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ENDOGENOUS TIMING IN A MIXED DUOPOLY WITH VERTICALLY RELATED MARKETS*

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Abstract

We examine an endogenous timing game in a mixed oligopoly by focusing on the vertical linkages. Our main findings are as follows. First, under discriminatory input pricing, public (private) leadership emerges in a price-setting (quantity-setting) mixed oligopoly. This results contrast with one-tier mixed oligopoly, where a simultaneous-move in Bertrand competition (Bárcena-Ruiz, 2007) or a sequential-move with multiple equilibria in Cournot competition (Pal, 1998) emerges. Second, with downstream Bertrand competition, firms' profit and

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consumer surplus rankings are reversed between uniform and discriminatory input pricing. Finally, banning (allowing) price discrimination on imported inputs is socially desirable under downstream Bertrand (Cournot) competition.

Keywords: endogenous-timing, observable delay game, mixed duopoly, vertically related market, discriminatory input pricing
JEL Classification Codes: D21, H44, L13

I. *Introduction*

Most of the literature on oligopolistic competition treats the timing of firms' moves (simultaneous or sequential) as exogenous. Firms simultaneously choose quantity (resp. price) in a Cournot (resp. Bertrand) game; by contrast, one firm chooses first, and the other, having observed this, reacts to it in a Stackelberg game. As market outcomes differ depending on whether firms make quantity or price decisions simultaneously or sequentially, determining the timing of moves is important. In their seminal paper, Hamilton and Slutsky (1990) allowed firms to choose the order of moves in an observable delay game and showed that a sequential game occurs in equilibrium if both firms have upward-sloping reaction functions while a simultaneous game occurs if both have downward-sloping reaction functions. The issue of endogenous timing of moves becomes more prominent when applied to a mixed oligopoly in which state-owned public firm and private firms coexist.[1] This is because the differences in the objectives of public firm and private firms — social welfare maximizing and profit maximizing, respectively — may change firm behavior and hence, affect the timing of their moves.

Using an observable delay game, Pal (1998) demonstrated that when firms produce a homogenous good, they decide quantities sequentially in a mixed oligopoly. By contrast, Bárcena-Ruiz (2007) examined the same issue in a price-setting mixed oligopoly and demonstrated that simultaneous moves would occur in equilibrium.[2] More recently, by introducing various factors influencing market outcomes, the literature on endogenous timing in the mixed oligopoly has become richer and more diverse (Matsumura, 2003; Lu, 2006; Matsumura and Ogawa, 2010; Heywood and Ye, 2009; Bárcena-Ruiz and Garzon 2010; Bárcena-Ruiz and Sedano, 2011; Capuano and De Feo, 2010; Tomaru and Kiyono, 2010; Lee and Xu, 2018; Haraguchi and Hirose, 2018).

However, the existing literature assumes a simple one-tier market, and hence, has devoted

[1] According to Hirose and Matsumura (2019), in Japanese financial markets, both public leadership and private leadership emerge in different times. Until the 1980s, public firms had a leading role in the industry. A major change began to take place starting in 2000s, as the Koizumi Cabinet (2001-2006) pursued privatization of public firms under the slogan of "What the private can do must be entrusted to the private". Many of the government-owned firms became privatized and major public institutions were substantially downscaled during this period. That is, the public firms should play a complementary role to private firms, which can be regarded as the private leadership model. However, with the advent of new public financial institutions, public institutions recently began to lead Japanese markets again. In addition, in some developing countries (e.g., China), the public firm is much stronger than the private firms, in which situation the public firm is likely to be the leader in the market (Pi et al., 2018).

[2] The results of both Pal (1998) and Bárcena-Ruiz (2007) on mixed oligopoly are in sharp contrast to those for a private oligopoly, where quantity competition yields simultaneous-move (Cournot equilibrium) and price competition yields sequential-move choice (Stackelberg equilibrium).

scant consideration to the impact of vertical linkage in a two-tier market on market outcomes. The vast majority of products are manufactured in multiple production stages of the so-called vertical production chain from raw material to final product. Indeed, in developing countries, coexistence of public and private firms producing final goods from a downstream sector is commonly observed and many manufacturers in the downstream sector depend on imported inputs in the production process of final goods. This could be due to the higher quality of foreign inputs with embedded technology or because certain foreign inputs may not be perfectly substitutable by domestic inputs. Such inputs may include natural resources (e.g., petroleum, natural gas, steel, coal) and transport equipment (e.g., commercial aircraft, mass rapid transit, or railway). For example, in the domestic liquefied natural gas (LNG) market in Korea, the state-owned KOGAS competes with two private firms, POSCO and SK E&S, while the LNG market mostly depends on imports. The Japanese gas market, which is also dependent on imports, can be regarded as a mixed oligopoly consisting of the LP gas service and city-gas service, where city-gas prices and service areas are heavily regulated by the government (Satoh, 2015). As in other developing countries, India's aviation market is a typical mixed oligopoly. India has three full-service airlines: the state-owned Air India and two privately owned airlines, Jet Airways and Tata SIA Airlines. These Indian airlines import more than 70% of their aircraft from Airbus.

In this study, we examine the endogenous order of moves in a mixed oligopoly by focusing on the interaction between upstream and downstream markets. To do this, we consider a vertically-related industry with one foreign upstream manufacturer and two domestic downstream firms — one public and one private — producing differentiated final goods. The upstream foreign monopolist produces an essential intermediate-input that is sold to both domestic firms downstream. The trade between the upstream and downstream firms is conducted through either uniform pricing or discriminatory pricing by the upstream supplier, implying that the upstream monopolist has all the bargaining power in the market.

The main findings of our paper are as follows. First, firms' order of moves depends on whether the foreign upstream supplier uses uniform or discriminatory pricing. Adoption of discriminatory input pricing under downstream Bertrand (resp. Cournot) competition leads to public (resp. private) leadership as a sequential game in equilibrium. These results sharply contrasts with those of a one-tier mixed oligopoly, in which a simultaneous-move equilibrium emerges in a price-setting (Bárcena-Ruiz, 2007) and sequential-move[3] multiple equilibria in a quantity-setting (Pal, 1998) mixed oligopoly. The first finding is largely related to the fact that with input price discrimination under downstream Bertrand competition, the upstream monopolist handicaps the private firm by charging higher input price but subsidizes the public firm by charging a lower input price than uniform price (i.e., private firm's relative cost handicap against the public firm). Intuitively, this can be explained via the shape of derived demand for input of each firm.

It is well known that the cost pass-through ratio is lower for the oligopolistic firm than for the firm in the competitive market. In our model, if firms face input price increase, the profit maximizing private firm, which is forced to absorb the cost increase at the expense of its own profits, will pass only a fraction of the rise in input price to the retail price (i.e., low pass-

[3] Matsumura and Ogawa (2010) used the concept of risk dominance suggested by Harsanyi and Selten (1988) to show that private leadership is robust when firms compete in terms of quantity in a one-tier mixed duopoly.

through ratio). On the other hand, welfare maximizing public firm, taking into consideration the effects on consumer surplus and private firms' profits, sets its price close to marginal cost, and the increase in input price is simply passed on to the retail price (i.e., high pass-through ratio). As a result, the decrease in final-product sales (input demand under fixed coefficient production function) due to the higher input price is greater for the public firm than for the private firm, which in turn implies that the derived demand curve of the public firm for input is flatter than that of the private firm: i.e., public firm's input demand function is more elastic than the private firm's input demand function. Under discriminatory input pricing, the upstream monopolist offers a lower input price to the public firm to encourage and a higher input price to the private firm to discourage aggressive behavior, thereby earning higher profits than otherwise. In addition, the cost handicap of the private firm is greatest when it is the leader and smallest when it is the follower. The private firm, in order to reduce this cost handicap, strategically chooses to be the follower in the market, which results in public leadership in equilibrium.[4]

Second, the standard rankings for firm's profits and consumer surplus under a one-tier mixed oligopoly are reversed when discriminatory input pricing is adopted in a two-tier mixed oligopoly. Third, upstream supplier's nationality does make a difference for the timing of moves by firms. With a domestic upstream manufacturer, the endogenous order of moves in the downstream mixed oligopoly is consistent between a two-tier and a standard one-tier mixed oligopoly in the downstream market, as long as nonnegative profit constraint is introduced when downstream market is characterized by Cournot competition.

The contributions of this paper are two-fold. First, our results show the importance of interaction between upstream and downstream markets in determining the timing of moves in a mixed oligopoly. We demonstrate that the standard conclusion of the order moves in a one-tier mixed oligopoly is reversed in the two-tier mixed oligopoly when discriminatory input pricing is adopted by upstream firms. Our analysis complements the literature on first- and second-mover advantages (Lee et al., 2017; Amir and Jin, 2001; Amir and Stepanova, 2006; Hoppe, 2000) by analyzing the role of downstream firms in the leader-follower relationship in vertically related markets. Second, our analysis has important implications for competition policy. Price discrimination has long been a contentious issue in competition policy. In many countries, price discrimination is prohibited in the market by government regulations, which forbid dominant firms from charging different buyers different prices for the same product. According to our analysis, the competition mode[5] (i.e., Cournot or Bertrand) in the downstream market matters in determining whether input price discrimination by the foreign upstream manufacturer should be banded as a matter of policy. Banning price discrimination for imported inputs is desirable

[4] With downstream Cournot competition, the private firm has a cost handicap only when it is the follower (i.e., public firm leadership). The private firm, by strategically choosing to be the leader in the market, can remove the cost handicap in input prices and increase its profits than otherwise. The public firm takes the strategy to be the follower, because it makes the private firm choose a larger output, which is also welfare-improving. Therefore, private leadership emerges in equilibrium under downstream Cournot competition.

[5] There are two most common models to describe the strategic interaction between firms in an oligopolistic market: that is, Cournot competition and Bertrand competition. The main difference between the two is whether firms compete in terms of quantity or price. Examples of Cournot competition would be petroleum and natural gas, chemicals, textile, aircraft, shipping containers, and healthcare industry. Examples of Bertrand competition would be airlines, tobacco products, cell phone services, pharmaceutical products, and most of personal service industries. On the other hand, Bloomfield (2018) categorizes 48 industries as Cournot versus Bertrand using three different measures for the mode of competition.

when downstream firms compete à la Bertrand. With the enforcement of uniform pricing for imported inputs, a simultaneous-move equilibrium emerges, leading to higher social welfare and firm profits compared to the public-leadership scenario with discriminatory input pricing. By contrast, government intervention in input pricing is unnecessary with downstream competition à la Cournot. Allowing discriminatory input pricing by the foreign upstream monopolist in effect blocks the possibility of realizing multiple equilibria with public leadership, which is socially less efficient than private leadership.

The remainder of the paper is organized as follows. Section II outlines a simple two-tier mixed duopoly model. Section III examines three types of fixed-timing games in a mixed duopoly: a simultaneous game, public leadership, and private leadership. Section IV compares the market outcomes of these three subgames for both uniform and discriminatory input pricing. Section V analyzes endogenous timing in a mixed-duopoly observable delay game, while Section VI applies the analysis to quantity competition in the downstream market. In Section VII, we examine whether the upstream firm's nationality makes a difference by analyzing a case where the upstream supplier is a domestic firm. Finally, Section VIII provides our concluding remarks.

II. *Model*

Consider a vertical market structure consisting of an upstream foreign monopolist, denoted by firm M, and two downstream firms, denoted by firm 0 and firm 1, respectively. The upstream firm produces an intermediate-input with zero marginal production cost and exports these intermediate-inputs to the downstream firms domestically. The domestic downstream industry is a mixed oligopoly, where one public firm (firm 0) and one private firm (firm 1) produce differentiated final products solely for sale to domestic consumers. The public firm's objective is to maximize social welfare, whereas the private firm's objective is to maximize its own profits. The production technology that links the upstream and downstream manufacturing is one of fixed proportions: one unit of intermediate input is required to produce one unit of final product.

