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JOB DESIGN AND INCENTIVES IN HIERARCHIES WITH TEAM PRODUCTION*

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Abstract

The purpose of this paper is to examine how tasks should be allocated in a simple hierarchy consisting of an organizational designer and subordinates, in the framework of a principal-agent relationship with moral hazard. Those who are assigned to tasks choose costly and unobservable inputs into the tasks. There is a verifiable and "informative" signal that measures the joint performance: The principal and the agents engage in team production. It is shown that when the principal cannot perform all the tasks and hence must delegate at least one task for some exogenous reasons, she may choose to group all the tasks into an agent's job. Without cost substitutes or complements, the optimal task allocation is either delegating only the task with the smallest responsiveness of effort to incentives, or delegating all the tasks to an agent: There is a substantial non-convexity in delegation decision. It is shown that "complete delegation" is desirable if the team performance is sufficiently easy to measure, or the effort responsiveness to incentives at each task is sufficiently high.

Keywords: Principal-agent relationships, moral hazard, team production, job design, delegation.

I. *Introduction*

An indispensable feature of hierarchical organizations is delegation of decision making authority to subordinates. This feature, in its simplest form, can be captured by a principal-agent relationship in which one party, an owner of a production function, hires another party and delegates authority to control the production to the latter. In the standard analysis, the sole role of the principal is to provide her agents with effort incentives through design of compensation, monitoring, and other control instruments. It is only the agents who engage in productive activities. However, there is no a priori reason *all* the activities must be delegated to the agents: the principal, even after delegating some tasks to the agents, could leave other activities under her control in order to affect production. The output of a production worker

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is determined not only by his effort but also by his supervisor's help such as advising and coordination. Sales depend on salespersons' sales activities as well as advertising strategy by managers. A multidivisional corporation often maintains some functions such as finance, R&D, and personnel at the corporate level rather than pushes all activities down the division level.

This paper is concerning task allocation in a simple hierarchy that consists of an organizational designer and its subordinates. I call one of the parties a "principal" who is a risk neutral residual claimant of the returns from production and is entitled to allocate tasks and design contractual terms for subordinates called "agents." The paper does not offer any explanation for the emergence of the agency relationship itself: I assume that for some exogenous reasons, the principal cannot perform all the tasks. This assumption itself is a weak, uncontroversial one, given that all the literature on agency relationships also presumes this implicitly, and that in most situations in practice, limited attention, the lack of expertise, or other factors force the principal to decentralize something. It is then an important decision for the principal to allocate tasks between agents and herself: Which of the tasks should be delegated to the subordinates? How much delegation is desirable?

Answers to such job design questions depend on many factors. First, there are technological factors. The principal will delegate tasks that require expertise available only to the agents. Creating specialization by assigning each agent one task may be more effective than grouping a large set of tasks into a job and assigning it to just one agent. On the other hand, several tasks may have technological complementarities and hence if one of them is to be allocated to an agent, the other should be delegated together.

A second factor is information and incentives. One advantage of delegating tasks is that the subordinates possess private information relevant to the tasks and hence task delegation enables such information to be utilized for decision making. However, delegation creates incentive problems as preference incongruities between the principal and the agents exist (Holmström 1984). Furthermore, if the principal can elicit the agents' information via revelation mechanisms, delegation is of no value: the communication-based centralization always performs the best, and delegation without communication performs as well only in special cases (Melumad and Reichelstein 1987). Several recent papers thus examine the value of delegation under the assumption of incomplete contracts with costly communication or no commitment to mechanisms.[1]

This paper also focuses on incentive effects of task allocation and assumes away the technological factors from the model. However, the source of the incentive problems is moral hazard: Agents do not possess any private information about the state of nature. No incentive problem would thus arise if the principal could perform all the tasks by herself. This motivates me to assume exogenously that something must be delegated. In the model, however, assigning more tasks to agents does not necessarily make the incentive problem more serious, but mitigates a problem associated with preference incongruities between the principal and the agents. When the principal must delegate something to the agent, she may choose to delegate all the tasks as an agent's job and to specialize in design of contracts.[2]

[1] See, for example, Melumad and Mookherjee (1989) and Melumad, Mookherjee, and Reichelstein (1995).

[2] This inquiry into the question of optimal job design is similar in spirit to Riordan and Sappington (1987). They consider a production process with two stages. The first stage is always performed by an agent who possesses relevant information privately. The second stage is either performed by the agent or by the principal. The second-stage information (which may be correlated with the first-stage information) becomes known only to the

In my framework, the principal and the agents, when assigned tasks, choose costly inputs (efforts) into the tasks, and the effort to each task is observable only to the party who performs that task. The main feature of the model is that there is a verifiable and "informative" signal measuring the joint performance of the principal and the agents: In other words, the principal and the agents engage in *team production*. It is quite natural to assume that some aggregate measures are available, even if each activity cannot be measured separately, or direct and imperfect observation of each activity is not verifiable. Although not necessary to establish the results, it is assumed in the simplest model presented in the next section that the only available information for contracting is the aggregate performance measure which is determined jointly by the efforts of the principal and the agents, and by a noise term. Those who are assigned tasks then must be given incentives: If the principal performs some tasks, she must design incentive schemes such that not only her agents but also she herself behaves appropriately. A conflict will arise when the principal does not delegate all the tasks to the agents: inducing the agents to choose higher efforts at their tasks reduces effort incentives for the principal at her tasks because of the use of the team performance measure.[3] Complete delegation in which all the tasks are delegated to the agents mitigates this problem (with other costs, however).

I first study the allocation of two tasks between a principal and an agent who jointly determine team performance. I show that the principal, when she allocates one of the tasks to the agent, assigns him the task with the higher slope of the marginal cost of effort, or with the lower *responsiveness of effort to incentives*. The principal prefers engaging in the task where her reaction to incentives is larger. This *partial delegation* enables the agent to avoid part of the risk he dislikes, and the resulting reduction in effort is compensated for by the corresponding increase in effort from the principal. Since the principal is risk neutral, this assignment is superior to the other way in which the task with the higher effort responsiveness is delegated.

