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UNION, EFFICIENCY OF LABOUR AND ENDOGENOUS GROWTH*

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Abstract

This paper investigates the growth effect of unionization, with a special focus on the 'Efficiency Wage Hypothesis' and on the human capital formation financed publicly by taxation. The employment level becomes independent of the public policy parameters and of the labour market institution parameters when the government chooses the growth rate maximizing tax rate. The growth effect of unionisation is found to be independent of the labour union's orientation in its preference structure. It depends on the net efficiency of the worker. The growth effect of unionisation is different from the corresponding welfare effect.

Keywords: labour union, efficiency wage hypothesis, human capital formation, income tax, endogenous growth
JEL Classification Codes: J51, O41, J31, J24, H52, H21

I. *Introduction*

During recent years, many European countries have been suffering from the problems of high unemployment rate as well as of low economic growth rate. During 2003 - 2019, the unemployment rate in European Union remained much higher than the rest of the world. Also during this time, the annual percentage growth rate of GDP in the European Union was lower

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Table 1. Unemployment Rate and Annual Growth Rate of GDP during 2003 – 2019

Year	European Union		World	
	Unemployment Rate	Growth Rate	Unemployment Rate	Growth Rate
2003	9.65	0.93	6.19	2.96
2004	9.88	2.60	6.02	4.41
2005	9.57	1.93	5.90	3.91
2006	8.63	3.50	5.59	4.38
2007	7.45	3.15	5.35	4.32
2008	7.20	0.65	5.35	1.85
2009	9.12	-4.33	6.00	-1.67
2010	9.80	2.20	5.92	4.30
2011	9.84	1.84	5.80	3.14
2012	10.80	-0.74	5.77	2.52
2013	11.32	-0.06	5.77	2.66
2014	10.84	1.58	5.63	2.85
2015	10.02	2.36	5.64	2.88
2016	9.12	2.05	5.67	2.59
2017	8.14	2.73	5.57	3.26
2018	7.27	2.15	5.39	3.10
2019	6.67	1.52	5.40	2.47

Source: Unemployment, Total (% of total labor force) (modeled ILO estimate) ; GDP Growth (annual %), World Bank.

than that of the rest of the world. This is shown in the table 1.

Can unionisation in the labour market explain these high unemployment rates and low growth rates in European Union? The question receives importance because labour unions are very active in European countries. Table 2, given below, describes the labour union density rate[1] for a few European countries and for a few non-European countries for the year 2015; and it clearly shows the strong presence of labour unions in European countries.

However, the existing empirical literature does not converge to an unique answer to this question. Empirical works of Nickell and Layard (1999), Nickell (1997), Fitoussi et al. (2000), Nickell et al. (2005) etc. show a negative impact of unionisation on employment creation. This strengthens the pro-competitive market ideology of reducing labour market frictions to generate employment. Studies like Kim (2005), Pantuosco et al. (2001), Vedder and Gallaway (2002), Padovano and Galli (2003), Carmeci and Mauro (2003), Adjemian et al. (2010), Carmeci and Mauro (2002) etc. show a negative impact of labour unions on economic growth. On the other hand, Traxler and Brandl (2009), Monastiriotis (2007), Jaoul-Grammare and Terraz (2013) and OECD (2004) show that unionization does not have any significant impact on the growth of GDP. Asteriou and Monastiriotis (2004) finds a statistically significant positive relationship between unionism in the labour market and the rate of economic growth. These differences in empirical findings motivate researchers to start theoretical investigation about the role of unionization on unemployment and economic growth.

There exists a set of theoretical works[2] focusing on various channels through which labour

[1] This labour union density rate shows the number of union member employees as a percentage of the total number of employees.

TABLE 2. LABOUR UNION DENSITY RATE IN 2015

European Country	Labour Union Density Rate	Non-European Country	Labour Union Density Rate
Norway	52.5	Malaysia	8.8
Belgium	54.2	New Zealand	17.9
Sweden	67	Japan	17.4
Denmark	68.6	Canada	28.6
Finland	66.5	Republic of Korea	10.1
Iceland	91.6	United States	10.6
Italy	35.7	Australia	15

Source: Trade Union Density Rate (%) - Annual, ILO.

unions can affect economic growth. However, only a few contributions from that[3] set of works focus on the change in the efficiency of labour due to unionisation in the labour market. Unfortunately, no other work except Bhattacharyya and Gupta (2016) considers the role of empirically confirmed[4] 'Efficiency Wage Hypothesis'[5,6]. This is a serious limitation of the existing literature, because, this hypothesis points out a positive role of unionisation on the efficiency of labour. On the one hand, this hypothesis states that a higher wage rate leads to a higher efficiency level of the workers[7], but, on the other hand, a powerful labour union goes for a higher wage rate. A few works consider the role of efficiency wage on union firm bargaining[8] using a static framework. However, they do not analyse its role on economic growth in a dynamic framework.

Even Bhattacharyya and Gupta (2016) ignores the intertemporal accumulation of labour efficiency through investment in human capital accumulation. This is also a serious limitation because a vast empirical literature[9] points out the importance of human capital formation on economic growth and many theoretical models analyse how human capital formation explain endogenous growth in competitive labour markets. In many countries, the government spends a huge amount on education to raise the efficiency of workers.[10] Table 3 presented below shows

[2] This set consists of Palokangas (1996, 2004), Sorensen (1997), Bräuninger (2000b), Irmen and Wigger (2002/2003), Ramos-Parreño and Sánchez-Losada (2002), Lingens (2003a, 2003b, 2007), Chang et al. (2007), Adjemian et al. (2010), Lai and Wang (2010), Bhattacharyya and Gupta (2015, 2016), Ji et al. (2016), Grieben and Sener (2017), Chu et al. (2016), Boone (2000), Chang and Hung (2016) etc.

[3] For example, Lingens (2003b), Sorensen (1997), Bhattacharyya and Gupta (2016) and Ramos-Parreño and Sánchez-Losada (2002).

[4] See Peach and Stanley (2009) for meta-analysis on this topic.

[5] 'Efficiency wage hypothesis' is well-explored in the literature. For example, Solow (1979), Yellen (1984), Stiglitz (1976), Shapiro and Stiglitz (1984), Akerlof (1982, 1984), Akerlof and Yellen (1986) etc. can be seen.

[6] An earlier version of Palokangas (2004) paper, i.e., Palokangas (2003) incorporates 'Efficiency Wage Hypothesis' in the model. However, this version does not stress on the role of 'Efficiency Wage Hypothesis' while analysing effects of unionisation. Indeed in a footnote, this fact is accepted and written as "*However, the results in this paper hold even if the effort per worker is wholly inflexible......*". However, the published version of this paper, i.e., Palokangas (2004) does not consider the 'Efficiency Wage Hypothesis'.

[7] See sections 9.2 and 9.3 of Romer (2006).

[8] Some examples are Garino and Martin (2000), Marti (1997), Mauleon and Vannetelbosch (2003) and Pereau and Sanz (2006).

[9] See for example Barro (1991), Barro and Lee (1994), Jones (2002), Valadkhani (2003) etc.

[10] Government also spends for health care of the people to raise their efficiency. However, in this model we overlook the health aspect of workers. As a result, skill level becomes equivalent to the stock of human capital in this model.

TABLE 3. EXPENDITURE ON EDUCATION IN 2014

Countries	Education Expenditure as Percentage of Government Expenditure
United States	13.40
Belgium	12.01
Denmark	13.83
Finland	12.31
New Zealand	16.28
Germany	11.15
Australia	13.88
Austria	10.41
Italy	8.01
Japan	9.09
Netherlands	12.15
Norway	17.04
Poland	11.58
Spain	9.54
Sweden	15.33
Switzerland	15.46
United Kingdom	13.67

Source: Government Expenditure on Education, Total (% of government expenditure), World Bank.

percentages of total government expenditure allocated for education in a few developed countries for the year 2014.

From Table 3, government's priority towards education can be easily understood. The budgeted share of education varies from country to country in between 8% and 16%; and this justifies that. So one should study the effect of unionisation on economic growth with a special focus on the government's role on human capital accumulation.

A sustainable way of financing this public spending on education is taxation. There exists a set[11] of endogenous growth models focusing on the role of human capital formation on economic growth and analysing optimum tax policy to finance the educational expenditure. However, these models do not consider unionised labour markets.

Features of the unionised labour market are different from those of the competitive labour market; and the bargaining power of the labour union directly affects wage and employment. Moreover, the efficiency wage hypothesis establishes a direct relationship between wage and productivity. As a result, the optimum rate of income tax imposed to finance educational expenditure in an unionised economy should be different from that obtained in the competitive economy.[12]

[11] This set consists of Blankenau and Simpson (2004), Ni and Wang (1994), Corsetti and Roubini (1996), Chakraborty and Gupta (2009), Bandyopadhyay and Basu (2001), Tournemaine and Tsoukis (2015) etc.

[12] There exists a set of works analysing optimal income tax rate to finance productive public expenditure when the labour market is unionised. They are Chang and Chang (2015), Bhattacharyya and Gupta (2015), Raurich and Sorolla (2003), Kitaura (2010) etc. However, the optimal income tax rate to finance productive public expenditure in an unionised economy should be different from the optimal income tax rate to finance investment in human capital. The positive externality of productive public capital enjoyed by the private producers is independent of the number of employed workers. Contrary to this, the rise in the efficiency level of workers enjoyed by producers depends on the number of employed workers which in term is affected by unionisation in the labour market.

