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# The Effect of Reduction of Working Hours in Society Where People Have Both the Homogeneous Way to Enjoy Leisure and the Homogeneous Way to Take Leisure Time 

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# The effect of reduction of working hours in society where people have both the homogeneous way to enjoy leisure and the homogeneous way to take leisure time. 

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#### Abstract

The reduction of working hours policy is often grasped as the cause of the decrease of GDP. (see Hayashi and Prescott (2002). But we consider that reduction of working hours policy may cause the rise of GDP through the rise of leisure demand of household. While, we often hear the news that there occur serious congestion in many leisure facilities in Japan. Here, we should also consider that such congestion is related with two homogeneous character surrounded with leisure consumption in Japanese society, ( homogeneous or constant time zone of holiday (Obon, Oshogatsu, G.W.) and the homogeneous way to enjoy leisure ) So, we also confirm that we have to disperse the time zone of holydays and diversify the way to enjoy leisure in order to get the enough effect of reduction of working hours policy in Japan.


## 1. Introduction

In1990s, when Heisei recession started, simultaneously, legal statutory working hours was reduced from 48 hours to 40 hours per week in Japan.
Hayashi and Prescott(2002), from such a fact, point out that it is because of this reduction of legal statutory working hours that economic recession occurs by using real business cycle model without arguing the problem about monetary sector. The reduction of working hours mainly give born to the reason of economic recession with

[^0]out the problem of monetary sector by using RBC model. Certainly, in the usual real business cycle model, reduction of working hours decreases the labor input and decrease GDP and so, the utility of household also decrease. But while, in 1980s, Nakasone Yasuhiro Cabinmate have gave argument that reduction of working hours give born to leisure demand and the rise of GDP and should encourage the reduction of labor time. Should we note also that the reduction of working hours have an effect on the rise of demand of leisure and GDP? so, should we also argue such a demand side of the effect of reduction of working hours?

Certainly, in 1980s, there are some articles or models ${ }^{2}$ shown that, if the reduction of working hours make production at the point of the high productivity of labor realized through the shortening of work time per labor and deduce the rise of GDP.

But such arguments mainly forces on only the supply side. While, we force on the effect of the reduction of labor time on the demand side and try to argue about the effect of the reduction of working hours on GDP through the rise of leisure demand.

In order to do such an argument, we also try to divide the leisure time into the aggressive leisure time with consumption of leisure goods or services and the idol leisure time without consumption of leisure goods or services.
Moreover, in order that we describe the leisure as the aggressive time which is associated with demand of leisure goods and services, we cannot be depended totally on the use of the framework of perfect separate type utility function which is not suitable to analyze such an argument, because such a perfect separate type utility function does not divide the aggressive leisure time which is spent with consumption of goods or services and idol leisure time which is not spent with leisure consumption of goods or services distinctly. While, we have to classify leisure time into the aggressive time which give birth to leisure goods or services and idle time.
In order to do such an argument, we also refer home production model including non-separate type utility function as like Becker(1965) which grasp the behavior of household as the combination time and good.

But the original Becker(1965) model has fixed coefficient about the relationship between time and good, and has been mainly used mainly in partial equilibrium model which focus on behavior of household.

So, we try to modify the setting of original Becker(1965), we assume that household

[^1]get utility from both leisure time and leisure good or service separately but consumption of leisure goods or service accompany with the time so, household have to spend time to consume leisure goods or services.

In short, instead that household get utility from combination of time and goods as like Becker(1965), we assume that household get utility from both leisure time and leisure goods separately, which is near to approach of separate type utility function but we also assume that demand or consumption for leisure goods of household is assumed to be accompanied with consumption of time and try to come near thought of Becker(1965).
sometimes to be faced constraint from leisure time, which reflect thought Becker(1965).
Through such a setting we succeed in introducing the thought of Becker(1965) into general equilibrium including supply side and show that there is a situation that if the reduction of working hours cause the rise of GDP.

But in Japan there are some special circumstances that are surrounded with the effect of reduction of working hours and such a special circumstance may make the reduction of work weaker than the real effect. Such a special character in circumstance is often accompanied with the character of leisure demand itself.

So, in order to examine the effect of the reduction of work hours in Japan exactly, we should also notice the special character of leisure demand and the special character of environment around leisure demand in Japan and consider also how such a character of leisure and an environment give born to the modification of the effect of reduction of working hours.
First, we notice that leisure service has the character of the simultaneous realization of both consumption and production of leisure service.

As related with such a character of leisure service, we also notice that people often go to the leisure facility directly where leisure service is offered (Theme park, Athlete stadium Theater, Movie theater, Music Hall and so on ) in order to enjoy leisure service.

Here if the capacity of leisure facility is below the capacity which could accommodate people who desire to enjoy leisure, all the people could not go to in order to enjoy leisure and congestion problem occur.

Then even if the reduction of working hours policy is done in order to rise demand of leisure and GDP, such a policy could not realize enough the rise of leisure demand and GDP by the overcapacity or congestion problem.
p. 3

So, next, we should notice that it is also important to consider the capacity of leisure facility and the congestion problem which is caused from simultaneously realization of demand and supply in leisure service

Here in order that firm accommodate all the people who desire to enjoy leisure in its facility in order to avoid occurring congestion, firm should prepare the large capacity enough to accommodate the people holding the leisure demand.

In addition, usually leisure industry firm pay the cost of facility before entering and producing leisure good. So, such a congestion problem is connected with the entry cost and which influence the variety of leisure demand, and such a problem make firms entry or sure industry difficult.