The problem of the partial delegation—balancing the incentives between the principal and the agent—does not arise when the principal allocates both tasks to the agent. However, under such *complete delegation* the principal must increase the responsibility of the agent, and hence impose more risk on him. This tradeoff determines the optimal task allocation. In particular, under some conditions, the principal, when she must delegate something, may chooses to design an agent's job such that he performs both tasks.

Two factors, the measurement error of team performance and the responsiveness of effort to incentives, are important. If the team performance is sufficiently easy to measure, the cost to assign more responsibility to the agent under complete delegation is low, relative to the benefit to avoid the conflicting incentives between the principal and the agent, and hence complete delegation is optimal. If the responsiveness to incentives at *each* task is sufficiently high, the cost of the conflicting incentives under partial delegation is high, and hence complete

party who makes decisions at the second stage, and hence the use of private information for decision making is not a reason for delegation. In their model, the level and the sign of the correlation determine the optimal assignment mode. Of course, since their model is one of adverse selection, the logic behind their results is different from that in my paper.

[3] This problem can be mitigated if a third party is introduced as a residual claimant in order to "break the budget" (Holmstrom 1982). I assume that such a method is infeasible. One reason is the possibility of collusion between the third party and some of the team members (Eswaran and Kotwal 1984). Another justification for the assumption is that the joint benefit generated by the principal and the agents may not be observable by the third party. The benefit can be observable only to the owner of the assets, while some verifiable signal measuring the joint activities may exist.

delegation is better. However, if a player is not able to respond well at one task, it is better to delegate only that task to the agent with small burden and keep the other task under the principal's control with more responsibility assigned to her.

This comparison can be extended to a more general situation of finitely many tasks and finitely many agents. When the technological factors are excluded so that the activities are neither substitutes nor complements, the principal does not hire more than one agent. And the optimal task allocation is either to delegate only one task with the smallest effort responsiveness, or to structure the agent's job such that he performs all the tasks: There is a substantial non-convexity in delegation. Therefore, delegation may always create a problem of either too much delegation or too little delegation compared with the first-best situation where only the technological factors determine the task allocation.

The remainder of the paper is organized as follows. In Section II, the allocation of two tasks between a principal and agents who form a team is examined. Section III discusses extensions to several directions. I examine the cases in which: (i) activities are perfect substitutes; (ii) there are more than two tasks; (iii) the average team performance measure does not coincide with the expected benefit from team production; and (iv) each activity can be directly but imperfectly measured. Section IV is a concluding remark.

II. *Allocating Two Tasks in the Simple Hierarchy*

1. The Basic Model

Suppose that a principal owns two tasks indexed by $t=1, 2$. Input (effort) into task t is denoted as $a_t \in [0, \infty)$. The principal cannot perform all the tasks, and hence hires an agent, assigning him to at least one task. The principal and the agent are equally productive at each task. The party who is assigned to task t makes a choice of effort a_t. I assume that each task is indivisible, for example, because it requires use of assets that cannot be operated by several persons at the same time. The possibility of task sharing is hence excluded, and if one party is assigned to task t, the other party is not allowed to exert any positive effort on task t. The party incurs private cost $\hat{C}(a_1, a_2)$ which is assumed to be strictly increasing and strictly convex. The cost to perform only task 1 is denoted by $C_1(a_1):=\hat{C}(a_1, 0)$. Similarly, $C_2(a_2):=\hat{C}(0, a_2)$.

Activities at the two tasks generate expected benefit $f(a_1, a_2)$. In this section I assume that task-specific measures are not available, and for simplicity the only publicly observable performance measure is $x=f(a_1, a_2)+\epsilon$ where ϵ is Normally distributed with mean zero and variance $\sigma^2>0$.[4] The analysis does not depend on whether or not this measure is equal to the actual benefit to the organization: In the latter case, the actual benefit is not publicly observable. Throughout the paper, I focus on a special case that the expected team performance is linear in the activities, and I assume $f(a_1, a_2)=a_1+a_2$. One can transform more general linear technologies into this form by redefining the effort variables.

The principal is risk neutral and chooses a *task allocation mode* which is represented by a variable $d \in \{1, 2, 12\}: d=1$ implies that the principal delegates task 1 to the agent and

[4] Cases where the average performance does not coincide with the expected benefit or a verifiable signal of each activity is available will be discussed in Section 3.

performs task 2 by herself; under $d=2$ the principal delegates task 2 to the agent and engages in task 1; and $d=12$ implies that the principal assigns the agent both tasks. Recall that for some exogenous reasons (e.g., limited attention), the principal cannot perform both tasks by herself. Otherwise, the principal's best choice would be to allocate none of the tasks to the agent. When $d \in \{1, 2\}$, the task allocation mode is called *partial delegation*, and when $d=12$, it is said to be *complete delegation*.

Besides task allocation, the principal selects a pay scheme $w(\cdot)$ for the agent, which is a function of the verifiable performance measure x. I assume that $w(\cdot)$ is linear in $x: w(x) = \alpha x + \alpha_0$. The model presented here can be regarded as a reduced form of a dynamic model, as in Holmstrom and Milgrom (1987, 1991), in which optimal incentives can be provided with linear contracts in time aggregates of team performance.

The agent is risk averse with constant absolute risk aversion coefficient $r>0$. His von Neumann-Morgenstern utility function is given by $-\exp[-r(w-C_d(a_d))]$ if $d \in \{1, 2\}$; and $-\exp[-r(w-\hat{C}(a_1, a_2))]$ if $d=12$. The agent's certainty equivalent is then calculated as

$$\begin{array}{ll} \alpha(a_1+a_2)+\alpha_0-C_d(a_d)-\dfrac{1}{2}r\sigma^2\alpha^2 & \text{if } d\in\{1, 2\} \\ \alpha(a_1+a_2)+\alpha_0-\hat{C}(a_1, a_2)-\dfrac{1}{2}r\sigma^2\alpha^2 & \text{if } d=12 \end{array}$$

The timing of the events and decisions is as follows. The principal chooses and commits herself to a contractual decision of task allocation mode d and $w(\cdot)$. The agent then decides whether to accept them or not. If he rejects, he will receive the reservation wage w_0. If he accepts, the agent and the principal simultaneously and independently select efforts for their tasks. Then output x publicly becomes known and $w(x)$ is paid to the agent.

