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A factor-augmented MIDAS approach

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Forecasting GDP growth using stock returns in Japan: A factor-augmented MIDAS approach*

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Abstract

Asset prices reflect expectations of future economic conditions. In this study, we use the property of asset prices, especially stock prices, to forecast the GDP growth rate in Japan. For optimal use of the rich time-series and cross-sectional information of stock prices, we combine MIDAS (mixed-data sampling) regression and factor analysis to examine which dimensions of information contribute to the accuracy of the GDP growth rate forecast. Our results show that the use of factors significantly improves forecast accuracy and that extracting factors from a broader set of stock prices further improves accuracy. This highlights the important role of cross-sectional stock market information in forecasting macroeconomic activity.

Keywords: Forecasting; MIDAS regression; factor model; stock returns.

JEL classification: C22; C53; E37.

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1 Introduction

Asset prices are a mirror reflecting expectations of future economic conditions as they are determined in a forward-looking manner based on currently available information. Forecasting literature has supported this forward-looking nature of asset prices by demonstrating that the forecast accuracy for macroeconomic fluctuations is improved using the information in asset markets (e.g., Stock and Watson, 2003; Forni et al., 2003; Andreou et al., 2013).[1] In asset markets, many asset prices are recorded with high frequency, and this rich data environment can be used for forecasting macroeconomic activities. This study explores how such rich information in the asset market contributes to the improvement of macroeconomic forecasts, focusing particularly on the relationship between GDP growth rate and Japanese stock prices.

More precisely, information in the stock market is spread across both time-series and cross-sectional dimensions. We handle these two-dimensional information by combining mixed data sampling (MIDAS) regression and factor model approach, following recent literature of Marcellino and Schumacher (2010) and Andreou et al. (2013). In forecasting literature, those two methods were originally developed separately. MIDAS regression, originally introduced by Ghysels et al. (2007), handles data sampled at different frequencies in a single equation using a function that optimally transforms higher-frequency data into lower-frequency data. Intuitively, giving optimal weights attached to each high-frequency observation by "letting the data speak for itself" will help improve forecast ability compared to using the time-aggregated data. Conversely, the factor model approach for forecasting macroeconomic activity is adopted in Stock and Watson (2002) and Boivin and Ng (2005) for the single-frequency case. These studies demonstrate that a multivariate model outperforms a univariate model in forecast accuracy. Considering this strand of literature, this study employs the MIDAS model augmented by factors (hereinafter referred to as the Factor MIDAS model) in forecasting GDP growth rate with stock prices.

This study addresses the following three main questions: First, do stock prices re-

[1] In another strand of literature, asset prices, particularly for stocks, are used as a proxy for capturing news shock in the time series macroeconomic analysis. For example, Beaudry and Portier (2006) regard innovations in stock prices as an anticipated TFP shock, while Fisher and Peters (2010) use stock returns of military contractors to proxy for capturing anticipated government spending shock.

ally contain the relevant information for future GDP growth in Japan? Second, which dimension of information, time-series or cross-sectional, contributes to a more accurate forecast of GDP growth rate? Third, will the ongoing COVID-19 pandemic change the role of stock prices as a leading indicator of economic activities? Hence, eleven alternative forecast models, distinguished by their information sets, are compared with the naïve forecast model in predictive ability. Five of 11 models are MIDAS specifications that regress quarterly GDP growth rate on monthly stock returns. Three of these five models are MIDAS models augmented by factors extracted from the set of individual stock returns. Furthermore, the three Factor MIDAS models are classified according to the set of stocks used for extracting factors (components of Nikkei225, components of TOPIX, and all listed firms). In contrast, six of the 11 models are single-frequency models using quarterly aggregated stock returns or quarterly factors as regressors. As in the MIDAS model, three of six models exploit the cross-sectional information augmented by the factors, and factors are estimated from three sets of stocks. One of the quarterly models is a simple AR model without stock returns that only contains lagged GDP growth rate to assess the role of stock returns. Finally, the naïve forecast of the benchmark is obtained from the in-sample mean of GDP growth rate. The forecast ability of each model is evaluated by the relative root mean square error (RMSE) to the benchmark model.

We provide the following answers to the above questions. First, we find that stock returns are informative in predicting the future GDP growth rate before the COVID-19 pandemic. All the models with stock returns outperform those without stock returns and the naïve forecast model. Second, results highlight the importance of cross-sectional information in the stock market for the forecast of GDP growth rate in Japan. Specifically, the forecast accuracy of the models with factors (both MIDAS and quarterly specifications) always exceeds that of the corresponding models without factors. In contrast, the enrichment of time-series information does not always contribute to more accurate forecasts. Only one MIDAS model shows the improvement of the forecast compared with its quarterly counterpart. Moreover, the best predicting performance is obtained in the quarterly model with factors extracted from a set of stock returns of all listed firms. Forecast accuracy then tends to improve as the set of stock prices increases, that is, in the order of Nikkei225, TOPIX, and all listed firms.[2] These findings emphasize the contribution of cross-sectional information to the forecast of GDP growth rate. Third, as mentioned, the predictive ability of stock returns for GDP growth rate seems to be lost after the COVID-19 pandemic. Including the sample period after the first quarter of 2020, all models failed to provide forecasts that are significantly better than the naïve forecast. The possible reasons for this lack of predictive power of stock prices may be attributed to massive stock market intervention by the Bank of Japan (BOJ)'s ETF (exchange traded funds) purchases and unprecedented macroeconomic fluctuations. Since the outbreak of the COVID-19 pandemic, particularly in 2020, the BOJ had conducted massive money injection to the stock market through the large-scale purchases of ETF to cope with a sharp drop in stock prices. Moreover, such interventions from the BOJ may have distorted the price formation mechanism of stock prices. In contrast, GDP growth rates themselves have also fluctuated dramatically after the COVID-19 outbreak, making forecasting difficult. The GDP growth rate has been taking positive and negative values in turn after the first quarter of 2020, reflecting the infection situation and intensity of regulation. These two specific situations under the COVID-19 pandemic may cause a loss of predictive power in stock returns to GDP growth rate, which had been reliably observed before the pandemic.