Here we consider that the problem of the capacity of leisure facility or the entry cost is depended on how wide the time zone for holiday is taken in society
If holiday is concentrated into the constant range of time zone, there are tendency that is possible that people rash into the leisure facility at the same time and congestion tend to occur.
While, if holiday is distributed into the various time zone,
many people does not rash into the leisure facility at the same time and
leisure firm do not have to prepare the large facility in order to accommodate people.

For example, if there are 40 peoples who lives in one city and can take 2 holiday days a week, the half of the all the 40 peoples, ( 20 peoples) dividedly could go to the leisure facility a holiday a week. and leisure firm has only to prepare leisure facility which accommodate 20 peoples in order to accommodate all the 40 peoples.
While if there are 40 peoples who lives in one city and can take only 1 holiday days a week, all the peoples go to leisure facility a holyday and leisure firm has to prepare facility which accommodate 40 peoples in order to accommodate all the 40 peoples.
In japan where the time zone oh the holiday is concentrated into the constant time zone, (Obon, Oshogats, Golden week.), each leisure industry firm has to face the situation that many peoples rush to his own facility within the constant time and the congestion tend to occur

So, the government also have to disperse or lengthen the time zone of holiday in order to make working hours reduction policy effect

It is meant that the policy of the disperse or lengthen of have also effect of GDP

Here, even though the holiday is concentrated in the constant time zone, if the p. 4
peoples know the way to enjoy various leisure and the preference of household for leisure goods or services is variant, such a congestion is to be avoided. Because household enjoy leisure with spallation from each other in the leisure facility.

But, in Japan, , the way to enjoy leisure is homogeneous, so it remains still important to consider capacity and congestion of leisure facility in holiday.

So finally we should notice the variant of the both of the way to enjoy leisure and the way to time zone of holiday in order to consider the effect of the working hour reduction policy.

In order to describe the variant of the way to enjoy leisure, we assume the preference of household for leisure demand is assumed to be represented by Dixit and Stiglitz(1977), which
use variety type utility function. In short, we assume that household have the utility from preference of variety like Dixit and Stiglitz(1977) about leisure goods.

And additionally, we assume that firm which composite such a variety of leisure good have to entry pay cost for the construction of leisure facility whose capacity increase according to capacity in entering leisure industry.
Through this setting, we consider also circumstance around the reduction of working hour.

Through the next seven sections, we develop the theoretical analyze (basic model. the effect of working hour reduction by including the thought of Becker(1965), the effect of the variant of holyday including the argument of Dixit and Stiglitz(1977), and the combination of the working hour reduction and the variant of holiday including the relationship between the variant of leisure goods and the variant of time zone of holyday)

Finally, we estimate our argument.

## 2. Behavior of Household

At first, in economy the number of households assumed to be $\bar{N}$ and household assumed to get from the usual good, leisure time and leisure goods as the below utility function.

$$
\begin{equation*}
U=\alpha \log X+\beta \log \Gamma+\gamma \log T \tag{1}
\end{equation*}
$$

Where $X$ is consumption of the usual good, $\quad \Gamma$ is parameter about the utility of leisure goods and Tis consumption of leisure time
Moreover, we assume that the below relationship between $\Gamma$ and $Y_{i}$ is satisfied.
p. 5

$$
\begin{equation*}
\Gamma^{\rho}=\sum_{i=1}^{n} Y_{i}^{\rho} \tag{2}
\end{equation*}
$$

Where $n$ is the number of leisure good household can consume, $Y_{i}$ is the consumption of each leisure and $\rho$ is parameter about the relationship between the relationship between $\Gamma$ and $Y_{i}$. each household distribute initial endowed time $\bar{L}$ into leisure time $T$ and labor time $L$.

$$
\begin{equation*}
L+T=\bar{L} \tag{3}
\end{equation*}
$$

In addition, we assume that each household distributes his revenue getting from offering labor and capital to buy usual good and leisure goods and initial endowed time is $\bar{L}$ and initial endowed capital is $\bar{K}$. so, we get the below budget constraint equation.

$$
\begin{equation*}
p_{X} X+\sum_{i} p_{Y_{i}} Y_{i}+w T=w \bar{L}+r \bar{K}=M \tag{4}
\end{equation*}
$$

where $p_{X}, p_{Y i}, w, r$ are respectively the price of usual good, the price of leisure good, wage, interest rate. As the number of households is $\bar{N}$, all the time endowed in economy is to be shown as $\overline{N L}$ and all the capital endowed in economy is to be shown as $\bar{N} \bar{K}$

Each household offer his labor to industry producing usual good $L_{X}$ or to industry producing leisure goods $L_{Y}$ similarly, each Household offer his capital to industry producing usual good $K_{X}$ or industry producing leisure good $K_{Y}$
So, the next equations about distribution of labor are satisfied.

$$
\begin{align*}
& N_{X}+N_{Y}=\bar{N}  \tag{5}\\
& L_{X}=N_{X} L  \tag{6}\\
& L_{Y}=N_{Y} L \tag{7}
\end{align*}
$$

where $L_{X}, L_{y}$ are recently labor into industry of usual goods ,industry of leisure goods.
$N_{X}, N_{Y}$ the number of household engaged to industry producing the usual good and the number of household engaged to industry producing the leisure good. While, the next equations about distribution of capital are satisfied.

$$
\begin{equation*}
K_{X}+K_{Y}=\bar{N} \bar{K} \tag{8}
\end{equation*}
$$

From (1), (2), (3), (4) and first order condition of household, the subjective equilibrium of household is shown as next.

