Essays on Fiscal-Monetary Interactions and Unconventional Monetary Policies

by

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Submitted in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in
Economics

Graduate School of Economics
Hitotsubashi University
2022

Acknowledgments

First of all, I would like to express my sincere gratitude to my adviser Etsuro Shioji for his continuous support and encouragement throughout my undergraduate, graduate, and Ph.D. studies. Without his continuous support, I would have never completed this dissertation.

I am also indebted to my committee members Tokuo Iwaisako, Takashi Kano, Masaya Sakuragawa, and Motohiro Sato. They gave me invaluable advice that has significantly improved my work.

I am deeply grateful to Masashi Hino, So Kubota, Kazushige Matsuda, Takeshi Ojima, Makoto Saito, Yuta Takahashi, Hiroshi Morita, and Naoki Takayama for helpful discussions and suggestions.

Finally, I would like to thank my family and friends for their support and encouragement.

Hidekazu Niwa

March, 2022

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Introduction

1.1 Motivation

Since the onset of the global financial crisis in 2008, the fiscal and monetary authorities in developed countries have experienced dramatic changes. There are two observations on recent developments in fiscal and monetary policies that motivate studies in this dissertation. First, the debt-to-GDP ratio has been growing in most developed countries. For example, in the United States, the public debt-to-GDP ratio has sharply increased to a historically high level. Furthermore, according to the Congress Budget Office's report, this ratio is expected to keep growing (see Figure 1.1). As of 2022, the U.S. government has not announced a clear plan for a fiscal adjustment that would stabilize the debt-to-GDP ratio. While inflation rates in the most developed countries have been relatively low and steady, if these fiscal situations remain unchanged, one of consequences is that public confidence in the fiscal authority's willingness to stabilize the debt-to-GDP ratio is lost, and the central bank is forced to adjust its policy to ensure the sustainability of public debt, which could have significant effects on inflation and economic activities.[1]

Second, in recent years major central banks have faced low levels of economic activity and low inflation rates. To stimulate the economy, they have set their policy interest rates to

[1] Bernake (2010) and Shirakawa (2012), for example, stress the importance of maintaining public confidence in fiscal sustainability to stabilize financial markets and the macroeconomy.

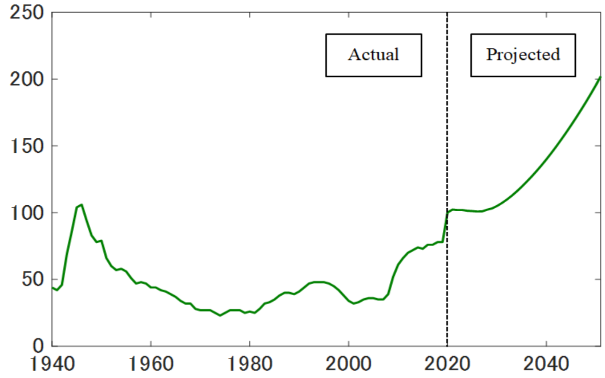

Figure 1.1. The U.S. federal debt-to-GDP ratio. Source: Congress Budget Office (2020)

zero. After the short-term nominal interest rate reached its zero lower bound (ZLB), some unconventional monetary policy measures have been introduced in an attempt to influence the current state of the economy. The Federal Reserve and the Bank of Japan have relied on forward guidance by announcing that they intend to keep short-term nominal interest rate low in the future. In addition to this, central banks have engaged in massive purchases of long-term government bonds (e.g., the Federal Reserve's Large Scale Asset Purchase Programs and the Bank of Japan's Quantitative and Qualitative Monetary Easing (QQE)). These operations have expanded the size of central banks' balance sheets and lengthened the duration of assets held by the Bank of Japan (see Figure 1.2). This has raised concern that central banks would incur losses on their balance sheets anytime the policy rate increases. Some have pointed out that these losses could require a recapitalization from the fiscal authority, especially in countries like Japan, where the central bank holds a lot of risky assets on its balance sheet.[2] If the fiscal authority is unable or unwilling to provide sufficient financial support for the central bank due to political reasons, possible actions by the central bank would be constrained by its financial condition.

[2] For projections for losses that the Bank of Japan could incur on its balance sheet in a process of normalizing the ongoing programs, see Iwata et al. (2014, 2016, 2018) and Fujiki and Tomura (2015, 2017)

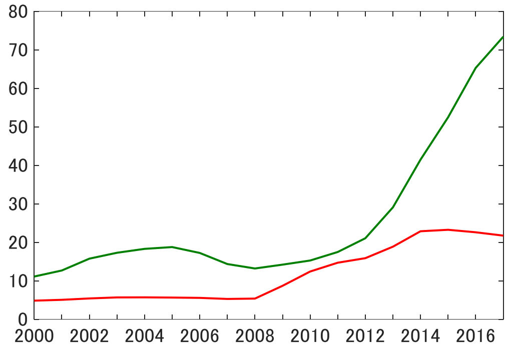

Figure 1.2. The central bank's total assets to GDP. Green line: Japan. Red line: the U.S. Sources: the Bank of Japan, the Federal Reserve Bank of St. Louis (Federal Reserve Economic Data).

In standard models for monetary policy analysis, however, the above two issues do not arise because it is implicitly assumed that (1) the fiscal authority commits to covering possible losses on the central bank's balance sheet, and (2) it also commits to adjusting the present discounted value of primary surpluses to ensure solvency of the consolidated government.[3]

1.2 Objective of the Dissertation

In this dissertation we study the effects that unconventional monetary policy measures, especially purchases of long-term bonds, have on the economy departing from the two standard assumptions. The issue of unconventional monetary policy measures has already been addressed by the seminal work of Eggertsson and Woodford (2003). They study monetary policy at the ZLB using a New Keynesian model in which the ZLB becomes binding temporarily due to an exogenous negative shock to the natural interest rate. Implications of their analysis for policy are twofold. First, purchases of long-term bonds by a central bank per se should have no effect on the economy. Second, an effective way a central

[3] To be precise, the assumption of the consolidated government is valid if in addition the central bank is assumed to commit to transferring seigniorage revenues to the fiscal authority.

bank can take to mitigate deflation and a recession caused by the existence of the ZLB is to commit to keeping the short-term nominal interest rate zero for a while even after the economy recovers in the future.[4]

Indeed, at the ZLB central banks have made efforts to communicate policy intentions to the public. It is also worth noting that massive purchases of long-term bonds, which itself should be neutral, were introduced in the hope that they would lead to changes in expectations about the future conduct of monetary policy. A well-known channel through which purchases of long-term bonds may influence the economy at the ZLB is via a signal about the future path of nominal interest rates, reducing long-term interest rates (e.g., Woodford, 2012; Bernanke, 2020).

Eggertsson and Woodford (2003) demonstrate these results under the two standard assumptions. However, as discussed in subsection 1.1, it is worth reconsidering how purchases of long-term bonds affect the economy in a setting where interactions between the fiscal and monetary authorities have equilibrium implications. In each chapter of this dissertation, we investigate other channels through which purchases of long-term bonds stimulate the economy at the ZLB and/or possibilities that the operation would be destabilizing. Some recent studies, explained in detail in the following subsection, have already analyzed the effect of purchases of long-term bonds from the perspective of fiscal-monetary interactions (Bhattarai et al., 2015; Berriel and Mendes, 2015; Benigno and Nisticò, 2020). However, their focus is mainly on the mechanism through which the operations increase output and inflation at the ZLB, providing a rationale for the aforementioned view that purchases of long-term bonds generate a signal. The contribution of this dissertation is to show that without appropriate coordination between the fiscal and monetary authorities purchases of long-term bonds would backfire.