Methodologically, this study adopts the Almon lag polynomial among several functional forms of MIDAS specifications (Ghysels et al., 2007; Ghysels, 2016). This is because the MIDAS model with Almon lag polynomial can be estimated by OLS. Additionally, the factors are estimated by the principal components approach based on the Stock and Watson (2002)'s EM algorithm. This allows us to estimate the factors from the dataset with missing observations. Forecast ability is almost independent on the MIDAS specification and factor estimation methods (e.g., Forni et al., 2003; Boivin and Ng, 2005; Marcellino and Schumacher, 2010).

Several forecasting studies have emphasized the importance of exploiting the asset prices, mixed-frequency data, and a large amount of information summarized in factors. All of which are considered in this study. Stock prices have the information to improve

[2] As detailed in Section 3, components of Nikkei225, TOPIX, and all listed firms in this study are 225, 2180, and 5294, respectively.

the forecast ability on GDP growth rate in Aylward and Glen (2000), Ferrara and Marsilli (2013) and Andreou et al. (2013), while the benefits of the MIDAS model compared with single-frequency forecast model has also been reported in Ghysels et al. (2007), Clements and Galvão (2008), Clements and Galvão (2009), and Foroni et al. (2015). Moreover, the pioneering works of Stock and Watson (2002) and Boivin and Ng (2005) highlighted that the multivariate forecasts using the factors outperform the univariate ones and subsequent works of Marcellino and Schumacher (2010) and Andreou et al. (2013) show that the combination of MIDAS and factor analysis provides a more accurate forecast. We follow this strand of literature in the presented study and conduct comparison analysis to clarify important information in the forecast of GDP growth rate. We believe no studies have explained the role of cross-sectional information in the entire stock market in forecasting GDP growth. Stock prices are an informative large dataset that is relatively easy to access.[3] Hence, our findings, highlighting the role of heterogeneous information in the stock market, have important practical implications for the forecasting analysis of real economic activities.[4]

The remainder of the paper is organized as follows. Section 2 explains the structure of MIDAS models employed in this study. Section 3 details the dataset used for the analysis, and Section 4 discusses the design of forecasting. Section 5 presents the forecasting results. Finally, Section 6 concludes the paper.

[3]The other possible financial variables are the prices of private and government bonds. However, most bonds are traded over-the-counter and not on the market. Hence only reference statistical prices are available. Furthermore, unlike stock prices, the same company or public institution issues multiple bonds. Their prices constantly change based on the period to maturity, so it is difficult to use raw data of bond prices for the analysis.

[4]The forecasting analysis for the Japanese economy has relatively less literature compared with the analyses for the U.S. and the euro area, only conducted by Shintani (2005), Urasawa (2014), Bragoli (2017), and Chikamatsu et al. (2021).

2 MIDAS regression model

2.1 Basic MIDAS model

The MIDAS model with augmented distributed lag (ADL-MIDAS) for forecasting h-steps ahead of quarterly variable y^q_{t+h} using monthly data can be formulated as

$$y^q_{t+h} = \mu^h + \sum_{j=1}^{p_y} \rho_j y^q_{t-j-1} + \beta^h \sum_{j=1}^{p_x} \sum_{i=0}^{k-1} \omega^{\Theta^h}_{i+(j-1)*k} x^m_{t-j-1,k-i} + u^h_{t+h}, \tag{1}$$

where $x^m_{t,k-i}$ denotes the monthly variable at the i-th month counting backward from the end of quarter t and k is fixed at 3 in the monthly/quarterly regression because of three observations in one quarter, and the quarterly lags for y^q_t and x^m_t are represented by p_y and p_x, respectively. By directly incorporating monthly data into the MIDAS model without time-aggregation, the information in the time-series dimension can be used more effectively for the forecast than in a single-frequency model. Particularly, weights of each monthly observation $\omega^{\Theta^h}_{i+(j-1)*k}$ are specified as a function of a low dimension vector of parameters $\Theta^h = (\theta^h_1, \theta^h_2, \cdots, \theta^h_P)$. By assuming that $P \ll p_x \times k$, the MIDAS model can avoid a parameter proliferation problem, even if it contains many lagged monthly variables. Here, we adopt the Almon lag polynomial specification for MIDAS weight as in Pettenuzzo et al. (2016) because of its simplicity.[5]

Almon lag weight can be provided as

$$\beta^h \omega^{\Theta^h}_{i+(j-1)*k} = \sum_{p=1}^{P} \theta^h_p (i + (j-1) * k)^{p-1}. \tag{2}$$

Then, inserting equation (2) into equation (1) leads to the following formulation for the

[5]There are other types of MIDAS weights: exponential Almon polynomial, beta polynomial, step function, and unrestricted (e.g., Appendix of Ghysels, 2016). Except for unrestricted MIDAS (UMIDAS), a nonlinear estimation technique is required to estimate the model. Besides this practical difficulty, no substantial difference in forecasting ability can be found among the MIDAS specifications (e.g., Marcellino and Schumacher, 2010; Foroni et al., 2015). Hence, we adopt the Almon lag polynomial by which the estimation can be conducted by OLS.

parameters $\Theta^h = (\theta_1^h, \theta_1^h, \cdots, \theta_P^h)$:

$$y_{t+h}^q = \mu^h + \sum_{j=1}^{p_y} \rho_j y_{t-j-1} + \sum_{j=1}^{p_x} \sum_{i=0}^{k-1} \sum_{p=1}^{P} \theta_p^h (i + (j-1) * k)^{p-1} x_{t-j-1,k-i}^m + u_{t+h}^h. \tag{3}$$