$$
\begin{align*}
& X=\frac{\alpha M}{p_{X}}  \tag{9}\\
& Y_{i}=\left(\frac{\beta M}{p_{Y i} \Gamma^{\rho}}\right)^{1 /(1-\rho)}  \tag{10}\\
& T=\frac{\gamma M}{w} \tag{11}
\end{align*}
$$

p. 6

So, total edemand function in economy is shown as next.

$$
\begin{align*}
& \bar{N} X=\frac{\alpha \bar{N} M}{p_{X}}  \tag{12}\\
& \bar{N} Y_{i}=\bar{N}\left(\frac{\beta M}{p_{Y i} \Gamma^{\rho}}\right)^{1 /(1-\rho)}  \tag{13}\\
& \bar{N} T=\frac{\gamma \bar{N} M}{w} \tag{14}
\end{align*}
$$

## 3. Behavior of firms (of leisure goods or services and of

## usual goods) ${ }^{3}$

Next we describe about supply side. In economy, we assume that there are variety of leisure goods in economy. According to Dixit and Stiglitz(1977), we assume that each leisure good is produced by each independent firm and productive function is liner and the element is only labor, which represent the fact that leisure industry generally have strong labor-intensive in leisure industry. Productive function is liner and the productive element is only labor which represent the character of labor-intensive in leisure industry.

$$
\begin{equation*}
Y=b_{2} L_{Y} \tag{15}
\end{equation*}
$$

where $b_{2}$ is parameter about productivity of leisure industry. From equation (13), the profit of each leisure is shown as below.

$$
\begin{equation*}
\pi_{i}=p_{y i} \bar{N} Y-w L_{y i}=\left[b_{2}\left(L_{y_{i}}\right)\right]^{\rho}\left(\frac{\beta M \bar{N}}{\Gamma^{\rho}}\right)-w L_{Y i} \tag{16}
\end{equation*}
$$

As each firm maximizes the above profit, the distribute of labor into each leisure firm is shown as the below.

$$
\begin{equation*}
\frac{\rho b_{2}^{\rho}\left(L_{y i}\right)^{\rho}}{L_{Y i_{i i}}} \frac{\beta M \bar{N}}{\Gamma^{\rho}}=\frac{\rho\left(Y_{i}\right)^{\rho}}{L_{Y i_{i i}}} \frac{\beta M \bar{N}}{\Gamma^{\rho}}=w \tag{17}
\end{equation*}
$$

While, the rent each firm can get $Z$ is also shown as the below.

$$
\begin{equation*}
\frac{(1-\rho)\left(Y_{i}\right)^{\rho} \beta M \bar{N}}{\Gamma^{\rho}}=Z \tag{18}
\end{equation*}
$$

In order to get this rent, each firm try to entry into leisure industry and through the behavior of each firm, expand the variety of leisure survive expand

But before entering leisure industry, each firm should pay entry cost for construction of leisure facticity which represent the fact that leisure industry firm should construct facility where the
p. 7
people enjoy leisure service before starting to producing leisure goods.
${ }^{3}$ 。As the magnitude of such a facility has usually increase as the magnitude of the capacity, the entry cost is also assumed to increase as the magnitude of capacity. So, we assume the magnitude of capital firm should prepare before entering leisure industry $K_{Y}$ increase as the increase of capac $C$ shown as the below.

$$
\begin{equation*}
K_{Y}=C^{\theta} \tag{19}
\end{equation*}
$$

where $\theta$ is parameter about the relationship between the capital $K_{Y}$ and capacity $C$.
we multiply the interest r against the both side on the (19) and we get the entry cost into leisure industry as the below.

$$
\begin{equation*}
r K_{Y}=r C^{\theta} \tag{20}
\end{equation*}
$$

Each firm compare the rent with the entry cost and decide whether it entry leisure industry or not.
The entry of firm succeed till the differentiation between the rent and the cost become zero which decide the number of the varieties in leisure goods.
On the other hand, firms which produce the usual good has Cobb Douglas productive technology function as the below.

$$
\begin{equation*}
X=K_{x}^{b_{1}} L_{x}^{a_{1}} \tag{21}
\end{equation*}
$$

We assume that each firm which producing usual goods is under the in perfect competition. So, the profit of the firm which produce the usual good is show as the below.

$$
\begin{equation*}
\pi_{X}=P_{X} X-w L_{x}-r K_{x}=P_{x} K_{x}^{b_{1}} L_{x}^{a_{1}}-w L_{x}-r K_{x} \tag{22}
\end{equation*}
$$

from the first condition for the maxi minimum of the profit, the equations of distribution of labor and capital in industry producing usual goods are shown as the below.

$$
\begin{align*}
& w L_{X}=a_{1} \alpha M \bar{N}  \tag{23}\\
& r K_{X}=b_{1} \alpha M \bar{N} \tag{24}
\end{align*}
$$

## 4. The distribution of initial endowed capital and labor <br> without time constraint and congestion

As each firm which produce each leisure good has same productive function and same entry cost, the production of each firm of leisure industry equal in the equilibrium So equation (2) can be translated to the below.
then, the enation is translated as the below.