In this paper, we develop a simple endogenous growth model with a special focus on the 'Efficiency Wage Hypothesis' and on the government's role on human capital accumulation. Our objective is to analyse the effect of unionisation on economic growth as well as on the optimum tax policy when educational expenditure is financed by taxation on labour income. Unionisation is defined as an exogenous increase in the relative bargaining power of the labour union. This relative bargaining power is not endogenously determined in this model; and it is independent of the union density rate. So unionisation in the labour market does not necessarily imply an increase in the union density rate; and, in this sense, our theoretical definition of unionisation is not perfectly consistent with the empirical definition of unionisation i.e., an exogenous increase in the labour union density rate. However, a significant number of existing theoretical models on unionisation define unionisation as an exogenous increase in the relative bargaining power[13]. We follow that tradition ignoring this inconsistency problem and then use both the 'Efficient Bargaining' model of McDonald and Solow (1981) and the 'Right to Manage' model of Nickell and Andrews (1983) to solve the negotiation problem between the employers' association and the labour union.

The incorporation of the 'Efficiency Wage Hypothesis' and the government's financing of educational expenditure in this model helps us to derive a few interesting results. First, for a given tax rate on labour income, unionisation in the labour market lowers the number of employed workers, raises the wage rate and also the efficiency level of the workers. Employment level also varies inversely with the tax rate on labour income and with the unemployment benefit rate. However, when the government imposes the growth rate maximising tax rate on labour income, then the number of employed workers becomes independent of the labour union's bargaining power, labour income tax rate and the rate of unemployment benefit. The employment level corresponding to the growth rate maximising tax rate varies inversely only with the elasticity of labour efficiency with respect to human capital; and this employment level always falls short of the full employment level. These results are independent of the nature of orientation of the labour union. Secondly, this growth rate maximising tax rate on labour income varies positively with the elasticity of worker's efficiency with respect to human capital as well as with the budget share of educational expenditure. However, this tax rate varies inversely with the degree of unionisation in the labour market. Thirdly, the growth rate maximising tax rate on labour income is higher than the corresponding welfare maximising tax rate; and the welfare effect of unionisation is stronger than the growth effect of unionisation if the labour union is not employment oriented. Lastly, the growth effect of unionisation is independent of the orientation bias of the labour union in its preference structure; and it depends only on how unionisation affects workers' efficiency. This effect on efficiency consists of a positive effort effect and an ambiguous human capital accumulation effect. All these results are independent of the choice of the bargaining model. However, there is a condition to ensure a positive growth effect of unionisation; and the condition states that the elasticity of worker's efficiency with respect to the wage premium should be greater than that elasticity with respect to human capital. In the case of 'Efficient Bargaining' model, it is a sufficient condition but not a necessary one. In the case of 'Right to Manage' model, this condition is necessary as well as sufficient.

The result about the employment level corresponding to the growth rate maximising tax

[13] See, for example, Palokangas (1996, 2004), Sorensen (1997), Chang et al. (2007), Lingens (2003a, 2003b, 2007), Ji et al. (2016), Adjemian et al. (2010), Grieben and Sener (2017), Chu et al. (2016), Chang and Hung (2016) etc.

rate on labour income is very interesting because it implies that the government has the power to cancel out the negative effect of some labour market distortions on the level of employment when its objective is to maximise the long run growth rate. Few works show that there exists an optimal amount of friction to maximise growth or welfare or output[14]. However, our paper does not point out any optimal amount of friction. Rather, it shows that, whatever be the level of friction, i.e., the bargaining power of the labour union, there exists an unique employment level corresponding to the growth rate maximisation; and this employment level is always less than full employment level.

Rest of the paper is organized as follows. In section II, we describe the basic model with 'Efficient Bargaining'. In section III, we analyse the existence, uniqueness and stability of the balanced growth equilibrium. We also analyse properties of growth rate maximising tax rate and the growth effect of unionisation in this section. In section IV, the same issues are dealt with a 'Right to Manage' model. The paper is concluded in Section V.

II. *The Model*

1. Production of the Final Good

The representative competitive firm[15] produces the final good, Y, with the following production function[16].

$$Y = AK^{\alpha}(eL)^{\beta}\bar{K}^{\gamma}; \qquad \alpha, \beta, \gamma, \alpha+\beta \in (0, 1). \tag{1}$$

Here $A > 0$ is a time independent technology parameter and K denotes the amount of capital used by the representative firm. eL represents firm's effective employment in efficiency unit where L stands for the number of workers employed and e stands for the efficiency per worker.[17] $\bar{K}$ symbolizes the average quantity of capital stock of all firms in the economy; and $0 < \gamma < 1$ ensures that the external effect of capital is positive. The Cobb — Douglas production function satisfies private diminishing returns. However, social returns to scale may not be diminishing. Decreasing returns to private inputs in the production function results into a positive profit in equilibrium; and this profit is the rent in the bargaining to be negotiated between the employers' association and the labour union. Following Chang et al. (2007), we assume that a given amount of land is necessary to set up a firm; and as a result, the number of firms remain unchanged even in the presence of positive profit because land is a fixed factor of production.[18]

We assume that the net efficiency per worker, e, depends on his accumulated stock of efficiency, e_1, as well as on his effort level, e_2. Efficiency stock of a worker, e_1, varies

[14] See for example, Palokangas (2004, 2009), Mazumder (2017).

[15] Following Chang et al. (2007), here also free entry assumption of perfect competition is restricted by the existence of a fixed factor land. Necessity of this assumption will be discussed in a little while.

[16] Chang et al. (2007) does not consider efficiency of workers, e. In their model, $e \equiv 1$. Otherwise, this production function is identical to that in Chang et al. (2007).

[17] It is assumed that all workers have identical efficiency level.

[18] Number of firms is normalized to unity.

positively with his level of human capital. This is consistent with the assumptions made in Lucas (1988), Uzawa (1965), Caballé and Santos (1993), Bucci (2008), Docquier et al. (2008) etc. His effort level, e_2, varies positively with his net wage relative to his net reservation income. This keeps consistency with the assumption made by the 'Efficiency Wage Hypothesis'.[19] For simplicity, we assume that a worker's net reservation income is the after tax unemployment benefit given to an unemployed worker. So the worker's net efficiency, e, is given by[20]

$$e = e_1 e_2. \tag{2}$$

Here

$$e_1 = h^{\eta} \quad \text{with} \quad 0 < \eta < 1; \tag{2.a}$$

and

$$e_2 = \left(\frac{[1-\tau]w}{[1-\tau]b}\right)^{\delta} = \left(\frac{w}{b}\right)^{\delta} \quad \text{with} \quad 0 < \delta < 1. \tag{2.b}$$

Here h and w denote the level of human capital and the wage rate respectively, and b stands for the amount of unemployment benefit given to an unemployed worker. η and δ represent elasticities of net efficiency with respect to the stock of human capital and with respect to the relative wage rate respectively; and they are assumed to be positive fractions. Chang et al. (2007) does not distinguish between labour time and labour efficiency. So, in Chang et al. (2007), $e \equiv e_1 \equiv e_2 \equiv 1$, i.e., $\eta = \delta = 0$.

The firm maximises profit, π, given by

$$\pi = Y - wL - rK, \tag{3}$$

where r represents rental rate on capital.

The capital market is perfectly competitive, and so the equilibrium value of the rental rate on capital is determined by the supply-demand equality in this market. The inverted demand function for capital is obtained from the firm's profit maximization exercise, and it is given by

$$r = \alpha A K^{\alpha-1} (eL)^{\beta} \bar{K}^{\gamma} = \frac{\alpha Y}{K}. \tag{4}$$

2. The Government

The government finances investment in human capital accumulation (educational expenditure) as well as the benefit given to unemployed workers. To finance these expenditures, a proportional tax on the wage income as well as on the unemployment benefit is imposed at the rate τ. The budget is balanced at each point of time. The total tax revenue is allocated to these two types of expenditures in an exogenously given proportion.[21] For the sake of simplicity, it is

[19] See footnotes 6 and 8.

[20] Danthine and Kurmann (2006) model uses similar functional form.

[21] We do not consider any productive role of unemployment benefit in this model. So endogenous determination of this proportion by maximising the economic growth rate is beyond the scope of this model. The growth rate

also assumed that one unit of educational expenditure of the government can create one unit of additional human capital. So we have

$$\lambda[\tau wL+\tau b(1-L)]=h; \tag{5}$$

and

$$(1-\lambda)[\tau wL+\tau b(1-L)]=b(1-L). \tag{6}$$

Here $(1-L)$ is the unemployment level as we normalise the total labour endowment to unity; and λ and $(1-\lambda)$ are two fractions of tax revenue allocated to finance educational expenditure and unemployment allowances respectively.

We consider taxation only on labour income but not on capital income. This is done due to three reasons. First, both the channels of expenditure provide benefits to workers but not to capitalists. Beneficiaries should bear the burden of taxation. Secondly, taxation on capital income makes the analysis complicated. Thirdly, capital income taxation reduces the net marginal productivity of capital and thereby reduces the rate of growth. So, a growth rate maximising government may not impose a tax on capital income. A set of works on public economics literature consisting of Bräuninger (2000a, 2005), Crossley and Low (2011), Landais et al. (2010), Davidson and Woodbury (1997) etc. also considers taxation only on wage income to finance unemployment benefit scheme.

3. The Labour Union and Efficient Bargaining

In this model, the labour union derives utility from the net wage premium of the worker and from the number of members of the union. The net wage premium is defined as the difference between the after tax bargained wage rate and the after tax unemployment benefit rate[22]. We consider closed shop labour union; and as a result, all employed workers are members of the union. This assumption may be inconsistent with the empirical findings because labour union density rate is found to be less than 100% in all countries mentioned in table 2. These empirical findings clearly imply that non-unionised workers exist in those countries; and so the assumption of an open shop labour union is more meaningful in this theoretical model. Few theoretical works develop interesting analysis of membership dynamics in open shop labour union models.[23] However, bargaining solutions about wage and employment are not affected in those models when the fraction of non-unionised workers is exogenously given and when only unionised workers bargain while non-unionised workers are takers of that solution.

The utility function of the labour union is given by

$$u_T=[(1-\tau)w-(1-\tau)b]^m L^n=(1-\tau)^m(w-b)^m L^n \quad \text{with} \quad m, n>0. \tag{7}$$

Here u_T symbolizes the level of utility of the labour union. Two parameters, m and n represent

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maximising share of unemployment benefit is always zero in this model.