We define $(P \times N)$ matrix Q and the $(N \times 1)$ vector of the lagged monthly data X_t^m, respectively, as follows:

$$Q = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 2 & \cdots & N \\ 0 & 1 & 2^2 & \cdots & N^2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & 2^{P-1} & \cdots & N^{P-1} \end{bmatrix} \text{ and } X_t^m = \begin{bmatrix} x_{t,k}^m \\ x_{t,k-1}^m \\ \vdots \\ x_{t,1}^m \\ x_{t-1,k}^m \\ \vdots \\ x_{t-(p_x-1),1}^m \end{bmatrix}, \tag{4}$$

where $N(\equiv p_x \times k)$ is the total number of weights. Using equation (4), the model (3) can be simply rewritten as follows:

$$y_{t+h}^q = \mu^h + \sum_{j=1}^{p_y} \rho_j y_{t-j-1}^q + \Theta^h \tilde{X}_t^m + u_{t+h}^h, \tag{5}$$

where $\tilde{X}_t^m = Q X_t^m$ is a $(P \times 1)$ vector of transformed monthly regressors. As emphasized in Ghysels (2016) and Pettenuzzo et al. (2016), parameters in equation (5) can be estimated by OLS because the Almon lag polynomial transforms the MIDAS specification into a linear regression model.

We apply this MIDAS model to forecast quarterly GDP growth using monthly stock returns in Japan. Specifically, we estimate equation (5) using the market returns of the Nikkei225 average and TOPIX (Tokyo stock price index), two of Japan's leading stock indices, as monthly stock returns. Additionally, we estimate the model including only the quarterly average of these indices, as specified by the following:

$$y_{t+h}^q = \mu^h + \sum_{j=1}^{p_y} \rho_j y_{t-j-1}^q + \sum_{j=1}^{p_x} \psi_j x_{t-j-1}^q + u_{t+h}^h, \tag{6}$$

where x_t^q is a quarterly stock returns aggregated in advance, thus not making full use of the time series information in equation (6). The comparison of the forecast accuracy between equations (5) and (6) highlights the importance of the high-frequency data in the forecasting of GDP growth rate.

2.2 Factor-augmented MIDAS model

As the market returns comprise the (weighted) average of stock prices of specific firms, information used in the basic MIDAS model is likely to be only a small part of the original information in the stock market. This is because thousands of individual stock price movements can be found behind market indices. Thus, this study further augments the MIDAS model with factors extracted from a set of individual stock returns (i.e., the Factor MIDAS model), aiming to fully exploit the large number of individual movements of stock prices. By combing the MIDAS model with factors, we use the information not only in the time-series dimension but also the cross-sectional dimension and explore whether additional cross-sectional information can contribute to improving the forecast accuracy.

Denoting the r-th monthly factor at the i-th month counting backward from the end of quarter t by $f^m_{r,t-j,k-i}$, the Factor MIDAS model can be formulated as follows:

$$y^q_{t+h} = \mu^h + \sum_{j=1}^{p_y} \rho_j y^q_{t-j-1} + \sum_{r=1}^{R} \beta^h_r \sum_{j=1}^{p_x} \sum_{i=0}^{k-1} \omega^{\Theta^h_r}_{i+(j-1)*k} f^m_{r,t-j-1,k-i} + u^h_{t+h}. \tag{7}$$

where the number of the factors is denoted by R, and the parameters β^h_r and Θ^h_r are assumed to be different across the factors, allowing the different weights on the factors measured at monthly basis. Parallel to the basic model, equation (7) can be transformed using the Almon lag weight as follows:

$$y^q_{t+h} = \mu^h + \sum_{j=1}^{p_y} \rho_j y^q_{t-j-1} + \sum_{r=1}^{R} \Theta^h_r \tilde{F}^m_{r,t} + u^h_{t+h}, \tag{8}$$

where $\Theta^h_r = (\theta^h_{r,1}, \theta^h_{r,1}, \cdots, \theta^h_{r,P})$ and $\tilde{F}^m_{r,t} = [f^m_{r,t,k}, f^m_{r,t,k-1}, \cdots, f^m_{r,t,1}, f^m_{r,t-1,k}, \cdots, f^m_{r,t-(p_x-1),1}]'$. Importantly, equation (8) can be also estimated by OLS as in the basic MIDAS model, if the factors are given.

This study uses a two-step procedure in estimating the Factor MIDAS model: First, factors are extracted from a set of individual stock returns by principal component. These are then used to estimate the Factor MIDAS model.[6] While the second step of the estimation is conducted via simple OLS as noted, the first step of extracting factors is somewhat more complicated as the dataset of individual stock returns contains the missing values owing to lack of trade or entry and exit from the stock market. To handle the missing value problem in factor estimation, this study employs EM algorithm proposed by Stock and Watson (2002), where factors are estimated by iterating the replacement of missing values by fitted values and estimation of factors and loadings using principal component eigenvalue calculation. Appendix A presents the detailed estimation procedure of the factors.

For comparison purposes, this study further considers the model with the quarterly factors extracted from a set of quarterly aggregated individual stock returns, such as the following:

$$y^q_{t+h} = \mu^h + \sum_{j=1}^{p_y} \rho_j y^q_{t-j-1} + \sum_{r=1}^{R} \sum_{j=1}^{p_x} \psi_{r,j} f^q_{r,t-j-1} + u^h_{t+h}, \tag{9}$$

where $f^q_{r,t}$ is the r-th factor extracted from quarterly aggregated individual stock returns. This specification exploits cross-sectional information, but not the time-series information sufficiently in terms of using quarterly stock returns. Therefore, the comparison among the forecasts derived from equation (5), (6), (8), and (9) reveals the extent to which time-series and cross-sectional information in the stock market contribute to the forecast GDP growth rate.

3 Data

As noted, this study regresses the quarterly GDP growth rate in Japan on either monthly market stock returns or factors extracted from a set of individual stock returns and their lagged variables and a constant. The quarterly sample period is from 1994Q2 to 2021Q3.

[6]The two-step procedure of factor forecasting is commonly used in the literature, such as Stock and Watson (2002) and Boivin and Ng (2005). Moreover, Marcellino and Schumacher (2010) report that the forecast results are independent of the factor estimation technique.