The principal's problem is to choose a task allocation mode, a pay scheme, and instructions for efforts in order to maximize her certainty equivalent (the expected profit), subject to the constraints that the principal assures the reservation wage for the agent (the participation constraint) and that both the principal and the agent follow the efforts specified by the principal (the incentive compatibility constraints). For example, under partial delegation $d=1$, the principal's problem can be stated as follows:

$$\max_{a_1, a_2, \alpha, \alpha_0} (a_1+a_2)-\alpha(a_1+a_2)-\alpha_0-C_2(a_2)$$

subject to

$$\begin{aligned} & a_1 \in \arg\max_{a'} \alpha(a'+a_2)+\alpha_0-C_1(a')-\frac{1}{2}r\sigma^2\alpha^2 \\ & a_2 \in \arg\max_{a'} (a_1+a')-\alpha(a_1+a')-\alpha_0-C_2(a') \\ & \alpha(a_1+a_2)+\alpha_0-C_1(a_1)-\frac{1}{2}r\sigma^2\alpha^2 \geq w_0 \end{aligned}$$

In the problem, the fixed salary component α_0 simply plays a role of surplus transfer between the principal and the agent. I thus can reformulate the problem such that the principal's objective is to maximize the total certainty equivalent $TCE(a_1, a_2, \alpha, d)$, which is the certainty equivalent of the joint surplus of the principal and the agent, subject to only the incentive compatibility constraints. The total certainty equivalent is given as follows:

$$TCE(a_1, a_2, \alpha, d) = \begin{cases} a_1 + a_2 - C_1(a_1) - C_2(a_2) - \dfrac{1}{2} r\sigma^2\alpha^2 & \text{if } d \in \{1, 2\} \\ a_1 + a_2 - \hat{C}(a_1, a_2) - \dfrac{1}{2} r\sigma^2\alpha^2 & \text{if } d = 12 \end{cases}$$

Then under partial delegation mode $d \in \{1, 2\}$, the principal's problem is simplified as follows:

$$\max_{a_1, a_2, \alpha} \; TCE(a_1, a_2, \alpha, d)$$

subject to

$$\alpha - C_d'(a_d) = 0$$
$$(1-\alpha) - C_p'(a_p) = 0$$

where $p \neq d$. And the principal's problem under complete delegation $d = 12$ is stated as follows:

$$\max_{a_1, a_2, \alpha} \; TCE(a_1, a_2, \alpha, 12)$$

subject to

$$(a_1, a_2) \in \arg\max_{a_1', a_2'} \alpha(a_1' + a_2') - \hat{C}(a_1', a_2').$$

Let $c = (d, \alpha)$ be the generic contract offered by the principal, and α_d^* be the optimal share rate under mode d. I say the principal prefers a task allocation mode d to another mode d' if $c = (d, \alpha_d^*)$ yields higher total certainty equivalent than $c' = (d', \alpha_{d'}^*)$. The optimal task allocation mode is mode d such that the total surplus under $c = (d, \alpha_d^*)$ is the highest. To find the optimal task allocation mode, I first compare the two partial delegation modes, and then proceed to compare the better partial delegation with complete delegation.

2. Which Task Should Be Delegated?

One can compute the optimal share rates under each of the partial delegation modes easily. The optimal share rates α_d^* under partial delegation $d = 1, 2$, are given by

$$\alpha_1^* = \frac{C_2''}{C_1'' + C_2'' + r\sigma^2 C_1'' C_2''}$$
$$\alpha_2^* = \frac{C_1''}{C_1'' + C_2'' + r\sigma^2 C_1'' C_2''} \tag{1}$$

As expected, both share rates are decreasing in risk aversion r and risk σ^2. The share rate α_d^* is also decreasing in C_d'', the slope of the marginal cost at the agent's task d, and increasing in C_p'' for $p = 1, 2, p \neq d$.

The second derivatives of the cost function have important interpretations. From the incentive compatibility constraints, under partial delegation mode $d \in \{1, 2\}$, the agent's *responsiveness of effort to incentives* at task d is given by $\partial a_d / \partial \alpha = 1/C_d''$, and the principal's effort responsiveness at task p, $p \neq d$, is $\partial a_p / \partial(1-\alpha) = 1/C_p''$. The optimal share rate for the agent should be higher, the more responsive the agent is to incentives at his task or the less responsive the principal is at the other task.

Also note that (i) $\alpha_d^* < 1$, that is, each agent's supply of effort is less than the first-best level that satisfies $1 = C_d'(a_d)$; and (ii) $\alpha_1^* + \alpha_2^* < 1$—under each partial delegation mode, the principal is given higher incentives at her task than the agent when he is assigned the same task under the other partial delegation mode.

Assume that the second derivatives of the cost function are constant: $C_t(\cdot)$ are quadratic for all tasks $t \in \{1, 2\}$. The responsiveness of effort to incentives is thus constant under either partial delegation mode. If $1/C_1'' < 1/C_2''$, one obtains $\alpha_1^* < \alpha_2^*$. This implies that allocating task 1 to the agent has a risk sharing advantage while it has a disadvantage of reduced effort at his task. However, the effort incentives given to the principal are higher under $d=1$ since $1-\alpha_1^* > 1-\alpha_2^*$. The organization with mode $d=1$ has thus the advantage of more effective incentives for the principal. Let B_d^* be the expected benefit under contract (d, α_d^*). Then one can compute the difference in the optimal level of the expected team benefit between the two partial delegation modes as

$$B_1^* - B_2^* = \left(\frac{1}{C_2''} - \frac{1}{C_1''}\right)(1 - \alpha_1^* - \alpha_2^*) > 0.$$

That is, $d=1$ is superior to $d=2$ in terms of both risk sharing and effort incentives under the initial assumption. I report this result in the following proposition.