[4] See also Jung et al. (2005), Adam and Billi (2006), Nakov (2008), and Werning (2011) for other relevant studies

1.3 Related Literature

This subsection reviews the literature closely related to our study. The first is the literature on the Fiscal Theory of the Price level (FTPL). The foundation of this literature is developed by Leeper (1991), Sims (1994), Woodford (1995, 2001), and Cochrane (2001). They stress the importance of a role that expectations about the future conduct of fiscal policy play in determining the equilibrium price level. As explained in Section 1.1, in standard models for monetary policy analysis, the fiscal authority adjusts primary surpluses to maintain the sustainability of public debt. Therefore, the central bank achieves independent control of inflation by setting the short-term nominal interest rate appropriately.[5] However, if the fiscal authority does not make such adjustments, the equilibrium price level must be determined so as to maintain the sustainability of public debt. In the terminology of Leeper (1991), this is the case of an active fiscal policy, in which the real value of public debt is endogenously determined given public confidence in the government. Note that in standard models the fiscal policy is passive in the terminology of Leeper (1991).

Second, our study is related to the recent theoretical literature that sheds light on a relationship between central bank solvency and financial arrangements between the fiscal authority and the central bank (Reis, 2013, 2015; Bassetto and Messer, 2013; Hall and Reis, 2015; Del Negro and Sims, 2015; Cavallo et al., 2019; Benigno and Nisticò 2020; Benigno, 2020). Suppose that the fiscal authority can credibly commit to covering possible losses on the central bank's balance sheet, as it does in standard models. In this case the central bank can never become insolvent independently from the overall government. However, in reality there is no guarantee that the fiscal authority always recapitalizes the central bank if necessary. The literature emphasizes a separation of the budget constraints of the two

[5] This is the case, for example, in the standard New Keynesian model in which monetary policy is typically assumed to follow a Taylor rule.

authorities. One of important features of their analysis is to focus on the intertemporal budget constraint that the central bank faces to address the issue of central bank solvency.

Finally, most closely related to our work are Bhattarai et al. (2015), Berriel and Mendes (2015), and Benigno and Nisticò (2020). The optimal policy at ZLB proposed by Eggertsson and Woodford (2003), among others, involves a time-inconsistent commitment to the future conduct of monetary policy. This is due to the fact that as soon as the economy recovers, the benevolent central bank finds it optimal to renege on its promise to keep the nominal interest rate zero. Berriel and Mendes (2015) and Bhattarai et al. (2015) show that purchases of long-term bonds resolve this time-inconsistency problem at least to some extent. The central bank that holds long-term bonds has an incentive to lower the nominal interest rate even after the economy sufficiently recovers in order to avoid large balance sheet losses. Benigno and Nisticò (2020) uncover conditions under which purchases of long-term bonds are not neutral, focusing on a rule according to which the central bank makes transfers to the fiscal authority and a rule according to which the fiscal authority adjusts lump-sum taxes.

The motivation to pay particular attention to the financial stability of central banks is that the fiscal authority may be unable and unwilling to cover losses on the central bank's balance sheet due to political reasons. In this case, the central bank must conduct policy ensuring its solvency. Sims (2000), Ueda (2003), and more recently Cavallo et al. (2019) point out that a central bank that is exposed to the risk of losses on its balance sheet may be subject to such political constraints. Moreover, Bunea et al. (2016) investigate the policies implemented by the central banks worldwide in terms of accounting rules and rules according to which they make transfers to the fiscal authorities based on a questionnaire that the European Central Bank sends to central banks. Their analysis and discussion imply that, for central banks, receiving recapitalization from the fiscal authority entails political and/or institutional difficulties especially in developed countries.

By law, when incurring losses on its balance sheet, the Federal Reserve is permitted to make zero remittances and record deferred assets as negative liabilities on its balance sheet that can be paid back by future profits. Even with such legislation is in place, possible losses on the Federal Reserve's balance sheet were politically controversial in 2013. After the onset of the global financial crisis in 2008, the Federal Reserve implemented multiple rounds of large-scale asset purchase programs. In U.S. House Committee on Financial Services in February 2013, questions about possible losses that the Federal Reserve could incur after liftoff from the ZLB were put to the then Fed chairman Bernanke.[6] Moreover, in the Federal Open Market Committee's meeting of March 2013, some participants expressed concern about possible losses on the Federal Reserve's balance sheet and a resulting decline in remittances to the Treasury. [7]

The separation between the balance sheets of the fiscal authority and the central bank is also important when studying monetary policy issues in Japan. For example, Okina (1998,172) states that "A clause in the old Bank of Japan Law whereby the government was obliged to compensate for any losses incurred by the BOJ was deleted in compiling the current Bank of Japan Law. Under the current Bank of Japan Law, any profits are transferred to the government coffers, while any losses incurred are borne by the BOJ." He then raises the question about adopting the assumption of the consolidated government in a situation where the Bank of Japan is exposed to the risk of losses on its balance sheet. In addition, Ueda (2003) states that "Since the enforcement of the new Bank of Japan's Law in 1998, the degree of the Bank's financial integration with the government has been reduced" and that

[6] "Monetary Policy and the State of The Economy, Hearing before the Committee on Financial Services, U.S. House of Representatives, One Hundred Thirteenth Congress, First Session, February 27, 2013" CHRG-113hhrg80869.pdf (govinfo.gov)
[7] "Minutes of the Federal Open Market Committee", March 19-20, 2013
The Fed - Monetary Policy: (federalreserve.gov)

"The Bank needs to implement its monetary policy carefully so that its financial condition does not impede the fulfillment of price stability on either front." [8]

Their discussion is particularly relevant when considering the Bank of Japan's strategy to exit from the QQE. As the scale of the Bank of Japan's balance sheet has expanded, possible losses on it and measures for dealing with them have attracted attention. For example, at the Monetary Policy Meeting held in April 2013, a policy board member commented that "it might be worthwhile to examine the feasibility of an arrangement in which the Bank's losses would be covered by the government." [9] Moreover, studies that simulate possible losses on the Bank of Japan's balance sheet suggest that the Japanese government and the Bank of Japan must make an explicit agreement on how to share possible losses between them before the Bank of Japan exits from the QQE (Fujiki and Tomur, 2015, 2017; Iwata et al., 2014, 2018). [10]

1.4 Structure of the Dissertation

This dissertation is structured as follows. In chapter 2, we briefly review the FTPL. In chapter 3, we examine how purchases of long-term bonds influence the economy at the ZLB by using the framework of the FTPL. One of key assumptions in this chapter is that the fiscal authority does not make fiscal adjustments needed to stabilize public debt. We demonstrate that, with an appropriate institutional arrangement between both authorities in place, the operation has an expansionary effect even without the signaling channel. Another key assumption to show this is that the fiscal authority commits to covering possible future significant losses on the

[8] Ueda (2003), "The Role of Capital for Central Banks"
https://www.boj.or.jp/en/announcements/press/koen_2003/ko0402b.htm/
[9] Bank of Japan (2013), "Minutes of the Monetary Policy Meeting on April 3 and 4, 2013"
https://www.boj.or.jp/en/mopo/mpmsche_minu/minu_2013/g130404.pdf
[10] We discuss the other issues often pointed out in the debate on the financial stability of the Bank of Japan after exiting the QQE in Chapter 4. An example is that the results of our analysis would critically depend on whether the assets of the Bank of Japan are evaluated by their book value or their market value.

central bank's balance sheet. In this setup, the consolidated government's budget must incur large losses at a time of liftoff of the nominal interest rate, which results in an increase in inflation via the mechanism highlighted by the FTPL. This prospect leads the private sector to expect higher future inflation even when the current nominal interest rate is stuck at the ZLB, stimulating inflation and output today.

This result has an important implication for monetary policy at the ZLB. When the economy falls into the ZLB again in a next recession, the central banks can stimulate the economy without struggling to communicate their future policy intentions to the public. The management of expectations at the ZLB has presented a serious challenge for central banks. In light of the current fiscal situations in developed countries, purchases of long-term bonds can be an alternative tool for central banks to influence the macroeconomy during a next deep recession.

Chapter 4 develops a theory to consider solvency of the whole government sector and that of the central bank in a unified way. Specifically, we analyze a model with two key assumptions: (1) fiscal policy is active, and (2) the central bank has the responsibility to maintain its financial stability. In this model, possible actions by the central bank at a time of liftoff from the ZLB can be subject to not only a solvency condition of the whole government but also its own solvency condition. The model is used to ask: How does a lack of public confidence in fiscal sustainability constrain the Bank of Japan's strategy to exit from the QQE?