Moreover, its monthly counterpart is from 1994M04 to 2021M09. Quarterly GDP data is derived from the *System of National Accounts*, published by the Cabinet Office, Government of Japan. GDP data in accordance with 2008 SNA is released only from 1994Q1; hence, the sample period starts from 1994Q2 owing to taking its log differentials. The time series of GDP used in this paper is the published data of *Quarterly Estimates of GDP (The 2nd preliminary)*, which is available in January 2022. All stock returns are collected from the *Nikkei NEEDS-FinancialQuest database*, provided by Nikkei Media Marketing Incorporated. Market stock returns used in the basic MIDAS model are the log differentials of monthly closing prices in Nikkei225 and TOPIX. Nikkei225, a representative index of Japanese stocks, is an average stock price index of 225 stocks selected from the 1st section of the Tokyo Stock Exchange (TSE), and TOPIX, another representative index, is an average stock price index of all stocks listed in the 1st section of TSE.

Individual stock returns for the components of Nikkei225 and TOPIX are used in estimating the Factor MIDAS model. In addition to the components of two representative indices, individual stock returns for the firms listed or formerly listed on any stock markets are also used to extract the factors. As of January 28, 2022, when data were downloaded, the number of individual stock returns in each set was 225 for the Nikkei225, 2180 for the TOPIX, and 5294 for all listed firms, respectively. However, information from all the firms cited above is not always used in estimating the factors owing to the missing values. For instance, the Nikkei225 components in this study are those listed in the index at the time of data collection (January 2022). Since some of the components are changed once yearly, stocks that had not been included in the index in the past have missing values. Similarly, firms listed in TOPIX and all listed firms are likely to have missing values in their stock returns as some firms may exit from the stock market owing to bankruptcy, or others may be newly listed in the middle of the sample period. As mentioned above and detailed in Appendix A, the Stock and Watson (2002)'s EM algorithm allows us to estimate the factors from the dataset with missing values; however, the firms which have too many missing values in their stock returns should be removed from the dataset to ensure factor estimation stability.[7] Specifically, the series of data that contains more than

[7]In addition to "missing" observations, we also treat outliers as missing observations. In the same

a quarter of missing values for the estimation period is not used in estimating the factor.

Finally, quarterly stock returns in the comparison model are constructed as the log differentials of the 3-month average of closing prices for each month. The quarterly individual stock returns are also constructed in the same manner. We then extract quarterly factors from them. Hence, models using quarterly stock returns correspond to a special case of the MIDAS model wherein the same weights are assigned to each monthly observation.

4 Forecast design

The performance of each model is assessed by a recursive out-of-sample forecasting, where the estimation and forecast are repeated with updated estimation and evaluation periods. Specifically, the initial estimation period covers 1994Q2 to 2011Q4, and the evaluation period is the period after 2012Q1. After initial estimation and forecasting, 2012Q1 data are added to the estimation, and the evaluation period is shifted to one-quarter ahead. This process is iterated until the evaluation period reaches the end of the sample period. In the actual forecasting exercises, two cases are considered for the end of the sample period: 2019Q4 and 2021Q3 (i.e., before and after COVID-19, respectively). This is because, during the pandemic, stock prices have consistently risen owing to monetary intervention with exception of a large initial drop. Hence, whether such a huge policy intervention to the stock market changes the role of stock returns as a leading indicator for real activities is of interest.

The gap between the forecasts and the actual GDP growth rate is measured by the root-mean-square error (RMSE). For the h-period ahead forecasts, we compute the RMSE by the following:

$$RMSE(h) = \sqrt{\frac{1}{(T_2 - h) - T_1 + 1} \sum_{t=T_1}^{T_2-h} (y_{t+h} - \hat{y}_{t+h})^2}, \tag{10}$$

where y_{t+h} is actual GDP growth rate and $\hat{y}_{t+h}$ is h-period ahead out-of-sample forecasts

manner, as in Stock and Watson (2002), observations with deviations from the median exceeding 10 times the quartile range are replaced by missing observations as outliers.

obtained from the model. Moreover, T_1 and T_2 correspond to the end of the initial estimation period and the end of the final evaluation period, respectively. By changing the specification of the models and the available data, we compute the 12 forecasts from the following:

(1) MIADS model using Nikkei225

(2) MIDAS model using TOPIX

(3) MIDAS model with factors from the components of Nikkei225

(4) MIDAS model with factors from the components of TOPIX

(5) MIDAS model with factors from all listed firms

(6) The model without stock returns

(7) The model using quarterly Nikkei225

(8) The model with quarterly TOPIX

(9) The model with quarterly factors from the components of Nikkei225

(10) the model with factors from the components of TOPIX

(11) The model with factors from all listed firms

(12) In-sample mean

In what follows, the naïve forecasts obtained from (12) in-sample mean is regarded as a benchmark, and the relative RMSE for each model is computed to evaluate forecast accuracy. This model is considered to perform well compared with naïve forecasts when the relative RMSE takes a value less than 1. Additionally, the significance of the difference in the relative RMSE is tested by Diebold and Mariano (1995)'s test as modified by Harvey et al. (1997).

5 Forecasting results

Table 1 reports the relative RMSEs of each model to in-sample mean forecast with respect to one-period ahead GDP growth for two sample periods. The numbers of the lag lengths (p_y, p_x), factors (R), and the parameters to specify the weights (P) are chosen so the RMSE for each model is minimized for all possible combinations of them with the maximum value of those being four. Hence, our results are the best-performing ones for each specification.