On the consumption side, there is a continuum of consumers of the same type whose utility function is linear and separate in the numeraire good. The representative consumer maximizes $U-p_0x_0-p_1x_1$, where $x_i\geq 0$, $i=0, 1$, is the amount of good i and p_i its price. The function U is assumed to be quadratic, strictly concave and symmetric in x_0 and x_1: $U=a(x_0+x_1)-\frac{[(x_0)^2+2bx_0x_1+(x_1)^2]}{2}$, where parameter a is a positive constant, and $b\in(0, 1)$ denotes the degree of product substitutability, that is, the higher the value of b, higher will be the degree of substitutability between products. Given this utility function, the direct and indirect demand functions for good i can be derived as follows:

$$x_i=\frac{a(1-b)-p_i+bp_j}{1-b^2},\ p_i=a-x_i-bx_j;\ i, j=0, 1,\ i\neq j, \tag{1}$$

The profit function for each downstream firm is given by

$$\pi_i(\mathbf{p};\ w_i)=(p_i-w_i-c_i)x_i(\mathbf{p}), \tag{2}$$

where $\mathbf{p}\equiv(p_0, p_1)$, w_i is the price of intermediate-input charged to the downstream firm i, and c_i is the per-unit production cost of firm i. For simplicity, we assume that domestic firms have the same production technology, that is, $c_0=c_1=c$. By assuming zero marginal production cost in the intermediate-input production, the profit of the upstream manufacturer (firm M) is given by $\pi_M=\sum_{i=0,1} w_i x_i$. The social welfare for the domestic country is given by

$$W(\mathbf{p}, \mathbf{w})=\sum_{i=0,1} \pi_i(\mathbf{p}; w_i)+CS(\mathbf{p}) \tag{3}$$

where $\mathbf{w}\equiv(w_0, w_1)$. The first and second terms of the right-hand side (RHS) in Eq. (3) represent producer surplus and consumer surplus $CS=U-p_0x_0-p_1x_1$, respectively. We make the following assumption throughout the paper.

Assumption: $b\in(0, \bar{b})$, where $\bar{b}=\frac{2}{3}$ for the Bertrand competition and $\bar{b}=\frac{3}{4}$ for the Cournot competition in the downstream market.

Assumption requires that the closeness between products is not too high and guarantees that both domestic firms will produce a positive quantity[6] of the final-good in all cases under consideration. We use the observable delay game of Hamilton and Slutsky (1990), where firms first choose the timing of their actions. There are two possible time periods for action and each firm chooses its action in only one of the two periods. Our model involves three decision-making stages. In stage 1, each of the downstream firms simultaneously chooses whether to set its price in period 1 ($T=1$) or in period 2 ($T=2$). There are three possible regimes with respect to the order of firm moves based on the price decision — simultaneous-move, public firm leader, and private firm leader. If the two firms' choices are consistent — public firm (resp. private firm) chooses period 1 and the other chooses period 2, the basic game played is a sequential game with public firm (resp. private firm) as the leader. Otherwise, if both firms choose period 1 or period 2, they receive equilibrium payoffs in a simultaneous-move game. In stage 2, the upstream manufacturer (firm M) sets the price for inputs based on either discriminatory pricing or uniform pricing. In the last stage of the game, firms select their prices knowing when the other firm will make its price choice. Our objective is to solve the subgame perfect Nash equilibrium (SPNE) of this extended game with observable delay using backward induction.

[6] In discriminatory input price setting, if the products are sufficiently close substitutes, then there might be cases where the upstream monopolist charges an exorbitantly high input price to one of the downstream firms that it becomes inactive and the other downstream firm produces like a monopolist. In our model, as it turns out, if $b\geq\bar{b}$, upstream firm can oust away the private firm and let the public firm produce like a monopolist. We focus on the case where both firms produce positive output to see the strategic interaction of downstream firms in the mixed market (i.e., $b<\bar{b}$). But this assumption on b will be relaxed later and we show that the qualitative nature of our results remains unchanged irrespective of the value of b.

III. *Mixed Duopoly with Bertrand Competition*

We first examine three types of fixed-timing games in a mixed duopoly — simultaneous-move, public leadership, and private leadership. We then examine the endogenous timing game.

1. Simultaneous-Move Game

In the last stage of the game, the first order condition of each downstream firm is given as:

$$\frac{\partial W}{\partial p_0}=(p_0-c-w_0)\frac{\partial x_0}{\partial p_0}+(p_1-c-w_1)\frac{\partial x_1}{\partial p_0}=0\Leftrightarrow R_0(p_1;\ \mathbf{w})=(c+w_0)+b(p_1-c-w_1), \quad (4.1)$$

$$\frac{\partial \pi_1}{\partial p_1}=x_1+(p_1-c-w_1)\frac{\partial x_1}{\partial p_1}=0\Leftrightarrow R_1(p_0;\ w_1)=\frac{1}{2}[a(1-b)+bp_0+c+w_1], \quad (4.2)$$

where R_i is the reaction function of firm i. In Eq. (4.1), $(p_1-(c+w_1))\frac{\partial x_1}{\partial p_0}(>0)$ represents a profit-driven effect for the private firm. When the public firm chooses its price level, it takes into consideration the fact that an increase in its own price over its marginal cost will increase the private firm's output (and thus, profits too) via substitution effect, and hence, will increase social welfare. Therefore, the public firm under Bertrand competition[7] will set its price strictly higher than the marginal cost; $[p_0]_{Bertrand}=(c+w_0)+r(p_1-c-w_1)>(c+w_0)$. The intersection of the two reaction curves gives the equilibrium values of retail prices and quantities at this stage of the game (the superscript "S" denotes the simultaneous-move):

$$p_0^S(\mathbf{w})=\frac{ab(1-b)+2(c+w_0)-b(c+w_1)}{2-b^2},\ p_1^S(\mathbf{w})=\frac{a(1-b)+(1-b^2)(c+w_1)+b(c+w_0)}{2-b^2}, \quad (5.1)$$

$$x_0^S(\mathbf{w})=\frac{(a-c-w_0)-b(a-c-w_1)}{1-b^2},\ x_1^S(\mathbf{w})=\frac{(a-c-w_1)-b(a-c-w_0)}{(1-b^2)(2-b^2)}, \quad (5.2)$$

where $\left|\frac{\partial x_0^S}{\partial w_0}\right|=\frac{1}{1-b^2}>\frac{1}{(1-b^2)(2-b^2)}=\left|\frac{\partial x_1^S}{\partial w_1}\right|$, suggests that the derived demand for inputs of firm 0 is more elastic than the derived demand for the inputs of firm 1. The maximization problem of firm M in stage 2 is $\text{Max}_{w_0,\,w_1}\pi_M^S(\mathbf{w})\left(\equiv\sum_{i=0,\,1} w_i x_i^S(\mathbf{w})\right)$. By solving $\frac{\partial \pi_M^S}{\partial w_0}=\frac{\partial \pi_M^S}{\partial w_1}=0$ simultaneously, we obtain equilibrium input prices in the downstream simultaneous-move game as follows (the superscript "*" denotes the equilibrium under discriminatory input pricing):

$$\frac{(4-b-b^2)\Gamma}{8-5b^2+b^4}=w_0^{S*}<w_1^{S*}=\frac{(2-b^2)(2+b-b^2)\Gamma}{8-5b^2+b^4}, \quad (6)$$

[7] Under Cournot competition, the welfare-maximizing public firm chooses its own output, taking the rival's output as given. Therefore, the public firm behaves like a welfare maximizing monopolist, and will produce its output based on the traditional marginal cost pricing condition, that is, $[p_0]_{Cournot}=c+w_0$.

TABLE 1. MARKET EQUILIBRIUMS UNDER EACH REGIME: DOWNSTREAM PRICE COMPETITION

	Simultaneous Move	Public Leadership	Private Leadership	
			$b\in\left(0, \frac{2}{3}\right)$	$b\in\left[\frac{2}{3}, 1\right)$
Input Price	$w_0^{S*}=\frac{(4-b-b^2)\Gamma}{8-5b^2+b^4}$ $w_1^{S*}=\frac{(2-b^2)(2+b-b^2)\Gamma}{8-5b^2+b^4}$	$w_0^{Pub*}=\frac{(16-2b-18b^2+b^3+5b^4)\Gamma}{32-41b^2+13b^4}$ $w_1^{Pub*}=\frac{(16+4b-23b^2-4b^3+8b^4+b^5)\Gamma}{32-41b^2+13b^4}$	$w_0^{Pri*}=\frac{(1-b)(4+3b)\Gamma}{8-9b^2}$ $w_1^{Pri*}=\frac{2(1-b)(2+3b)\Gamma}{8-9b^2}$	$w_0^{Pri*}=\frac{\Gamma}{2}$ $w_1^{Pri*}=\frac{(2-b)\Gamma}{2}$
Output Price	$p_0^{S*}=\frac{(4+b-4b^2+b^4)a+(4-b-b^2)c}{(8-5b^2+b^4)}$ $p_1^{S*}=\frac{(6-b-4b^2+b^4)a+(2+b-b^2)c}{(8-5b^2+b^4)}$	$p_0^{Pub*}=\frac{\begin{bmatrix}(16+2b-23b^2-b^3+8b^4)a\\+(16-2b-18b^2+b^3+5b^4)c\end{bmatrix}}{(32-41b^2+13b^4)}$ $p_1^{Pub*}=\frac{\begin{bmatrix}(24-6b-31b^2+7b^3+10b^4-2b^5)a\\+(8+6b-10b^2-7b^3+3b^4+2b^5)c\end{bmatrix}}{(32-41b^2+13b^4)}$	$p_0^{Pri*}=\frac{(4+b-6b^2)a+(4-b-3b^2)c}{8-9b^2}$ $p_1^{Pri*}=\frac{(6-b-6b^2)a+(2+b-3b^2)c}{8-9b^2}$	$p_0^{Pri*}=\frac{a+c}{2}$ $p_1^{Pri*}=\frac{a(2-b)+bc}{2}$
Quantities	$x_0^{S*}=\frac{(4+b-b^2)\Gamma}{(1+b)(8-5b^2+b^4)}$ $x_1^{S*}=\frac{(2-b-b^2)\Gamma}{(1+b)(8-5b^2+b^4)}$	$x_0^{Pub*}=\frac{(2-b^2)(8+3b-5b^2-2b^3)\Gamma}{(1+b)(32-41b^2+13b^4)}$ $x_1^{Pub*}=\frac{(1-b)(2-b^2)(4+3b)\Gamma}{(1+b)(32-41b^2+13b^4)}$	$x_0^{Pri*}=\frac{(4-3b)(a-c)}{8-9b^2}$ $x_1^{Pri*}=\frac{(2-3b)\Gamma}{8-9b^2}$	$x_0^{Pri*}=\frac{a+c}{2}$ $x_1^{Pri*}=0$
Profit	$\pi_0^{S*}=b(1-b^2)x_0^{S*}x_1^{S*}$ $=\frac{b(4+b-b^2)(2-3b+b^3)\Gamma^2}{(1+b)(8-5b^2+b^4)^2}$ $\pi_1^{S*}=(1-b^2)(x_1^{S*})^2$ $=\frac{(1-b)^3(2+b)^2\Gamma^2}{(1+b)(8-5b^2+b^4)^2}$	$\pi_0^{Pub*}=\frac{b(1-b^2)}{(2-b^2)}x_0^{Pub*}x_1^{Pub*}$ $=\frac{b(1-b)^2(4+3b)(2-b^2)(8+3b-5b^2-2b^3)\Gamma^2}{(1+b)(32-41b^2+13b^4)^2}$ $\pi_1^{Pub*}=(1-b^2)(x_1^{Pub*})^2$ $=\frac{(1-b)^3(8+6b-4b^2-3b^3)^2\Gamma^2}{(1+b)(32-41b^2+13b^4)^2}$	$\pi_0^{Pri*}=bx_0^{Pri*}x_1^{Pri*}$ $=\frac{b(4-3b)(2-3b)\Gamma^2}{(8-9b^2)^2}$ $\pi_1^{Pri*}=(x_1^{Pri*})^2$ $=\frac{(2-3b)^2\Gamma^2}{(8-9b^2)^2}$	$\pi_0^{Pri*}=0$ $\pi_1^{Pri*}=0$
Welfare	$CS^{S*}=\frac{(10-7b^2+b^4)\Gamma^2}{(1+b)(8-5b^2+b^4)^2}$ $W^{S*}=\frac{(14-16b^2+7b^4-b^6)\Gamma^2}{(1+b)(8-5b^2+b^4)^2}$	$CS^{Pub*}=\frac{(2-b^2)^2(40+12b-55b^2-20b^3+19b^4+8b^5)\Gamma^2}{(1+b)(32-41b^2+13b^4)^2}$ $W^{Pub*}=\frac{\begin{pmatrix}224+16b-560b^2-50b^3+522b^4+57b^5\\-215b^6-28b^7+33b^8+5b^9\end{pmatrix}\Gamma^2}{(1+b)(32-41b^2+13b^4)^2}$	$CS^{Pri*}=\frac{(1-b)(10-9b^2)\Gamma^2}{(8-9b^2)^2}$ $W^{Pri*}=\frac{2(1-b)(7-9b^2)\Gamma^2}{(8-9b^2)^2}$	$CS^{Pri*}=\frac{\Gamma^2}{8}$ $W^{Pri*}=\frac{\Gamma^2}{8}$

Note: $\Gamma\equiv a-c$

where $\Gamma \equiv a-c$. Substituting w_0^{S*} and w_1^{S*} into market variables, we obtain the equilibrium values for those variables as summarized in Table 1.