In a baseline analysis, we demonstrate the following two results. First, under the case of a passive fiscal policy, the central bank can maintain its solvency solely by allowing the price level to increase to an arbitrarily high level, regardless of the amount of losses it incurs due to a decline in the price of long-term bonds. Therefore, in this case, possible actions by the central bank are not constrained by its financial situation. Second, under the case of an active fiscal policy, the central bank that holds long-term bonds above a certain threshold cannot freely raise the future path of nominal interest rates at a time of liftoff from the ZLB.

Next, we explore richer implications of our study for policy, assuming an active fiscal policy. To do this, we extend the baseline model by specifying a rule according to which the central bank controls the short-term nominal interest rate after liftoff from the ZLB. First, we study the case of a passive monetary policy in the sense of Leeper (1991), as fiscal policy is assumed to be active. We then show that under certain conditions, the central bank can achieve its inflation target after liftoff at least in the long run. In this case, however, inflation right after liftoff must undershoot the central bank's target inflation. This means that large-scale purchases of long-term bonds at the ZLB would not lead to an increase in inflation expectations at the ZLB.

Second, we study the case of an active monetary policy; we assume that the Taylor principle is satisfied. The main result is that if the central bank follows the Taylor principle, under certain conditions, it fails to prevent the economy from converging to the deflationary steady state after liftoff. By using recent Japanese data, we also provide numerical examples to show that the Bank of Japan cannot stabilize inflation at a positive level after liftoff. The result implies that a central bank that engages massive purchases of long-term bonds would not achieve its inflation target after liftoff by actively controlling the short-term nominal interest rate. It is also worth noting that purchases of long-term bonds would not lead to an increase in inflation expectations at the ZLB regardless of whether the Taylor principle is satisfied.

Finally, we incorporate a possibility that the fiscal authority partially defaults on outstanding government bonds at a time of liftoff from the ZLB. We introduce a mechanism through which the equilibrium default rate is determined along the lines of Uribe (2006) and consider a one-off default at a time of liftoff. This study is motivated by Kocherlakota (2011).

He shows that even under the case of an active fiscal policy the central bank achieves independent control of inflation if it is willing to allow the fiscal authority to default on its bonds. We show that the central bank that is exposed to the risk of losses on its balance sheet cannot achieve its inflation target by following the Taylor principle even if it is willing to allow the fiscal authority to default on its bonds. This result implies that even if government bonds are defaultable, large-scale purchases of long-term bonds by the central bank would undermine the ability of monetary policy to stabilize inflation.

The main message of this chapter is that a lack of public confidence in fiscal sustainability, along with a commitment of the fiscal authority not to provide financial support for the central bank, undermines the ability of the central bank that purchases long-term bonds at the ZLB to stabilize inflation after liftoff. Moreover, if at the ZLB the public understands such a consequence of operations, it would have negative effects on the current state of the economy. Therefore, when considering the effects of purchases of long-term bonds, we should pay attention to the relationship between the fiscal authority and the central bank.

Chapter 5 develops a dynamic general equilibrium model augmented with a particular type of political economic aspect of fiscal policymaking. In the model used in the literature on fiscal-monetary interactions, fiscal policy is decided by a single policymaker in a centralized manner. This assumption, however, is not necessarily realistic. In reality, several interest groups are involved in a process of fiscal policymaking. Therefore it would be a strong assumption that all of them can coordinate to achieve certain conduct of fiscal policy. In the politico-economic literature, a common pool problem that arises when fiscal policymaking is influenced by fragmented interest groups is regarded as one of main causes of socially excessive public spending or public debt stock.

Thus motivated, we construct a New Keynesian model in which two fragmented interest groups influence tax policy. Interactions among the interest groups and the central bank through the consolidated government budget constraint have implications for equilibrium dynamics. This government structure induces a common pool problem once the interest groups expect the central bank to accommodate their free-riding behaviors. The main objective of this chapter is to study how coordination failure between interest groups distorts the optimal conduct of fiscal and monetary policy. Particular attention is paid to the roles the central bank is forced to play in a stabilization process.

Using the above model we first study how the economy in which the debt stock is initially above a steady-state level is stabilized over time. The results of numerical analysis are summarized as follows. First, it is socially optimal to increase both the labor income tax and inflation in order to decrease government debt. A benevolent planner would choose the timing of the tax collection and inflation to maximize social welfare. Because of the presence of long-term bonds, variations in future interest rates can be used to optimally choose the timing of tax collection and inflation.

Second, in a non-cooperative game between the interest groups, a resulting response of the tax rate becomes negative. Against the inflationary pressure from accumulated debt, interest groups find it optimal to lower the tax rate. They aim to attenuate upward pressure on group-specific inflation by decreasing marginal costs. The interest groups do not fully internalize that their actions affect economy-wide inflation through their budgetary effects. This free-riding activity puts socially excessive downward pressure on marginal costs and slows down debt stabilization excessively. The former in turn puts downward pressure on current inflation, whereas the later delays the timing of inflation. Overall, the free-riding activity of the interest groups causes a positive response of inflation excessively gradual. Then, the central bank is forced to make a response of the nominal interest rate excessively gradual. This is required to smooth the real interest rate and thus output gap given a gradual response of inflation.

We next introduce the possibility that the economy falls into the ZLB. As in the baseline case an initial debt stock is above a steady-state level. The difference now is that the economy temporarily falls into the ZLB due to a large negative shock to the natural interest rate. While the nominal interest rate is temporarily stuck at zero, the economy eventually converges to a steady state. We study how a negative demand shock, along with the presence of the ZLB, changes results in the baseline. The main results are twofold. First, if the interest groups coordinate, the operation has an expansionary effect on the economy at the ZLB. Second, coordination failure between the interest groups weakens this expansionary effect of purchases of long-term bonds.

The results in this dissertation suggest that whether or not purchases of long-term bonds by a central bank have favorable effects on the economy, at least in the short run or the long run, largely depends upon how the fiscal and monetary authorities coordinate. Especially, without appropriate coordination, the operation would be destabilizing. The literature on fiscal-monetary interactions initiated by the seminal work of Sargent and Wallace (1981) and developed by the studies mentioned above on the FTPL has emphasized that fiscal and monetary policies are linked through the government budget constraint. Their contributions have taught us that if the assumption that the fiscal authority takes responsibility for stabilizing public debt does not hold, the central bank's actions that are thought to be effective in stabilizing inflation through the lens of standard models would backfire. The results in this dissertation suggest that their findings are still important in addressing the issues of unconventional monetary policy measures.

Chapter 2

Review of the Fiscal Theory of the Price Level

2.1 Introduction

This chapter reviews the fiscal theory of the price level (FTPL). Our goal here is to briefly explain the following three points. First, how is an equilibrium determined in a simple model in which prices are perfectly flexible and all government bonds are one-period? Second, what is the intuition for equilibrium determination? Third, how does introducing long-term bonds or nominal price rigidities change results in the simplest case? We provide a textbook-style explanation by using a toy model. The discussion in this chapter is mainly based on Woodford (1996, 2001), Cochrane (2001, 2005, 2022), Iwamuma and Watanabe (2004), and Shioji (2018).

2.2 Two-period Model

We work with a deterministic two-period model with sticky prices. The two periods are $t = 0$, 1. The economy is populated by a representative household, a continuum of firms in the unit interval, the fiscal authority, and the central bank. In period 0, firms face adjustment costs in changing their prices so that they are rigid. In period 1, they can change their prices at no costs so that prices are perfectly flexible. The fiscal authority issues government bonds, which are held by the household. At the beginning of period 0, the household holds one-period and long-term government bonds. As explained in Section 1.3, the important assumption in the FTPL is that the fiscal authority does not adjust primary surpluses to maintain government solvency. The central bank controls the short-term nominal interest rate. Finally, it should be noted that the model described below encompasses the simplest case in which prices are perfectly flexible and all government bonds are one-period as a special case.