Table 1: Relative RMSE ratio (one-period ahead forecasts: $h = 1$)

	(a) Excl. COVID-19 period		(b) Incl. COVID-19 period	
	relative RMSE	(P, R, p_y, p_x)	relative RMSE	(P, R, p_y, p_x)
Basic MIDAS model				
(1) Nikkei225	0.915*	(3,–,3,2)	0.978	(1,–,1,1)
(2) TOPIX	0.919*	(3,–,3,2)	0.988	(1,–,1,1)
Factor MIDAS model				
(3) Nikkei225	0.901**	(3,1,3,2)	0.970	(1,1,1,1)
(4) TOPIX	0.891**	(3,2,3,2)	0.951	(1,4,1,1)
(5) All listed firms	0.901*	(3,2,3,2)	0.941	(1,2,1,1)
Quarterly model				
(6) Without stock returns	0.954**	(–,–,3,–)	1.060	(–,–,1,–)
(7) Nikkei225	0.920*	(–,–,3,1)	0.992	(–,–,1,–)
(8) TOPIX	0.917*	(–,–,3,1)	0.998	(–,–,1,–)
Quarterly factor model				
(9) Nikkei225	0.895**	(–,1,3,1)	0.990	(–,1,1,3)
(10) TOPIX	0.886**	(–,3,3,2)	1.000	(–,2,1,1)
(11) All listed firms	0.878*	(–,3,3,4)	0.997	(–,2,1,1)

Notes: The table shows the relative RMSEs to in-sample mean forecast for two sample periods: (a) excluding COVID-19 pandemic and (b) including COVID-19 pandemic. The null hypothesis that the prediction error in each model is greater than the error in in-sample mean forecast is tested by Diebold and Mariano (1995) modified by Harvey et al. (1997). The 5% and 10% significance levels are denoted by $**$ and $*$, respectively.

This table reveals the following five results. First, we find that all forecast models considered here work well before the COVID-19 pandemic in the sense of reducing forecast errors significantly compared with naïve forecasts but not at all after the pandemic. The relative RMSEs in the sample excluding the COVID-19 period take the values less than one with statistically significant differences. Conversely, significant improvements in the forecasts cannot be observed at all in the estimation using the sample including the COVID-19 period. This implies that, after the COVID-19 pandemic, the role of stock prices as forecasters for economic activities has changed. This finding contrasts with the period of the global financial crisis reported by Ferrara and Marsilli (2013), who document that stock prices contributed to the forecast accuracy of GDP growth rate in the euro area during the 2008–2009 global financial crisis; however, this is considered reasonable given that the origin of the current economic turmoil is not the financial market. Moreover, this finding is consistent with a disconnection between real economy and stock prices which have been observed during the pandemic. While the COVID-19 pandemic caused serious damage to the real economy, the Nikkei225 reached a high of over 30,000 yen for the first time in 30 years in February 2021 owing to the massive monetary easing. Drastic fluctuations in GDP growth rate during the pandemic period in response to infection status may also have been another factor that made forecasting difficult.[8] In the following discussion, we focus mainly on results obtained in the pre-COVID-19 period and examine whether the information in the time series or cross-sectional direction is more effective in improving forecasting performance.

Second, the models including stock returns in some form perform better than the model without stock returns. This result suggests that stock prices have relevant information about future GDP growth rate, which is consistent with Aylward and Glen (2000), Andreou et al. (2013) and Ferrara and Marsilli (2013).

Third, using high-frequency data does not necessarily contribute to improving forecasting ability, at least in the presented analysis. No significant difference in forecast accuracy between the MIDAS model and the corresponding quarterly model can be found; rather, the quarterly models, except for the case of using Nikkei225 without factors, show better performance in point estimates. This suggests that the advantage of having more time-series information may not outweigh the disadvantage of making the model more complex. Moreover, the number of the factors in the quarterly models (both for TOPIX and all listed firms) is three. In contrast, that in the MIDAS model is two. For confirmation of the role of the third factor, we check the forecasting ability of the quarterly

[8] The third possible reason for stock prices losing predictive power to GDP after the pandemic is the heterogeneous impact of COVID-19 on each firm or each industry. While the aviation industry has been severely hit by travel restrictions, the IT industry, which provided systems for working from home, has seen an improved business performance. Such heterogeneous movement of stock prices across industries attributed to the sector-specific COVID-19 shock may have caused the gap between stock prices and real economic activities.

model with two factors and find that the MIDAS model outperforms the quarterly model when the factors extracted from firms listed on TOPIX; however, when information on all listed firms is exploited, the quarterly model still provides better forecasts than MIDAS model.[9] Consider that the additional cross-sectional information contained in the third factor is critical in the sense of compensating for the lack of time-series information in the comparison between MIDAS and quarterly models, then Factor MIDAS model of using all listed firms provide better performance than the quarterly model having only two factors. However, this is not the case. Hence, the noise in the high-frequency stock returns for each firm, which may be irrelevant to macroeconomic activity, may affect undesirable effects on the forecasting when the factors are extracted considering too much information for both time-series and cross-sectional dimensions.

Fourth, the use of factors improves the forecasting performance compared with that of the market index. This suggests that relevant information about future GDP growth rates may have been lost by cross-sectional aggregation. Relative RMSEs in the Factor MIDAS model are all smaller than these counterparts in the basic MIDAS model. This is the same as in the case of quarterly models. Except for the case of Nikkei225, more than the two-factor model is chosen as the best performance model for each specification. This highlights the fact that incorporating information about the heterogeneity of stock prices into forecasts is beneficial. The forecasting abilities of the Factor MIDAS model using TOPIX, and all listed companies have improved from the one- to the two-factor models, and the same thing can be observed for the forecast accuracy of the quarterly factor model from the one- to the three-factor models. Therefore, the heterogeneous information aggregated into factors may have contributed to the improvement of the forecast accuracy.[10] More interestingly, even in the one-factor model of Nikkei225, the

[9]To be concrete, the relative RMSE of quarter factor model using TOPIX components in the case of $(P, R, p_y, p_x) = (-, 2, 3, 2)$ is equal to 0.916 while that of quarterly factor model using all listed firms in the case of $(P, R, p_y, p_x) = (-, 2, 3, 4)$ is 0.888.