Proposition 1 *For $t, s \in \{1, 2\}$, $t \neq s$, the principal prefers mode $d=t$ to mode $d=s$ if and only if $1/C_t'' < 1/C_s''$.*

At first sight, this result may be surprising: the agent should perform the less responsive task to incentives. Under the assumption of quadratic costs, the task delegated to the agent has a higher marginal cost of effort. However, the result is clear and intuitive once one recognizes that the principal as well must be given incentives. It is more economical to make the risk neutral principal exert high effort in response to incentives than the risk averse agent.

This simple result has several implications. First, it explains why workers at a hierarchically higher position can have discretion about more aspects of their work, facing more intense incentives, than those at a lower position. Second, the lack of strong explicit incentives in actual contracts may be partly explained by the contract designer's task allocation decision. The employer may delegate to employees tasks at which they cannot respond to incentives very much, and hence the contracts contain few incentive provisions.

3. The Optimal Task Allocation Mode

I now compare the optimal partial delegation to complete delegation. Assuming interior solutions $a_1 > 0$ and $a_2 > 0$, the incentive compatibility constraints under complete delegation mode $d=12$ are as follows:

$$\alpha = \hat{C}_1(a_1, a_2)$$
$$\alpha = \hat{C}_2(a_1, a_2)$$

where $\hat{C}_t := \partial\hat{C}/\partial a_t$. The optimal share rate α_{12}^* must satisfy

$$r\sigma^2 \alpha_{12}^* = \frac{(\hat{C}_{11} - \hat{C}_{12}) + (\hat{C}_{22} - \hat{C}_{12})}{\hat{C}_{11}\hat{C}_{22} - \hat{C}_{11}^2}(1 - \alpha_{12}^*)$$

where $\hat{C}_{ts}=\partial^2\hat{C}/(\partial a_t \partial a_s)$.

One can show that when two tasks are complementary in the cost function ($\hat{C}_{12}<0$), making it more negative raises the optimal share rate, and hence stronger incentives can be provided for the agent under complete delegation. Since the optimal values under partial delegation do not depend on the cross-partial derivatives, more complementary tasks favor complete delegation more: delegating both tasks together to the agent is more likely to be desirable. On the other hand, when $\hat{C}_{12}$ increases to become positive and hence two tasks are substitutes, the optimal share rate is reduced, and partial delegation is more likely to be superior. This observation is similar to the point made by Holmstrom nad Milgrom (1991, pp. 32-33). However, in their setting, there is no team performance measure: One of the activities can be measured with noise while the other cannot be measured at all. Then, under effort substitutes, increasing the share rate for the measurable task causes the agent to substitute effort away from the other task. In my setting, the same effect occurs because of team production.

To exclude these technological factors and clarify the main tradeoff, I focus on the case in which no interaction between two tasks exists: I assume $\hat{C}(a_1, a_2)=C_1(a_1)+C_2(a_2)$.[5] The case of perfect substitutes will be discussed in Section III. I continue to assume that C_t'' is constant for $t=1, 2$.

The optimal share rate under $d=12$ is then computed as

$$\alpha_{12}^*=\frac{C_1''+C_2''}{C_1''+C_2''+r\sigma^2 C_1''C_2''}. \tag{2}$$

Note that it is higher than α_1^* and α_2^*. This is because under the complete delegation, the share rate provides the agent with effort incentives for both task 1 and task 2, while increasing α_t^* yields stronger incentives for task t but weaker incentives for the other task: More authority for decision making should accompany more responsibility. Complete delegation $d=12$ has a clear advantage over partial delegation that under the former mode the effect of the share rate on a task is in the same direction as that on the other task. However, the agent bears more risk under $d=12$ and hence it is more costly in terms of risk sharing. The optimal delegation mode is thus determined by the relative importance of the agent's incentives and risk sharing, as the following proposition reports.

Proposition 2 *The optimal task allocation mode is complete delegation* $d=12$ *if* $r\sigma^2<\min\{1/C_1'', 1/C_2''\}$. *Partial delegation is optimal if the inequality is reversed.*

The proof is in Appendix. The proposition states that without cost complementarities, the principal prefers delegating both tasks together to the agent if the responsiveness of effort to incentives at *either* task is sufficiently high, or the team performance is relatively easy to measure. On the other hand, if the team performance measure is not informative enough, the principal leaves one of the tasks under her control, the task with the higher responsiveness of incentives to efforts (that is, the task with higher $1/C_t''$).

[5] This additive cost function also ensures that making the agent who works for a task exert a little bit of effort to an additional task does not requires effort away from the first task. With this assumption, one can exclude the fixed cost of engaging multiple tasks analyzed in Holmstrom and Milgrom (1991) and Itoh (1991, 1992). The separability is not essential, however. See Itoh (1991, 1994).

To see the underlying structure leading this result more closely, let $\bar{a}_t(\alpha, d)$ be the effort supply function for task t obtained from the incentive compatibility constraints given d, and $TCE^0(\alpha, d) = TCE(\bar{a}_1(\alpha, d), \bar{a}_2(\alpha, d), \alpha, d)$. Suppose that the choice of mode d is restricted to $\{1, 12\}$. One can show $\partial TCE^0/\partial\alpha|_{d=12} > \partial TCE^0/\partial\alpha|_{d=1}$: The share rate and the degree of delegation are complementary. Furthermore, $TCE^0(\alpha,\ d;\ -C''_t,\ -r,\ -\sigma^2)$ is *supermodular*:[6] The optimal share rate and task allocation mode $(\alpha^*(\theta),\ d^*(\theta))$ are then nondecreasing in any component of the parameter vector $\theta = (-C''_t,\ -r,\ -\sigma^2)$. Increasing any one of the parameters reduces the return from increasing the share rate, which effect in turn diminishes the relative value of complete delegation to the optimal partial delegation and vice versa. That is, the effects reinforce each other.

4. Introducing More Agents

In this subsection, I extend the previous model by introducing the possibility that the principal can hire another agent. Then the principal has one more option in terms of task allocation modes, that is, she can assign one agent task 1 and the other agent task 2. I assume that the agents are identical in the sense that they are equally productive at each task and have the same preference (identical risk aversion). Also suppose that the reservation wage is zero so that just hiring another agent is costless. Denote the agent who is assigned task t by agent t. I call this task allocation mode the *specialized delegation*, denoted as $d = 1/2$.