2. Sequential-move Game: Public Firm Leadership

In this case, the public firm sets price level p_0 to maximize social welfare anticipating the private firm's response. The maximization problem of the public firm is $\mathrm{Max}_{p_0} W^{Pub}(p_0;\mathbf{w})$ $(\equiv W(p_0, R_1(p_0, w_1);\mathbf{w}))$, where superscript '*Pub*' denotes the public leadership and $R_1(p_0;w_1)$ is given by Eq. (4.2). Solving the first order condition of welfare maximization, we obtain

$$p_0^{Pub}(\mathbf{w}) = \frac{ab(1-b)+2(2-b^2)(c+w_0)-b(c+w_1)}{4-3b^2},$$
$$p_1^{Pub}(\mathbf{w}) = \frac{(2-2b-b^2+b^3)a+b(2-b^2)(c+w_0)+2(1-b^2)(c+w_1)}{4-3b^2}, \quad (7.1)$$

$$x_0^{Pub}(\mathbf{w}) = \frac{(2-b^2)^2(a-c-w_0)-b(3-2b^2)(a-c-w_1)}{(4-3b^2)(1-b^2)},$$
$$x_1^{Pub}(\mathbf{w}) = \frac{(2-b^2)\{(a-c-w_1)-b(a-c-w_0)\}}{(4-3b^2)(1-b^2)}. \quad (7.2)$$

When compared to the simultaneous case, $p_i^{Pub}(\mathbf{w}) < p_i^S(\mathbf{w}) (i=0, 1)$ holds. This implies that the welfare-maximizing public firm as the price leader can increase social welfare by setting a price lower than the price in the simultaneous-move game. The follower (private firm) will lower its price too because prices are strategic complements. In stage 2, the maximization problem of firm M is $\mathrm{Max}_{w_0,w_1} \sum_{i=0,1} w_i x_i^{Pub}(\mathbf{w})$, where $x_i^{Pub}(\mathbf{w})$ is given by Eq. (7.2). By solving $\frac{\partial \pi_M^{Pub}}{\partial w_0} = \frac{\partial \pi_M^{Pub}}{\partial w_1} = 0$ simultaneously, we obtain the equilibrium input prices under public leadership as follows:

$$\frac{(16-2b-18b^2+b^3+5b^4)\Gamma}{32-41b^2+13b^4} = w_0^{Pub*} < w_1^{Pub*} = \frac{(16+4b-23b^2-4b^3+8b^4+b^5)\Gamma}{32-41b^2+13b^4}, \quad (8)$$

Equilibrium input prices w_i^{Pub*} will lead to the equilibrium market outcomes under downstream public leadership in a sequential-move game as summarized in Table 1.

3. Sequential-move Game: Private Firm Leadership

In this game, the maximization problem of the private firm is $\mathrm{Max}_{p_1} \pi_1^{Pri}(p_1;\mathbf{w})$ $(\equiv \pi_1(p_1, R_0(p_1;\mathbf{w}); w_1))$, where $R_0(p_1;\mathbf{w})$ is given in Eq. (4.1). Solving $\frac{\partial \pi_1^{Pri}}{\partial p_1} = 0$ gives the following (superscript "*Pri*" denotes private leadership):

$$p_0^{Pri}(\mathbf{w}) = \frac{ab(1-b)+(2-b^2)(c+w_0)-b(c+w_1)}{2(1-b^2)},$$
$$p_1^{Pri}(\mathbf{w}) = \frac{a(1-b)+(1-2b^2)(c+w_1)+b(c+w_0)}{2(1-b^2)}, \quad (9.1)$$

$$x_0^{Pri}(\mathbf{w})=\frac{(a-c-w_0)-b(a-c-w_1)}{1-b^2},\ x_1^{Pri}(\mathbf{w})=\frac{(a-c-w_1)-b(a-c-w_0)}{2(1-b^2)}, \qquad (9.2)$$

In contrast to the simultaneous case, we get $p_i^{Pri}(\mathbf{w})>p_i^{S}(\mathbf{w})$, implying that the private firm tends to set a higher price than that in the simultaneous-move game when it is the price leader. This reflects the fact that that private firms, profit maximizers, aim to reduce market competition because they are profits maximizers. As in the previous case, the maximization problem of the upstream manufacturer is $\mathrm{Max}_{w_0,\,w_1}\sum_{i=0,\,1} w_i x_i^{Pri}(\mathbf{w})$, where $x_i^{Pri}(\mathbf{w})$ is given by Eq. (9.2). By solving $\frac{\partial \pi_M^{Pri}}{\partial w_0}=\frac{\partial \pi_M^{Pri}}{\partial w_1}=0$ simultaneously, we obtain equilibrium input prices under private leadership as follows:

$$\frac{(1-b)(4+3b)\Gamma}{8-9b^2}=w_0^{Pri*}<w_1^{Pri*}=\frac{2(1-b)(2+3b)\Gamma}{8-9b^2}, \qquad (10)$$

Using $w_0^{Pri*}$and w_1^{Pri*}, we obtain the equilibrium market outcomes under downstream private leadership, as shown in Table 1. Here, it should be noted that the equilibrium output of the private firm is positive $\left(x_1^{Pri*}=\frac{(2-3b)\Gamma}{8-9b^2}>0\right)$ only when $b<\frac{2}{3}$, implying that if the goods are close substitutes $\left(\text{i.e., } b\in\left[\frac{2}{3},\ 1\right)\right)$, then only the public firm will serve the market because $x_1^{Pri*}=0$.

The following lemma regarding equilibrium input prices (Eqs. 6, 8, and 10) is immediate.

Lemma 1: (1) $w_1^{I*}>\frac{a-c}{2}>w_0^{I*}$ for $I=S,Pub,Pri$,

(2) $w_1^{Pri*}>w_1^{S*}>w_1^{Pub*}$ and $\Delta w^{Pri*}>\Delta w^{S*}>\Delta w^{Pub*}>0$, where $\Delta w^{I*}\equiv w_1^{I*}-w_0^{I*}$.

Proof: The proof is easily obtained with simple calculations based on Eqs. (6), (8), and (10).

Lemma 1(1) implies that the upstream supplier charges the private firm a higher input price and the public firm a lower price compared to $\frac{a-c}{2}$[8]; that is, the price-discriminator (upstream supplier) handicaps the private firm while subsidizing the public firm through input pricing. This is explained by the fact that the derived demand for the inputs of firm 0 is more sensitive than that of firm 1 to changes in its own input price and in the input price charged to the rival firm (private firm); that is, $\left|\frac{\partial x_0^I}{\partial w_0^I}\right|>\left|\frac{\partial x_1^I}{\partial w_1^I}\right|$ and $\left|\frac{\partial x_0^I}{\partial w_1^I}\right|>\left|\frac{\partial x_1^I}{\partial w_0^I}\right|$ for $I=S,Pri,Pub$. In this case, the upstream monopolist wishes the downstream public firm to behave aggressively while the private firm to behave less aggressively in the market. The upstream monopolist offers a lower input price to the public firm to encourage and a higher input price to the private firm to discourage aggressive behavior, thereby earning higher profits than otherwise.

The cost handicap of the private firm is greatest when it is the price leader and smallest

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[8] $\frac{a-c}{2}$ is the input price level under uniform input pricing, which will be discussed later immediately.

when it is the follower (Lemma 1(2)). The derived demand for the inputs of the private firm, when it is the price leader, is inelastic with respect to the input price (w_1), in contrast to the simultaneous-move case; that is, $\left|\frac{\partial x_1^{Pri}}{\partial w_1}\right| = \frac{1}{2(1-b^2)} < \frac{1}{(1-b^2)(2-b^2)} = \left|\frac{\partial x_1^S}{\partial w_1}\right|$.[9] This leads to $w_1^{Pri*} > w_1^{S*}$ because w_1 has an identical effect on the public firm's output in both private leadership and the simultaneous-move case $\left(\left|\frac{\partial x_0^{Pri}}{\partial w_1}\right| = \left|\frac{\partial x_0^S}{\partial w_1}\right| = \frac{b}{(1-b^2)}\right)$. As in private leadership, the derived demand for the inputs of the private firm, when the public firm is the price leader, is inelastic with respect to w_1, in contrast to the simultaneous-move case $\left(\text{i.e., } \left|\frac{\partial x_1^{Pub}}{\partial w_1}\right| = \frac{2-b^2}{(1-b^2)(4-3b^2)} < \frac{1}{(1-b^2)(2-b^2)} = \left|\frac{\partial x_1^S}{\partial w_1}\right|\right)$, which tends to $w_1^{S*} < w_1^{Pub*}$. By contrast, the public firm's demand for inputs is less sensitive to a change in w_1 with the public firm as the price leader compared to simultaneous-move case $\left(\text{i.e., } \left|\frac{\partial x_0^{Pri}}{\partial w_1}\right| = \frac{b(3-2b^2)}{(1-b^2)(4-3b^2)} < \frac{b}{(1-b^2)} = \left|\frac{\partial x_0^S}{\partial w_1}\right|\right)$, which tends to $w_1^{S*} > w_1^{Pub*}$. As the latter dominates the former, $w_1^{S*} > w_1^{Pub*}$ holds.

IV. *Comparisons*

1. Uniform Input Pricing

Before we compare the market outcomes in the three regimes, we consider the benchmark case where the upstream manufacturer adopts uniform pricing.[10] In this case, the upstream manufacturer will maximize the following expression to determine the input price: $\text{Max}_w \sum x_i^I(w, w)$, where $x_i^I(w, w)$ is given by Eqs. (5.2), (7.2), and (9.2) with $w_0 = w_1 = w$. Solving $\frac{d\pi_M^I(w)}{dw} = 0$ for w yields $w^{S+} = w^{Pub+} = w^{Pri+} = \frac{\Gamma}{2}$, where "+" denotes equilibrium in uniform pricing. The input prices imposed on downstream firms are the same irrespective of the state of the firm. This implies that, as long as uniform pricing is adopted, the presence of a vertically related upstream sector does not substantially affect the endogenous timing of firms' moves in a mixed oligopoly. The following lemma confirms the results in Bárcena-Ruiz (2007), who examined the same issue in a one-tier mixed oligopoly.

Lemma 2: The following holds in equilibrium under uniform input pricing:

[9] Under price competition, the private firm, as Stackelberg leader, chooses a higher price to increase its profits than in the simultaneous-move game. Therefore, private leadership in the price competition, compared to simultaneous-move game, reduces market competition, which leads to lower cost pass-through ratio of the private firm in the private leadership. Since input price is the cost of products, this implies that the derived demand for input of the private firm in the private-leadership is less elastic than in the simultaneous-move game.

[10] From the perspective of profit maximization, the upstream supplier would clearly prefer price discrimination to uniform pricing in the presence of asymmetric downstream firms. However, government regulations sometimes prohibit price discrimination in the input market; therefore, the upstream monopolist must charge the same price for all downstream firms.

$p_1^{Pri+} > p_1^{S+} > p_1^{Pub+}$, $p_0^{Pri+} > p_0^{S+} > p_0^{Pub+}$, $\pi_1^{Pri+} > \pi_1^{S+} > \pi_1^{Pub+}$, $\pi_0^{Pri+} > \pi_0^{S+} > \pi_0^{Pub+}$, $CS^{Pub+} > CS^{S+} > CS^{Pri+}$, and $W^{Pub+} > W^{S+} > W^{Pri+}$.