22 In Irmen and Wigger (2002/2003), Lingens (2003a) and Lai and Wang (2010), the difference between the bargained wage rate and the competitive wage rate is an argument in the labour union's utility function. Contrary to this, in Adjemian et al. (2010) and Chang et al. (2007), the difference between the after tax bargained wage rate and the net unemployment benefit is an argument in the labour union's utility function. So our paper belongs to the second group.

23 See Kidd and Oswald (1987), Gupta (1997), Jones and Mackenna (1994) etc.

elasticities of labour union's utility with respect to the wage premium and with respect to the number of members respectively. If $m>(<)(=)n$, then the labour union is said to be wage oriented (employment oriented) (neutral). Chang et al. (2007) contains a brief discussion of these parameters.

We now consider an 'Efficient Bargaining' model where both the wage rate and the number of employed workers are determined mutually by the labour union and the employer's association. To obtain these results of bargaining, we maximize the 'generalised Nash product' function given by

$$\psi=(u_T-\overline{u_T})^{\theta}(\pi-\overline{\pi})^{(1-\theta)} \quad \text{satisfying} \quad 0<\theta<1. \tag{8}$$

Here $\overline{u_T}$ and $\overline{\pi}$ represent the reservation utility level of the labour union and the reservation profit level of the firm respectively. $\overline{u_T}$ and $\overline{\pi}$ are assumed to be zero as, bargaining disagreement stops production and hence employment. θ represents the relative bargaining power of the labour union.

Unionisation in the labour market is defined as an exogenous increase in the relative bargaining power of the labour union, i.e., in the value of θ. Here θ is a parameter. Its value is exogenously given and it is independent of time. So unionisation in this model is defined in a narrow sense like other theoretical models on unionisation.[24] In the empirical literature, unionisation may be defined as an increase in the labour union density rate because entire labour market is not unionised. In our model, there is no unorganised labour market; and so we cannot analyse the effect of a change in the union density rate. In many dual economy models[25] developed in the literature on Development Economics, we find coexistence of formal unionised labour market and the informal unorganised labour market; and those models are suitable to analyse this effect. In the present model, we analyse only the effect of an increase in the bargaining power of the labour union; and such an exogenous increase may take place when the government adopts labour market reforms with pro labour laws.

Now, using equations (3), (7) and (8), we obtain

$$\psi=\{(1-\tau)^m(w-b)^m L^n\}^{\theta}\{Y-wL-rK\}^{(1-\theta)}. \tag{9}$$

Here ψ is to be maximised with respect to w and L. First order conditions of maximization are given by

$$\frac{\theta m}{w-b}+\frac{(1-\theta)}{[Y-wL-rK]}\left\{\frac{\beta\delta Y}{w}-L\right\}=0; \tag{10}$$

and

$$\frac{\theta n}{L}+\frac{(1-\theta)}{[Y-wL-rK]}\left\{\frac{\beta Y}{L}-w\right\}=0. \tag{11}$$

Using equations (4) and (11) we obtain

$$\frac{wL}{Y}=\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}. \tag{11.a}$$

[24] See works mentioned in footnote 13.

[25] See Calvo (1978), Quibria (1988) etc.

Equation (11.a) shows that the labour share of income is independent of capital stock and it varies positively with the relative bargaining power of the labour union.[26] If the labour union has no bargaining power, i.e., if $\theta=0$, then this labour share of income is equal to its competitive share, i.e. β. However, if the labour union is a monopolist, i.e., if $\theta=1$, then it takes away all the income left after paying rental cost of capital; and hence the labour share of income is equal to $(1-\alpha)$.

Using equations (1), (2), (2.a), (2.b), (4), (6), (10) and (11), we obtain[27]

$$L^{*}=\frac{[1-(1-\lambda)\tau]}{[1-(1-\lambda)\tau]+\Theta_1(1-\lambda)\tau}; \tag{12}$$

and

$$w^{*}=b\Theta_1. \tag{13}$$

Here,

$$\Theta_1=\frac{[\theta n(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]}{[\theta(n-m)(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]}. \tag{14}$$

Θ_1 represents the negotiated wage rate relative to the unemployment benefit rate. Equations (13) and (14) clearly imply that the bargained wage rate is independent of the policy parameters like tax rate, τ, and the expenditure allocation ratio, λ. We assume the denominator of Θ_1 to be positive. Its numerator is always positive. This ensures that $0<L^{*}<1$. When the labour union is neutral or employment oriented, i.e., when $m\leq n$, the denominator of Θ_1 is always positive. However, when the labour union is wage oriented, i.e., when $m>n$, $\Theta_1>0$ implies that the preference of labour union can not be highly biased for wage premium. The right hand side of equation (14) makes it clear that $\Theta_1>1$[28], which further implies that $w^{*}>b$. Now, from equations (2.b) and (13), we obtain the effort level per worker as given by

$$e_2^{*}=(\Theta_1)^{\delta}. \tag{15}$$

Equation (12) shows that L^{*} varies inversely with Θ_1. As Θ_1 is increased, the labour union negotiates for a higher wage given the unemployment allowance rate; and so the number of employed workers is reduced. It can be easily shown that

$$\frac{\partial L^{*}}{\partial \tau}=-\frac{(1-\lambda)\Theta_1}{\{[1-(1-\lambda)\tau]+\Theta_1(1-\lambda)\tau\}^{2}}<0; \tag{16}$$

and

$$\frac{\partial L^{*}}{\partial \lambda}=\frac{\tau\Theta_1}{\{[1-(1-\lambda)\tau]+\Theta_1(1-\lambda)\tau\}^{2}}>0. \tag{17}$$

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[26] $\dfrac{\partial\left(\frac{wL}{Y}\right)}{\partial\theta}=\dfrac{n(1-\alpha-\beta)}{(1-\theta+\theta n)^{2}}>0.$

[27] See appendix A for derivation of optimal w and L.

[28] If the denominator of Θ_1 is positive, then Θ_1 is greater than unity as the numerator of Θ_1 is obviously greater than the denominator of Θ_1.

As the tax rate is increased, marginal utility of net wage premium of the labour union is decreased. So, given the unemployment benefit rate, b, the labour union wants a higher wage rate to raise its marginal utility of net wage premium; and the employer wants to lower the number of employed workers in this case. By the similar logic, the number of employed workers is reduced when the budgetary allocation to finance unemployment benefit is increased given the tax rate.

Now from equation (14), we obtain

$$\frac{\partial \Theta_1}{\partial \theta}=\frac{m\beta(1-\alpha-\beta)(1-\delta)}{[\theta(n-m)(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}>0; \tag{18}$$

and, from equation (15), we have

$$\frac{\partial e_2^*}{\partial \theta}=\delta(\Theta_1)^{\delta-1}\frac{\partial \Theta_1}{\partial \theta}>0. \tag{19}$$

Equation (18) implies that the labour union claims a higher wage relative to the alternative income (unemployment benefit) of the worker when its bargaining power is increased. As a result of this, equation (19) implies that the effort level per worker varies positively with the degree of unionisation.

Now, from equations (12) and (18), we obtain

$$\frac{\partial L^*}{\partial \theta}=-\frac{[1-(1-\lambda)\tau](1-\lambda)\tau}{\{[1-(1-\lambda)\tau]+\Theta_1(1-\lambda)\tau\}^2}\frac{\partial \Theta_1}{\partial \theta}<0. \tag{20}$$

Equation (20) shows that, given other parameters, the negotiated number of employed workers varies inversely with the degree of unionisation in the labour market irrespective of the labour union's orientation bias in its preference structure. This is so because unionisation raises the negotiated wage rate as well as the ratio of that wage rate to the unemployment benefit rate; and, as a result, effort level per worker is increased[29]. This rise in the wage rate raises the cost per worker and the increase in the worker's effort level substitutes the number of employed workers. As a result, number of employed workers is declined due to the unionisation in the labour market. The negative effect of unionisation on the employment level is empirically supported[30].

We summarize these results in the following proposition.

Proposition 1: *(i) Given other parameters, an exogenous increase in the tax rate on labour income and / or in the rate of unemployment benefit lowers the number of employed workers irrespective of the orientation bias in its preference structure of the labour union. (ii) Unionisation in the labour market, given other parameters, raises the wage rate as well as the effort level of the representative worker but lowers the number of employed workers irrespective of the preference of the labour union.*

It may be noted that the relative bargaining power of the labour union, θ, is a time independent parameter in this model; and we do not introduce any story about how unionisation measured

[29] See equation (19).

[30] See the Introduction section.

by θ may change over time endogenously. Unionisation in the labour market itself is a dynamic process and thus changes over time with growth and fluctuation in the economy and with labour policies of the government. So one may think of a more general dynamic model where θ itself changes over time. In this case, the rate of increase in θ taking place over time may be assumed to vary positively with the union density rate. In a closed shop labour union model, the level of employment itself is a measure of the union density rate when the labour endowment is exogenously given. This may be a way by which we can introduce the positive effect of an increase in the union density rate on the relative bargaining power of the labour union when it changes over time. Such an extended analysis may partly remove the inconsistency between the theoretical definition and the empirical measure; and is always welcome in a dynamic model. However, the present model assumes θ to be a time independent parameter.

4. The Household Sector

This sector consists of a big family with many employed members and unemployed members staying together. All members are identical in terms of preferences. The consumption decision and labour supply decision are taken by this household sector for all members. Many of the existing works follow this big family approach[31] to model the household sector behaviour.

The household sector derives instantaneous utility only from consumption of the final good[32]. She maximises her discounted present value of instantaneous utility over the infinite time horizon subject to her budget constraint and the labour allocation constraint which shows labour allocation between earning wage and earning unemployment allowance. So the dynamic optimisation problem of the household sector is defined as follows.

$$Max\int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma}e^{-\rho t}dt \qquad \text{with} \qquad \sigma>0, \tag{21}$$

subject to, $\dot{K}=(1-\tau)wL+rK+\pi+(1-\tau)b(1-L)-c;$ (22)

and $0\leq L\leq \text{Min}\{1, L^*\}$

with $K(0)=K_0$ (K_0 is historically given).