The principal's problem under $d = 1/2$ is defined as follows:

$$\max_{a_1, a_2, \alpha_1, \alpha_2} a_1 + a_2 - C_1(a_1) - C_2(a_2) - \frac{1}{2} r\sigma^2\alpha_1^2 - \frac{1}{2} r\sigma^2\alpha_2^2$$

subject to

$$\begin{aligned} \alpha_1 - C'_1(a_1) &= 0 \\ \alpha_2 - C'_2(a_2) &= 0 \end{aligned}$$

where α_t is the agent t's share rate at task t. Although this new mode has a disadvantage in terms of risk bearing, flexibility increases in the sense that the two incentive compatibility constraints are independent of one another. The optimal share rates, denoted by $\hat{\alpha}_1$ and $\hat{\alpha}_2$, are well known and computed as

$$\begin{aligned} \hat{\alpha}_1 &= \frac{1}{1 + r\sigma^2 C''_1} \\ \hat{\alpha}_2 &= \frac{1}{1 + r\sigma^2 C''_2} \end{aligned} \tag{3}$$

Note that $\alpha^*_{12} > \hat{\alpha}_t > \hat{\alpha}^*_t$ for $t = 1, 2$.

The following proposition establishes that the principal never chooses to hire another agent because the specialized delegation is always dominated by complete delegation, and hence the introduction of multiple agents does not alter the conclusion in the previous analysis.

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[6] Let a vector z be defined by $z = (\alpha, d,\ -C''_t,\ -r,\ -\sigma^2)$, and let $d \in \{t, 12\}$. Then TCE^0 is supermodular if $\partial^2 TCE^0/(\partial z_i \partial z_j) \geq 0$ and $\partial TCE^0/\partial z_i|_{d=12} \geq \partial TCE^0/\partial z_i|_{d=t}$ for all $i,\ j \in \{1, 3, 4, 5\}$ and $i \neq j$. See, for example, Topkin (1998) for details.

Proposition 3 *The principal prefers $d=12$ to $d=1/2$.*

Proof Suppose $\hat{\alpha}_1 \geq \hat{\alpha}_2$. Define $\bar{\alpha}=\hat{\alpha}_1$. Let $\bar{a}_t$ be the effort at task t under contract $(12, \bar{\alpha})$, and $\hat{a}_t$ be the optimal effort at task t under contract $(1/2, \hat{\alpha}_1, \hat{\alpha}_2)$. Then clearly $\bar{a}_1=\hat{a}_1$. And since $\bar{\alpha}=\hat{\alpha}_1 \geq \hat{\alpha}_2, \bar{a}_1 \geq \bar{a}_2$. Furthermore, the risk premium is greater under $(1/2, \hat{\alpha}_1, \hat{\alpha}_2)$ than under 12, $\bar{\alpha}$). The principal hence prefers $d=12$ to $d=1/2$. When $\hat{\alpha}_1 \leq \hat{\alpha}_2$, define $\bar{\alpha}=\hat{\alpha}_2$ and follow the same steps. □

When the activities are neither substitutes nor complements, the principal has no interest in hiring multiple agents because specialized delegation simply raises the risk premium terms without increasing the expected benefit from complete delegation. Since introducing the possibility of cost substitutes or complements does not alter the optimal share rates under $d=1/2$, specialized delegation is never optimal when two tasks are complementary. On the other hand, when they are substitutes, the advantage of complete delegation is reduced, and hence there is a possibility that specialized delegation is optimal.[7] The first analysis in the next section will present such a case.

III. *Extensions*

1. The Case of Attention Allocation

Propositions 2 and 3 in the previous section depend on the assumption that there is no externality in the activities at two tasks. The introduction of either cost substitutes or complements only affects complete delegation, and the effects are straightforward as I explained. However, it is instructive to see how the results change in the extreme case that two activities are perfect substitutes.

Suppose that $\hat{C}(a_1, a_2)=C(a_1+a_2)$. Then since $C_1(a)=C_2(a)=C(a)$, the principal is indifferent between two partial delegation modes $d=1$ and $d=2$. The optimal share rate under partial delegation is given by $\alpha^*=1/(2+r\sigma^2 C'')$. Concerning the optimal share rate under complete delegation $d=12$, the allocation between two tasks does not matter, and the agent's problem is to choose the total effort $a=a_1+a_2$ to satisfy the incentive compatibility constraint $\alpha-C'(a)=0$. The principal's problem is to choose (α, a) to maximize $a-C(a)-(r/2)\sigma^2\alpha^2$ subject to the incentive compatibility constraint. The optimal share rate α_{12}^* is calculated as $\alpha_{12}^*=1/(1+r\sigma^2 C'')$. Comparing this with the optimal share rates under specialized delegation in (3) with $C_1''=C_2''=C''$, one can observe $\alpha_{12}^*=\hat{\alpha}_1=\hat{\alpha}_2$. Since under specialized delegation, the principal can choose the share parameter and the effort at *each* task such that $a_t-C(a_t)-(r/2)\sigma^2\alpha_t^2$ is maximized, obviously specialized delegation is better than complete delegation.

Proposition 4 *When two activities are perfect substitutes, there exists a unique value $m<1$ such that specialized delegation is optimal when $r\sigma^2<m/C''$ and partial delegation is optimal when $r\sigma^2>m/C''$.*

Proof From the argument given above, complete delegation is never optimal. I first show that if $r\sigma^2 \geq 1/C''$, then partial delegation is preferred to specialized delegation. Define $\bar{\alpha}=\hat{\alpha}_1$. Then

[7] See Itoh (1994) for more on this.

$1-\bar{\alpha}=1-\hat{\alpha}_1\geq\hat{\alpha}_2$ by the assumption $r\sigma^2\geq 1/C''$. Let $\bar{a}_t$ be the effort at task t under partial delegation with the share rate $\bar{\alpha}$, and $\hat{a}_t$ be the optimal effort at task t under specialized delegation. Then clearly $\bar{a}_1=\hat{a}_1$. And since $C'(\bar{a}_2)=1-\bar{\alpha}\geq\hat{\alpha}_2=C'(\hat{a}_2)$, $\bar{a}_2\geq\hat{a}_2$ holds. Furthermore, the risk premium terms are greater under specialized delegation than under partial delegation. The principal hence prefers partial delegation to specialized delegation.