[^2]\[

$$
\begin{equation*}
\Gamma^{\rho}=n Y_{i}^{\rho} \tag{25}
\end{equation*}
$$

\]

Moreover, the equation about distribution of labor and the rent of each leisure firm can be translated equation (19 and (20) to the below.

$$
\begin{align*}
& \frac{\rho}{L_{Y_{i_{i}}}} \frac{\beta M \bar{N}}{n}=w  \tag{26}\\
& (1-\rho) \frac{\beta M \bar{N}}{n}=Z \tag{27}
\end{align*}
$$

So, total labor initially endowed in economy $\overline{N L}$ is distributed to the labor for production of usual good, the production of leisure and the consumption of leisure time as the below.

$$
\begin{align*}
& L_{X}=\frac{a_{1} \alpha}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}  \tag{28}\\
& n L_{Y i}=\frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L} \text { (all the leisure indstries) }  \tag{29}\\
& L_{Y i}=\frac{1}{n} \frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L} \text { (each leisure industry) }  \tag{30}\\
& \bar{N} T=\frac{\gamma}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L} \tag{31}
\end{align*}
$$

So, when we divide the both side of the above equation (30) are divided by $\bar{N}$, and get leisure time $T i$ and labor time Lby using the result. Shown as the below.
$n$ whic each household consume leisure good or service and labor time of each household devoteis shown are as the below.

$$
\begin{align*}
& T=\frac{\gamma}{\rho \beta+a_{1} \alpha+\gamma} \bar{L}  \tag{32}\\
& L=\bar{L}-\frac{\gamma}{\rho \beta+a_{1} \alpha+\gamma}  \tag{33}\\
& \bar{L}=\frac{\rho \beta+a_{1} \alpha}{\rho \beta+a_{1} \alpha+\gamma} \\
& \bar{L}
\end{align*}
$$

We the above equation (33) by equation(28)or(29 by (33) )and we get the number of labor who work in the industry of usual goods $N_{X}$ and the industry of leisure goods $N_{Y}$ as the below.

$$
\begin{align*}
& N_{X}=\frac{a_{1} \alpha}{\rho \beta+a_{1} \alpha} \bar{N}  \tag{34}\\
& N_{Y}=\frac{\rho \beta}{\rho \beta+a_{1} \alpha} \bar{N} \tag{35}
\end{align*}
$$

So, We get also production of each leisure good and total production of leisure goods are shown as the below.

$$
\text { p. } 9
$$

$$
\begin{align*}
& Y_{i}=b_{2} L_{Y i}=b_{2}\left(\frac{1}{n} \frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right)  \tag{36}\\
& n Y_{i}=n b_{2} L_{Y i}=b_{2}\left(\frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right) \tag{37}
\end{align*}
$$

Next, we consider the distribution of capital.
First, from (27), we get the rent of leisure industry is show as the below.

$$
\begin{equation*}
(1-\rho) \beta M \bar{N}=n Z \tag{38}
\end{equation*}
$$

In equilibrium, as the entry of 1 firms into leisure industry succeed till such a rent is equal to the entry cost, the next equation is satisfied.

$$
\begin{equation*}
(1-\rho) \beta M \bar{N}=n Z=n r K_{Y i} \tag{39}
\end{equation*}
$$

We will consider about the number of variety of leisure good which is dependent with the value of $n K_{Y i}$ in next section.
So, the total capital in economy $\bar{N} \bar{K}$ is distributed into the production of the usual good and the production of leisure goods the next.

$$
\begin{gather*}
K_{X}=\frac{b_{1} \alpha}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K}  \tag{40}\\
n K_{Y i}=\frac{(1-\rho) \beta}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K} \quad \text { (all the leisure industries) }  \tag{41}\\
K_{Y i}=  \tag{42}\\
\frac{1}{n} \frac{(1-\rho) \beta}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K} \quad \text { (each leisure industry) }
\end{gather*}
$$

So the production in economy is decided as the next.

$$
\begin{equation*}
X=\left(\frac{a_{1} \alpha}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right)^{a_{1}}\left(\frac{b_{1} \alpha}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K}\right)^{b_{1}} \tag{43}
\end{equation*}
$$

So, the real GDP is decided as the next.

$$
\begin{equation*}
G D P=\left(\frac{b_{1} \alpha}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K}\right)^{b_{1}}\left(\frac{a_{1} \alpha}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right)^{a_{1}}+b_{2}\left(\frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right) \tag{44}
\end{equation*}
$$

## 5. The Necessary Time needed for the demand of leisure service and time constraint (The effect of reduction of

p. 10

## working hours )

Next, we consider the effect of working hour reduction policy and how this reduction policy is influenced from the time zone of holiday and capacity of leisure facility
Usually, Household should consume time as well as leisure good when they enjoy leisure. If household face the constraint to time, it cannot enjoy leisure.

So, ffrst, We assume that household consume time $L_{y c}$ when they consume leisure $Y$ as the below.

$$
\begin{equation*}
L_{y c}=\omega Y \tag{45}
\end{equation*}
$$

This setting inherits the thought of Becker(1965) but we also consider traceability.
We can take as $Y$ in the right hand, whether each leisure demand $Y_{i}$ of each household or all households or all the leisure demand $n Y_{i}$ of each household or all households.
Correspondently, we can get the necessary time needed for each leisure service of each household or all households $L_{y c}=\omega Y_{i}$ or the necessary time needed for all the leisure services of each household or all households $L_{y c}=\omega n Y_{i}$
Based on this setting, we can divide leisure time of household , $T$ into the aggressive leisure time with consumption of leisure services and the idol time without consumption of leisure services. For example, from (37) we get optimal production of leisure services or goods

$$
n Y_{i}=n b_{2} L_{Y i}=b_{2}\left(\frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right)
$$

So, from(45), we get also the necessary time needed for such a leisure services or goods shown as the below.

$$
\begin{equation*}
L_{y c}=\omega n Y_{i}=\omega n b_{2} L_{Y i}=\omega b_{2}\left(\frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \bar{N} \bar{L}\right) \tag{46}
\end{equation*}
$$