[10]Estimated factor loadings in the respective models are shown in Figures B.1 and B.2 of Appendix B. Factor loadings are obtained in the final estimation of out-of-sample forecasting and normalized so that the total sum of the absolute value of factor loadings is equal to 100. Factor loadings of the first factor in all specifications show the same sign (and roughly the same values). This suggests that the first factor represents the average movement of stock prices. Conversely, while the second and third-factor loadings show different signs and magnitudes for each firm, they often take on the same sign continuously when seen from the x-axis direction. As the firms plotted in the x-axis are aligned by industry owing to the data source, this sequence of factor loadings with the same sign indicates that stock price fluctuations in specific industries are aggregated in each factor.

model with factor improves the forecast accuracy over the model with the market indicator aggregated in advance. Figure 1 presents the quarterly stock returns of Nikkei225 and the first factor derived from the quarterly stock returns of the firms listed in Nikkei225, in the final exercise of out-of-sample forecasting.[11] Overall, both time series show the same movement; however, a gap is sometimes observed between the two series.

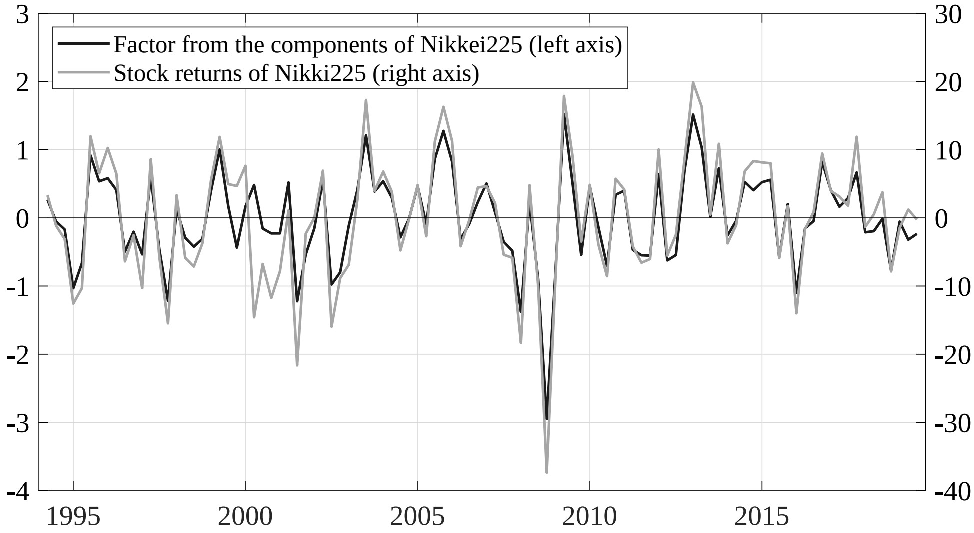

Figure 1: Nikkei225 and the first factor obtained from its components

Notes: This figure plots both the first factor obtained from the components of Nikkei225 and stock returns of Nikkei225 (market aggregated index). The series of the factor, shown here, is obtained in the final estimation period for 1994Q2 to 2019Q4.

Additionally, Figure 2 depicts the estimated factor loadings in the quarterly factor model and the weights assigned to each stock comprising the Nikkei225. Factor loadings are normalized so that the total sum of the absolute value of factor loadings equals 100 for comparison purposes with the weights for Nikkei225. The total number of the firms used for extracting the factor is 178 and not 225 owing to the missing observations as mentioned above. The compositional weights of the 225 stocks in Nikkei225 are quite skewed as is well-known. From Figure 2, we confirm that the estimated weights in the factor model are substantially less biased compared with the ones in the Nikkei225 index, suggesting the importance of utilizing the information as unbiased as possible considering

[11] Although the estimated factor and its loadings are different depending on the estimation period, we demonstrated the result obtained in the final estimation of the out-of-sample forecasting. This is because final estimation makes use of the most information.

forecast accuracy.

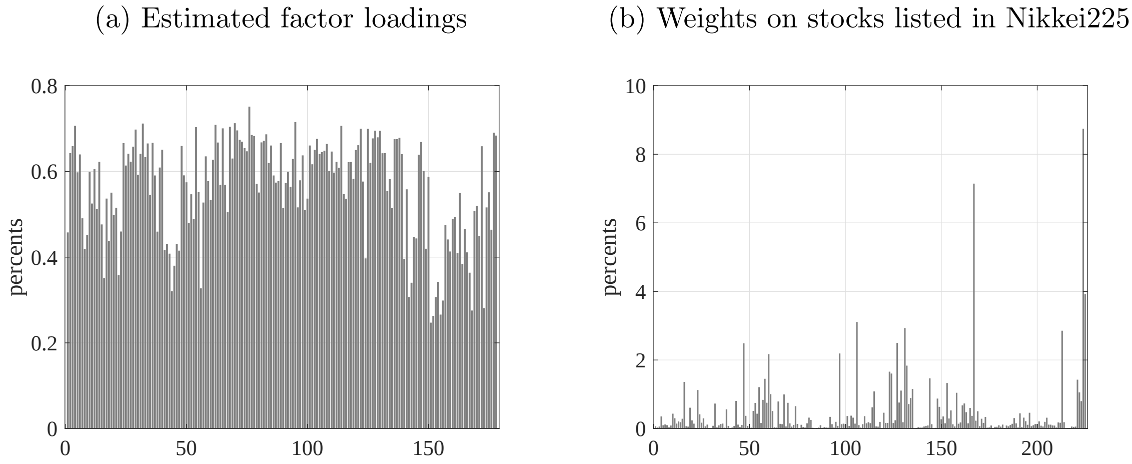

Figure 2: The Factor loadings and weights assigned to each stock of Nikkei225

Notes: The figures show estimated factor loadings for the first factor obtained from quarterly stock returns of the Nikkei225 (Figure 2a) and the weights assigned to stocks listed in Nikkei225(Figure 2b). Weights of the Nikkei225 are the values adopted at the end of January 2022. From Figure 2b, we find that a large weighting is assigned to several firms (e.g., DAIKIN (3.1%), Softbank group (3.8%), Tokyo Electron (7.1%), and Fast Retailing (8.7%)). Compared with the weights of Nikkei225, no substantial difference in the estimated factor loadings can be found.