If $r\sigma^2C''$ goes to zero, the optimal value approaches to the first-best level under specialized delegation since $\hat{\alpha}_t\to 1$ for $t=1, 2$. However, under partial delegation, the first-best cannot be attained since $\alpha^*\to 1/2$. Therefore, there exists a value $m<1$ such that specialized delegation is optimal if $r\sigma^2C''<m$. Finally, although in either mode the optimal value increases as $r\sigma^2C''$ decreases, it increases faster under specialized delegation because $0>d\alpha^*/d(r\sigma^2C'')>d\hat{\alpha}_t/d(r\sigma^2C'')$ and there is an additional risk premium term under specialized delegation. Therefore, for $r\sigma^2C''>m$, partial delegation is optimal. □

When two activities are perfect substitutes, an agent, when assigned to both tasks, behaves as if he allocates his attention only to one of the activities, while by hiring two agents, the principal enables each agent to specialize and to allocate all of his attention to his task. Therefore, the specialized delegation is always better than complete delegation, which result is in stark contrast with Proposition 3 under independent activities.

The optimal task allocation is thus either partial delegation or specialized delegation, while the result is qualitatively similar to Proposition 2: The principal delegates all the activities to agents if the responsiveness of effort to incentives is sufficiently high or the team performance measure is easy to measure.

2. Multiple Tasks: A Non-convexity in Delegation Decision

I extend the model in the previous section by considering more than two tasks. Suppose there are N tasks and let $T=\{1, \ldots, N\}$ be the set of tasks. Let $D\subseteq T$ be the set of activities delegated to the agent. D can be any subset of T except $\emptyset$. The principal then engages in activities in $D^c=T\backslash D$. The private cost to perform task $t\in T$ is $C_t(a_t)$, which I assume quadratic. I exclude substitutability and complementarity, and hence the total cost to perform tasks in D is given by $\sum_{t\in D} C_t(a_t)$. The expected benefit is $f(a_1, \ldots, a_N)=\sum_{t=1}^{N} a_t$, and the team performance measure is $x=f(a_1, \ldots, a_N)+\epsilon$. Since the principal never hires more than one agent as in the previous basic model, each task allocation mode is represented by a delegation set D. Given D, the principal solves

$$\max_{\alpha, a_1, \ldots, a_N} \sum_{t=1}^{N} a_t-\sum_{t=1}^{N} C_t(a_t)-\frac{1}{2}r\sigma^2\alpha^2$$

subject to

$$\alpha-C_t'(a_t)=0, \quad t\in D;$$
$$(1-\alpha)-C_t'(a_t)=0, \quad t\in D^c.$$

The optimal share rate α_D^* is calculated as

$$\alpha_D^*=\frac{\sum_{t\in D}\prod_{s\neq t} C_s''}{\sum_{t=1}^{N}\prod_{s\neq t} C_s''+r\sigma^2\prod_{t=1}^{N} C_t''}. \tag{4}$$

Clearly, $0<\alpha_D^*<1$ for all $D\subseteq T$, and if $D\subset D'$ ($D\subseteq D'$ and $D\neq D'$), then $\alpha_D^*<\alpha_{D'}^*$.

The following lemma highlights the non-convexity of task allocation in this model.

Lemma 1 *The optimal task allocation mode must be either $D=\{t\}$ for some $t\in T$ or $D=T$.*

Proof Suppose D contains at least two tasks and $D\neq T$. Suppose that $\alpha_D^*\geq 1/2$. Define $\bar{\alpha}=\alpha_D^*$, and consider mode T with the share rate $\bar{\alpha}$. Let a_t^* be the effort chosen under contract (D, α_D^*), and $\bar{a}_t$ be the effort under $(T, \bar{\alpha})$. Then clearly $a_t^*=\bar{a}_t$ for $t\in D$. For $t\in D^c$, since $1-\alpha_D^*\leq 1/2\leq \bar{\alpha}$, $a_t^*\leq\bar{a}_t$ holds. Thus, the total certainty equivalent under $(T, \bar{\alpha})$ is at least as high as that under (D, α_D^*). Since $D\neq T$ and the optimal share rate under mode T satisfies $\alpha_T^*>\alpha_D^*=\bar{\alpha}$, (T, α_T^*) is strictly preferred to (D, α_D^*).

Next, suppose $\alpha_D^*\leq 1/2$. In this case, I consider mode $D'=\{s\}$ for some $s\in D$, with share rate $\bar{\alpha}$ as defined above. Then $\bar{a}_s=a_s^*$. For $t\in D\backslash\{s\}$, since $1-\bar{\alpha}\geq\alpha_D^*$, $\bar{a}_t\geq a_t^*$. Finally, for $t\in D^c$, $\bar{a}_t=a_t^*$ because $1-\bar{\alpha}=1-\alpha_D^*$. Thus, the total certainty equivalent under $(D', \bar{\alpha})$ is at least as high as that under (D, α_D^*). Since the optimal share rate under D' satisfies $\alpha_{D'}^*<\alpha_D^*=\bar{\alpha}$, mode D' is strictly preferred to mode D. □

The lemma states that the principal either delegates to the agent just one task or designs the agent's job such that the agent engages in all the tasks. This result implies that there is a bias toward under-delegation or over-delegation. In the current model where there is no externality among activities, the task allocation does not matter when the agent's efforts are observable. When some activities are mutually complements, they should be allocated together to one party from the purely technological perspective. However, when the incentive problems exits, all of those mutually complementary activities may not be grouped as a job. On the other hand, suppose that all activities are mutually substitutes so that the technological consideration leads to very fine job design in which each of the principal and the agents is assigned to just one task. With the incentive considerations, the principal may prefer allocating all the tasks to just one agent unless the activities are sufficiently substitutable.