Here c is the level of consumption of the household sector; and σ and ρ are two parameters representing the elasticity of marginal utility with respect to consumption and the rate of discount respectively. All factors are owned by the household sector and entire savings is invested. Capital stock does not depreciate over time. The right hand side of equation (22) represents the savings of the household sector.

The household sector owns one unit of labour endowment; and L^* is the level of employment opportunities created by the bargaining solution. Bargaining takes place between

[31] See for example Chang et al. (2007), Chang and Hung (2016), Domenech and Garcia (2008), Eriksson (1997), Bhattacharyya and Gupta (2015, 2016), Chu et al. (2016), Maiti and Bhattacharyya (2020), Ji et al. (2016) etc.

[32] For technical simplicity, we rule out the labour leisure choice of the household sector and assume that she does not derive utility from her human capital stock.

the labour union and the employer's association; and the household sector is a taker of that solution. The household sector knows that her employment possibility can not exceed that solution. So the household sector's labour allocation to the production sector can not exceed the minimum of her endowment level and the bargained level of employment; and this is implied by the labour allocation constraint

$$0 \leq L \leq \mathrm{Min}\{1,\ L^*\}.$$

Here utility function is independent of L; and equation (22) shows that $\dot{K}$ varies positively with L when $W > b$. So solving this dynamic optimisation problem with respect to L, we find that optimum labour allocation to the production sector is always at the upper boundary, i.e., optimum $L = \mathrm{Min}\{1,\ L^*\}$, if $W > b$.

However, bargaining solution in the labour market, as analysed in subsection II.3, confirms that $W^* > b$ and $L^* < 1$. Hence optimum $L = L^*$; and so the labour supply function of the household sector to the production sector is given by

$$L = L^*.$$

It means that the household sector supplies only that amount of labour which is required by the bargaining solution.

Here c is another control variable and K is the state variable in this problem. Solving this dynamic optimisation problem with respect to c and K, we derive the rate of growth of consumption as given by[33]

$$g = \frac{\dot{c}}{c} = \frac{\alpha A K^{\alpha-1} (eL^*)^{\beta}\, \overline{K}^{\gamma} - \rho}{\sigma}. \tag{23}$$

III. *Steady State Equilibrium*

The symmetric steady state balanced growth equilibrium satisfies following properties:

(i) $\frac{\dot{c}}{c} = \frac{\dot{K}}{K} = \frac{\dot{h}}{h} = \frac{\dot{Y}}{Y} = \frac{\dot{w}^*}{w^*} = \frac{\dot{\pi}}{\pi} = \frac{\dot{b}}{b} = g;$

(ii) $K = \overline{K}$; and

(iii) r, L^*, τ, λ, e_2^* and g are time independent.

To ensure that h, K and Y grow at the same rate, i.e., to satisfy property (i), we further assume that $\gamma = 1 - \alpha - \beta\eta$. This implies that the production function satisfies the property of social constant returns to scale in terms of the augmenting factors.

Using equations (1), (2), (2.a), (5), (6), (11.a), (15), (22), (23), and putting $\gamma = 1 - \alpha - \beta\eta$, $L = L^*$ and $K = \overline{K}$, we obtain

[33] See appendix B for derivation of equation (23).

$$g=\frac{\dot{c}}{c}=\frac{\alpha AL^{*\beta}\left(\frac{h}{K}\right)^{\beta\eta}[\Theta_1]^{\beta\delta}-\rho}{\sigma}; \tag{24}$$

$$g=\frac{\dot{h}}{h}=\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]A\left(\frac{K}{h}\right)^{1-\beta\eta}L^{*\beta}[\Theta_1]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)}; \tag{25}$$

and

$$g=\frac{\dot{K}}{K}=AL^{*\beta}\left(\frac{h}{K}\right)^{\beta\eta}[\Theta_1]^{\beta\delta}\left[1-\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right]-\frac{c}{K}. \tag{26}$$

We define two new variables M and N such that $M=(c/K)$ and $N=(h/K)$. So using equations (24), (25) and (26), we obtain

$$\frac{\dot{M}}{M}=\frac{\alpha AL^{*\beta}(N)^{\beta\eta}[\Theta_1]^{\beta\delta}-\rho}{\sigma} -AL^{*\beta}(N)^{\beta\eta}[\Theta_1]^{\beta\delta}\left[1-\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right]+M; \tag{27}$$

and

$$\frac{\dot{N}}{N}=\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]A(N)^{\beta\eta-1}L^{*\beta}[\Theta_1]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)} -AL^{*\beta}(N)^{\beta\eta}[\Theta_1]^{\beta\delta}\left[1-\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right]+M. \tag{28}$$

In the steady state growth equilibrium, $\frac{\dot{M}}{M}=\frac{\dot{N}}{N}=0$; and this implies that

$$\frac{\alpha AL^{*\beta}(N)^{\beta\eta}[\Theta_1]^{\beta\delta}-\rho}{\sigma}=\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]A(N)^{\beta\eta-1}L^{*\beta}[\Theta_1]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)}. \tag{29}$$

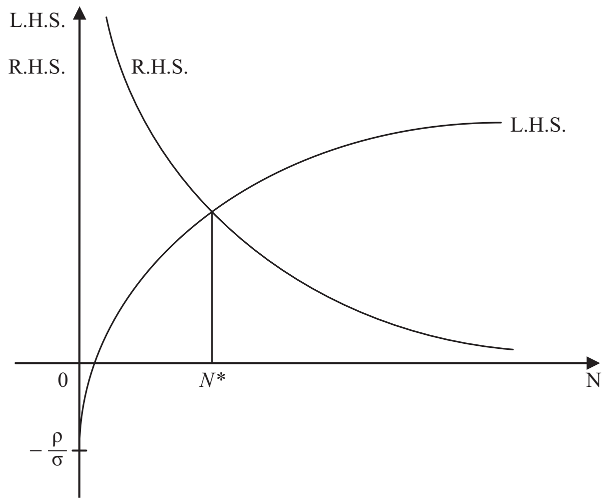
1. Existence and Stability

Equation (29) is solely a function of N. We now turn to show the existence and uniqueness of the steady state equilibrium; i.e., a unique solution of equation (29). For this purpose, we use a diagram. In figure 1, L.H.S. and R.H.S. of equation (29) are measured on the vertical axis and N on the horizontal axis.

The L.H.S. curve is positively sloped and is concave to the horizontal axis with a point of intersection on that axis. However, the R.H.S. curve is negatively sloped, convex to the origin and asymptotic to both axes. The unique point of intersection of these two curves with $N=N^*$ shows the existence of the unique steady state growth equilibrium.

To analyse the local stability of this equilibrium, we use equations (27) and (28). The mathematical sign of the Jacobian determinant, given by

FIGURE 1. EXISTENCE OF A UNIQUE STEADY STATE EQUILIBRIUM



$$|J| = \begin{vmatrix} \frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial M} & \frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial N} \\ \frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial M} & \frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial N} \end{vmatrix},$$

is to be evaluated. It can be easily shown that[34]

$$|J| = -\Big[(1-\beta\eta)\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]A(N)^{\beta\eta-2}L^{*\beta}[\Theta_1]^{\beta\delta}}{[1-(1-\lambda)\tau](1-\theta+\theta n)} + \frac{\beta\eta\alpha AL^{*\beta}(N)^{\beta\eta-1}[\Theta_1]^{\beta\delta}}{\sigma}\Big] < 0; \quad (30)$$

and the negative sign of $|J|$ implies that the two latent roots of the J matrix are of opposite sign. This implies that the unique steady state growth equilibrium is saddle point stable.

2. Growth Rate Maximising Tax Rate

We assume that the government determines the rate of tax on labour income maximising the rate of growth in the steady state equilibrium[35]; and we now turn to analyse properties of

[34] See appendix C for derivation of equation (30).

[35] Usually it is assumed that the objective of the government is to maximise social welfare. However, for technical simplicity, here we consider growth rate maximization. Agénor and Neanidis (2014) also focuses on growth rate maximisation rather than on welfare maximisation on the ground that, in practice, imperfect knowledge about household

growth rate maximising tax rate. Substituting (h/K) from equation (25) into equation (24), we obtain

$$(\rho+\sigma g)[g]^{\frac{\beta\eta}{1-\beta\eta}}=A^{\frac{1}{1-\beta\eta}}\alpha L^{*\frac{\beta}{1-\beta\eta}}[\Theta_1]^{\frac{\beta\delta}{1-\beta\eta}}\left\{\lambda\tau\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right\}^{\frac{\beta\eta}{1-\beta\eta}}. \tag{31}$$

The L.H.S. of equation (31) is a monotonically increasing function of g. So the tax rate, which maximises the R.H.S. of equation (31), also maximises the growth rate. So, from equations (12) and (31), we obtain the growth rate maximising tax rate[36] as given by

$$\tau^*=\frac{\eta}{[(1-\eta)\Theta_1+\eta](1-\lambda)}. \tag{32}$$

Equation (32) shows that τ^* varies positively with η and λ. If human capital is not productive, i.e., if $\eta=0$, then no tax should be imposed on labour income in order to maximise the growth rate. Equation (31) clearly shows that the rate of growth varies inversely with the tax rate when $\eta=0$. This is so because L^* varies inversely with τ. Again, from equation (32), we obtain

$$\frac{\partial\tau^*}{\partial\theta}=-\frac{\eta(1-\eta)}{[(1-\eta)\Theta_1+\eta]^2(1-\lambda)}\frac{\partial\Theta_1}{\partial\theta}<0. \tag{33}$$

Equation (33) shows that the growth rate maximising tax rate varies inversely with the degree of unionisation in the labour market. The intuition behind this result is as follows. The change in the tax rate has two opposite effects on the growth rate. The first effect works negatively by reducing the employment level and the second effect works positively by increasing human capital accumulation. These two effects balance each other at $\tau=\tau^*$. Now, a rise in θ itself lowers the employment level; and so the negative effect becomes more powerful following this change. So the growth rate maximising tax rate is decreased to bring back the balance.