Exactly, this mean the aggressive leisure time with leisure consumption or leisure production.
While, from(31), we get optimal leisure time shown as the below.

$$
\bar{N} T=\frac{\gamma}{\rho \beta+a_{1} \alpha+\gamma}+\bar{N} \bar{L}
$$

So, when $\bar{N} T>L_{y c}$, we get idol leisure time without leisure services or leisure goods shown as the below.

$$
\begin{equation*}
\bar{N} T-L_{y c}=\frac{\gamma}{\rho \beta+a_{1} \alpha+\gamma} \bar{N} \bar{L}-\omega b_{2}\left(\frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \bar{N} \bar{L}\right) \tag{47}
\end{equation*}
$$

Based on this setting of the leisure time, we examine the effect of reduction of legal working time
p. 11
against the rise of leisure demand and the rise of GDP.
Next, we assume that legal working hours is $\tilde{L}$
In Japan, when firm set longer labor time than the decided legal working hour, firm should pay premium wage, while firm often have motivation to make laborer to engage long working time in order to make a collection of laborer's education cost
So, if legal working hours decrease, firm tend to decrease the labor time and household shift hs or her working time to leisure time.

Here if the next relationship between $T^{*}$ and $L_{y c}$ is satisfied, household keeps or has the leisure time enough to enjoy optimal leisure service in optimal equilibrium. So, household can enjoy full leisure good or service and leisure time.

$$
\begin{equation*}
n \omega Y^{*}<T^{*}<\bar{L}-\tilde{L} \tag{48}
\end{equation*}
$$

Under legal working hours, household keep enough time to enjoy the optimal consumption of leisure time and the time for optimal consumption of leisure goods. So, household solve the usual maximization problem and get the usual optimal solution.
Under such a situation, even if legal working hours $\tilde{L}$ decrease in order to rise GDP, the demand of the leisure time and leisure good does not increase.

While, If the relationship between $T^{*}$ and $L_{y c}$ is satisfied, household does not keep the optimal consumption of leisure time and the time for optimal consumption of leisure goods.

$$
\begin{equation*}
\bar{L}-\widetilde{L}<n \omega Y^{*}<T^{*} \tag{49}
\end{equation*}
$$

So, household cannot solve usual maximize problem and solve modified maximized problem. In short,household try to consume as many leisure goods as possible under such a constraint, the below relationship is satisfied.

$$
\begin{equation*}
n L_{Y C}=n \omega Y=1-\tilde{L} \tag{50}
\end{equation*}
$$

The labor demand of leisure good industry is decided by minimization between the solution of the above equation and legal working time,

$$
\begin{equation*}
L_{Y}=\min \left(\grave{L} \cdot \frac{1-L}{\omega} \frac{1}{b_{2}}\right) \tag{51}
\end{equation*}
$$

Here, we find that labor demand of leisure good industry is shorter than legal working hour when the productivity of leisure industry is high.
p. 12

So, From the above relationship, all the demand for leisure goods in economy, in which $\bar{N}$ households exist, is shown as the below.

$$
\begin{equation*}
Y=\bar{N} \frac{1-\tilde{L}}{\omega} \tag{52}
\end{equation*}
$$

The labor demand is decided by minimization between the solution of the above equation and legal working time as the below.

$$
\begin{equation*}
L_{Y}=\min \left(\grave{L} \cdot \frac{1-\tilde{l}}{\omega} \frac{1}{b_{2}}\right) \tag{53}
\end{equation*}
$$

While, labor of the usual good industry is shown as the below,

$$
\begin{equation*}
L_{x}=\min \left(\dot{n} \grave{L} . N \grave{L}-\frac{1-\widetilde{L}}{\omega} \frac{1}{b_{2}}\right) \tag{54}
\end{equation*}
$$

Here, as we found that labor demand of leisure good industry is shorter than legal working hour when the productivity of leisure industry is high. the potential labor demand of the usual good industry is larger than legal working hour. As a result, the labor into the usual good industry

$$
\begin{equation*}
X=\left(\frac{b_{1} \alpha}{(1-\rho) \beta+b_{1} \alpha} \bar{K}\right)^{b_{1}}\left(\bar{N} \tilde{L}-\bar{N} \frac{\overline{1}-\tilde{L}}{\omega} \frac{1}{b_{2}}\right)^{a_{1}} \tag{55}
\end{equation*}
$$

Considering the above situation, we find that, when legal working hours decrease by $\Delta \tilde{L}$, if the below condition is satisfied, the increase of production of leisure industry outweigh the decrease of production of usual goods industry and GDP rise.(Figure 3)

$$
\begin{equation*}
\frac{\bar{N}}{\omega} \Delta \tilde{L}>\left|a_{1}\left(\frac{b_{1} \alpha}{(1-\rho) \beta+b_{1} \alpha} \bar{K}\right)^{b_{1}}\left(\tilde{L}\left(\bar{N}-\bar{N} \frac{1-\tilde{L}}{n \omega} \frac{1}{b_{2}} \frac{1}{\widetilde{L}}\right)\right)^{a_{1}-1}\left(\bar{N}+\frac{\bar{N}}{n \omega} \frac{1}{b_{1}}\right)(-\Delta \tilde{L})\right| \tag{56}
\end{equation*}
$$

## 6. Time zone of holiday and the effect of the variant of

 time zone of holidayMoreover, we consider that it is also important to consider how long or wide the length of the leisure time zone is to be taken.
p. 13

Even if the working hour reduction policy about each household is done, under such a situation that the zone of holiday is constant or short or narrow, congestion tend to occur in leisure facility and the effect against the rise of GDP could not enough operate.