2.2.1 Households

The representative household has the following utility function

$$\log(C_0) - N_0 + \beta[\log(C_1) - N_1], \tag{2.1}$$

where C_t is an aggregate of consumption, N_t is labor supplied, and $\beta \in (0,1)$ is the discount factor. The aggregate consumption C_t is defined as

$$C_t \equiv \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{2.2}$$

where $c_t(j)$ denotes the quantity of good $j \in [0,1]$ consumed by the household. $\theta > 1$ parameterizes the elasticity of substitution across goods. The aggregate price index is

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}, \tag{2.3}$$

where $p_t(j)$ denotes the price of good j.

At the beginning of period 0, the household holds one-period nominal bonds B_{-1} and long-term bonds D_{-1}. Long-term government bonds outstanding at the beginning of period 0 pay one dollar in period 0 and ρ dollars in period 1. In a special case with $\rho = 0$, all government bonds outstanding in period 0 are one-period. The price of a one-period government bond is denoted by $1/(1 + i_t)$, where i_t is the short-term nominal interest rate, and that of long-term bonds is denoted by Q_t. The household earns labor income $N_t W_t$, where W_t is the nominal wage, receives profits $Z_t(j)$ from firm j, and pays lump-sum taxes T_t. Since all government bonds newly issued in period 0 are one-period, the budget constraint is given by

$$P_0 C_0 + \frac{B_0}{1+i_0} \leq B_{-1} + (1+Q_0)D_{-1} + W_0 N_0 + \int_0^1 Z_0(j)dj - P_0 T_0, \tag{2.4}$$

The budget constraint in period 1 can be written as

$$P_1 C_1 + \frac{B_1}{1+i_1} \leq B_0 + (\rho \ + Q_1)D_{-1} + W_1 N_1 + \int_0^1 Z_1(j)dj - P_1 T_1. \tag{2.5}$$

The household is also subject to a constraint that prevents it from dying with debt at the end of period 1:

$$\frac{B_1}{1+i_1} \geq 0. \tag{2.6}$$

2.2.2 Firms

There is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$. Firm j produces goods using production technology

$$y_t(j) = n_t(j)\,. \tag{2.7}$$

where $n_t(j)$ is the labor hired. Firm j faces a downward-sloping demand curve given by

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\theta} Y_t. \tag{2.8}$$

where $Y_t \equiv \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}$ denotes aggregate output. In period 0, firms face adjustment costs in changing their prices. Following Rotemberg (1982), firm j faces price adjustment costs:

$$\frac{\gamma}{2}\left(\frac{p_0(j)}{p_{-1}(j)}-1\right)^2 Y_0. \tag{2.9}$$

γ is the parameter that controls the degree of nominal price rigidities. When $\gamma = 0$, firms can change their prices at no costs so that prices are perfectly flexible in both periods.

The profits of firm j in period 0 is then expressed as

$$Z_0(j) = \left[p_0(j)y_0(j) - W_0 y_0(j) - P_0\frac{\gamma}{2}\left(\frac{p_0(j)}{p_{-1}(j)}-1\right)^2 Y_0\right]. \tag{2.10}$$

As in period 1, there are no price adjustment costs, profits in this period can be rewritten as

$$Z_1(j) = [p_1(j)y_1(j) - W_1 y_1(j)\,]. \tag{2.11}$$

2.2.3 Government

The fiscal authority imposes lump-sum taxes on the household and issues bonds, one-period bonds B_t^F and long-term bonds D_t^F, respectively. The flow government budget constraints in each period are given by

$$(1+Q_0)D_{-1}^F + B_{-1}^F = \frac{B_0^F}{1+i_0} + P_0 T_0, \tag{2.12}$$

$$(\rho+Q_1)D_{-1}^F + B_0^F = \frac{B_1^F}{1+i_1} + P_1 T_1. \tag{2.13}$$

They can be rewritten in real terms:

$$\frac{1}{P_0}[(1+Q_0)D_{-1}^F + B_{-1}^F] = \frac{b_0^F}{1+i_0} + T_0, \tag{2.14}$$

$$\frac{1}{P_1}[(\rho+Q_1)D_{-1}^F + B_0^F] = \frac{b_1^F}{1+i_1} + T_1, \tag{2.15}$$

where $b_t^F \equiv B_t^F/P_t$ is the real value of one-period government bonds.

The central bank controls the short-term nominal interest rate i_t.

2.2.4 Equilibrium conditions

This subsection describes the equations that characterize the equilibrium allocation. The household maximizes its lifetime utility (2.1) subject to the budget constraints (2.4) and (2.5) and the no-Ponzi game condition (2.6). The Euler equation in period 0 is given by

$$\frac{1}{1+i_0} = \beta \frac{C_0}{C_1} \Pi_1^{-1}, \tag{2.16}$$

where $\Pi_t \equiv P_t / P_{t-1}$ is the gross inflation. The optimality condition for labor supply is

$$\frac{W_t}{P_t} = C_t \text{ for } t = 0,1 \tag{2.17}$$

Given the no-arbitrage condition between one-period and long-term bonds, the price of long-term bonds must satisfy

$$Q_0 = \frac{1}{1+i_0}(\rho + Q_1), \tag{2.18}$$

$$Q_1 = 0 \tag{2.19}$$

Finally, since household has no incentive to save in period 1, household optimization requires that the following terminal condition holds:

$$\frac{B_1}{1+i_1} = 0 \tag{2.20}$$

Next, firms' profits maximization is considered. Firm j sets its price in each period $\{p_0(j), p_1(j)\}$ to maximize profits (2.10) and (2.11) subject to the downward-sloping demand curve (2.8). Deriving the first-order conditions in each period and focusing on a symmetric equilibrium where $p_t(j) = P_t$, the following conditions are obtained:

$$(1-\theta) + \theta C_0 = \gamma \Pi_0 (\Pi_0 - 1), \tag{2.21}$$

$$C_1 = 1 - \theta^{-1}. \tag{2.22}$$

Equation (2.21) shows that when $\gamma = 0$, C_0 is also fixed at $1 - \theta^{-1}$. The Phillips curve in period 1 is vertical as prices are perfectly flexible.

Clearing in the goods market requires that

$$\left[1 - \frac{\gamma}{2}(\Pi_0 - 1)^2\right] Y_0 = C_0 \tag{2.23}$$

$$Y_1 = C_1. \tag{2.24}$$

Equilibrium in the bonds market requires that

$$B_t^F = B_t \qquad \text{for} \qquad 0, 1 \tag{2.25}$$

$$D_{-1}^F = D_{-1}. \tag{2.26}$$

To complete the characterization of the equilibrium we need to specify rules according to which fiscal and monetary policies are chosen. We assume that the fiscal authority pre-commits to a certain sequence of primary surpluses as follows:

$$T_t = \bar{T}_t, \qquad \text{for} \qquad 0, 1 \tag{2.27}$$

This means that the fiscal authority does not make fiscal adjustments needed to maintain government solvency, unlike in standard models. This assumption plays a critical role in the FTPL as explained in Chapter 1. The central bank also exogenously sets the short-term nominal interest rate in period 0:

$$i_0 = \bar{\imath}_0 \,. \tag{2.28}$$

Since the fiscal authority does not adjust primary surpluses, government solvency condition is relevant; endogenous variables must be determined so as to ensure government solvency. Combining flow government budget constraints in each period, (2.14) and (2.15), and imposing the terminal condition (2.20), the government solvency condition in period 0 is obtained:

$$\frac{1}{P_0}[(1+Q_0)D_{-1}+B_{-1}] = \bar{T}_0 + \left(\frac{\Pi_1}{1+i_0}\right)\bar{T}_1$$

$$= \bar{T}_0 + \beta\left(\frac{C_0}{C_1}\right)\bar{T}_1 \tag{2.29}$$

When deriving equation (2.29), we have used the bonds market clearing condition (2.25) and (2.26) and the Euler equation (2.16) that describes the relationship between the real interest rate and the path of consumption. Imposing the terminal condition (2.20) on government budget constraint (2.15) gives the government solvency condition in period 1:

$$\frac{1}{P_1}[(\rho+Q_1)D_{-1}+B_0] = \bar{T}_1\,. \tag{2.30}$$

The equilibrium in this model is a collection of processes $\{Y_t, C_t, \Pi_t, Q_t\}_{t=0}^{1}$ that satisfy (2.18), (2.19), (2.21)-(2.24), (2.29), and (2.30), given the predetermined variables $\{B_{-1}, D_{-1}, P_{-1}\}$ and policy variables $\{\bar{T}_0, \bar{T}_1, \bar{\iota}_0\}$. We need to choose a value for P_{-1} to uniquely determine the equilibrium path of the inflation rate.