Fifth and most importantly, richer cross-sectional information tends to improve forecast accuracy. Specifically, models with the factors from the components of TOPIX and all listed firms provide better forecast performance than those using Nikkei225 components. Moreover, the quarterly model with the factors from all listed firms performs best overall. Hence, much more rich variations of information for the cross-sectional direction are key in forecasting the GDP growth rate.

To summarize all the results, forecasting analyses support the use of cross-sectional information in the stock market to forecast GDP growth rate. Specifically, augmentation of forecast models with factors provides better forecasts on future GDP growth rates. Moreover, forecast accuracy improves with a larger set of stock prices from which the factors are extracted. Conversely, this study fails to detect the significant contribution of the enrichment of time-series dimension in forecasting quarterly GDP growth rate. Additionally, comparison analysis between two sample periods implies that stock prices may have lost their role as a leading indicator of real economic activity during the COVID-

19 pandemic.

6 Conclusion

This study has explored whether stock prices contain relevant information on future GDP growth rates. The Factor-MIDAS model is employed to exploit the time-series and cross-sectional information in the stock market and clarify the contribution of stock market information to the macroeconomic forecast. Although the analysis has conducted in a parsimonious way in the sense of using the information only in the stock market, this study provides new insights on the role of stock prices as a leading indicator of real economic activities. Our findings emphasize the role of cross-sectional heterogeneity in contributing to a more accurate GDP growth rate forecast. Additionally, stock prices contain relevant information about future macroeconomic activity in the first place. We find that forecasting ability improves using factors extracted from a larger set of individual stock prices. In contrast, enrichment of time-series information does not necessarily contribute to the improvement of forecast accuracy at least in the current monthly and quarterly specifications. Furthermore, stock prices as a leading indicator of economic activity may have been lost during the COVID-19 pandemic, owing to the drastic fluctuation in GDP growth rate and massive monetary easing. Overall, these findings support that asset prices are a mirror of expectations about the future economic condition and that these expectations are accurate to some extent. The data-rich environment of the stock market can hence be highly useful in predicting future economic conditions, possibly helping policymakers manage their policies in advance of actual economic fluctuations. Simultaneously, however, excessive intervention in the financial market can lead to the loss of the role of stock prices as a forecaster of the real economy.

As an extension of this study, considering the case of a combination of daily stock returns and quarterly GDP growth rates in the forecasting would be possible. Moreover, our method can be also applied to the relationship between stock prices and GDP components. These tasks are then left to future work.

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A Appendix: Factor estimation method

Factors are estimated from the dataset with missing observations using the EM algorithm proposed by Stock and Watson (2002). Let vector $Z_t = (z_{1t}, z_{2t}, \cdots, z_{Mt})'$ be defined as containing the normalized stock returns of firm i in period t as the i-th element. The standard factor representation can then be specified as follows:

$$Z_t = \Lambda F_t + \xi_t, \tag{A.1}$$

where $\Lambda = (\lambda_1, \cdots, \lambda_M)'$ is the $(M \times R)$ factor loadings, $F_t = (f_{1,t}, \cdots, f_{R,t})'$ is the $(R \times 1)$ vector of factors, and ξ_t is the $(M \times 1)$ vector of idiosyncratic disturbance. As emphasized, elements of Z_t are missing. First, the missing observations are replaced with random variables drawn from $N(0,1)$ and the new matrix is defined as $\hat{Z}_t^{(0)}$. Since $\hat{Z}_t^{(0)}$ is the balanced dataset without missing observations, the factors $\hat{F}_t^{(0)}$ and loadings $\hat{\Lambda}^{(0)}$ can be estimated using the usual principal component eigenvalue calculation. By setting $j = 0$, we update the elements of $\hat{Z}_t^{(j+1)}$ as $\hat{Z}_{it}^{(j+1)} = \hat{\lambda}'^{(j)}_i \hat{F}_t^{(j)}$ if Z_{it} is missing and $\hat{Z}_{it}^{(j+1)} = Z_{it}$ otherwise. $F_t^{(j+1)}$ and $\hat{\Lambda}^{(j+1)}$ are the reestimated by using the updated dataset $\hat{Z}_t^{(j+1)}$. This iteration of the algorithm is repeated until the estimated factors are converged sufficiently.