Proof: Given in Appendix 1.

Considering that private firms would want to reduce market competition (i.e., raise their prices) as profit maximizers while public firm would want to raise market competition (i.e., reduce its price) as a welfare maximizer, Lemma 2 is straightforward. When the private firm is the price leader, it will set a price higher than the price in the simultaneous-move because it knows that the public firm (the follower) will also raise its price because of strategic complementarity in prices (i.e., $p_1^{Pri+} > p_1^{S+}$ and $p_0^{Pri+} > p_0^{S+}$). By contrast, when the public firm is the price leader, it will set a price lower than the price in the simultaneous case because the public firm knows that the private firm will also lower its price (i.e., $p_0^{Pub+} < p_0^{S+}$ and $p_1^{Pub+} < p_1^{S+}$). As a result, market competition is stronger (resp. weaker) when the public (resp. private) firm is a leader than otherwise. Since the two firms set higher (resp. lower) prices under private (resp. public) leadership than under simultaneous-move game, the profits of the firms will be higher (resp. lower) and consumer surplus will be lower (resp. higher), and welfare will be lower (resp. higher) under private (resp. public) leadership than when firms' decisions are set simultaneously (i.e., $\pi_i^{Pri+} > \pi_i^{S+} > \pi_i^{Pub+}$, $CS^{Pub+} > CS^{S+} > CS^{Pri+}$, and $W^{Pub+} > W^{S+} > W^{Pri+}$).

2. Discriminatory Input Pricing

By comparing the market outcomes under discriminatory input pricing, we obtain the following proposition (superscript "*" denotes equilibrium in discriminatory input pricing).

Lemma 3: $x_0^{I*} - x_1^{I*} > x_0^{I+} - x_1^{I+} > 0$ for $I = S, Pub, Pri$.

Proof: From Eqs. (5.2), (7.2) and (9.2), we have $x_0^S - x_1^S = \dfrac{(1-b)^2(a-c-w_0^S) + (1+b-b^2)\Delta w^S}{(1-b)(2-b^2)}$, $x_0^{Pub} - x_1^{Pub} = \dfrac{(2-3b+b^3)(a-c-w_0^{Pub}) + (2+b-b^2)\Delta w^{Pub}}{(1-b)(4-3b^2)}$, and $x_0^{Pri} - x_1^{Pri} = \dfrac{(1-b)(a-c-w_0^{Pri}) + (1+2b)\Delta w^{Pri}}{2(1-b^2)}$. Since $\Delta w^I = 0$ under uniform pricing, we have $x_0^{I+} - x_1^{I+} > 0$. In addition, since $x_0^I - x_1^I$ is a positive function in Δw^I and a negative function in w_0^I, it follows that $x_0^{I*} - x_1^{I*} > x_0^{I+} - x_1^{I+}$ because $\Delta w^{I*} > \Delta w^{I+} (=0)$ and $w_0^{I*} < w^+ \left(= \dfrac{a-c}{2} \right)$. ■

Lemma 3 implies that since the upstream supplier handicaps the private firm and subsidizes the public firm in discriminatory input pricing, production shifts from the private firm to the public firm. Therefore, the production gap between the public and private firms widens in comparison with uniform input pricing. From Table 1, the following proposition is immediate.

Proposition 1: (1) $p_1^{Pri*} > p_1^{S*} > p_1^{Pub*}$,
(2) $x_1^{Pub*} > x_1^{S*} > x_1^{Pri*}$ and $x_0^{Pri*} - x_1^{Pri*} > x_0^{S*} - x_1^{S*} > x_0^{Pub*} - x_1^{Pub*} > 0$.

$$(3)\ x_0^{Pub*}+x_1^{Pub*}>x_0^{S*}+x_1^{S*}>x_0^{Pri*}+x_1^{Pri*},\ \ (4)\ \pi_1^{Pub*}>\pi_1^{S*}>\pi_1^{Pri*}.$$

Proof: See Appendix 2.

In Proposition 1, $p_1^{Pri*}>p_1^{S*}>p_1^{Pub*}$ is straightforward. Note that $p_1^{Pri+}>p_1^{S+}>p_1^{Pub+}$ holds under uniform input pricing. Since input prices are passed through to the output prices, $w_1^{Pri*}>w_1^{S*}>w_1^{Pub*}$(Lemma 1(2)) implies $p_1^{Pri*}>p_1^{S*}>p_1^{Pub*}$. The cost handicap of the private firm against the public firm is largest when the private firm is the price leader and smallest when it is the follower ($\Delta w^{Pri*}>\Delta w^{S*}>\Delta w^{Pub*}$), which leads to $x_1^{Pri*}<x_1^{S*}<x_1^{Pub*}$and $\pi_1^{Pub*}>\pi_1^{S*}>\pi_1^{Pri*}$. In addition, aggregate output is largest when the public firm is the leader, lower in the simultaneous case, and, lowest when it is the follower. Compared to the ranking under uniform pricing, $\pi_1^{Pub*}>\pi_1^{S*}>\pi_1^{Pri*}$ seems somewhat paradoxical. Unlike in uniform input pricing, the private firm's cost disadvantage (i.e., handicap in input price) against the public firm is largest when it is the price leader and lowest when it is the follower. As a result, the profit ranking of the private firm under discriminatory input pricing is the opposite of the input price ranking, $w_1^{Pri*}>w_1^{S*}>w_1^{Pub*}$.

Next, we turn to the price ranking of the public firm. By comparing the equilibrium price of good 0 under different regimes presented in Table 1, we obtain the following lemma:

Lemma 4: Suppose discriminatory pricing in the upstream sector; the following results hold:

$$\begin{cases} p_0^{Pri*}>p_0^{S*}>p_0^{Pub*} & \text{if} \quad b\in(0,\ 0.447) \\ p_0^{S*}>p_0^{Pri*}>p_0^{Pub*} & \text{if} \quad b\in(0.447,\ 0.582) \\ p_0^{S*}>p_0^{Pub*}>p_0^{Pri*} & \text{if} \quad b\in\left(0.582,\ \dfrac{2}{3}\right) \end{cases}$$

Proof: See Appendix 3.

If b is sufficiently low (i.e., $b<0.447$), the price ranking of the public firm under discriminatory input pricing ($p_0^{Pri*}>p_0^{S*}>p_0^{Pub*}$) is consistent with that under uniform pricing. This is straightforward, considering that if the goods are sufficiently independent, then the gap in equilibrium input prices across different regimes almost vanishes.[11] However, as b increases, p_0^{Pub*} becomes relatively high while p_0^{Pri*} becomes relatively low, resulting in a reversal of the public firm's price ranking.

This is largely related to the pricing behavior of the public firm as a welfare maximizer. Recall that the public firm sets its price by adding a profit-driven effect for the private firm to its marginal cost (i.e., $[p_0]_{Bertrand}=(c+w_0)+b(p_1-c-w_1)$, see Eq. (4.1)). This profit driven effect, $b(p_1-c-w_1)$, is stronger as x_1(i.e., price-cost margin) increases. In Proposition 1(2), we have $x_1^{Pub*}>x_1^{S*}>x_1^{Pri*}$. As goods become closer substitutes (an increase in b), the leader-follower relationship exerts a greater influence on the market outcome, expanding the output gap across different regimes. This results in a relative decrease in x_1^{Pri*} and a relative increase in x_1^{Pub*}, which in turn leads to a relative decrease in p_0^{Pri*} and a relative increase in p_0^{Pub*}.

As for the consumer surplus and total surplus, the following proposition can be obtained.

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[11] If b approaches zero, we find that $\lim_{b\to 0} w_0^{S*}=\lim_{b\to 0} w_0^{Pri*}=\lim_{b\to 0} w_0^{Pub*}=\dfrac{a-c}{2}$.

Proposition 2: Suppose discriminatory pricing in the upstream sector; the following holds true:

$$(1)\begin{cases} CS^{Pub*}>CS^{S*}>CS^{Pri*} & \text{if} \quad b\in(0,\ 0.587) \\ CS^{Pub*}>CS^{Pri*}>CS^{S*} & \text{if} \quad b\in(0.587,\ 0.641) \\ CS^{Pri*}>CS^{Pub*}>CS^{S*} & \text{if} \quad b\in\left(0.641,\ \frac{2}{3}\right) \end{cases}, \quad (2)\ W^{Pub*}>W^{S*}>W^{Pri*}.$$

Proof: See Appendix 4.

The consumer surplus ranking is almost the opposite of the public firm's output price ranking in Lemma 4. Since $x_0^{I*}>x_1^{I*}$ in Proposition 1(2), p_0^{I*} has a greater impact on consumer surplus than p_1^{I*} in all cases. Whereas the consumer surplus ranking varies depending on b, the social welfare ranking remains constant and coincides with that under uniform input pricing. This can be explained as follows. Note that $W=f(x_0+x_1)+d(x_0-x_1)-\pi_M$, where $f(\cdot)=(a-c)(x_0+x_1)-\frac{1+b}{4}(x_0+x_1)^2$, $d(\cdot)=-\frac{1-b}{4}(x_0-x_1)^2$, and $\pi_M=\sum_{i=0,\,1}w_ix_i$. Here, $x_0^{Pub*}+x_1^{Pub*}>x_0^{S*}+x_1^{S*}>x_0^{Pri*}+x_1^{Pri*}$ and $x_0^{Pub*}-x_1^{Pub*}>x_0^{S*}-x_1^{S*}>x_0^{Pri*}-x_1^{Pri*}$ in Proposition 1 (2) makes $W^{Pub*}>W^{S*}>W^{Pri*}$ more likely since $f'(\cdot)>0$ and $d'(\cdot)<0$ while $\pi_M^{Pub*}>\pi_M^{S*}>\pi_M^{Pri*}$[12] makes it less likely. It emerges that the former effect dominates the latter, resulting in the standard welfare ranking.

What is noteworthy here is that the standard leader-follower rankings under uniform input pricing are reversed for consumer surplus if b is sufficiently high. In Proposition 2 (1), if $b\in\left(0.641,\ \frac{2}{3}\right)$, then $CS^{Pri*}>CS^{Pub*}$, which contrasts with the conventional view that public leadership in price is more beneficial to the consumers than private leadership. Despite the reversal in the leader-follower rankings for the consumer, the welfare ranking remains the same in both discriminatory pricing and uniform input pricing.

V. *Endogenous Timing Game*

1. CASE 1: $b\in\left(0,\ \frac{2}{3}\right)$

We now discuss the first-stage choice in an endogenous timing game between two downstream firms that produce positive quantities in the market. Table 2 provides the payoff matrix of the observable delay game in a mixed oligopoly. Each firm i simultaneously chooses whether to move early ($T=1$) or late ($T=2$). First, we examine the case of uniform input pricing in the upstream sector. From Lemma 2, the following lemma is immediate.

Lemma 5: Suppose that the downstream firms compete in price. (1) Under uniform input pricing, a "simultaneous-move" occurs, where both firms choose their prices in period 1. (2) However, the equilibrium outcome is inefficient.

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[12] Notice that $\pi_M^{I*}=\sum_{i=0,\,1}w_i^{I*}x_i^{I*}=\frac{a-c}{2}\sum_{i=0,\,1}x_i^{I*}$ holds under discriminatory input pricing. Considering this, $\sum x_i^{Pub*}>\sum x_i^{S*}>\sum x_i^{Pri*}$ in Proposition 1(3) implies $\pi_M^{Pub*}>\pi_M^{S*}>\pi_M^{Pri*}$.

Proof: In Lemma 2, $W^{Pub+} > W^{S+} > W^{Pri+}$ for firm 0 and $\pi_1^{Pri+} > \pi_1^{S+} > \pi_1^{Pub+}$ for firm 1 imply setting prices in $T=1$ is the dominant strategy for both firms. This leads to (1). Equilibrium payoff (W^{S+}, π_1^{S+}) is not efficient because $W^{Pub+} > W^{S+}$ and $\pi_1^{Pri+} > \pi_1^{S+}$ in Lemma 2. This leads to (2). ■

Table 2: Payoff Matrix in a Mixed Duopoly

Firm 1 / Firm 0	$T=1$	$T=2$
$T=1$	$W^{S\Psi}$, $\pi_1^{S\Psi}$	$W^{Pub\Psi}$, $\pi_1^{Pub\Psi}$
$T=2$	$W^{Pri\Psi}$, $\pi_1^{Pri\Psi}$	$W^{S\Psi}$, $\pi_1^{S\Psi}$

Note: $\Psi=+$ for uniform pricing while $\Psi=*$ for discriminatory pricing.