2.3 The Simplest Case in Which $\gamma = \rho = 0$

Using the above model, we study how the equilibrium is determined. First of all, the simplest case in which $\gamma = \rho = 0$ is considered; we assume that prices are perfectly flexible and all government bonds are one-period. This is a simplified version of models considered by Leeper (1991), Sims (1994), and Woodford (2001). They study flexible-price models of endowment economy.[1]

2.3.1 Equilibrium determination

[1] To be precise, they consider infinite-horizon models in which money is demanded.

In a case with $\gamma = 0$, the goods market clearing condition in period 0 (2.23) can be rewritten as

$$Y_0 = C_0 \tag{2.31}$$

Since $\gamma = 0$ implies that $C_0 = 1 - \theta^{-1}$, the government solvency condition in period 0 (2.30) can be expressed as

$$\frac{1}{P_0}(D_{-1} + B_{-1}) = \bar{T}_0 + \beta\bar{T}_1. \tag{2.32}$$

This is the key condition uniquely determining the equilibrium price level. Since in the case of flexible-prices, the real interest rate is fixed at $\beta^{-1} - 1$, the presented discounted value (PDV) of primary surpluses $\bar{T}_0 + \beta\bar{T}_1$ is unchanged. The nominal value of outstanding government bonds $B_{-1} + D_{-1}$ is predetermined in period 0. We can thus uniquely determine the current price level P_0 to satisfy the government solvency condition (2.32). In other words, P_0 is determined to equate the real value of outstanding government bonds and the PDV of primary surpluses. The important point here is that as the fiscal authority does not adjust primary surpluses, the endogenous variables (the current price level in this simplest case) should adjust so as to maintain government solvency in the equilibrium.

2.3.2 Intuition I: Government bonds as net wealth

What is the intuition for the equilibrium determination? To examine this, we focus on the optimizing decision of the household. Combining the flow budget constraints in each period (2.4) and (2.5) gives the household's intertemporal budget constraint

$$\begin{aligned} &C_0 + \beta\left(\frac{C_0}{C_1}\right)C_1 \leq B_{-1} + D_{-1} \\ &+W_0N_0 + \int_0^1 Z_0(j)dj + \beta\left(\frac{C_0}{C_1}\right)\left(W_1N_1 + \int_0^1 Z_1(j)dj\right) - \left[\bar{T}_0 + \beta\left(\frac{C_0}{C_1}\right)\bar{T}_1\right]. \end{aligned} \tag{2.33}$$

Moreover, imposing the optimality conditions of the household (2.17) and (2.20), the optimality conditions of firms (2.21) and (2.22), and goods market clearing conditions (2.24) and (2.31), we obtain the following condition:

$$\frac{1}{P_0}(D_{-1}+B_{-1})+(1+\beta)(\theta-1)-(\bar{T}_0+\beta\bar{T}_1)=C_0+\beta C_1 \tag{2.34}$$

Note that equation (2.34) is one of the equilibrium conditions as it contains information about the optimality conditions of the private sector.

The condition (2.34) is informative about how an increase in outstanding government bonds $D_{-1}+B_{-1}$ affects the optimizing decision of the household. Recall that since the fiscal authority is assumed not to adjust primary surpluses regardless of the amount of bonds outstanding, the PDV of primary surpluses $\bar{T}_0+\beta\bar{T}_1$ is unchanged. An increase in outstanding bonds then leads to an increase in the PDV of lifetime income of the household, which is given by the left-hand side of the condition (2.34), and then stimulates the household's demand for goods. In other words, an increase in outstanding bonds induces a positive wealth effect on the household. This expands aggregate demand and thereby requiring a rise in the price level P_0. Since the aggregate supply of goods is fixed, the equilibrium condition (2.34) is restored solely by a change in the price level P_0.

The above discussion confirms that when the fiscal authority does not adjust primary surpluses, the Ricardian equivalence does not hold. Indeed, in the FTPL, an increase in outstanding bonds induces a change in the optimizing decision of the household and therefore affects the equilibrium price level.

2.3.3 Intuition II: Stock analogy discussed by Cochrane (2005)

In this section, we introduce another explanation for the economic mechanism behind the adjustment in the price level in the FTPL, which is highlighted in Section 2.3.1. Cochrane

(2005) draws an analogy between the FTPL in which the equilibrium price level is endogenously determined so as to maintain government solvency and the theory of stock price determination.[2] More specifically, he argues that government bonds, including monetary base, share a similar property with stocks, which is the security that private corporations issue with a promise of future dividends. As well-known, the stock price is determined to equate its market value (stock price × number of stocks) and the PDV of future dividends. This implies that stock prices reflect how market participants evaluate the ability of the corporation to make profits in the future. For example, a decrease in the PDV of future dividends leads to a decline in stock price.

One of the main messages of Cochrane (2005) is that the same logic determines the price of government bonds (the inverse of the price level) in the FTPL. He writes that "The fiscal theory of the price level recognizes that nominal debt, including the monetary base, is a residual claim to government primary surpluses, just as Microsoft stock is a residual claim to Microsoft's earnings" (p.502). As explained in the previous section, in the FTPL, the price of government bonds is endogenously determined so as to equate the real value of government bonds and the PDV of primary surpluses. In this sense, the price of government bonds reflects how the public evaluates the government's ability (or willingness) to raise primary surpluses in order to return goods to bond holders in the future. Indeed, the government solvency condition in period 0 (2.32) shows that when the PDV of primary surpluses decreases, the real value of government bonds declines (the price level increases). In other words, in the FTPL, the real value of government bonds is endogenously determined by public confidence in the government.

2.4 Case with Long-Term Bonds

[2] His idea is also explained by Iwamura and Watanabe (2004), Shioji (2018), and Cochrane (2022).

Next, the case in which $\gamma = 0$ and $\rho > 0$ is considered; we assume that long-term bonds are outstanding at the beginning of period 0. Cochrane (2001) and Woodford (2001) study the FTPL with long-term bonds. For simplicity, prices are assumed to be perfectly flexible as in the previous case.

2.4.1 Equilibrium determination

When $\rho > 0$, the government solvency condition can be written as

$$\frac{1}{P_0}[(1+Q_0)D_{-1}+B_{-1}] = \frac{1}{P_0}\left[\left(1+\frac{\rho}{1+\bar{\iota}_0}\right)D_{-1}+B_{-1}\right] \\ = \bar{T}_0 + \beta\bar{T}_1 \tag{2.35}$$

As prices are perfectly flexible, the real interest rate is fixed, and so is the PDV of primary surpluses. The important point in the case with long-term bonds is that the nominal value of outstanding government bonds $(1+Q_0)D_{-1}+B_{-1}$ is no longer predetermined in period 0. The reason is that the price of long-term bonds Q_0 depends on the current short-term nominal interest rate $\bar{\iota}_0$. Then, the current price level P_0 is uniquely determined to satisfy the government solvency condition (2.35) given the policy variables $\{\bar{T}_0, \bar{T}_1, \bar{\iota}_0\}$ and the predetermined variables $\{B_{-1}, D_{-1}\}$. It is worth noting that when long-term bonds are outstanding, not only fiscal policy but also monetary policy plays a role in determining the price level since it affects the price of long-term bonds.

2.4.2 Numerical illustration

This section presents a numerical example to show how a change in the price of long-term bonds affects the dynamics of the equilibrium price level. We adopt β =0.5 and θ =10. Outstanding bonds in period 0 are given by B_{-1} =1 and $D_{-1} = 1$. The price level in period − 1 is set to $P_{-1} = 1$. Primary surpluses in periods 0 and 1 are set to $\bar{T}_0 = \bar{T}_1 = 1$.

