

FIGURES
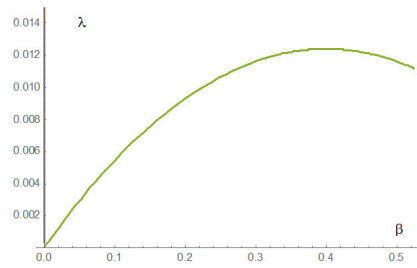
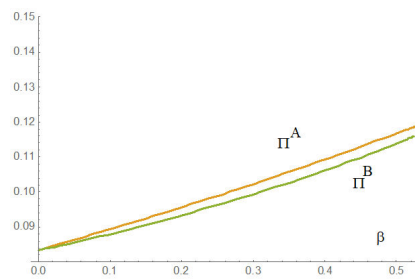Figure 1. $\lambda^*(\beta)$ at Equilibrium B

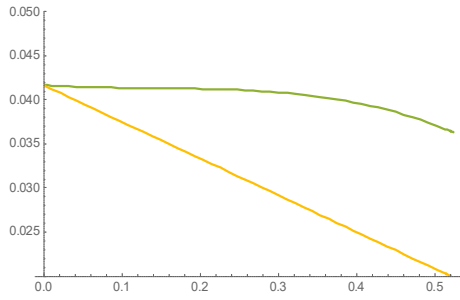Figure 2: Equilibrium profits (Eq. A vs Eq B)

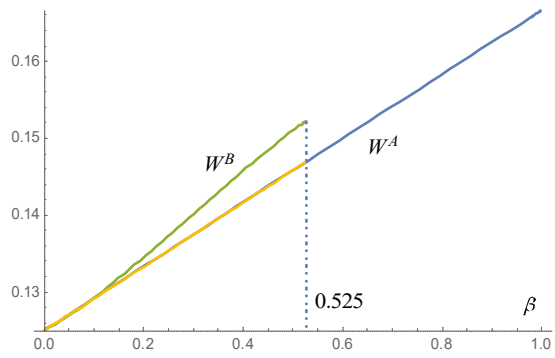Figure 3: Expected consumers' surplus (Eq. A vs Eq B)

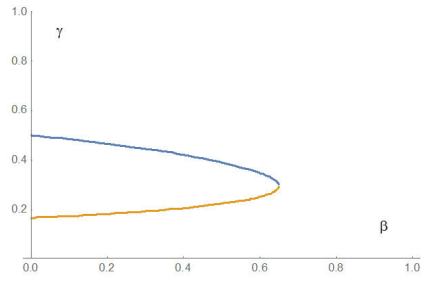Figure 4: Social Welfare at the two MPE (Eq. A vs Eq. B)

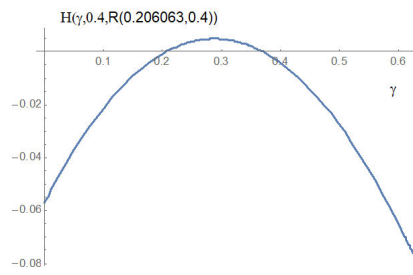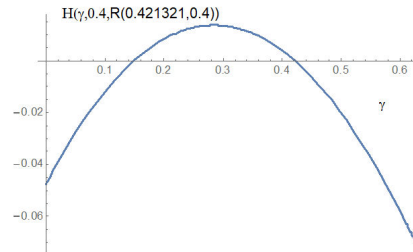Figure 5. Candidate interior equilibrium cut-off parameters

Figure 6. $H(\gamma, 0.4, 0.00286051)$

Figure 7. $H(\gamma, 0.4, 0.0123984)$