Using the lemma, I can obtain the optimal task allocation mode in this multi-task model as follows.

Proposition 5 *If $r\sigma_2<\min_t\{1/C_t''\}$, complete delegation $(D=T)$ is optimal. If $r\sigma_2>\min_t\{1/C_t''\}$, then the optimal task allocation is $D=\{t^*\}$ where $t^*\in\arg\min_t\{1/C_t''\}$.*

The proof is provided in Appendix. The result is a straightforward extension of the result in the case of two tasks (Proposition 2). Complete delegation is more likely to be optimal as the responsiveness of effort to incentives at each task is higher or the team performance measure is less noisy. If complete delegation is suboptimal, the principal chooses to delegate to the agent only the task with the smallest responsiveness of effort to incentives.

3. A Generalization in the Team Performance Measure

In the model presented in the previous section, I assumed that the team performance measure x is of the form $f(a_1, a_2)+\epsilon$, the sum of the expected benefit and a noise term. This assumption enabled me to highlight the important effect of the responsiveness of effort to incentives at the tasks in the previous section. Relaxing this assumption does not alter the previous results qualitatively, while it introduces other factors in the determination of the optimal task allocation.

Suppose that $f(a_1, a_2)=a_1+a_2$ and $x=\mu_1 a_1+\mu_2 a_2+\epsilon$ where μ_t are positive constants for $t=1, 2$. It is assumed that the expected benefit and the average team performance are not necessarily equal. For example, while the benefit from the joint production depends on production costs and quality both of which are affected by the inputs from the principal and the agent, the only former are publicly observable and hence contractible. Other elements of the model are the same as those in the previous section.

Higher μ_t implies that the team performance measure is more informative concerning the activity at task t, given variance σ^2. The responsiveness of effort to incentives at task t is then given by $R_t:=\mu_t/C_t''$: As the marginal change of the team performance measure with respect to a_t increases, the performer's response to incentives increases.

Under this setting, it is straightforward to calculate the optimal share rates α_d^* for task delegation modes $d\in\{1, 2, 12\}$:

$$\begin{aligned}\alpha_1^* &= \frac{R_2^{-1}}{\mu_1 R_2^{-1}+\mu_2 R_1^{-1}+r\sigma^2 R_1^{-1}R_2^{-1}}\\ \alpha_2^* &= \frac{R_1^{-1}}{\mu_1 R_2^{-1}+\mu_2 R_1^{-1}+r\sigma^2 R_1^{-1}R_2^{-1}}\\ \alpha_{12}^* &= \frac{R_1^{-1}+R_2^{-1}}{\mu_1 R_2^{-1}+\mu_2 R_1^{-1}+r\sigma^2 R_1^{-1}R_2^{-1}}.\end{aligned} \tag{5}$$

The parameters μ_t have two effects on the optimal share rates. Increasing μ_t raises the effort responsiveness to incentives at task t, which effect increases the optimal share rate under partial delegation mode $d=t$. On the other hand, by the incentive compatibility constraints $\alpha\mu_t-C_t'(a_t)=0$ under mode $d=t$, higher μ_t implies that the same incentive intensity can be attained with smaller α, which effect reduces the risk premium. The second effect hence reduces the optimal share rate α_t^*. Therefore, increasing μ_t may increase or decrease the optimal share rate under partial delegation mode $d=t$. This second effect disappears for the incentive intensity $\alpha_t^*\mu_t$: It is increasing in μ_t. Similarly, the optimal share rate α_{12}^* under complete delegation may be increasing or decreasing in μ_t, $t=1, 2$. However, the second effect does not completely disappear for the incentive intensities $\alpha_{12}^*\mu_1$ and $\alpha_{12}^*\mu_2$.

Two additional insights can be obtained from the introduction of the μ_t parameters.[8] First, the optimal partial delegation mode is determined not only by the effort responsiveness to incentives but also by the informativeness of the team performance measure. One can show that for t, $s\in\{1, 2\}$ and $t\neq s$, partial delegation mode t is preferred to mode s if and only if $\mu_t^2+r\sigma^2 C_t''>\mu_s^2+r\sigma^2 C_s''$. For simplicity, suppose $C_1''=C_2''$. Then partial delegation mode $d=t$ is preferred to $d=s$ if and only if $\mu_t>\mu_s$. Under the condition $\mu_t>\mu_s$, the responsiveness of effort to incentives is higher at task t than at task s, and hence mode $d=s$ has an advantage since the principal can engage in the more responsive task. Furthermore, since $\alpha_t^*>\alpha_s^*$, one might think that the principal's incentive intensity would be lower under $d=t$ than under $d=s$ as in the basic model. This is not true, however. Since the principal's incentive intensity under mode $d=t$ is $1-\alpha_t^*\mu_s$, using (5), one can obtain $1-\alpha_1^*\mu_2=1-\alpha_2^*\mu_1$: The principal's incentive intensity is equal between two partial delegation modes when $C_1''=C_2''$. Therefore, delegating the more informative task is more desirable although the agent engages in the task with the

[8] Since the derivation of the results is not interesting, I omit the presentation of the formal analysis.

higher effort responsiveness to incentives.

The second additional insight is concerning the optimality of complete delegation.[9] One can show that the optimal task allocation is complete delegation if and only if

$$r\sigma^2 < \min\{(2\mu_2-\mu_1)R_1,\ (2\mu_1-\mu_2)R_2\}.$$

Besides the already discussed effects of the responsiveness to incentives and the measurement cost, the condition suggests that for complete delegation to be optimal, the informativeness of the team performance be sufficiently similar between two tasks. For example, if $\mu_1 > 2\mu_2$, partial delegation $d=1$ is better than complete delegation because the latter mode can motivate both the agent and the principal without incurring much risk on the agent. Assigning to an agent two tasks that have very different signal-quality in terms of the activities is costly because the team performance measure must be utilized for incentives at both tasks.[10]

4. Monitoring Each Activity

The driving force of the results in the paper is the use of team performance measures. I assumed that a team performance measure is the only available information for contracts. Although observing each activity separately may be possible (with errors), such measures are often not verifiable, and hence non-contractible.[11]

Even if direct observations of the activities are verifiable, my results do not change qualitatively as long as there is a verifiable and informative (in the sense of Holmström (1979)) signal of the team production. The conflicting objective of motivating both the principal and the agent under partial delegation modes then arises and leads to the possibility of delegating all the tasks to the agent.