Figure 2.1 displays a numerical example of the price level in periods 0 and 1. We report the results for alternative values of $\bar{\imath}_0$. When the nominal interest rate is increased, the price of long-term bonds declines. This leads to a decrease in the price level in period 0 and an increase in the price level in period 1. This result suggests that when long-term bonds are outstanding, the government can choose the timing of inflation needed to maintain its solvency by changing the price of bonds. For example, lowering the price of long-term bonds, which corresponds to a higher price level in the next period, reduces the reliance on current inflation.

An economic mechanism through which a decline in the price of bonds leads to a lower price level is clear in light of the fact that, in the FTPL, government bonds are net wealth for the household. When the fiscal authority does not adjust primary surpluses, a decline in the price of long-term bonds held by the household induces a negative wealth effect. This reduces aggregate demand and therefore lowers the price level.

2.5 Case with Nominal Rigidities

Finally, the case in which $\gamma > 0$ and $\rho = 0$ is considered; namely we assume that prices are rigid in period 0. We analyze this case following Woodford (1996), which is the first study that incorporates the FTPL framework into a New Keynesian model. When prices are rigid, a fluctuation of aggregate demand is not absorbed entirely by a change in the price level. This also requires a variation in the level of real economic activities and then in the real interest rate. For simplicity, it is assumed that all government bonds are one-period.

2.5.1 Equilibrium determination

When $\gamma > 0$ and $\rho = 0$, the government solvency condition in period 0 is given by

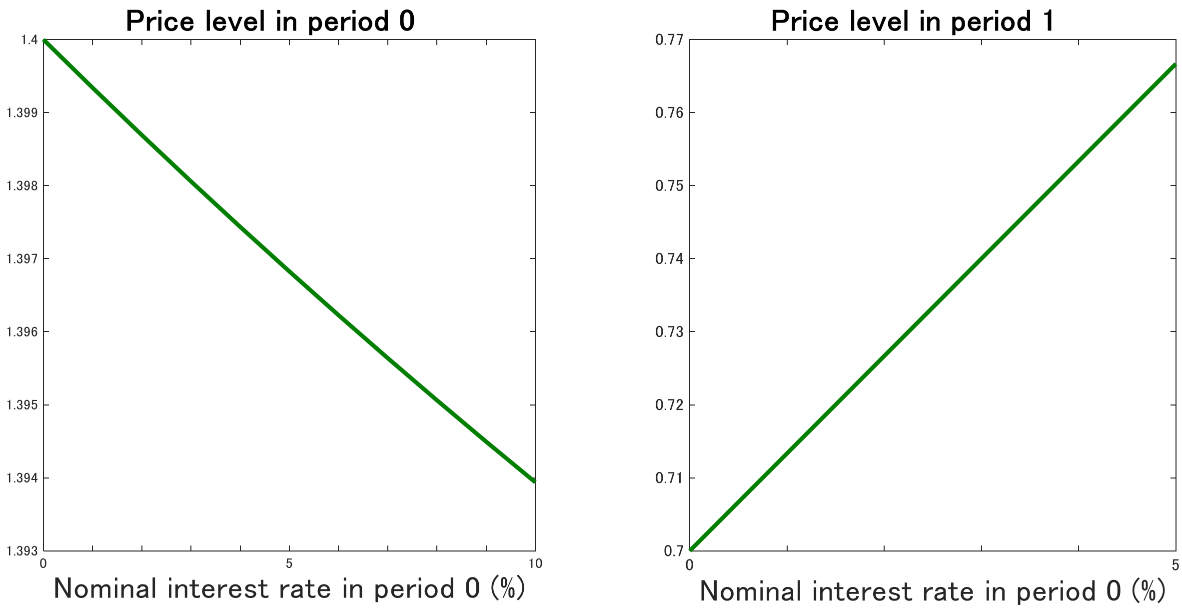

Figure 2.1. A numerical example of the price level in periods 0 and 1 at alternative values of $\bar{\imath}_0$.

$$\frac{1}{P_0}[D_{-1} + B_{-1}] = \bar{T}_0 + \left(\frac{\Pi_1}{1 + i_0}\right)\bar{T}_1$$
$$= \bar{T}_0 + \beta\left(\frac{C_0}{C_1}\right)\bar{T}_1 \tag{2.36}$$

In a case with $\rho = 0$, the nominal value of outstanding bonds $D_{-1} + B_{-1}$ is predetermined in period 0. The important point is that, in contrast to the previous two cases, due to nominal rigidities the government solvency condition cannot be satisfied solely by a change in the price level P_0. A change in the real interest rate is also needed. Given that the level of consumption in period 1 C_1 is fixed, not only the price level P_0 but also consumption C_0 should adjust to satisfy the government solvency condition (2.36).

Therefore in the case with nominal rigidities, the equilibrium price level is not uniquely determined solely by the government solvency condition, unlike in the two previous cases. We need another equilibrium condition to be combined with the government solvency condition to determine the price level and consumption. Phillips curve (2.21) describes the relationship between inflation and consumption in period 0. The price level P_0 and consumption C_0 are jointly determined to satisfy both the Phillips curve (2.21) and the government solvency condition (2.36), given the policy variables $\{\bar{T}_0, \bar{T}_1, \bar{\imath}_0\}$ and the predetermined variables $\{B_{-1}, D_{-1}, P_{-1}\}$.

2.5.2 Numerical illustration

This section presents a numerical example to show how a change in the degree of nominal rigidities affects inflation and the real interest rate in period 0. We use the same values for β, θ, B_{-1}, D_{-1}, $\bar{T}_0$, $\bar{T}_1$, and P_{-1} as in Section 2.4.2, and adopt $\rho = 0.1$.

Figure 2.2 displays the price level, the real interest rate, the net inflation rate, and consumption in period 0. We report results for ten values of γ, $\gamma \in \{0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$. The numerical result shows that when prices are stickier (i.e., as γ becomes larger), a larger decline in the real interest rate is needed to maintain government solvency. Given more sluggish adjustments in the price level, the government must put more reliance on a decline in the real interest rate to maintain its solvency.

It is also worth noting that consumption increases as prices are stickier. As explained in the FTPL, the outstanding government bonds put upward pressure on aggregate demand through the positive wealth effect. Given more sluggish adjustments in prices, an increase in consumption is needed to ensure goods market clearing.

2.6 Concluding Remarks

This chapter has reviewed the FTPL using a simple model. We studied how the equilibrium is determined in three cases: (i) the simplest case in which prices are perfectly flexible and all government bonds are one-period, (ii) the case with long-term bonds, and (iii) the case with nominal rigidities. The important point common to the three cases is that in the FTPL, the