It may be noted that both Kitaura (2010) and Bhattacharyya and Gupta (2015) show a positive relationship between the degree of unionisation and the growth rate maximising tax rate in their models. However, in between 1980s and late 1990s, for many OECD countries, union power has reduced whereas the tax rate on total labour income has risen[37]. The present model considers taxation on labour income and provides a plausible explanation for government's choice of raising the tax rate on labour income in response to a decline in the bargaining power of the labour union. Neither Kitaura (2010) nor Bhattacharyya and Gupta (2015) highlights this point.

These properties of the growth rate maximising tax rate on labour income are summarised in the following proposition.

Proposition 2: *The growth rate maximising tax rate on labour income, on the one hand, varies positively with the elasticity of workers' efficiency with respect to the human capital as well as with the budget share of the educational expenditure; and, on the other hand, varies inversely with the degree of unionisation in the labour market.*

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sector preferences makes it easier to measure their income level rather than their welfare level.

[36] We assume that the second order condition is satisfied.

[37] See tables 3 and 4 of Nickell et al. (2005).

Incorporating the value of τ^* from equation (32) in equation (12), we obtain

$$L^* = 1 - \eta. \tag{34}$$

Equation (34) shows that the employment level varies inversely with the elasticity of efficiency with respect to human capital when the government imposes the growth rate maximising tax rate. This employment level is independent of the level of unemployment benefit and the degree of unionisation in the labour market. However, $0 < \eta < 1$ implies that the equilibrium (growth rate maximising) level of employment falls short of the full employment level. This is a very interesting result as the common notion is that the unemployment benefit policy affects the level of unemployment. This implies that the government has the power to cancel out the negative effects of unionisation and the unemployment benefit rate on the level of employment. If the objective of the government is to maximise the rate of growth, then it can do that through the choice of the tax rate. Unemployment benefit policy has no role to play when the government maximises the growth rate through the choice of the tax rate. Another point to note is that a few works show the existence of optimal amount of friction.[38] However, our result does not imply that there should exists an optimal amount of friction. It shows that there exists a unique growth rate maximising level of unemployment whatever be the level of friction.

The intuition behind this result is as follows. Both unionisation and unemployment benefit has two different effects on employment. One is the direct effect, and the other is the indirect effect operating through the change in the growth rate maximising tax rate. Equations (20) and (33) show that both L^* and τ^* vary directly with unionisation parameter, θ; and equation (16) shows that L^* varies inversely with the tax rate, τ^*. As a result, these two effects of unionisation on L^* cancel out each other; and thus the employment level corresponding to the growth rate maximising tax rate becomes independent of the degree of unionisation. Similarly, equations (17) and (32) show that both L^* and τ^* vary inversely with the fraction of tax revenue used to finance unemployment benefit, i.e., $(1-\lambda)$. However, equation (16) shows that L^* varies inversely with τ^*; and these two effects of unemployment benefit on L^* cancel out each other. As a result, employment level corresponding to the growth rate maximising tax rate becomes independent of the rate of unemployment benefit. Equation (34) also shows that L^* varies inversely with η. This is so because a higher value of η indicates a higher level of efficiency of the worker; and the efficiency gain per worker always substitutes the number of employed workers.

These results are stated in the following proposition.

Proposition 3*: When the government imposes the growth rate maximising tax rate on labour income, then the level of employment becomes independent of the degree of unionisation in the labour market as well as of the rate of unemployment benefit, but varies inversely with the elasticity of efficiency with respect to the human capital. However, this level of employment corresponding to the growth rate maximisation falls short of the full employment level.*

The welfare level of the household sector, ω, is defined as her discounted present value of instantaneous utility over the infinite time horizon. It is obtained from equations (1), (3), (6), (11.a), (21), (22) and (23) and is given by[39]

[38] See the discussion in the Introduction section.

[39] See appendix D for derivation of equation (35).

$$\omega=\frac{K_0^{1-\sigma}\left[\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau}{[1-(1-\lambda)\tau]}\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}-g\right]^{1-\sigma}}{(1-\sigma)[\rho-g(1-\sigma)]}+constant. \quad (35)$$

We assume $1>\sigma$ and $\rho>g(1-\sigma)$. Since the level of initial consumption, c_0, is positive, so

$$\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau}{[1-(1-\lambda)\tau]}\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}>g.$$

Here, we do not derive the welfare maximising income tax rate on labour income for technical complexity. Rather, we check whether the growth rate maximising income tax rate on labour income, given by equation (32), is identical to the welfare maximising tax rate on labour income or not if the later exists. For this purpose, we differentiate ω with respect to the tax rate, τ, and evaluate the derivative at $\tau=\tau^*$ and obtain

$$\left.\frac{\partial\omega}{\partial\tau}\right|_{\tau=\tau^*}=-\frac{K_0^{1-\sigma}\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{\frac{[\theta n(1-\alpha)+\beta(1-\theta)]\lambda}{(1-\theta+\theta n)[1-(1-\lambda)\tau^*]^2}\right\}[\rho-g^*(1-\sigma)]^{-1}}{\left[\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{1-\frac{\lambda\tau^*}{[1-(1-\lambda)\tau^*]}\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)}\right\}-g^*\right]^{\sigma}}<0. \quad (36)$$

Here $g^*=g|_{\tau=\tau^*}$. Equation (36) implies that the welfare maximising tax rate on labour income is lower than the growth rate maximising tax rate. This is so because, given the budgetary allocation of tax revenue, the initial level of consumption of the household sector falls with an increase in the tax rate on labour income. Economic growth rate, g^*, in the steady state equilibrium does not depend on the level of initial consumption[40]. So the growth rate maximising tax rate, τ^*, does not take into account this negative effect of taxation on initial consumption. On the other hand, the welfare level depends on the level of initial consumption; and so the welfare maximising tax rate on labour income takes into account this negative effect. This result is stated in the following proposition.

Proposition 4*: The welfare maximising tax rate on labour income, if it exists, is lower than the corresponding growth rate maximising tax rate.*

3. Growth and Welfare Effect of Unionisation

We now turn to analyse the effect of an increase in the relative bargaining power of the labour union, θ, on the endogenous growth rate, g^*, when the government imposes the growth rate maximising tax rate on labour income[41]. Using equations (31) and (32), we obtain

$$(\rho+\sigma g^*)[g^*]^{\frac{\beta\eta}{1-\beta\eta}}=A^{\frac{1}{1-\beta\eta}}\alpha(1-\eta)^{\frac{\beta}{1-\beta\eta}}[\Theta_1]^{\frac{\beta\delta}{1-\beta\eta}}\left\{\frac{\eta\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\eta)\Theta_1(1-\lambda)(1-\theta+\theta n)}\right\}^{\frac{\beta\eta}{1-\beta\eta}}. \quad (37)$$

From equation (37), we have

[40] See equation (31).

[41] Since we can not derive the welfare maximising labour income tax rate analytically, so we are unable to derive the growth effect of unionisation when the government charges the welfare maximising labour income tax rate.

$$\left[\frac{\sigma g^*}{(\rho+\sigma g^*)}+\frac{\beta\eta}{1-\beta\eta}\right]\frac{\frac{\partial g^*}{\partial\theta}}{g^*}=\left(\frac{\beta\eta}{1-\beta\eta}\right)\left\{\frac{n(1-\alpha-\beta)}{(1-\theta+\theta n)[\theta n(1-\alpha)+\beta(1-\theta)]}\right\}$$
$$-\frac{\beta^2 m\eta(1-\alpha-\beta)(1-\delta)}{(1-\beta\eta)[\theta(n-m)(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}$$
$$+\frac{\beta^2 m\delta(1-\alpha-\beta)(1-\delta)}{(1-\beta\eta)[\theta(n-m)(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}. \qquad (38)$$

Equation (38) shows that the growth effect of unionisation is ambiguous. It consists of two effects — (i) the effort effect and (ii) the human capital accumulation effect. The first effect is operated through the change in the effort level of the worker. It is positive when $0<\delta<1$; and is captured by the third term in the R.H.S. of equation (38). The second effect is operated through the change in the rate of human capital accumulation. It is ambiguous in sign and is captured by the first term as well as by the second term in the R.H.S. of equation (38). On the one hand, unionisation in the labour market raises the income share of labour and thereby enlarges the tax base[42]. This positive effect is captured by the first term in the R.H.S. of equation (38). However, on the other hand, unionisation lowers the growth rate maximising tax rate; and this negative effect is captured by its second term. So the net effect of unionisation on the generation of tax revenue is ambiguous. Since a fixed fraction of the tax revenue is spent to finance the human capital accumulation, so its effect on human capital accumulation is also ambiguous. If the human capital is not productive, i.e., if $\eta=0$, then only the positive effort effect remains and the unionisation always raises the rate of economic growth. Similarly, if the effort level is independent of the wage rate, i.e., if $\delta=0$, then the third term of the equation (38) vanishes and then the growth effect of unionisation depends only on the human capital accumulation effect. However, if we ignore the entire dynamic 'Efficiency Wage Hypothesis', i.e., if we assume that $\delta=\eta=0$, then all the three terms vanish from the R.H.S. of equation (38) and hence unionisation does not affect the growth rate of the economy.

This result is valid regardless of the nature of orientation of the labour union. This happens because unionisation does not affect the level of employment in this model when the government chooses the growth rate maximising tax rate; and, as a result, growth effect of unionisation completely depends on its effect on the net efficiency level of the worker. In Chang et al. (2007), growth effect is completely determined by the employment effect.