From simultaneously realization of production and consumption of leisure service, household sometimes go to the leisure facility directly.

Here If many household rash to go to same leisure facility at the same time, leisure facility may hold the problem of overcapacity. Then, though many households want to enjoy leisure-service, all the household could not maybe enjoy leisure service fully. (see Figure1)

Such a situation also has to make household modify time schedule or consumption plan
Such a overcapacity problem is serious especially in Japan where the holiday is concentrated in the constant time zone (Goldenweek, Obon, Oshogats) .

In order to describe such a situation, we assume that time zone where household can enjoy leisure service in economy (the length of time zone of holyday) is $\tilde{H}$.
Then, each household consume $\omega Y$ within the range of the time zone $\tilde{H} \dot{\mathrm{~m}}$ order to consume leisure service $Y$.

When the length or range of the time zone of holiday $\tilde{H}$ is long or wide, each household can allow for choice about the leisure time for leisure service $\omega Y$ within the long or wide range of $\tilde{H}$ and congestion in leisure facility does not tend to occur. while, When the length or range of the holiday time zone $\tilde{H}$ is short or narrow, each household has to choose the leisure time for leisure service $\omega Y$ within the short range of $\tilde{H}$ and congestion in leisure facility tend to occur. . (see Figure2)

In short, Each household consume $\omega Y$ in the range of the time zone $\tilde{H} \dot{\mathrm{~m}}$ order to consume leisure service $Y$. the below relationship is satisfied.

$$
\begin{equation*}
\tilde{m} L_{y c}=\tilde{m} \omega Y_{i}=\tilde{H} \tag{57}
\end{equation*}
$$

Where $\widetilde{m}$ is the ratio of leisure time for leisure good demand of each household against all the time zone household can enjoy leisure. Under the large ration $\widetilde{m}$, each household can allow for choice about the leisure time for leisure service without congestion. while, under the small ration $\widetilde{m}$, each household has to choose the leisure time and congestion in leisure facility tend to occur.

In order to addolorato the household who have leisure demand, the firm can divide all the number of household who want to enjoy leisure good by the ratio $\widetilde{m}$, and the firm has only to prepare the facility whose capacity can accommodate the number of all the number of household who want to enjoy leisure good by the ratio $\widetilde{m}$,

So, the below equation about the necessity cost for entry into leisure industry (19) is translated into the next.
p. 14

$$
\begin{equation*}
K_{Y}=C^{\theta}=(\bar{N} / \tilde{m})^{\theta} \tag{58}
\end{equation*}
$$

From and , the equation about $\tilde{m}$ is translated into the below.

$$
\begin{equation*}
\tilde{m}=\frac{n\left(\rho \beta+a_{1} \alpha+\gamma\right)}{\omega b_{2} \rho \beta \bar{L}} \tilde{H} \tag{59}
\end{equation*}
$$

Moreover, We divide the rent of all leisure industry by the entry cost and we get the number of leisure goods is decided in equilibrium as the below.

$$
\begin{equation*}
n^{*}=\left[\frac{(1-\rho) \beta}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K}\right]^{\frac{1}{1-\theta}}\left[\frac{\left(\rho \beta+a_{1} \alpha+\gamma\right) \tilde{H}}{\omega b_{2} \rho \beta \bar{L} \bar{N}}\right]^{\frac{\theta}{1-\theta}} \tag{60}
\end{equation*}
$$

We find that, when the time zone of holiday $\tilde{H}$ expand, the entry cost decreases and the number of variety of leisure goods increase. We also find that the utility of household is shown as the below.

$$
\begin{align*}
& U=\alpha \log \left(\frac{b_{1} \alpha}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K}\right)^{b_{1}}\left(\frac{a_{1} \alpha}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right)^{a_{1}}+ \\
& \beta \log \left(\left[\frac{(1-\rho) \beta}{(1-\rho) \beta+b_{1} \alpha} \bar{N} \bar{K}\right]^{\frac{1}{1-\theta}}\left[\frac{\left(\rho \beta+a_{1} \alpha+\gamma\right) \tilde{H}}{\omega b_{2} \rho \beta \bar{L} \hat{N}}\right]^{\frac{\theta}{1-\theta}}\right)^{\frac{1}{\rho}} b_{2}\left(\frac{\rho \beta}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right) \\
& +\gamma \log \left[\frac{\gamma}{\rho \beta+a_{1} \alpha+\gamma} \overline{N L}\right] \tag{61}
\end{align*}
$$

## 7. The combination of the effect of working hour reduction and the effect of variant of holiday

We argued about the effect of working hour reduction and the variant of the time zone of holiday in the previous section.
First, we found that, when legal working hour $\tilde{L}$ is short or narrow and household does not keep the leisure time enough to enjoy optimal leisure service the working hour reduction policy may have effect against the rise of GDP.

But, next, we found that, after the capacity of leisure facility is decided, such a reduction policy sometimes may lead to the congestion problem.
p. 15

So, we argued that the holiday variant policy also has to be done in order to keep the effect of the working reduction policy
In order to describe such an argument, we show the below equation about the congestion effect again

$$
\begin{equation*}
\tilde{m} L_{y c}=\tilde{m} \omega Y_{i}=\tilde{H} \tag{62}
\end{equation*}
$$

Though the operation of working hours reduction policy under long legal working hour, potential leisure demand and potential leisure time for leisure demand $L_{y c}$ increase. Under the situation that the time zone of holiday $\tilde{H}$ is constant, each leisure facility cannot accommodate all the household who want to enjoy leisure.