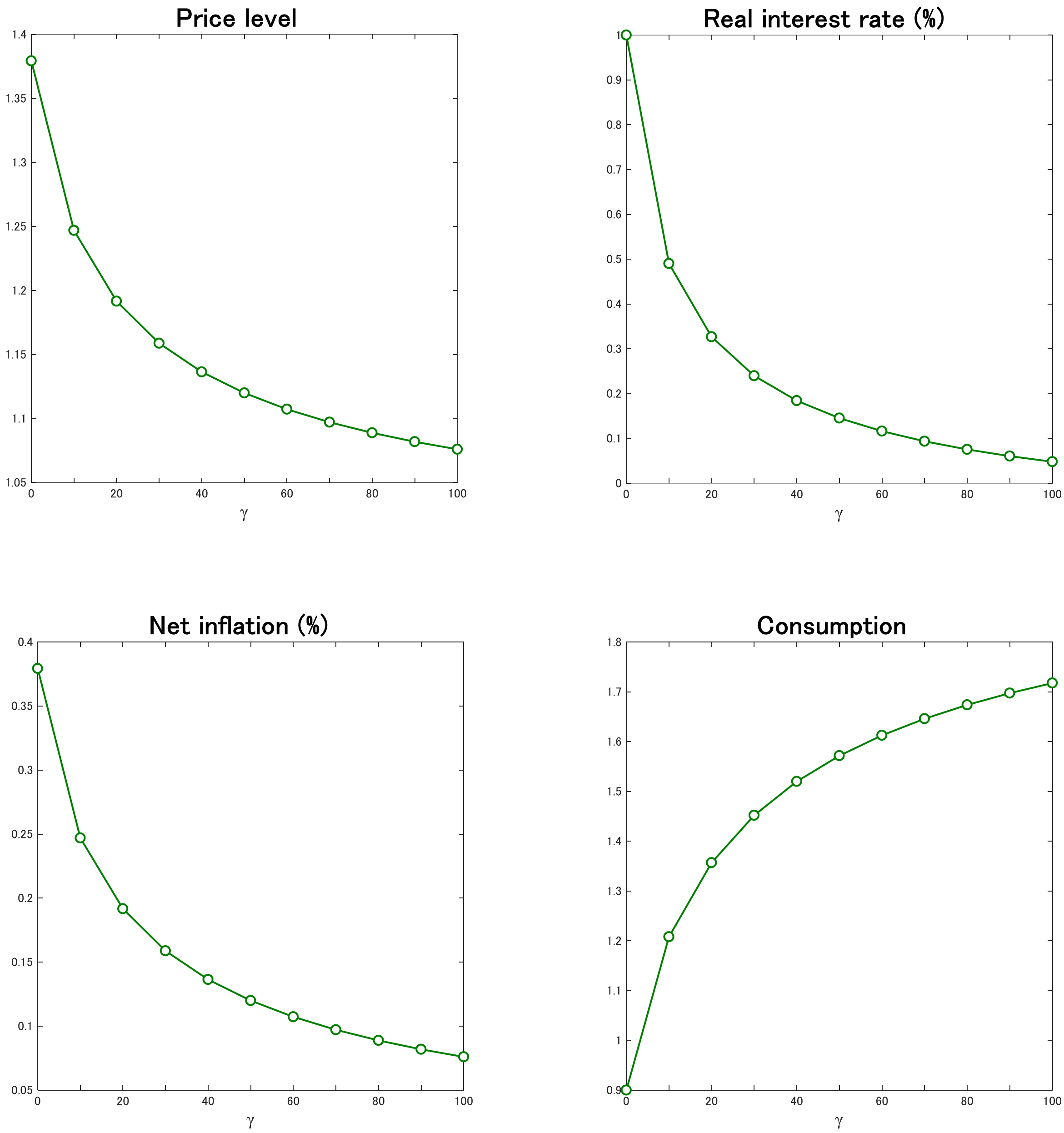

Figure 2.2. A numerical example of the price level, the real interest rate, the net inflation rate, and consumption in period 0 at alternative values of γ.

Ricardian equivalence does not hold so that an increase in outstanding bonds induces a positive wealth effect on households and therefore affects the equilibrium allocations. Again, the assumption that the fiscal authority does not make fiscal adjustments needed to maintain government solvency plays a critical role.

In Chapters 3 and 4, we explain our models based on the discussion in this chapter. Chapter 3 studies a three-period model with sticky prices in which the fiscal authority issues long-term bonds. In Chapter 4, we consider a flexible-price model of an endowment economy and extend the FTPL with long-term bonds.

Chapter 3

Central Bank's Balance-Sheet Policy in a Non-Ricardian Regime: An Expansionary Effect without a Signal about Future Interest Rates

3.1 Introduction

3.1.1 Objective of this study

As discussed in Chapter 1, one of the well-known channels through which purchases of long-term bonds may influence the economy at the zero lower bound (ZLB) is via a signal about the future path of nominal interest rates, reducing long-term interest rates. This chapter demonstrates that purchases of long-term bonds have an expansionary effect on the economy at the ZLB even without any changes in expectations regarding the future path of nominal interest rates.

This is essential to explore because the signaling channel might be unreliable. The management of expectations at the ZLB has presented a serious challenge for central banks. Whether the signaling channel operates well essentially depends on how the public interprets a central bank's future policy intentions. Many would accept the view that the Federal Reserve was successful in signaling its future policy intentions. Some studies have examined the effects of the Federal Reserve's Large Scale Asset Purchase Programs and found evidence that supports this view (e.g., Krishnamurthy and Vissing-Jorgensen, 2011; Bauer and Rudenbusch, 2014; Swanson and Williams, 2014; and Del Negro et al., 2015). Nevertheless, when the economy falls into the ZLB in a next deep recession, central banks would have difficulties in managing expectations again. One example is the recent experience in Japan. The Bank of Japan has engaged in the large-scale purchases of long-term government bonds since 2013, but Japanese inflation has been below its 2% target.[1]

3.1.2 Model environment

This study examines the effects of purchases of long-term bonds using a model with ZLB à la Eggertsson and Woodford (2003). The ZLB becomes binding due to an exogenous negative shock to the natural interest rate, and this shock disappears with a certain probability in each period. Furthermore, the central bank is assumed to set the nominal interest rate equal to the natural interest rate as long as it is positive. Accordingly, the nominal interest rate increases as soon as the natural interest rate returns to positive territory. Because of uncertainty regarding the timing of liftoff of the nominal interest rate from zero, the purchases of long-term bonds expose the central bank to the risk of losses on its balance sheet due to an unexpected decline in the price of long-term bonds.

Note that uncertainty regarding the timing of liftoff from the ZLB plays an essential role in this study. The important point of our study is that purchases of long-term bonds expose the central bank to the risk of losses on its balance sheet due to an unexpected decline in the

[1] For example, Kuroda (2016) stated that "In order to overcome deflation that has lasted for 15 years and achieve the 2 percent price stability target, the Bank of Japan has conducted large-scale monetary easing by introducing 'Quantitative and Qualitative Monetary Easing (QQE)' in April 2013 and 'QQE with a Negative Interest Rate' in January 2016. […] On the price front, a measure of underlying inflation—the year-on-year rate of change in the consumer price index (CPI) for all items less fresh food and energy—has remained positive for almost three years. Japan's economy is no longer in deflation, which is commonly defined as a sustained decline in prices. However, the price stability target of 2 percent has not been achieved."

price of bonds. Suppose that at the ZLB, the agents know that in a next period the nominal interest rate liftoffs from the ZLB and the price of long-term bonds decline. Since the price of bonds at the ZLB contains all relevant information about future events, the operation does not expose the central bank to the risk of losses on its balance sheet. Then, in our model, the operation does not affect the macroeconomy.

This study particularly focuses on the fiscal implications of possible future losses on the central bank's balance sheet. To do this, we make two key assumptions about the institutional arrangement between the fiscal authority and the central bank. First, the fiscal authority commits to covering possible future losses on the central bank's balance sheet, so that they directly translate into liabilities of the consolidated government.[2] Purchases of long-term bonds thus change the expected path of total consolidated government liabilities.

Second, fiscal-monetary policy is in a non-Ricardian regime, in which the fiscal authority commits to a certain sequence of fiscal surpluses. As aforementioned, the central bank sets the short-term nominal interest rate according to a simple rule. Under this regime endogenous variables are determined to maintain government solvency. Therefore, purchases of long-term bonds, which change the future path of government liabilities, can affect the equilibrium dynamics.

In this setup, purchases of long-term bonds have an expansionary effect on the economy at the ZLB. Suppose that when the economy falls into the ZLB due to a negative shock to the natural interest rate, the central bank purchases long-term bonds, exposing its balance sheet to the risk of losses. At a time of liftoff, the risk materializes and the central bank incurs losses on its balance sheet. This leads to an expansion in total government liabilities, which under a non-Ricardian regime results in an increase in inflation and output. Purchases of long-term bonds at the ZLB thus lead the private sector to expect higher future inflation, which reduces

[2] Here, we assume that the budget constraints of the fiscal authority and the central bank are consolidated.

the real interest rate, and thereby increasing inflation and output at the ZLB.

For the sake of simplicity, this study illustrates this mechanism using a three-period model with sticky prices. As prices are rigid, purchases of long-term bonds can affect real activity. A richer model does not essentially change the results of this study. In the first period, the economy falls into the ZLB due to a negative shock to the natural interest rate and the central bank purchases long-term bonds. In the second period, the nominal interest rate liftoffs from zero, the risk of losses on the central bank's balance sheet materializes, and then total government liabilities expand. This results in an increase in inflation in the second period and thus has an expansionary effect on the economy in the first period.