Combining the second term and the third term in the R.H.S. of the equation (38), we have

$$\left[\frac{\sigma g^*}{(\rho+\sigma g^*)}+\frac{\beta\eta}{1-\beta\eta}\right]\frac{\frac{\partial g^*}{\partial\theta}}{g^*}=\left(\frac{\beta\eta}{1-\beta\eta}\right)\left\{\frac{n(1-\alpha-\beta)}{(1-\theta+\theta n)[\theta n(1-\alpha)+\beta(1-\theta)]}\right\}$$
$$+\frac{\beta^2 m(1-\alpha-\beta)(1-\delta)(\delta-\eta)}{(1-\beta\eta)[\theta(n-m)(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}. \qquad (38.a)$$

This combined second term in the R.H.S. of equation (38.a) shows that the positive work effort

[42] See footnote 28.

effect dominates the negative component of the human capital accumulation effect if the elasticity of worker's efficiency with respect to the wage premium rate, δ, is higher than the elasticity of worker's efficiency with respect to the stock of human capital, η. So, the growth effect of unionisation in this case is always positive because the other component of human capital accumulation effect is always positive. However, the converse is not necessarily true. So, $\delta > \eta$ is a sufficient condition but not a necessary condition to ensure a positive growth effect of unionisation in the labour market. These results are summarised in the following proposition.

Proposition 5*: The growth effect of unionisation is independent of the orientation bias in the preference structure of the labour union when the government imposes the growth rate maximising tax rate on labour income. This growth effect is always positive if the elasticity of worker's efficiency with respect to human capital is not higher than the elasticity of worker's efficiency with respect to wage premium.*

It is interesting to know about the empirical magnitude of these elasticities. The meta-regression analysis of Peach and Stanley (2009) finds the wage productivity elasticity to be 0.18; and it represents δ in our model. Mamuneas et al. (2006) shows that output elasticity with respect to human capital varies across countries. For the sample of high income economies, the unweighted average is 0.19, whereas, for the sample of developing economies, the unweighted average is 0.13. So, according to our theoretical finding, the growth effect of unionisation is marginally positive for high income economies and is negative for developing countries. However, these are average elasticities of two sample groups of countries and the elasticities for individual countries may vary from these average values. So for individual countries, this effect may be different from the group-average effect even if they belong to the same group. In the introduction section, we have already mentioned about existing empirical works which find ambiguous growth effects of unionisation. So our theoretical result is not contrary to the existing empirical findings.

Now, we analyse the effect of the unionisation in the labour market on the welfare level of the household sector, ω, when the government imposes the growth rate maximising tax rate on labour income. For this purpose, we use equations (32) and (35); and obtain

$$\left.\frac{\partial\omega}{\partial\theta}\right|_{\tau=\tau^*}=\frac{\partial g^*}{\partial\theta}\frac{\left[\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{1-\frac{\lambda\eta[\theta n(1-\alpha)+\beta(1-\theta)]}{\Theta_1(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\}-g^*\right]^{1-\sigma}}{K_0^{\sigma-1}[\rho-g^*(1-\sigma)]}\left\{\frac{1}{[\rho-g^*(1-\sigma)]}+\frac{\left[\frac{\sigma}{\alpha}\left\{1-\frac{\lambda\eta[\theta n(1-\alpha)+\beta(1-\theta)]}{\Theta_1(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\}-1\right]}{\left[\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{1-\frac{\lambda\eta[\theta n(1-\alpha)+\beta(1-\theta)]}{\Theta_1(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\}-g^*\right]}\right\}$$
$$-\frac{\left\{\frac{(n-m)(1-\alpha-\beta)\theta n[\theta n(1-\alpha-\beta)+2\beta(1-\delta)(1-\theta+\theta n)]+\beta^2(1-\delta)(1-\theta+\theta n)^2[n(1-\delta)-m]}{(1-\theta+\theta n)^2[\theta n(1-\alpha-\beta)+\beta(1-\delta)(1-\theta+\theta n)]^2}\right\}}{\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}^{-1}\left[\frac{\lambda\eta(1-\alpha-\beta)}{(1-\eta)(1-\lambda)}\right]^{-1}K_0^{\sigma-1}\left[\left\{\frac{\rho+\sigma g^*}{\alpha}\right\}\left\{1-\frac{\lambda\eta[\theta n(1-\alpha)+\beta(1-\theta)]}{\Theta_1(1-\eta)(1-\lambda)(1-\theta+\theta n)}\right\}-g^*\right]^{\sigma}[\rho-g^*(1-\sigma)]}. \tag{39}$$

Equation (39) shows that the welfare effect of unionization consists of two parts. One of them is the growth effect of unionisation and it is captured by the first term in the R.H.S. of equation (39). The second effect comes from the change in the initial consumption level of the household sector due to a change in the tax payment, and it is captured by the second term in that R.H.S.. This second effect is ambiguous in sign. This is so because, on the one hand, unionisation lowers the tax rate and thereby increases the disposable income[43] of the household sector. On the other hand, unionisation raises the income share of labour and thereby enlarges the tax base.[44] So if $m \geq n$, then the effect on tax rate dominates the other effect on tax base and hence the initial consumption effect becomes positive. So the welfare effect of unionisation is stronger than its growth effect when the labour union is not employment oriented. The major result is stated in the following proposition.

Proposition 6*: The welfare effect of unionization is different from its growth effect when the government charges growth rate maximizing tax rate on labour income; and is stronger than the growth effect if* $m \geq n$*, i.e., if the labour union is not employment oriented.*

In the model of Chang et al. (2007), growth effect, as well as the welfare effect of unionisation, solely consist of the employment effect of unionisation, which depends only on the orientation of the labour union. However, there is no employment effect in our model, and hence the growth effect is independent of the orientation of the labour union. The welfare effect is also not similar to the growth effect. So, our results are different from those existing in the literature as the role of the dynamic efficiency function of workers is not considered there.

IV. *The 'Right to Manage' Model*

In this section, we use 'Right to Manage' model instead of 'Efficient Bargaining' model to analyse the negotiation process between the firms' association and the labour union. In this case, the employers' union and the labour union bargain only over the wage rate; and the firm solely determines the number of employed workers from its labour demand function obtained from its profit maximisation exercise. So, from equations (1), (2), (2.a), (2.b) and (3), we obtain the inverted labour demand function of the representative firm as given by

$$w = [\beta A K^{\alpha} \overline{K}^{\gamma} L^{\beta-1} h^{\beta\eta} b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}. \tag{40}$$

So the firms' association and the labour union jointly maximise the 'generalised Nash product' function given by equation (9) with respect to w only subject to the equation (40). Using the first order condition of maximisation and equations (1), (2), (2.a), (2.b), (4), (6) and (40), optimum values of L and w are obtained as[45]

$$L^{**} = \frac{[1-\tau(1-\lambda)]\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}-[1-\tau(1-\lambda)]\theta m(1-\beta)(1-\alpha-\beta)} < 1; \tag{41}$$

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[43] This can be easily understood from the term $\lambda\tau/[1-(1-\lambda)\tau]$ in the R.H.S. of the equation (35).

[44] This can be easily understood from the term $[\theta n(1-\alpha)+\beta(1-\theta)]/(1-\theta+\theta n)$ in the R.H.S. of the equation (35).

[45] Derivations of equations (41), (42) and (44) are shown in appendix E. We assume that the second order condition of maximisation is satisfied.

and

$$w^{**}=\beta AK^{\alpha}\overline{K}^{\tau}h^{\beta\eta}L^{**\beta-1-\beta\delta}\tau^{-\beta\delta}(1-L^{**})^{\beta\delta}(1-\lambda)^{-\beta\delta}[1-\tau(1-\lambda)]^{\beta\delta}. \tag{42}$$

We assume the following parametric restriction to be valid in order to ensure that $L^{**}>0$.

$$\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}>\theta m(1-\beta)(1-\alpha-\beta).$$

This restriction implies that the labour union can not be highly wage oriented. In this model too, L^{**} varies inversely with θ when τ and λ are given. This is shown by

$$\frac{\partial L^{**}}{\partial\theta}=-\frac{[1-(1-\lambda)\tau](1-\lambda)\tau\beta m(1-\beta)^{2}(1-\alpha-\beta)(1-\delta)}{[\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}-[1-\tau(1-\lambda)]\theta m(1-\beta)(1-\alpha-\beta)]^{2}}<0. \tag{43}$$

Similarly, it is very easy to check that L^{**} varies inversely with τ and λ. The intuitions for these results are similar to those provided in 'Efficient Bargaining' model.

Now, from equations (2.b), (6) and (41), the effort level of the representative worker is obtained; and it is given by

$$e_{2}^{**}=\left[\frac{[1-(1-\lambda)\tau](1-L^{**})}{(1-\lambda)\tau L^{**}}\right]^{\delta}$$

$$=\left[\frac{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}\right]^{\delta}. \tag{44}$$

From equation (44), we have

$$\frac{\partial e_{2}^{**}}{\partial\theta}=\frac{\delta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)\}^{\delta-1}\beta m(1-\beta)^{2}(1-\alpha-\beta)(1-\delta)}{[\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)]^{\delta+1}}>0. \tag{45}$$

Equation (45) implies that the effort level of the worker varies positively with the degree of unionisation in the labour market and the intuition behind this result is similar to that of the corresponding results in the previous model. Since, in this model, the government's budget balancing equations as well as the household sector's behaviour are identical to those given in the 'Efficient Bargaining' model, so the existence and stability properties of the steady state equilibrium derived in that model will remain unchanged here.