Ofcourse, the measurement error in each direct observation emerges as a new important determinant for task allocation. Suppose that $x=f(a_1, a_2)+\epsilon=a_1+a_2+\epsilon$, $y_1=a_1+\eta_1$, and $y_2=a_2+\eta_2$ where ϵ, η_1, η_2 are Normally and independently distributed with mean zero and variances σ_ϵ^2, $\sigma_{\eta_1}^2$, and $\sigma_{\eta_2}^2$, respectively. The agent's pay scheme is $w(x, y_1, y_2)=\alpha x+\beta_1 y_1+\beta_2 y_2+\alpha_0$. It is then easy to see that the optimal share rate α_t^* under partial delegation $d=t$, $t=1, 2$, is increasing in $\sigma_{\eta_t}^2$: If the observation of activity at task t is noisier, more incentives are provided to the agent based on the team performance measure.[12] And if $C_1''=C_2''$, then the comparison between two partial delegation modes is determined by the informativeness of individual observations of two activities: For $t, s\in\{1, 2\}$, $t\neq s$, $d=t$ is preferred to $d=s$ if and only if $\sigma_t^2<\sigma_s^2$. The principal prefers to allocate the agent the task with the better observation of the activity.

The optimal share rate α_{12}^* under $d=12$ is also increasing in $\sigma_{\eta_1}^2$ and $\sigma_{\eta_2}^2$. However, the marginal increase of the share rate with regard to $\sigma_{\eta_t}^2$ is higher under mode $d=t$ than under

[9] The argument in Proposition 3 can be applied to show that specialized delegation is never optimal in this model.

[10] The similar result in Holmstrom and Milgrom (1991) is due to the problem of attention allocation in contrast to the team performance problem in the current paper.

[11] Such nonverifiable measures could be utilized, however. See Demski and Sappington (1993) and Hermalin and Katz (1991).

[12] On the other hand, $\sigma_{\eta_s}^2$, $s\neq t$, does not affect the optimal share rate α_t^*. Since higher β_s increases the risk premium and reduces the principal's incentives at task s, $\beta_s=0$ is chosen under mode $d=t$.

$d=12$. It is not easy to see how the change in the informativeness of the signal of each activity affects the optimal task allocation mode. I conjecture that complete delegation is more likely to be optimal as the activity at each task is harder to measure, because more informative signals of the individual activities reduce the cost associated with the conflicting incentives in partial delegation.

IV. *Concluding Remarks*

This research can be viewed as an example that follows a recent trend in the economics of organization: Incentives can be provided not only through pay schemes but also through other control instruments such as task allocation, ownership of relevant assets, and the restrictions on the ways jobs are conducted. In this paper, it is shown that delegating most tasks to a subordinate may be a device to mitigate an incentive problem associated with the use of a verifiable and informative team performance measure. Without cost substitutes or complements, the optimal task assignment is non-convex: Either to keep most decision making under direct control or to delegate all decision making to the subordinate is optimal. The latter complete delegation mode is more likely to be preferred as the team performance is easier to measure, tasks are similar in terms of the informativeness of the team performance measure concerning the activities, and the effort responsiveness to incentives at each task is higher.

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Appendix

Proof of Proposition 2

Suppose $1/C_1'' < 1/C_2''$ and compare $d=1$ and $d=12$. Let TCE_d^* be the optimal value of the total certainty equivalent under mode d. Then

$$TCE_{12}^* - TCE_1^* = \frac{1}{2C_1''C_2''}\{C_2''(\alpha_{12}^* - \alpha_1^*)(2-(\alpha_{12}^*+\alpha_1^*)(1+r\sigma^2C_1''))$$
$$+C_1''(1-(\alpha_{12}^*-\alpha_1^*))(\alpha_{12}^*+\alpha_1^*-1)\}.$$

Substituting α_{12}^* and α_1^* from (1) and (2) yields

$$TCE_{12}^* - TCE_1^* = \frac{1-r\sigma^2C_1''}{2(C_1''+C_2''+r\sigma^2C_1''C_2'')} > 0 \Leftrightarrow r\sigma^2 < 1/C_1''.$$

□

Proof of Proposition 5

I first show that among the task allocation modes such that only one task is delegated to the agent, the most preferred mode is $D=\{t^*\}$ where $t^* \in \arg\min_t \{1/C_t''\}$. Suppose $t' \notin \arg\min_t \{1/C_t''\}$ and consider mode $D'=\{t'\}$. Then by (4), $\alpha_D^* < \alpha_{D'}^*$. The risk premium is thus smaller under D than under D'. It is then straightforward to show that the expected benefit is also higher under D than under D'. Therefore D is preferred to D'.

Next compare D with mode T. Let a_t^D be the optimal effort at task t under mode D and a_t^T be the optimal effort under mode T. Then for each task $t \neq t^*$, it is easy to show

$$(a_t^T - C_t(a_t^T)) - (a_t^D - C_t(a_t^D)) \propto \alpha_T^* - (1-\alpha_D^*) \propto 1 - r\sigma^2 C_{t^*}''$$

where $\propto$ means "proportional to." For the remaining terms in the total certainty equivalent, one can calculate the difference as follows.

$$\left(a_{t^*}^T - C_{t^*}(a_{t^*}^T) - \frac{1}{2}r\sigma^2\alpha_T^*\right) - \left(a_{t^*}^D - C_{t^*}(a_{t^*}^D) - \frac{1}{2}r\sigma^2\alpha_D^*\right)$$
$$\propto (\alpha_T^* - \alpha_D^*)(2-(\alpha_T^*+\alpha_D^*)(1+r\sigma^2C_{t^*}''))$$
$$\propto 1 - r\sigma^2 C_{t^*}''.$$

Therefore, T is preferred to D if and only if $r\sigma^2 < 1/C_{t^*}''$. □