Lemma 5 coincides the results obtained by Barcena-Ruiz (2007) who examined a similar issue in a standard one-tier mixed oligopoly. This implies that the presence of vertically related markets does not substantially impact the endogenous order of moves in a mixed oligopoly as long as uniform input pricing is adopted. We now examine the order of firms' moves under discriminatory input pricing. The following proposition is obtained.

Proposition 3: If the upstream monopolist adopts discriminatory input pricing, then "public leadership" is the Nash equilibrium outcome, which is Pareto efficient.

Proof: $W^{Pub*} > W^{S*} > W^{Pri*}$ (Proposition 2(2)) implies $T=1$ is the dominant strategy for firm 0, while $\pi_1^{Pub*} > \pi_1^{S*} > \pi_1^{Pri*}$ (Proposition 1 (4)) implies $T=2$ is the dominant strategy for firm 1. Therefore, public leadership is SPNE. In addition, since $\pi_1^{Pub*} > \pi_1^{S*} > \pi_1^{Pri*}$ and $W^{Pub*} > W^{S*} > W^{Pri*}$, the equilibrium payoff (W^{Pub*}, π_1^{Pub*}) is an efficient one. ■

The intuition behind Proposition 3 is as follows. As in the uniform pricing regime, it is the dominant strategy for the public firm to set price at $T=1$. Public firm wants to be the leader in price because the greater welfare is obtained than otherwise. However, the private firm's strategy differs between discriminatory input pricing and uniform pricing. As stated in Lemma 1 (2), the private firm faces the highest cost handicap against the public firm when it is the price leader and the lowest when it is the follower, so that $\pi_1^{Pub*} > \pi_1^{S*} > \pi_1^{Pri*}$. Instead of setting the price at $T=1$, the private firm will choose $T=2$ as the dominant strategy, reducing the cost handicap imposed by the upstream supplier and consequently achieving higher profits than otherwise. This means that the private firm earns higher profits as the follower than it would as the leader or simultaneous mover.

Proposition 3 states that standard conclusions about endogenous timing in a mixed oligopoly altered if discriminatory input pricing is adopted. In addition, the public leadership, as equilibrium outcome of endogenous timing game, does not ensure the highest consumer surplus than other cases if goods are close substitutes, that is, $CS^{Pri*} > CS^{Pub*} > CS^{S*}$ if $b \in \left(0.641, \frac{2}{3}\right)$.

In establishing competition policy, it is crucial for the government to clarify the market structure. Our model suggests policy implication on whether to ban the price discrimination of the foreign upstream manufacturer or not. By comparing equilibrium payoffs in the uniform input pricing and discriminatory input pricing, we can obtain the following proposition on government policy option.

Proposition 4: Suppose firms compete in prices (Bertrand) in downstream mixed oligopoly. In this case, banning price discrimination on imported inputs is desirable in terms of both social welfare and profits of the private firm.

Proof: A simultaneous-move, where equilibrium payoffs are (W^{S+}, π_1^{S+}), occurs under uniform input pricing (Lemma 5), while public leadership, where equilibrium payoffs are (W^{Pub*}, π_1^{Pub*}), occurs under discriminatory input pricing (Proposition 3). From Table 1, we have

$$W^{S+}-W^{Pub*}$$
$$=\frac{b(1-b)\Gamma^2(512-448b-1024b^2+1143b^3+561b^4-1094b^5+102b^6+465b^7-175b^8-74b^9+40b^{10})}{8(2-b^2)^2(32-41b^2+13b^4)^2}>0,$$
$$\pi_1^{S+}-\pi_1^{Pub*}$$
$$=\frac{b(1-b)\Gamma^2(8+7b-15b^2-4b^3+6b^4)(64-8b-97b^2+8b^3+45b^4-2b^5-6b^6)}{4(2-b^2)^2(32-41b^2+13b^4)^2}>0. \quad \blacksquare$$

2. CASE 2: $b\in\left[\frac{2}{3}, 1\right)$

As mentioned in Footnote 3, if the private firm is Stackelberg leader in price and $b\in\left[\frac{2}{3}, 1\right)$, then only the public firm serves the market ($x_1^{Pri*}=0$). This can be explained as follows. Basically, private firms maximize profits and want to reduce market competition in an oligopoly market. When the private firm is the price leader, it will set a higher price and produce a lower output than under simultaneous case. In addition, the magnitude of cost handicap of the private firm is the largest when the private firm is the price leader. Compared to simultaneous–move case, this induces a production shift from the private to the public firm. This production shift would increase as the products become closer substitutes (i.e., an increase in b). In our model, a sufficiently large substitutability $\left(b\geq\frac{2}{3}\right)$ will lead to zero production by the private firm when the private firm is price leader ($x_1^{Pri*}=0$). The following proposition is obtained.

Proposition 5: If goods are close substitutes $\left(b\geq\frac{2}{3}\right)$ and the private firm is the leader in price, then the following equilibrium is obtained as a corner solution:

$$p_0^{Pri*}=\frac{a+c}{2},\ p_1^{Pri*}=\frac{2a-b\Gamma}{2},\ w_0^{Pri*}=\frac{\Gamma}{2},\ w_1^{Pri*}=\frac{(2-b)\Gamma}{2},$$
$$x_0^{Pri*}=\frac{a+c}{2},\ \pi_0^{Pri*}=\pi_1^{Pri*}=0,\ CS^{Pri*}=W^{Pri*}=\frac{\Gamma^2}{8}.$$

Proof: Solving $x_1^{Pri}(\mathbf{w})=0$ for w_1 in Eq. (9.2) yields $w_1=bw_0+(1-b)\Gamma$. By substituting this into $x_0^{Pri}(\mathbf{w})$ in Eq. (9.2), we get $x_0^{Pri}(w_0)=\Gamma-w_0$. And the maximization problem of upstream manufacturer is $\mathrm{Max}_{w_0} w_0 x_0^{Pri}(w_0)$, which yields $w_0^{Pri*}=\frac{\Gamma}{2}$ and $w_1^{Pri*}=\frac{(2-b)\Gamma}{2}$. Once w_0^{Pri*} and

w_1^{Pri*} are obtained, $p_0^{Pri*}=\frac{a+c}{2}$, $p_1^{Pri*}=\frac{2a-b\Gamma}{2}$, and $x_0^{Pri*}=\frac{a+c}{2}$ are given by Eqs. (9.1) and (9.2). Since $\pi_0^{Pri*}=bx_0^{Pri*}x_1^{Pri*}$ and $\pi_1^{Pri*}=(x_1^{Pri*})^2$, $x_1^{Pri*}=0$ implies $\pi_0^{Pri*}=\pi_1^{Pri*}=0$. Consumer surplus is $CS^{Pri*}=\frac{\Gamma^2}{8}$, which equals to social welfare because producer surplus is zero (i.e., $\sum_{i=0,1}\pi_i^{Pri*}=0$). ■

Then, what will be the firms' timing of moves at equilibrium if goods are close substitutes? In this regard, we can see that the rankings on social welfare in Proposition 2(2) and that on profits of the private firm in Proposition 1(4) still hold in $b\in\left[\frac{2}{3}, 1\right)$, implying that public leadership is SPNE in observable delay game even when $b\in\left[\frac{2}{3}, 1\right)$.

Lemma 6: Suppose that goods are close substitutes $\left(b\geq\frac{2}{3}\right)$ and firms compete in price. In this case, if discriminatory input pricing is adopted, "public leadership" is the Nash equilibrium in observable delay game, suggesting that Proposition 3 holds irrespective of the value of b.

VI. *The Case of Cournot Competition*

In this section, we examine the case of downstream Cournot competition. In the simultaneous-move case, the profit maximization problem of firm 1 is $\text{Max}_{x_1}\pi_1(\mathbf{x}; w_1)$, while that of firm 0 is $\text{Max}_{x_0}W(\mathbf{x}; \mathbf{w})$. Solving the respective first order condition yields $\phi_0(x_1; w_0)=a-bx_1-(c+w_0)$ and $\phi_1(x_0; w_1)=\frac{1}{2}[a-bx_0-(c+w_1)]$, where ϕ_i, $i=0, 1$, is firm i's reaction function.[13] Given $\mathbf{w}=(w_0, w_1)$, the equilibrium output under each leader-follower relationship is given by:

$$x_0^S(\mathbf{w})=\frac{2\{a-(c+w_0)\}-b\{a-(c+w_1)\}}{2-b^2},\ x_1^S(\mathbf{w})=\frac{\{a-(c+w_1)\}-b\{a-(c+w_0)\}}{2-b^2} \tag{11.1}$$

$$x_0^{Pri}(\mathbf{w})=\frac{(2-b^2)\{a-(c+w_0)\}-b\{a-(c+w_1)\}}{2(1-b^2)},\ x_1^{Pri}(\mathbf{w})=\frac{\{a-(c+w_1)\}-b\{a-(c+w_0)\}}{2(1-b^2)}. \tag{11.2}$$

$$x_0^{Pub}(\mathbf{w})=\frac{4\{a-(c+w_0)\}-3b\{a-(c+w_1)\}}{4-3b^2},\ x_1^{Pub}(\mathbf{w})=\frac{2[\{a-(c+w_1)\}-b\{a-(c+w_0)\}]}{4-3b^2}, \tag{11.3}$$

First, we examine the case where uniform input pricing is adopted. Solving the maximization problem, $\text{Max}_w\sum_{i=0,1}x_i^I(w, w)w$, where $x_i^I(w, w)$ is given in Eqs. (11.1)-(11.3) with $w_0=w_1=w$, gives $w^{S+}=w^{Pri+}=w^{Pub+}=\frac{\Gamma}{2}$. The equilibrium input price remains unchanged

[13] Note that marginal cost pricing is confirmed from the reaction function of the public firm (i.e., $p_0=c+w_0$).

irrespective of firms' role in the leader-follower relationship. Using input prices, we obtain the equilibrium market values under downstream Cournot competition as follows:

$$x_0^{S+}=\frac{(2-b)\Gamma}{2(2-b^2)},\ x_1^{S+}=\frac{(1-b)\Gamma}{2(2-b^2)},\ \pi_1^{S+}=\frac{(1-b)^2\Gamma^2}{4(2-b^2)^2},\ W^{S+}=\frac{(7-6b-2b^2+2b^3)\Gamma^2}{8(2-b^2)^2}, \tag{12.1}$$

$$x_0^{Pri+}=\frac{(2+b)\Gamma}{4(1+b)},\ x_1^{Pri+}=\frac{\Gamma}{4(1+b)},\ \pi_1^{Pri+}=\frac{(1-b)\Gamma^2}{16(1+b)},\ W^{Pri+}=\frac{(7+b)\Gamma^2}{32(1+b)}, \tag{12.2}$$

$$x_0^{Pub+}=\frac{(4-3b)\Gamma}{2(4-3b^2)},\ x_1^{Pub+}=\frac{(1-b)\Gamma}{4-3b^2},\ \pi_1^{Pub+}=\frac{(1-b)^2\Gamma^2}{(4-3b^2)^2},\ W^{Pub+}=\frac{(7-6b)\Gamma^2}{8(4-3b^2)}, \tag{12.3}$$

We are now ready to examine the first stage of the game — the endogenous order of moves in a mixed oligopoly. From Eqs. (12.1)-(12.3), the following lemma is obvious.

Lemma 7: Suppose that the downstream firms compete in quantity in a mixed duopoly. If the upstream monopolist adopts uniform input pricing, then both "public leadership" and "private leadership" are Nash equilibria.