Now, using equations (2), (2.a), (2.b), (5), (6), (23), (42) and (44), we obtain the balanced growth equation given by

$$(\rho+\sigma g)[g]^{\frac{\beta\eta}{1-\beta\eta}}=A^{\frac{1}{1-\beta\eta}}\alpha L^{**\frac{\beta(1-\delta)}{1-\beta\eta}}\left[\frac{(1-L^{**})}{(1-\lambda)}\right]^{\frac{\beta\delta}{1-\beta\eta}}[\beta\lambda]^{\frac{\beta\eta}{1-\beta\eta}}\left\{\frac{[1-(1-\lambda)\tau]}{\tau}\right\}^{\frac{\beta\delta-\beta\eta}{1-\beta\eta}}. \tag{46}$$

Using equations (41) and (46), we obtain the growth rate maximising tax rate given by

$$\tau^{**}=\frac{\eta\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\theta m(1-\beta)(1-\alpha-\beta)\}}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\eta\theta m(1-\beta)(1-\alpha-\beta)\}(1-\lambda)}. \tag{47}$$

From equation (47), we obtain

$$\frac{\partial\tau^{**}}{\partial\theta}=-\frac{\eta(1-\eta)\beta m(1-\beta)^{2}(1-\alpha-\beta)(1-\delta)}{\{\theta n(1-\alpha-\beta)(1-\beta\delta)+\beta(1-\delta)(1-\theta)(1-\beta)-\eta\theta m(1-\beta)(1-\alpha-\beta)\}^{2}(1-\lambda)}<0. \quad (48)$$

So the growth rate maximising tax rate varies inversely with the degree of unionisation. It is easy to check that, in this model too, the growth rate maximising tax rate on labour income varies positively with the elasticity of workers' efficiency with respect to human capital as well as with the budget share of the government to finance educational expenditure. The intuitions behind these properties of the growth rate maximising tax rate are similar to those of the previous model. Incorporating the value of τ^{**} from equation (47) in equation (41), we obtain a value of L^{**} equal to L^{*} as given in equation (34). So, the properties of the employment function corresponding to the growth rate maximisation obtained in 'Efficient Bargaining' model are also valid in this model.

Now, we turn to check the equivalence between the growth rate maximising labour income tax rate and the welfare maximising labour income tax rate. We use equations (1), (3), (6), (21), (22), (23) and (40); and thus obtain

$$\omega=\frac{K_0^{1-\sigma}\left[\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau\beta}{[1-(1-\lambda)\tau]}\right\}-g\right]^{1-\sigma}}{(1-\sigma)[\rho-g(1-\sigma)]}+constant. \quad (49)$$

We assume $1>\sigma$ and $\rho>g(1-\sigma)$. Since the initial consumption level, c_0, is always positive, so $\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left\{1-\frac{\lambda\tau\beta}{[1-(1-\lambda)\tau]}\right\}$ has to be greater than g. From equation (49), we obtain

$$\left.\frac{\partial\omega}{\partial\tau}\right|_{\tau=\tau^{**}}=-\frac{K_0^{1-\sigma}\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{\frac{\beta\lambda}{[1-(1-\lambda)\tau^{**}]^{2}}\right\}}{\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]^{\sigma}[\rho-g^{**}(1-\sigma)]}<0. \quad (50)$$

Equation (50) shows that, in this model too, the welfare maximising tax rate falls short of the growth rate maximising tax rate due to the negative effect of taxation on the initial consumption level.

Now, using equations (34), (46) and (47), we obtain

$$\left[\frac{\sigma g^{**}}{(\rho+\sigma g^{**})}+\frac{\beta\eta}{1-\beta\eta}\right]\frac{\frac{\partial g^{**}}{\partial\theta}}{g^{**}}=-\left(\frac{\beta[\delta-\eta]}{1-\beta\eta}\right)\left\{\frac{(1-\lambda)}{[1-(1-\lambda)\tau^{**}]}+\frac{1}{\tau^{**}}\right\}\frac{\partial\tau^{**}}{\partial\theta}\gtreqless 0 \quad iff \quad \delta\gtreqless\eta. \quad (51)$$

The equation (51) shows that the mathematical sign of the growth effect of unionisation depends solely on the mathematical sign of $(\delta-\eta)$. So, if the elasticity of worker's efficiency with respect to the wage premium, δ, is higher than (equal to) (lower than) the elasticity of worker's efficiency with respect to human capital, η, then unionisation in the labour market raises (does not affect) (lowers) the rate of economic growth. In 'Efficient Bargaining' model, the sign of growth effect of unionisation partially depends on the mathematical sign of $(\delta-\eta)$.

However, in 'Right to Manage' model, the sign of this growth effect fully depends on the mathematical sign of $(\delta-\eta)$. So, in this model, $\delta>\eta$ is a necessary as well as a sufficient condition to ensure a positive growth effect of unionisation; whereas, in 'Efficient Bargaining' model, it is only a sufficient condition but not a necessary one.

In order to analyse the welfare effect of unionisation, we use equation (49) and obtain

$$\left.\frac{\partial \omega}{\partial \theta}\right|_{\tau=\tau^{**}}=\frac{K_0^{1-\sigma}[\rho-g^{**}(1-\sigma)]^{-1}\frac{\partial g^{**}}{\partial \theta}}{\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\beta\tau^{**}}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]^{\sigma-1}}\left\{\frac{\left[\frac{\sigma}{\alpha}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-1\right]}{\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]}\right.$$

$$\left.+\frac{1}{[\rho-g^{**}(1-\sigma)]}\right\}-\frac{[\rho-g^{**}(1-\sigma)]^{-1}\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\frac{\lambda\beta}{[1-(1-\lambda)\tau^{**}]^2}\frac{\partial \tau^{**}}{\partial \theta}}{K_0^{\sigma-1}\left[\left\{\frac{\rho+\sigma g^{**}}{\alpha}\right\}\left\{1-\frac{\lambda\tau^{**}\beta}{[1-(1-\lambda)\tau^{**}]}\right\}-g^{**}\right]^{\sigma}}. \tag{52}$$

The equation (52) implies that here also the welfare effect of unionisation consists of the growth effect as well as of the initial consumption effect. The first term in the R.H.S. of equation (52) captures the growth effect of unionisation; and its second term captures the initial consumption effect. Here, the growth rate maximising tax rate varies inversely with the degree of unionisation. As a result, the disposable income of the household sector also varies inversely with the degree of unionisation; and so the initial level of consumption is always increased with a rise in the bargaining power of the union. Hence, the initial consumption effect of unionisation is always positive. So the welfare effect of unionisation is always stronger than its growth effect.

Important results derived in this section are summarized in the following proposition.

***Proposition** 7: When the government imposes the growth rate maximising tax rate on labour income, then, in 'Right to Manage' model of bargaining, unionisation in the labour market raises (does not affect) (lowers) the rate of growth if and only if the elasticity of worker's efficiency with respect to wage premium is higher than (equal to) (lower than) the elasticity of worker's efficiency with respect to human capital. The welfare effect of unionisation is always stronger than its growth effect.*

V. *The Conclusion*

This paper develops an endogenous growth model to analyse the effect of unionisation in the labour market on the long run economic growth rate. It has a special focus on human capital formation as well as on 'Efficiency Wage Hypothesis'. Efficiency of an worker varies positively not only with the net wage premium but also with the stock of human capital. A proportional tax on labour income finances educational expenditure and unemployment allowances in this model. We derive properties of the growth rate maximizing tax rate. We use both 'Efficient Bargaining' model of McDonald and Solow (1981) and 'Right to Manage' model of Nickell and Andrews (1983) to derive the outcome of negotiation between the labour union and the employers' association. Our analysis is important because the existing literature focuses neither on the role of 'Efficiency Wage Hypothesis' nor on the government's role on human capital formation while analyzing the growth effect of labour union.

We derive many interesting results. First, in each of these two alternative types of bargaining models, for a given tax rate on labour income, unionisation in the labour market lowers the number of employed workers but raises the wage rate as well as the effort level of the representative worker irrespective of the orientation of the labour union. The level of eemployment also varies inversely with the tax rate on labour income and with the rate of unemployment benefit but the wage rate is independent of the tax rate. However, when the government imposes the growth rate maximising tax rate on labour income, the level of employment becomes independent of the labour union's bargaining power and the rate of unemployment benefit; but varies inversely with the elasticity of efficiency with respect to the human capital. In fact, the level of unemployment is equal to the elasticity of efficiency with respect to human capital. So higher productivity in human capital accumulation function generates higher unemployment. Secondly, the growth rate maximising tax rate on labour income varies positively with the elasticity of workers' efficiency with respect to human capital as well as with the budget share of educational expenditure but varies inversely with the degree of unionisation in the labour market. The growth rate maximising tax rate on labour income is different from the corresponding welfare maximising tax rate in these two models. The Welfare effect of unionisation is also different from the growth effect of unionisation in each of these two models. Lastly, in the case of 'Efficient Bargaining' model, we derive a sufficient condition for the positive growth effect of unionization. The condition states that the elasticity of the worker's effort level with respect to the wage premium is higher than the elasticity of worker's efficiency with respect to the stock of human capital. However, it is not a necessary condition in this case. On the other hand, in the case of 'Right to Manage' model, this condition becomes necessary as well as sufficient.

However, our simple theoretical model does not consider many important aspects of reality. Issues like population growth, technological progress, the positive externality of public goods etc. are ignored for the sake of simplicity. We also do not consider the capital income taxation for analytical simplicity. We only focus on the publicly financed educational expenditure and do not consider the role of private education on human capital formation. To avoid complexity in the theoretical analysis, we assume a 'closed shop labour union', rather than the more common 'open shop labour union'. The labour union's simple utility function does not take care of its other priorities like workplace safety and environmental issues. We do not consider public good provision in government budget. We plan to do further research in future removing these limitations.