In order to get the effect of work reduction, government also has to disperse or lengthen the time zone of holiday $\tilde{H}$ in the right hand.
But, because of the cost of the adjustment of the time zone of holiday $\tilde{H}$, we consider that it is necessary to consider how long the length or range of the time zone of holiday $\tilde{H}$ should be.. Here, we also found that, when leisure demand distribute various leisure goods, congestion does not tend to occur. in economy which have various leisure goods.
So, next in order that we examine the relationship between the length or range of time zone of holiday $\tilde{H}$ which has to be adjusted and the variety of leisure goods, we notice that the below relationship which we can get from is satisfied,

$$
\begin{equation*}
\omega Y_{i}=\omega\left(\left(\frac{1-\tilde{L}}{n^{*} \omega}\right)\right)=\left(\frac{1-\tilde{L}}{n^{*}}\right) \tag{63}
\end{equation*}
$$

Under the above equation, when the legal working hours reduce by $\Delta \tilde{L}$, the below equation is satisfied.

$$
\begin{equation*}
\tilde{m} \omega Y_{i}=\tilde{m}\left(\frac{\Delta \tilde{L}}{n}\right)=\Delta \tilde{H} \tag{64}
\end{equation*}
$$

In short, when legal working hour $\Delta \tilde{L}$, in order to avoid the occurrence of congestion, government has to increase $\tilde{H}$ in the left hand. by $\tilde{m}(\Delta \tilde{L} / n)$

But, we find that, when the number of the variety of leisure goods $n$ is large, the range or length of the time zone of holiday $\tilde{H}$ has to be lengthen or widen a little. while, when the number of the variety of leisure goods $n$ is small, the range or length of the time zone of holiday $\tilde{H}$ has to be lengthen or widen large. In short, the lomgth or range the time zone of holiday accompanied with the reduction of working hours depend with the variety of keisure goods. here in japan the people do not have various way to enjoy leisure, so, the variant of time
zone of holiday should be widede.

## 8. Estimation

We estimate our argument about the effect of the reduction of working hour and the effect of the variant of holiday.
According to Emi (1974), We choose leisure related expenditure of household ${ }^{4}$ and all the expenditure of household from kakeichosa nenpou, the annual report on the family income and expenditure survey by statistics bureau management and coordination agency government of Japan.
We divide leisure related expenditure Lei1 by all the expenditure of household EXP and calculate lei 1 /exp, the ratio of leisure related expenditure.
We get also working hour '(Work) from maitsuki kinrou toukei chosa, monthly labor statics survey by ministry of health and labor.
We get also GNP from kokuminn Keizai keisan, national account of japan by Cabinet office and population from jinkou suikei, population census by statistics bureau management and coordination agency government of japan.

We divide GNP by population and calculate GNP /POPU.
Here, it is because Hayashi and Prescott(2000) use GNP and we accompany with their analyze that we use GNP instead of GDP.
Moreover, we apply Year Dummy which indicate 1 before 1980 and indicate 0 after 1980.

We should notice that the two days off a week system is introduced into many Japanese firms after 1980s. So, this Year Dummy is surrogate variable for variable of time zone of holiday

About the above variances, we pick up the data from in 1963 to in2001 as the object period.
So, we regress lei 1 lexp, on Work, GNP/POPU, Year and get the below estimated

[^3]p. 17
result.
$($ LEI $1 / E X P)(t)=0.424894-\left(0.94407 \times 10^{-4}\right)$ WORK $(t)+0.013485 \operatorname{YAR} 1(t)$
(-2.29)
$+(0.012479) G N P(t) / P O P U(t)$
(1.94)
\[

$$
\begin{equation*}
\bar{R}^{2}=0.856440 \quad D . W .=0.196723 \tag{65}
\end{equation*}
$$

\]

Estimated Coefficient on GNP /POPU shows significant plus and so, we recognize that leisure good is superior good for general Japanese household. While estimated coefficient on work shows significant minus and so, we recognize that reduction of working hours has an influence against the rise of demand of leisure good.

But we consider mainly the relationship between capacity of leisure facility and the congestion. So, we especially notice leisure which have the character of simultaneous realization of both consumption and production and the problem of the capacity of facility
So, we pick up, as Lei3, the leisure expenditure which is deeply related with use of the leisure facility by household ${ }^{5}$ And we divide such a leisure related expense with use of facility, Lei3 by all the expenditure and calculate Lei3/EXP.
Similarly to case of Lei1/EXP, we regress Lei3/EXP on Work, GNP/POPU, Year to estimate and as a result, we get the next estimated equation.

$$
\begin{align*}
& (\text { LEI } 3 / E X P)(t)=0.095842-\left(0.35078 \times 10^{-4}\right) \text { WORK }(t)-0.816845 \times 10^{-2} \text { YEAR } 1(t) \\
& \quad(4.50) \quad(-4.47) \\
& +(0.017553) G N P(t) / P O P U(t) \\
& \quad(10.81)  \tag{66}\\
& \bar{R}^{2}=0.986613 \quad \text { D.W. }=0.821405
\end{align*}
$$

We find the estimation result similar to the case on lei1.
In short, Estimated Coefficient on $G N P / P O P U$ shows significant plus and so, we also recognize that such leisure goods or service is also superior good. While, estimated

[^4]p. 18
coefficient on Work shows significant minus and so, we recognize that reduction of working hours has an influence against the rise of demand of such leisure goods or services.