Proof: $W^{Pri+}-W^{S+}=\dfrac{b^2(1-b)(4-b^2)\Gamma^2}{32(1+b)(2-b^2)^2}>0$ and $W^{Pub+}-W^{S+}=\dfrac{b^2(1-b)^2\Gamma^2}{8(2-b^2)^2(4-3b^2)}>0$ for firm 0 and $\pi_1^{Pri+}-\pi_1^{S+}=\dfrac{b^4(1-b)\Gamma^2}{16(1+b)(2-b^2)^2}>0$ and $\pi_1^{Pub+}-\pi_1^{S+}=\dfrac{b^2(1-b)^2(8-5b^2)\Gamma^2}{4(8-10b^2+3b^4)^2}>0$ for firm 1 are obtained from Eqs. (12.1) -(12.3). This implies that neither firm wants to determine its output in the same period as the rival firm. Therefore, both "public leadership" and "private leadership" are SPNE. ■

Lemma 7, which corresponds to Lemma 5 for Bertrand competition, confirms the results of Pal (1998). The intuition for Lemma 7 is as follows. Suppose all firms produce in period 1. In this case, if the public firm chooses period 2 instead and acts as the follower, then the output of the private firm increases ($x_1^{Pri+}>x_1^{S+}$) but that of the public firm decreases ($x_0^{Pri+}<x_0^{S+}$). This leads to an increase in total production $\left(\sum_i x_i^{Pri+}>\sum_i x_i^{S+}\right)$ so that social welfare is greater ($W^{Pri+}>W^{S+}$). Since the output of the private firm increases, its profit also increases ($\pi_1^{Pri+}>\pi_1^{S+}$). Alternatively, suppose that all firms produce in period 2. In this case, if the public firm moves into period 1 and acts as the leader, it would produce less output ($x_0^{Pub+}<x_0^{S+}$) and earn more profit ($\pi_0^{Pub+}>\pi_0^{S+}$) than it does in the simultaneous-move game. Since, in the Cournot model, quantities are strategic substitutes, this will lead to an increase in the private firm's profit ($\pi_1^{Pub+}>\pi_1^{S+}$) through output increase. An increase in producer surplus $\left(\sum_i \pi_i^{Pri+}>\sum_i \pi_i^{S+}\right)$ dominates the consumer surplus loss, resulting in an increase in social welfare ($W^{Pub+}>W^{S+}$).

Next, we turn to the case of discriminatory input pricing. Solving the maximization problem of the upstream manufacturer, $\text{Max}_{w_0, w_1}\sum_{i=0,1} w_i x_i^I(\mathbf{w})$, we obtain

$$w_i^{S*}=w_i^{Pri*}=\frac{a-c}{2},\ i=0,\ 1, \tag{13.1}$$

$$w_1^{Pub*}=\frac{(a-c)(16+4b-15b^2)}{32-25b^2}>\frac{a-c}{2}>\frac{2(a-c)(8-5b-5b^2)}{32-25b^2}=w_0^{Pub*} \tag{13.2}$$

As in Eq. (13.2), upstream manufacturer handicaps the private firm and subsidize the public firm using input pricing only when the public firm is the leader in the downstream Cournot competition. Since input prices for simultaneous-move and private leadership game are the same as those in uniform pricing, market outcomes for both cases under discriminatory input pricing are the same as those under uniform input pricing. The input prices under discriminatory pricing differ from those under uniform input pricing only when the market is under public leadership. By substituting Eq. (13.2) into market variables, we obtain equilibrium values for those variables under public leadership, as summarized in Table 3.[14]

Proposition 6: Suppose that downstream firms compete in quantity under discriminatory input pricing. Then, (1) both $\pi_1^{Pri*}>\pi_1^{S*}>\pi_1^{Pub*}$ and $W^{Pri*}>W^{S*}>W^{Pub*}$ holds. (2) In the observed delay game in Table 1, "private leadership" is the subgame-perfect Nash equilibrium (SPNE) in the mixed duopoly.

Proof: See Appendix 5.

Considering Eqs. (13.1) and (13.2), $\pi_1^{Pri*}>\pi_1^{S*}>\pi_1^{Pub*}$ in Proposition 6(1) is straightforward. Since $w_i^{S*}=w_i^{Pri*}=\dfrac{a-c}{2}$ for $i=0$, 1, $\pi_1^{S*}<\pi_1^{Pri*}$ holds as in uniform pricing. However, since the private firm has a cost handicap against the public firm due to the upstream supplier's input pricing (Eq. (13.2)), the private firm's profits is lower in the public leadership compared to simultaneous-moves (i.e., $\pi_1^{Pub*}<\pi_1^{S*}$). As to social welfare, $W^{S*}<W^{Pri*}$ holds as in uniform pricing because $w_i^{S*}=w_i^{Pri*}=\dfrac{a-c}{2}$. However, the public firm, as a Stackelberg leader, produces less output with public leadership than in the simultaneous case, resulting in a relative increase in the private firm's output. This tends to raise the price level of the public firm. Therefore, the overall price level under public leadership rises above that with simultaneous-move case, resulting in $CS^{Pub*}<CS^{S*}$. Social welfare, reflecting consumer surplus, is lower in public leadership compared to the simultaneous-move case, $W^{Pub*}<W^{S*}$.

The background intuition of Proposition 6(2) is as follows. In the uniform input pricing regime, both firms' strategy is to produce at different time periods than their rival chooses. That is, either public leadership or private leadership can be a Nash equilibrium (Lemma 7). We show that public leadership cannot be a SPNE under discriminatory input pricing in a sequential game. Note that the private firm faces a handicap against the public firm in terms of input price only when the market is under public leadership (Eq. (13.2)). If the market is under public leadership, the private firm, by moving to period 1, can remove the cost handicap in input prices and increase its profits (i.e., $\pi_1^{S*}>\pi_1^{Pub*}$). Similarly, public leadership is not attractive to the public firm either. The public firm has an incentive to move to period 2 (i.e.,

[14] Just as with price competition in the downstream sector, when downstream firms compete in quantity, a corner solution emerges. In the sub-game of a public leadership, if $b\in\left[\frac{4}{5}, 1\right)$, $x_0^{Pub*}>0$ and $x_1^{Pub*}=0$ are derived as a corner solution. That is, only the public firm will serve the market in equilibrium.

TABLE 3. MARKET EQUILIBRIUMS UNDER EACH REGIME: DOWNSTREAM QUANTITY COMPETITION

	Simultaneous Move	Private Leadership	Public Leadership	
			$b\in\left(0, \frac{4}{5}\right)$	$b\in\left[\frac{4}{5}, 1\right)$
Input price	$w_0^{S*}=w_1^{S*}=\frac{\Gamma}{2}$	$w_0^{Pri*}=w_1^{Pri*}=\frac{\Gamma}{2}$	$w_0^{Pub*}=\frac{2(8-b-5b^2)\Gamma}{32-25b^2}$ $w_1^{Pub*}=\frac{(16+4b-15b^2)\Gamma}{32-25b^2}$	$w_0^{Pub*}=\frac{\Gamma}{2}$ $w_1^{Pub*}=\frac{(2-b)\Gamma}{2}$
Output price	$p_0^{S*}=\frac{a+c}{2}$ $p_1^{S*}=\frac{a(3-b-b^2)+c(1+b-b^2)}{2(2-b^2)}$	$p_0^{Pri*}=\frac{a+c}{2}$ $p_1^{Pri*}=\frac{a(3-b)+c(1+b)}{4}$	$p_0^{Pub*}=\frac{a(16+2b-15b^2)+2c(8-b-5b^2)}{32-25b^2}$ $p_1^{Pub*}=\frac{3a(8-2b-5b^2)+2c(4+3b-5b^2)}{32-25b^2}$	$p_0^{Pub*}=\frac{a+c}{2}$ $p_1^{Pub*}=\frac{a(2-b)+bc}{2}$
Output	$x_0^{S*}=\frac{(2-b)\Gamma}{2(2-b^2)}$ $x_1^{S*}=\frac{(1-b)\Gamma}{2(2-b^2)}$	$x_0^{Pri*}=\frac{(2+b)\Gamma}{4(1+b)}$ $x_1^{Pri*}=\frac{\Gamma}{4(1+b)}$	$x_0^{Pub*}=\frac{2(8-5b)\Gamma}{32-25b^2}$ $x_1^{Pub*}=\frac{2(4-5b)\Gamma}{32-25b^2}$	$x_0^{Pub*}=\frac{\Gamma}{2}$ $x_1^{Pub*}=0$
Profit	$\pi_0^{S*}=0$ $\pi_1^{S*}=(x_1^{S*})^2=\frac{(1-b)^2\Gamma^2}{4(2-b^2)^2}$	$\pi_0^{Pri*}=0$ $\pi_1^{Pri*}=(1-b^2)(x_1^{Pri*})^2=\frac{(1-b)\Gamma^2}{16(1+b)}$	$\pi_0^{Pub*}=\frac{b}{2}x_0^{Pub*}x_1^{Pub*}=\frac{2b(8-5b)(4-5b)\Gamma^2}{(32-25b^2)^2}$ $\pi_1^{Pub*}=(x_1^{Pub*})^2=\frac{4(4-5b)^2\Gamma^2}{(32-25b^2)^2}$	$\pi_0^{Pub*}=0$ $\pi_1^{Pub*}=0$
welfare	$CS^{S*}=\frac{(5-2b-4b^2+2b^3)\Gamma^2}{8(2-b^2)^2}$ $W^{S*}=\frac{(7-6b-2b^2+2b^3)\Gamma^2}{8(2-b^2)^2}$	$CS^{Pri*}=\frac{(5+3b)\Gamma^2}{32(1+b)}$ $W^{Pri*}=\frac{(7+b)\Gamma^2}{32(1+b)}$	$CS^{Pub*}=\frac{4(40-28b-35b^2+25b^3)\Gamma^2}{(32-25b^2)^2}$ $W^{Pub*}=\frac{2(112-104b-80b^2+75b^3)\Gamma^2}{(32-25b^2)^2}$	$CS^{Pub*}=\frac{\Gamma^2}{8}$ $W^{Pub*}=\frac{\Gamma^2}{8}$

Note: $\Gamma\equiv a-c$

the simultaneous-move case) under public leadership. Although the public firm loses its "leadership" position, it can raise social welfare ($W^{S^*} > W^{Pub^*}$). On the other hand, when the market is under private leadership, no firm has an incentive to deviate from its strategy.

The above Proposition 6(2) is noteworthy from the following viewpoint. The indeterminacy of the firms' role with respect to the leader-follower relationship vanishes in a two-tier mixed oligopoly, in sharp contrast with Pal (1998) who demonstrated multiple equilibria in sequential games when both firms compete on quantity in a one-tier mixed oligopoly.

Should the government ban discriminatory input pricing by the foreign upstream monopolist? We offer the following proposition on government policy with respect to price discrimination.

Proposition 7: Suppose that firms compete in quantity (Cournot) in a downstream mixed oligopoly. In this case, allowing price discrimination on imported inputs is socially desirable.

Proof: We show that the equilibrium social welfare under discriminatory input pricing (W^{Pri^*}) is greater than or equal to the social welfare under uniform input pricing (W^{Pri+} or W^{Pub+}). Since $w_i^{Pri^*} = w^+ = \frac{a-c}{2}$ in Eq. (13.1), $W^{Pri^*} = W^{Pri+}$ holds. In addition, $W^{Pri^*} - W^{Pub+} = W^{Pri+} - W^{Pub+} > 0$ because $W^{Pri+} - W^{Pub+} = \frac{3b^2(1-b)\Gamma^2}{32(1+b)(4-3b^2)} > 0$ from Eqs. (12.2) and (12.3). ■

Proposition 7, which corresponds to Proposition 4 for the Bertrand competition, suggests that, in a quantity-setting mixed oligopoly, it is socially desirable for the domestic government not to intervene in the input pricing behavior (i.e., allow discriminatory input pricing) by the foreign upstream monopolist than enforcing uniform input pricing. This is because, if price discrimination was banned, then there is a possibility that firms' strategic behavior in endogenous timing game may result in socially undesirable one (that is, public firm leadership) of the two equilibria. Allowing discriminatory input pricing of the foreign upstream monopolist has the effect of blocking the possibility that the public firm leadership, which is socially less efficient than the private leadership, is realized as an equilibrium market structure.