Appendix

Appendix A: Derivation of the Optimal w and L

From equations (4) and (10), we obtain

$$\theta m[(1-\alpha)Y-wL]=(1-\theta)(w-b)\left\{L-\frac{\beta\delta Y}{w}\right\}. \tag{A.1}$$

Now, from the equation (6), we obtain

$$b(1-L)=\frac{(1-\lambda)\tau wL}{[1-(1-\lambda)\tau]}. \qquad \text{(A.2)}$$

Using the equations (A.1) and (A.2), we obtain

$$\theta m[(1-\alpha)Y-wL]=(1-\theta)\left(w-\frac{(1-\lambda)\tau wL}{[1-(1-\lambda)\tau](1-L)}\right)\left\{L-\frac{\beta\delta Y}{w}\right\}. \qquad \text{(A.3)}$$

Using the equations (11.a) and (A.3), we obtain

$$\frac{(1-L)[1-(1-\lambda)\tau]}{(1-\lambda)\tau L}=\Theta_1. \qquad \text{(A.4)}$$

From the equation (A.4), we obtain the equation (12) in the body of the article.
Incorporating the value of L^* from the equation (12) in the equation (A.2), we obtain the equation (13) in the body of the article. We assume that the second order conditions of maximisation are satisfied.

Appendix B: Derivation of the Equation (23)

Using equations (21) and (22), we construct the Current Value Hamiltonian as

$$H_c=\frac{c^{1-\sigma}-1}{1-\sigma}+\mu[(1-\tau)wL+rK+\pi+(1-\tau)b(1-L)-c]. \qquad \text{(B.1)}$$

Here μ is the co-state variable, function of time, t; and the transversality condition satisfies

$$\mu\geq 0 \text{ for all } t \text{ with } \lim_{t\to\infty}\mu\mathrm{K}=0.$$

Here

$$\frac{\partial H_c}{\partial L}=\mu[(1-\tau)w-(1-\tau)b]\geq 0 \quad \text{for} \quad w>b \text{ and } 0<\tau<1;$$

and hence optimum L is at the upper boundary in this case. Since $L^*<1$ and $w^*>b$, optimum $L=L^*$.

Maximising the Hamiltonian with respect to c and assuming that an interior solution exists, we obtain the following first order condition.

$$c^{-\sigma}-\mu=0. \qquad \text{(B.2)}$$

Again, from equation (B.1), we find that optimum time path of μ satisifes

$$\frac{\dot{\mu}}{\mu}=\rho-r. \qquad \text{(B.3)}$$

From equation (B.2), we have

$$\frac{\dot{\mu}}{\mu}=-\sigma\frac{\dot{c}}{c}. \qquad \text{(B.4)}$$

Using equations (B.3) and (B.4), we have

$$\frac{\dot{c}}{c}=\frac{r-\rho}{\sigma}. \tag{B.5}$$

Then using equations (B.5), (4) and $L=L^*$, we obtain equation (23) in the body of the paper.

Appendix C: Derivation of (i) the Jacobian determinant and (ii) the equation (32)

(i) The Jacobian determinant:
The Jacobian determinant is given below.

$$|J|=\begin{vmatrix} \dfrac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial M} & \dfrac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial N} \\ \dfrac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial M} & \dfrac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial N} \end{vmatrix}.$$

From the equations (27) and (28), we have

$$\frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial M}=\frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial M}=1;$$

$$\frac{\partial\left(\frac{\dot{M}}{M}\right)}{\partial N}=\frac{\beta\eta\alpha AL^{*\beta}[\Theta_1]^{\beta\delta}}{\sigma N^{1-\beta\eta}}-\frac{\beta\eta AL^{*\beta}[\Theta_1]^{\beta\delta}}{N^{1-\beta\eta}}\left[1-\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right];$$

and

$$\frac{\partial\left(\frac{\dot{N}}{N}\right)}{\partial N}=-\frac{(1-\beta\eta)\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]AL^{*\beta}[\Theta_1]^{\beta\delta}}{N^{2-\beta\eta}[1-(1-\lambda)\tau](1-\theta+\theta n)}$$
$$-\frac{\beta\eta AL^{*\beta}[\Theta_1]^{\beta\delta}}{N^{1-\beta\eta}}\left[1-\frac{\lambda\tau[\theta n(1-\alpha)+\beta(1-\theta)]}{(1-\theta+\theta n)[1-(1-\lambda)\tau]}\right].$$

Using these equations, we obtain the equation (30) in the body of the article.

(ii) The equation (32):
Putting the value of L^* in equation (31) from equation (12), we obtain

$$(\rho+\sigma g)[g]^{\frac{\beta\eta}{1-\beta\eta}}=A^{\frac{1}{1-\beta\eta}}\alpha[\Theta_1]^{\frac{\beta\delta}{1-\beta\eta}}.$$
$$\left\{\frac{[1-(1-\lambda)\tau]}{[1-(1-\lambda)\tau]+\Theta_1(1-\lambda)\tau}\right\}^{\frac{\beta}{1-\beta\eta}}\left\{\lambda\tau\frac{[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right\}^{\frac{\beta\eta}{1-\beta\eta}}. \tag{C.1}$$

Now taking log on both sides and differentiating it with respect to τ, we obtain

$$\frac{\sigma\frac{\partial g}{\partial \tau}}{(\rho+\sigma g)}+\frac{\frac{\beta\eta}{1-\beta\eta}}{g}\frac{\partial g}{\partial \tau}=\left(\frac{\beta}{1-\beta\eta}\right)\frac{-(1-\lambda)}{[1-(1-\lambda)\tau]}$$
$$-\left(\frac{\beta}{1-\beta\eta}\right)\frac{-(1-\lambda)+\Theta_1(1-\lambda)}{\{[1-(1-\lambda)\tau]+\Theta_1(1-\lambda)\tau\}}+\left(\frac{\beta\eta}{1-\beta\eta}\right)\frac{1}{\tau}$$
$$-\left(\frac{\beta\eta}{1-\beta\eta}\right)\left(\frac{-(1-\lambda)}{[1-(1-\lambda)\tau]}\right). \tag{C.2}$$

For maximising g with respect to τ, the R.H.S. of equation (C.2) is equalised with 0; and from that we solve the growth maximising rate of tax, τ^*, given by equation (32).

Appendix D: Derivation of the Equation (35)

From the equation (21), we obtain

$$\omega=\frac{c_0{}^{1-\sigma}}{[\rho-g(1-\sigma)](1-\sigma)}+constant. \tag{D.1}$$

Here, $c(0)=c_0$.
Now, from the equations (22) and (3), we obtain

$$\dot{K}=(1-\tau)wL+Y-wL+(1-\tau)b(1-L)-c. \tag{D.2}$$

Using the equations (D.2) and (6), we obtain

$$\dot{K}=(1-\tau)wL+Y-wL+\frac{(1-\tau)(1-\lambda)\tau wL}{[1-(1-\lambda)\tau]}-c. \tag{D.3}$$

Using the equations (D.3) and (11.a), we obtain

$$\dot{\mathrm{K}}=Y\left\{1-\frac{\tau\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right\}-c. \tag{D.4}$$

From the equation (D.4), we obtain

$$c_0=K_0\left\{\frac{Y_0}{K_0}\left[1-\frac{\tau\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right]-g\right\}. \tag{D.5}$$

Using the equations (D.5), (1) and (23), we obtain

$$c_0=K_0\left\{\left\{\frac{\rho+\sigma g}{\alpha}\right\}\left[1-\frac{\tau\lambda[\theta n(1-\alpha)+\beta(1-\theta)]}{[1-(1-\lambda)\tau](1-\theta+\theta n)}\right]-g\right\}. \tag{D.6}$$

Using the equations (D.1) and (D.6), we obtain the equation (35) in the body of the article.

Appendix E: Derivation of the Equations (41), (42) and (44)

Incorporating the inverted labour demand function of the representative firm from the equation (40) in the equation (9) and obtain

$$\psi=\{(1-\tau)^{m}\Big([\beta AK^{\alpha}\overline{K}^{\gamma}L^{\beta-1}h^{\beta\eta}b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}-b\Big)^{m}L^{n}\}^{\theta}$$

$$.\{(1-\beta)[\beta AK^{\alpha}\overline{K}^{\gamma}h^{\beta\eta}b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}L^{\frac{\beta(1-\delta)}{1-\beta\delta}}-rK\}^{(1-\theta)}. \tag{E.1}$$

Since the equation (40) shows a monotonic relationship between the w and L, so we maximise the equation (E.1) with respect to the L. Using this first order condition and the equation (4), we obtain

$$\frac{\frac{\theta m(\beta-1)}{1-\beta\delta}[\beta AK^{\alpha}\overline{K}^{\gamma}h^{\beta\eta}b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}L^{\frac{\beta(1+\delta)-2}{1-\beta\delta}}}{[\beta AK^{\alpha}\overline{K}^{\gamma}L^{\beta-1}h^{\beta\eta}b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}-b}+\frac{\theta n}{L}$$

$$+\frac{(1-\theta)(1-\beta)[\beta^{\beta\delta}AK^{\alpha}\overline{K}^{\gamma}h^{\beta\eta}b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}\frac{\beta(1-\delta)}{1-\beta\delta}L^{\frac{\beta-1}{1-\beta\delta}}}{(1-\alpha-\beta)[\beta^{\beta\delta}AK^{\alpha}\overline{K}^{\gamma}h^{\beta\eta}b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}L^{\frac{\beta(1-\delta)}{1-\beta\delta}}}=0. \tag{E.2}$$

From the equation (6), we have

$$b=\frac{(1-\lambda)\tau wL}{[1-(1-\lambda)\tau](1-L)}. \tag{E.3}$$

Now, using the equations (40) and (E.3), we obtain

$$b=\frac{(1-\lambda)\tau[\beta AK^{\alpha}\overline{K}^{\gamma}L^{\beta(1-\delta)}h^{\beta\eta}b^{-\beta\delta}]^{\frac{1}{1-\beta\delta}}}{[1-(1-\lambda)\tau](1-L)}. \tag{E.4}$$

Using the equations (E.2) and (E.4), we obtain the equation (41) in the body of the article. Now, using the equations (E.3) and (41), we obtain the equation (44) in the body of the article. We obtain the equation (42) in the main article using the equations (E.3) and (40).

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