Here, the $t$-value of coeffect on Work in this Lei 3 equation larger than that in the lei1 equation, so the reduction of work has an effect on the Lei 3 more than on Lei1.
So, we recognize that the effect of reduction of working hours against leisure demand with use of leisure facility is stronger.

And we also recognize that t -value on Year Dummy (indicate 1 the year from 1963 to 1979 and indicate 0 the year after 1980) is in the Lei3 equation is minus and the absolute value of $t$ value is larger than that on Lei1 equation

We note that two days off a week system was introduced Japanese firms from 1980s and households can get chance to choose time zone of holyday widely after 1980s. Each household go into leisure facility dispersedly and congestion does not tend to occur in economy.

Such this regression result reflects such a situation that household get circumstance that is suitable for the demand of leisure goods or services after 1980.
In previous theatrical section, we recognized that leisure demand with use of leisure facilities in more strongly related with the length or width of time zone of holyday and wee concern that the theoretical result is correct by the estimation.

But from D.W, (Durbin=Watson ratio) the two above estimation series may include correlations
So, in order to exclude correlations, we divide $G N P$ by work and introduce $G N P /$ work and we take series difference about lei3/exp. and get lei3/exp ( $\mathrm{t}-1$ ) which is the one year before about lei3/exp. And we regress lei3/exp. on $G N P /$ work, year and lei3/exp (t-1) and get the below estimated equation.

LEI $3 / \operatorname{EXP}(t)=0.807957 \times 10^{-2}+\left(0.348981 \times 10^{-3}\right)($ GNP $/($ WORK $)(t)$

$$
\begin{equation*}
+(0.890794)(\text { LEI } 3 / E X P)(t-1) \tag{1.62}
\end{equation*}
$$

$$
\begin{equation*}
\bar{R}^{2}=0.990024 \quad D . W .=2.18644 \tag{11.84}
\end{equation*}
$$

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The result means that Lei3 (leisure demand with use of leisure facility) increase as the increase of income $G N P$ and decrease of working hours.

So, we get, the estimation result including the previous estimation result with excluding correlation
Next, because we $G N P / W O R K$ almost mean wage, lei3 (demand of the leisure services and goods with use of leisure facility) increase as the rise of wage.

Becker(1965) demonstrate that household choose the good intensive combination more than the time intensive combination in the case that income holed by household is relatively larger than time holed by household.
So, from the thought of Becker(1965), we can consider Lei3 (leisure demand with use of leisure facility) as good intensive Lei3 (leisure demand with use of leisure facility)combination and we can grasp the situation that Japanese household get utility from good intensive combination(Lei3 (leisure demand with use of leisure facility) under situation that household cannot keep enough leisure time.

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(Figure 1) The relationship between the length of time zone of holidays and the magnitude of leisure facility (1) in case of $L_{y c}=\omega Y=H$

The length of time zone that is realized as holidays in society

$H$ : The time zone that is realized as holiday in society
$\bar{N}$ : the number of households existed in society
$\omega \mathrm{Y}$ : each time that each household need in order to enjoy leisure consumption
※we notice that, when only the narrow time zone of holiday is realized in society, many households visit leisure facility simultaneously.

So each leisure industry firm has to construct large leisure facility which could accommodate the many numbers of household members in order to respond the leisure demand of household.

For example, in society people take only one holyday a week in society, each firm has to construct the facility which accommodate all the $\bar{N}$ households in order to in order to offer leisure service to $\bar{N}$ households.
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(Figure 2) The relationship between time zone and the magnitude of leisure facility
(2) in case of $L_{y c}+L_{y c}=\omega Y+\omega Y \leq H$

$H$ : The time zone that is realized as holiday in society
$\bar{N}$ : the number of households existed in society
$\omega \mathrm{Y}$ : each time that each household in order to enjoy leisure consumption $Y$
※we notice that, when the sufficient or wide time zone of holiday is realized in society enough to make household enter into leisure facility separately each leisure firm sometimes have only to construct leisure facility which is smaller magnitude in order to respond the leisure demand of household.

For example, when household can take two holydays a week, each household choose which two day to visit leisure facility a week, then each leisure firm has only to construct leisure facility which accommodate $\bar{N} / 2$ households in order to offer leisure service to $\bar{N}$ households.

(Figure 3) the effect of reduction of legal working hour under the situation that household does not take leisure time enough to enjoy leisure services


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[^1]:    ${ }_{2}$ For example, see Rose, Mirowsky and Hurber (1983).
    p. 2

[^2]:    3 In this article we focus on the analysis of leisure we do not consider leisure services which is not with using leisure facility
    p. 8

[^3]:    ${ }^{4}$ As related expenditure related with leisure, we pick up public transportation (railway fares, bus fares, airplane fares), personal transportation (automobile, bicycle, automobile parts, ) recreational durable good (TV sets, Stereo photograph set, tape recorder, video tape recorder, computer and word professor, camera, video camera, piano, musical instrument, ) book and other reading material (newspaper, magazine, books and other readings)
    Sporting goods (ball ,golf,) TV game, film, film developed ,audio and video Disc, Cut flowers, Recreational services(,Hotel charges, package tours, ), in-room equipment ,social expenses, allowance, Admission fees (movie, concert, theater, sports watching ),cultural facility (art galley, museum)

[^4]:    5 As related expenditure related with leisure, we pick up public transportation (railway fares, bus fares, airplane fares), Recreational services(,Hotel charges, package tours, travel bag, stay , packed trips, ) athletic tool , admission fee to movie, concert, theater, watching spots (baseball, soccer, ),cultural facility (art gallery, museum)