VII. *Extension: Domestic Upstream Supplier*

To see how the upstream firm's nationality affects the main results, we assume in this section that the upstream supplier is a domestic firm. In this case, the social welfare is $SW = \frac{1}{2}\{(1-\gamma)(x_0^2+x_1^2)+(x_0+x_1)^2\} + \sum_{i=0,1}(p_i - c)x_i$, where the producer surplus is represented as the sum of the aggregated profits of vertically related firms with marginal cost c. First, we consider price competition. Given $\mathbf{w} = (w_0, w_1)$, the equilibrium output in the last stage of the game under each leader-follower relationship is given by

$$x_0^S = x_0^{Pri} = \frac{a-c}{1+b},\ x_0^{Pub}(w_1) = \frac{(4+b-3b^2-b^3)(a-c)+b(1+b)w_1}{(1+b)(4-3b^2)} \tag{14.1}$$

$$x_1^S(w_1) = \frac{a-c-(1+b)w_1}{(1+b)(2-b^2)},\ x_1^{Pri}(w_1) = \frac{(a-c)-(1+b)w_1}{2(1+b)},$$

$$x_1^{Pub}(w_1)=\frac{(2-b^2)(a-c)-2(1+b)w_1}{(1+b)(4-3b^2)}. \tag{14.2}$$

In the equations, the following points are noteworthy. First, the public firm produces a fixed amount of output irrespective of input prices $\left(x_0^S=x_0^{Pri}=\dfrac{a-c}{1+b}\right)$ with both simultaneous-moves and private-firm leadership.[15] Second, each firm's equilibrium output depends only on the input price charged to the private firm (w_1).

In the second stage of the game, the upstream supplier charges the private firm the monopoly price for the input $\left(w_1^{S*}=w_1^{Pri*}=\dfrac{(a-c)}{2(1+b)}\right)$ when the market structure is based on either simultaneous-moves or private leadership. However, when market is under public leadership, the upstream supplier imposes the upper bound input price to the private firm[16] $\left(\text{i.e., } w_1^{Pub*}=\dfrac{(2-b^2)(a-c)}{2(1+b)}\right)$, reducing the private firm's output to zero, so that only the public firm remains in the market. However, the possibility of private-firm production — even if no actual production takes place — forces the public firm to choose a larger output. Once equilibrium input prices are established, the equilibrium market values are obtained. As a result, when downstream firms engage in Bertrand competition, the social welfare and profit rankings for different leader-follower relationships are obtained as follows:

$$\pi_1^{Pri*}=\frac{\Gamma^2}{16(1+b)^2}>\frac{(1-b)\Gamma^2}{4(1+b)(2-b^2)^2}=\pi_1^{S*}>\pi_1^{Pub*}(=0), \tag{15.1}$$

$$W^{S*}=\frac{(23+32b-20b^2-32b^3+4b^4+8b^5)\Gamma^2}{8(1+b)^2(2-b^2)^2}>\frac{(23+32b)\Gamma^2}{32(1+b)^2}=W^{Pri*},$$

$$W^{S*}>\frac{(4+8b+3b^2)\Gamma^2}{8(1+b)^2}=W^{Pub*} \tag{15.2}$$

In above equations, the profit ranking of the private firm shows the same result as in Barcena-Ruiz (2007), but the welfare ranking is different. Unlike the welfare ranking ($W^{Pub*}>W^{S*}>W^{Pri*}$) in the one-tier mixed oligopoly analyzed by Barcena-Ruiz (2007), the welfare under public leadership is less than that in the simultaneous-move case. When the market is under public leadership, the negative effects of market distortion on social welfare caused by input price discrimination are large enough to reduce the welfare level below the simultaneous-move case.

We now consider Cournot competition. In this case, nonnegative profit constraint for the

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[15] If firm 0 behaves optimally, the reaction function $p_0[p_1]=c+b(p_1-c)$ is derived from $\dfrac{\partial SW}{\partial p_0}=0$, and substituting this into Eq. (2) yields the equilibrium output of firm 0, which is independent of input prices.

[16] Since $\dfrac{\partial q_0^{Pub}}{\partial w_1}>0$, it is advantageous for the upstream manufacturer to offer the private firm as high an input price as possible, thereby shifting all the output production from the private to the public firm, whose derived demand for intermediate input is inelastic.

public firm is required.[17] The nonnegative profit constraint is binding for all leader-follower relationships (i.e., $\pi_0^{S+}=\pi_0^{Pub+}=\pi_0^{Pri+}=0$). Given $\mathbf{w}=(w_0, w_1)$, the equilibrium output in the last stage game is

$$x_0^S(\mathbf{w})=x_0^{Pub}(\mathbf{w})=\frac{2\{a-(c+w_0)\}-b\{a-(c+w_1)\}}{2-b^2},$$
$$x_0^{Pri}(\mathbf{w})=\frac{(2-b^2)\{a-(c+w_0)\}-b\{a-(c+w_1)\}}{2(1-b^2)} \tag{16.1}$$
$$x_1^S(\mathbf{w})=x_1^{Pub}(\mathbf{w})=\frac{\{a-(c+w_1)\}-b\{a-(c+w_0)\}}{2-b^2},\ x_1^{Pri}(\mathbf{w})=\frac{\{a-(c+w_1)\}-b\{a-(c+w_0)\}}{2(1-b^2)}. \tag{16.2}$$

From the equations, the equilibrium output in the public leadership is the same as that in the simultaneous-move case (i.e., $x_i^{Pub}=x_i^S$, $i=0, 1$). Since non-negative profit constraint is binding, the public firm cannot make use of its strategic advantage of moving first, so that the market equilibrium is the same for public-firm leadership and the simultaneous case. In the second stage game, solving the maximization problem $\left(\text{i.e., } \underset{w_0,\, w_1}{\mathrm{Max}} \sum_{i=0,\,1} w_i x_i^I(\mathbf{w})\right)$ gives $w_i^{S*}=w_i^{Pub*}=w_i^{Pri*}=\dfrac{a-c}{2}$ for $i=0, 1$. Using equilibrium input prices, we obtain the following relationships for social welfare and firm 1's profits when downstream firms compete in quantity:

$$\pi_1^{Pri*}=\frac{(1-b)\Gamma^2}{16(1+b)}>\frac{(1-b)^2\Gamma^2}{4(2-b^2)^2}=\pi_1^{S*}=\pi_1^{Pub*}, \tag{17.1}$$

$$W^{Pri*}=\frac{(19+5b)\Gamma^2}{32(1+b)}>\frac{(19-14b-8b^2+6b^3)\Gamma^2}{8(2-b^2)^2}=W^{S*}=W^{Pub*}. \tag{17.2}$$

The private firm increases its output (as compared to the simultaneous case) when it is the leader because it knows that the follower (the public firm) will reduce its output, which means that $q_1^{Pri*}>q_1^{S*}=q_1^{Pub*}$ and $\pi_1^{Pri*}>\pi_1^{S*}=\pi_1^{Pub*}$. As to welfare ranking, $W^{S*}=W^{Pub*}$ holds because $q_i^{S*}=q_i^{Pub*}$ and $w_i^{S*}=w_i^{Pub*}$ for $i=0, 1$. $W^{Pri*}>W^{S*}(=W^{Pub*})$is explained as follows. Suppose that both firms produce in period 1 (i.e., $T=1$). In this case, if the public firm moves to $T=2$, then the output and profits of the private firm increase while those of the public firm remain unchanged (as compared to the simultaneous case), so that the producer surplus increases ($PS^{Pri*}>PS^{S*}$). Because total output increases, the consumer surplus tends to rise above the simultaneous-move level ($CS^{Pri*}>CS^{S*}$). As a result, $W^{Pri*}>W^{S*}(=W^{Pub*})$.

In view of Eqs. (15.1), (15.2), (17.1) and, (17.2), the following proposition is offered.

Proposition 8: Suppose that the upstream manufacturer is a domestic firm. When downstream

[17] If downstream firms behave optimally without constraints, the public firm produces up to the level where the price equals the marginal cost of the upstream manufacturer (i.e., $p_0=c$), resulting in negative profits for the public firm. In addition, the private firm is charged extremely high input prices by the upstream manufacturer and, thus, is barred from production. As a result, when both firms behave optimally without any constraint à la Cournot in the market, only the public firm remains in the market for all leader-follower relationships. It is useless to discuss mixed oligopoly in such a circumstance.

firms compete a là Bertrand (resp. Cournot) in an observable delay game, a simultaneous-move (resp. sequential-move) game is the SPNE in a mixed duopoly.

Proof: It is obvious from Eqs. (15.1) and (15.2) for Bertrand competition and from Eqs. (17.1) and (17.2) for Cournot competition.

The above proposition confirms the results of Pal (1998) and Barcena-Ruiz (2007) for Cournot and Bertrand competition, respectively. Therefore, if the upstream manufacturer is a domestic firm, the endogenous order of firms' moves is consistent between a two-tier mixed oligopoly with upstream monopolist and a standard one-tier mixed oligopoly, as long as nonnegative profit constraint is introduced when downstream market is under Cournot competition.

VIII. *Concluding Remarks*

Our main results are as follows. First, we have shown that, under discriminatory input pricing, the upstream supplier handicaps the private firm but subsidizes the public firm through input pricing. The cost handicap of the private firm is largest when the private (resp. public) firm is the leader in the downstream Bertrand (resp. Cournot) market. In addition, this cost handicap affects firms' move-timing decisions in an observable delay game, leading to public (resp. private) leadership in downstream Bertrand (resp. Cournot) competition. This result sharply contrasts with the simultaneous-move equilibrium in a price-setting (Bárcena-Ruiz, 2007) and sequential-move multiple equilibria in a quantity-setting (Pal, 1998) mixed oligopoly described in previous research. Second, when the downstream market is characterized by Bertrand competition, the rankings for firm's profit and consumer surplus are reversed but welfare ranking remains unchanged between uniform and discriminatory input pricing. Somewhat surprisingly, under discriminatory input pricing, the private firm's profits are highest (resp. lowest), in contrast to the ranking under uniform pricing, when the private firm is the follower (resp. leader) in the market. Third, the nationality of the upstream supplier makes a difference for the firms in terms of timing their moves. With a domestic upstream manufacturer, the endogenous order of firms' moves is consistent between a two-tier mixed oligopoly with upstream monopolist and a standard one-tier mixed oligopoly, as long as nonnegative profit constraint is introduced when downstream market is under Cournot competition. This implies that as long as the nonnegative profit constraint is satisfied, the endogenously determined order of firms' moves does not substantially change if the upstream monopolist in a vertically related market is a domestic firm. Fourth, our analysis has implications for competition policy. According to our analysis, policy makers need to consider the competition mode (i.e., Cournot or Bertrand) in the downstream market before making any decision on whether to ban input price discrimination by the foreign upstream manufacturer. That is, banning price discrimination for imported inputs is desirable, from the viewpoint of both social welfare and firms' profit, with downstream Bertrand competition, but input price discrimination is socially desirable with downstream Cournot competition.

The conclusions of our paper depends largely on critical assumptions to keep the model as simple and transparent as possible, such as linear demand function, constant marginal cost, full bargaining power of upstream manufacturer, simple vertical structure that upstream manufacturer belongs to foreign country. Especially noteworthy is that the public firm in our model is assumed as a simple welfare maximizer, and hence, we could not discuss the privatization problem that has been one of key issues in mixed oligopoly literature. One way to incorporate the privatization issue into the study is to introduce partial privatization approach formulated by Matsumura (1998) and analyze the optimal degree of privatization, which will provide a richer policy implications, particularly with respect to the interdependence between the public policy toward price discrimination in the upstream market and privatization policy in the downstream market. Our future follow-up to this study will focus on solving the above-mentioned problems.