B The estimated factor loadings

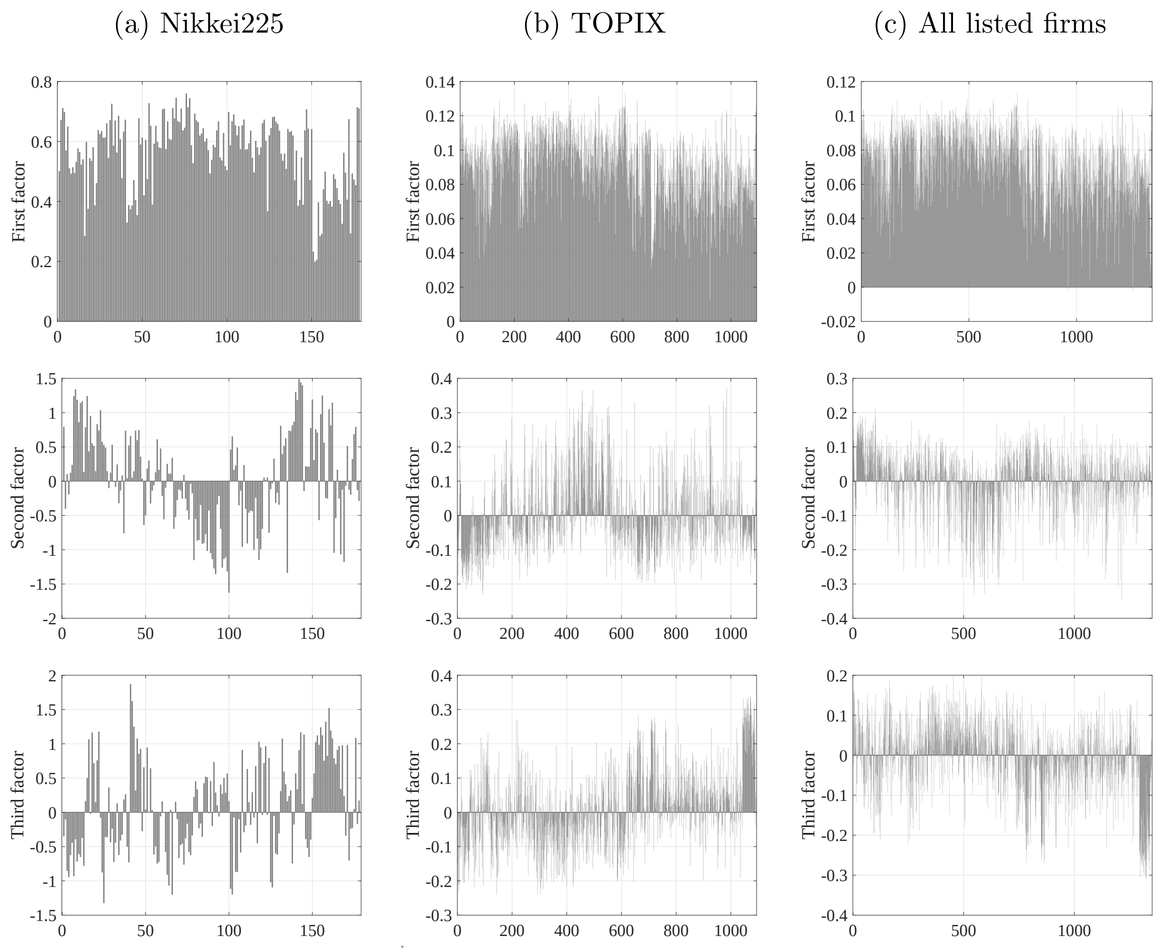

Figure B.1: The factor loadings in each Factor MIDAS model

Notes: The figure presents factor loadings for each factor in the respective factor MIDAS model. The x-axis indicates each firm listed in each group of stocks; the y-axis indicates the estimated factor loadings normalized so that the sum of absolute values for factor loadings is equal to 100, thus regarding the values taken on the y-axis as percentage.

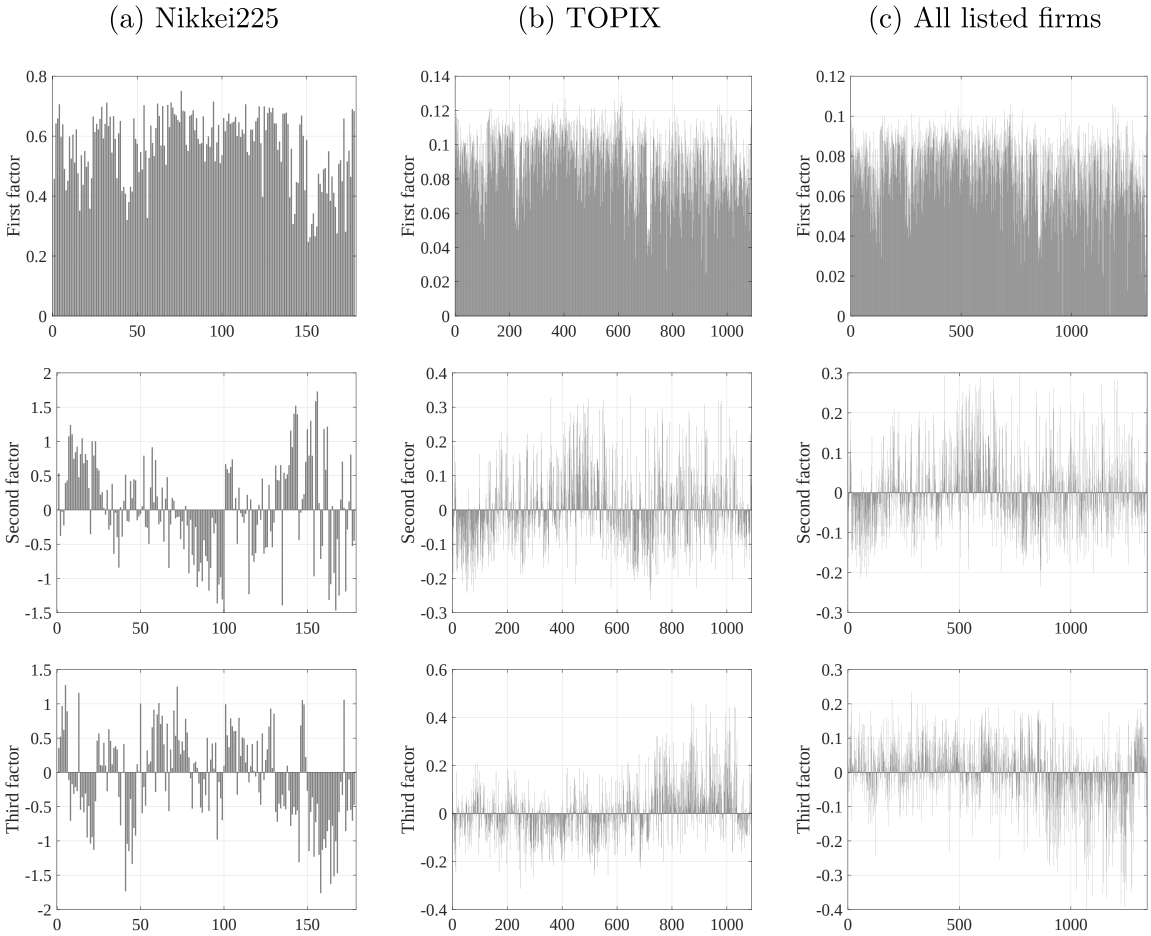

Figure B.2: The factor loadings in each Quarterly factor model

Notes: This figure shows the factor loadings for each factor in the respective quarterly factor model. The x-axis indicates each firm listed in each group of stocks; the y-axis indicates the estimated factor loadings normalized so that the sum of absolute values for factor loadings is equal to 100, thus regarding the values taken on the y-axis as percentage.