Appendix

1. Proof of Lemma 2

By substituting $w^{I+}=\dfrac{a-c}{2}(I=S,Pub,Pri)$ into Eqs. (5.1) and (5.2) for the simultaneous-move game, into Eqs. (7.1) and (7.2) for the public leadership, and into Eqs. (9.1) and (9.2) for the private leadership, we obtain the equilibrium outcomes in the three subgames under uniform input pricing as follows:

$$p_0^{S+}=\frac{2ab(1-b)+(2-b)(a+c)}{2(2-b^2)},\ p_1^{S+}=\frac{2a(1-b)+(1+b-b^2)(a+c)}{2(2-b^2)},$$

$$\pi_0^{S+}=\frac{b(1-b)(a-c)^2}{4(1+b)(2-b^2)},\ \pi_1^{S+}=\frac{(1-b)(a-c)^2}{4(1+b)(2-b^2)^2},\ CS^{S+}=\frac{(5-b-3b^2+b^3)(a-c)^2}{8(1+b)(2-b^2)^2},$$

$$W^{S+}=\frac{(7+b-7b^2-b^3+2b^4)(a-c)^2}{8(1+b)(2-b^2)^2},\ p_0^{Pub+}=\frac{a(4+b-4b^2)+c(4-b-2b^2)}{2(4-3b^2)},$$

$$p_1^{Pub+}=\frac{a(6-2b-4b^2+b^3)+c(2+2b-2b^2-b^3)}{2(4-3b^2)},\ \pi_0^{Pub+}=\frac{b(1-b)(4+b-3b^2-b^3)(a-c)^2}{4(1+b)(4-3b^2)^2},$$

$$\pi_1^{Pub+}=\frac{(1-b)(2-b^2)^2(a-c)^2}{4(1+b)(4-3b^2)^2},\ CS^{Pub+}=\frac{(5+b-3b^2-b^3)(a-c)^2}{8(1+b)(4-3b^2)},$$

$$W^{Pub+}=\frac{(7+b-5b^2-b^3)(a-c)^2}{8(1+b)(4-3b^2)},\ p_0^{Pri+}=\frac{a(2+3b)+c(2+b)}{4(1+b)},\ p_1^{Pri+}=\frac{a(3+2b)+c(1+2b)}{4(1+b)},$$

$$\pi_0^{Pri+}=\frac{b(a-c)^2}{8(1+b)^2},\ \pi_1^{Pri+}=\frac{(a-c)^2}{16(1+b)^2},\ CS^{Pri+}=\frac{(5+4b)(a-c)^2}{32(1+b)^2},\ W^{Pri+}=\frac{(7+8b)(a-c)^2}{32(1+b)^2},$$

Lemma 2 can be easily verified by a simple calculation using the above equilibrium values. ■

2. Proof of Proposition 1

(1) From Table 1, we have:

$$p_1^{Pri*}-p_1^{S*}=\frac{b^2(8-4b-8b^2-b^3+3b^4)\Gamma}{(8-9b^2)(8-5b^2+b^4)}>0 \text{ and}$$

$$p_1^{S*}-p_1^{Pub*}=\frac{b(1-b^2)^2(16-6b-13b^2+3b^3+2b^4)\Gamma}{(8-5b^2+b^4)(32-41b^2+13b^4)}>0, \text{ which yields } p_1^{Pri*}>p_1^{S*}>p_1^{Pub*}.$$

(2) From Table 1, we have:

$$x_1^{Pub*}-x_1^{S*}=\frac{b(1-b)(16-6b-7b^2+9b^3-b^4-3b^5)\Gamma}{(8-5b^2+b^4)(32-41b^2+13b^4)}>0 \text{ and}$$

$x_1^{S*}-x_1^{Pri*}=\dfrac{b^2(8+4b-8b^2-b^3-3b^4)\Gamma}{(1+b)(8-9b^2)(8-5b^2+b^4)}>0$, which yields $x_1^{Pub*}>x_1^{S*}>x_1^{Pri*}$ for $b\in\left(0,\ \dfrac{2}{3}\right)$.

Let $\Delta x^{I*}\equiv x_0^{I*}-x_1^{I*}(I=Pri,\ Pub,\ S)$. From Table 1, we have $\Delta x^{Pub*}=\dfrac{2(2-b^2)^2\Gamma}{(32-41b^2+13b^4)}$,

$\Delta x^{S*}=\dfrac{2\Gamma}{(8-5b^2+b^4)}$, and $\Delta x^{Pri*}=\dfrac{2\Gamma}{(8-9b^2)}$. Thus, $\Delta x^{Pri*}-\Delta x^{S*}=\dfrac{2b^2(4+b^2)\Gamma}{(8-9b^2)(8-5b^2+b^4)}>0$

and $\Delta x^{S*}-\Delta x^{Pub*}=\dfrac{2b^2(11-19b^2+9b^4-b^6)\Gamma}{(8-5b^2+b^4)(32-41b^2+13b^4)}>0$, implying $\Delta x^{Pri*}>\Delta x^{S*}>\Delta x^{Pub*}$.

(3) From Table 1, aggregate output, $\sum_{i=0,\ 1} x_i^{I*}$, is obtained as follows:

$\sum x_i^{Pub*}=\dfrac{2(12+2b-14b^2-3b^3+4b^4+b^5)\Gamma}{(1+b)(32-41b^2+13b^4)}$, $\sum x_i^{S*}=\dfrac{2(3-b^2)\Gamma}{(1+b)(8-5b^2+b^4)}$, and

$\sum x_i^{Pri*}=\dfrac{6(1-b)\Gamma}{(8-9b^2)}$. Thus,

$\sum x_i^{Pub*}-\sum x_i^{S*}=\dfrac{2b(1-b)^3(16+15b-4b^2-6b^3-b^4)\Gamma}{(8-5b^2+b^4)(32-41b^2+13b^4)}>0$ and

$\sum x_i^{S*}-\sum x_i^{Pri*}=\dfrac{2b^2(4-9b^2+3b^4)\Gamma}{(1+b)(8-9b^2)(8-5b^2+b^4)}>0$, implying $\sum x_i^{Pub*}>\sum x_i^{S*}>\sum x_i^{Pri*}$.

(4) We have $\pi_1^{Pub*}-\pi_1^{S*}=(1-b^2)(x_1^{Pub*}+x_1^{S*})(x_1^{Pub*}-x_1^{S*})>0$ because $x_1^{Pub*}-x_1^{S*}>0$ from Proposition 1(2), and

$\pi_1^{S*}-\pi_1^{Pri*}=\dfrac{b^3(256-288b-384b^2+388b^3+100b^4-115b^5+17b^6+3b^7-9b^8)\Gamma^2}{(1+b)(8-9b^2)^2(8-5b^2+b^4)^2}>0$,

implying $\pi_1^{Pub*}>\pi_1^{S*}>\pi_1^{Pri*}$. ■

3. Proof of Lemma 4

From Table 1 we get the following relations:

$$p_0^{Pri*}-p_0^{S*}=\frac{b^3(4-10b+b^2+3b^3)\Gamma}{(8-9b^2)(8-5b^2+b^4)}>[<]0 \text{ if } b\in(0,\ 0.447)\left[b\in\left(0.447,\frac{2}{3}\right)\right], \tag{A1}$$

$$p_0^{S*}-p_0^{Pub*}=\frac{b(1+b)(1-b)^2(16-12b-19b^2+6b^3+5b^4)\Gamma}{(8-5b^2+b^4)(32-41b^2+13b^4)}>0, \tag{A2}$$

$$p_0^{Pri*}-p_0^{Pub*}=\frac{b(16-28b-15b^2+27b^3+4b^4-6b^5)\Gamma}{(8-9b^2)(32-41b^2+13b^4)}>[<]0 \text{ if } b\in(0,\ 0.582)\left[b\in\left(0.582,\ \frac{2}{3}\right)\right], \tag{A3}$$

From Eqs. (A1), (A2), and (A3), $p_0^{Pri*}>p_0^{S*}>p_0^{Pub*}$ for $b<0.447$, $p_0^{S*}>p_0^{Pri*}>p_0^{Pub*}$ for $0.447<b<0.582$, and $p_0^{S*}>p_0^{Pub*}>p_0^{Pri*}$ for $0.582<b<\dfrac{2}{3}$. ■

4. Proof of Proposition 2

(1) The following relationship is obtained with respect to consumer surplus from Table 1:

$$CS^{Pub*}-CS^{S*}=\frac{b(1-b)^2(2-b^2)\begin{pmatrix}1536-320b-4032b^2+155b^3+3891b^4+315b^5\\-1713b^6-287b^7+345b^8+81b^9-27b^{10}-8b^{11}\end{pmatrix}\Gamma^2}{(8-5b^2+b^4)^2(32-41b^2+13b^4)^2}>0, \tag{A4}$$

$$CS^{S*}-CS^{Pri*}=\frac{b^2\begin{pmatrix}128-624b^2+888b^4\\-488b^6+109b^8-9b^{10}\end{pmatrix}\Gamma^2}{(1+b)(8-9b^2)^2(8-5b^2+b^4)^2}>[<]0 \text{ if } b\in(0,\ 0.587)\left[b\in\left(0.587,\ \frac{2}{3}\right)\right], \tag{A5}$$

$$CS^{Pub*}-CS^{Pri*}=\frac{b\begin{pmatrix}3072-1664b-15104b^2+4982b^3\\+30256b^4-5525b^5-31552b^6\\+2735b^7+18044b^8-542b^9\\-5364b^{10}+18b^{11}+648b^{12}\end{pmatrix}\Gamma^2}{(1+b)(8-9b^2)^2(32-41b^2+13b^4)^2}>[<]0 \text{ if } b\in(0,\ 0.641)\left[b\in\left(0.641,\ \frac{2}{3}\right)\right]. \tag{A6}$$

From Eqs. (A4) -(A6), it holds that $CS^{Pub*}>CS^{S*}>CS^{Pri*}$ for $b<0.587$, $CS^{Pub*}>CS^{Pri*}>CS^{S*}$ for $0.587<b<0.641$, and $CS^{Pri*}>CS^{Pub*}>CS^{S*}$ for $0.641<b<\frac{2}{3}$.

(2) From Table 1, we obtain:

$$W^{Pub*}-W^{S*}$$
$$=\frac{b(1+b)(1-b)^2(1024-640b-2432b^2+1778b^3+2416b^4-2000b^5-1298b^6+1123b^7+401b^8-310b^9-68b^{10}+33b^{11}+5b^{12})\Gamma^2}{(8-5b^2+b^4)^2(32-41b^2+13b^4)^2}>0,$$

and $W^{S*}-W^{Pri*}=\dfrac{b^2(128-400b^2+524b^4-361b^6+131b^8-18b^{10})\Gamma^2}{(1+b)(8-9b^2)^2(8-5b^2+b^4)^2}>0$, implying

$W^{Pub*}>W^{S*}>W^{Pri*}$.■

5. Proof of Proposition 6

(1) In Eq. (13.1), $w_i^{Pri*}=w_i^{S*}=\dfrac{a-c}{2}$, implying that if market is either in private leadership or in simultaneous-move, then the market equilibria under discriminatory input pricing are the same as those under uniform input pricing. Therefore, $W^{Pri*}=W^{Pri+}$ and $W^{S*}=W^{S+}$ holds. From the proof of Lemma 7, we have $W^{Pri+}>W^{S+}$, implying $W^{Pri*}>W^{S*}$. Next, we show that $W^{S*}>W^{Pub*}$. From Table 3, we obtain

$$W^{S*}-W^{Pub*}=\begin{cases}\dfrac{b(512-960b+192b^2+663b^3-486b^4+30b^5+50b^6)\Gamma^2}{8(64-82b^2+25b^4)^2}>0 \text{ if } b\in\left(0,\ \dfrac{4}{5}\right)\\[2ex] \dfrac{(1-b)^2(3-b^2)\Gamma^2}{8(2-b^2)^2}>0 \text{ if } b\in\left(\dfrac{4}{5},\ 1\right)\end{cases}$$

This leads to $W^{Pri*}>W^{S*}>W^{Pub*}$ for the domain of $b\in(0,\ 1)$.

(2) In Eq. (13.1), $w_i^{Pri*}=w_i^{S*}=\dfrac{a-c}{2}$, which leads to $\pi_1^{Pri*}=\pi_1^{Pri+}$ and $\pi_1^{S*}=\pi_1^{S+}$. In addition, we have $\pi_1^{Pri+}>\pi_1^{S+}$ in the proof of Lemma 7, which, in turn, implies $\pi_1^{Pri*}>\pi_1^{S*}$. Next, we will demonstrate

$\pi_1^{S*} > \pi_1^{Pub*}$. From Table 3, we obtain:

$$\pi_1^{S*} - \pi_1^{Pub*} = \begin{cases} \dfrac{b(512 - 1152b + 640b^2 + 369b^3 - 610b^4 + 225b^5)\Gamma^2}{4(64 - 82b^2 + 25b^4)^2} > 0 \text{ if } b \in \left(0, \dfrac{4}{5}\right) \\ \pi_1^{S*} > 0 \text{ if } b \in \left(\dfrac{4}{5}, 1\right) \end{cases}$$

Therefore, this leads to $\pi_1^{Pri*} > \pi_1^{S*} > \pi_1^{Pub*}$ for the domain of $b \in (0, 1)$. ■

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