3.1.3 Contracts with literature

This study is related to the literature that examines monetary policy at the ZLB. The literature investigates policy options to smooth welfare losses due to variations in output and inflation caused by a temporary negative shock to the natural interest rate over time using a New Keynesian model in which the nominal interest rate is subject to the ZLB constraint. To do this, the central bank needs to create a temporary overshooting of inflation right after a shock disappears, which involves welfare costs but can dampen a fall in output and inflation during periods of a negative demand shock.

The result of Eggertsson and Woodford (2003) implies that if the central bank can credibly commit to its future actions, forward guidance is effective. However, as explained in Section 1, the optimal monetary policy they propose involves a time-inconsistent commitment to the future conduct of monetary policy. This is because as soon as a negative chock disappears, the benevolent central bank finds it optimal to renege on its promise to keep the nominal interest rate low. Eggetrsson (2006) shows that if the central bank is discretionary, a negative demand shock causes a substantial fall in output and inflation.[3] He also proposes that increasing the stock of nominal government debt during periods of a negative demand shock is an effective way to resolve the time-inconsistency problem of forward guidance at least to some extent.[4] When taxation is costly, the benevolent government finds it optimal to use both taxes and inflation to reduce the real value of government bonds after a shock disappears. Bhattarai et al. (2015) and Berriel and Mendes (2015) demonstrate that purchases of long-term bonds serve as a commitment device. The central bank exposed to the risk of losses on its balance sheet has an incentive to lower the short-term nominal interest rates even after a negative shock disappears to avoid large balance sheet losses.

Most closely related to this study is Benigno and Nisticò (2020). They analyze conditions for which purchases of long-term bonds affect the macroeconomy, focusing on the transfer rules between the fiscal authority and the central bank and rules according to which the fiscal authority sets the path of lump-sum taxes. They show that when the fiscal authority does not ensure solvency of the central bank that incurs large balance sheet losses by adjusting lump-sum taxes, including the case of an active fiscal policy, purchases of long-term bonds matter for output and inflation in equilibrium. They further show that in such situations, purchases of long-term bonds at the ZLB change the optimal conduct of monetary policy after a demand shock disappears and thus influence the economy at the ZLB.[5]

The contribution of our work is to show that purchases of long-term bonds can influence the macroeconomy at the ZLB even without any changes in expectations about the future path of short-term nominal interest rates. Our result has an important implication for monetary policy at the ZLB; when the economy falls into the ZLB again in a next recession,

[3] Eggertsson and Woodford (2003), Jung et al. (2005), Adam and Billi (2007), and Werning (2011) also show this result before examining optimal monetary policy under commitment.
[4] Burgert and Schmidt (2014) also consider this possibility by using a more elaborate model.
[5] The main difference between Benigno and Nisticò (2020) and Berriel and Mendes (2015) and Bhattarai et al. (2015) is that Benigno and Nisticò (2020) assume that the central bank can credibly commit to its future actions.

the central banks can influence the macroeconomy without struggling to communicate their future policy intentions to the public.

3.1.4 Possible criticisms and our responses

Finally, some remarks are needed to counteract possible criticisms. First, some readers may argue that the assumption that the fiscal authority covers losses on the central bank's balance sheet is not necessarily realistic. As discussed in Chapter 1, receiving recapitalization from the fiscal authority entails political and/or institutional difficulties for central banks. However, we believe that the financial arrangement between the two authorities in this study is relevant for policies in the real world. For example, in November 2012, Her Majesty's Treasury explicitly agreed to cover possible losses on the Bank of England's balance sheet resulting from the Asset Purchase Facility (see MacLaren and Smith, 2013).

Second, we should counteract a criticism that from a long-term perspective intruding policies that expose the central bank to the risk of losses on its balance sheet would entail social costs, which could be large in some cases. An example is a concern that when the central bank continues to rely on financial support from the fiscal authority, the boundaries between the two authorities are violated (see e.g., Fukui, 2003; Plosser, 2012). This would threaten central bank independence and price stability in the long run. Such a risk would be avoided by imposing some restrictions on loss coverage. It is important to permit loss coverage only when this is due to a particular operation. A sunset provision would also be helpful. In this respect, the aforementioned agreement between Her Majesty's Treasury and Bank of England is precedent. Of course, there is the possibility that the government cannot credibly commit to such a rule. Therefore, when introducing the operation proposed in this study, it is necessary to compare benefit in the short run and costs in the long run.

Third, it should be noted that there is another well-known channel through which purchases of long-term bonds by the central bank may influence the economy at the ZLB: portfolio-balance effects (see e.g., Bernanke, 2020). From a theoretical perspective, to construct a model in which this channel operates, we must rely on some assumptions, such as the presence of preferred habitat investors and bond market segmentation that prevents households from taking full advantage of arbitrage opportunities. Moreover, there is no consensus on the quantitative importance of this channel, so that there is no guarantee that the portfolio-balance channel always operates well to have a significant effect.

3.1.5 Layout

The remainder of this chapter is structured as follows. Section 3.2 presents the model. Section 3.3 shows a numerical example to study how purchases of long-term bonds affect the macroeconomy at the ZLB. Finally, Section 3.4 concludes.

3.2 Model

In this study, we consider a three-period model with sticky prices. The three periods are $t = 0, 1, 2$. The economy is populated by a representative household, a continuum of firms in the unit interval, the fiscal authority, and the central bank. At times 0 and 1 firms face adjustment costs in changing their prices so that prices are rigid. At time 2 firms can change their prices at no costs so that prices are perfectly flexible. The fiscal authority issues one-period and long-term bonds, while the central bank issues reserves. An important assumption is that fiscal-monetary policy is in a non-Ricardian regime. The fiscal authority commits to a certain sequence of fiscal surpluses, and the central bank controls the nominal interest rate following a simple interest-rate rule. The fiscal authority also commits to covering possible losses on the central bank's balance sheet so that the budget constraints of the fiscal authority and the central bank are consolidated.

3.2.1 Households

The representative household has the following utility function:

$$\mathbb{E}_0[\xi_0\{\log(C_0) - N_0\} + \beta\xi_1\{\log(C_1) - N_1\} + \beta^2\xi_2\{\log(C_2) - N_2\}], \tag{3.1}$$

where C_t is an aggregate of consumption, N_t is labor supplied, ξ_t is a demand shock, $\mathbb{E}_0$ is the conditional expectation operator, and $\beta \in (0,1)$ is the discount factor.

The aggregate consumption C_t is defined as

$$C_t \equiv \left[\int_0^1 c_t(j)^{\frac{\theta-1}{\theta}} dj\right]^{\frac{\theta}{\theta-1}}, \tag{3.2}$$

where $c_t(j)$ denotes the quantity of goods $j \in [0,1]$ consumed by the household. $\theta > 1$ parameterizes the elasticity of substitution across goods. The price of goods j is denoted by $p_t(j)$. The aggregate price index is

$$P_t \equiv \left[\int_0^1 p_t(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}, \tag{3.3}$$

There are three types of government liabilities: one-period and long-term bonds issued by the fiscal authority, B_t and D_t respectively, and reserves, X_t, issued by the central bank. A long-term bond issued at time t pays one dollar from time $t+1$ onward for $t = 0, 1$. The central bank pays interest on reserves. The price of a one-period government bond is denoted by $1/(1+i_t)$, where i_t is the short-term nominal interest rate, and that of long-term bonds is denoted by Q_t.

The household earns labor income $N_t W_t$, where W_t is the nominal wage, receives profits $Z_t(j)$ from firm j, and is levied lump-sum taxes T_t by the fiscal authority. At the beginning of time 0, the household holds initial assets, B_{-1}, D_{-1} and X_{-1}, so that the budget constraint at this time is given by

$$P_0C_0 + \frac{B_0 + X_0}{1 + i_0} + Q_0D_0 = B_{-1} + X_{-1} + (1 + Q_0)D_{-1} + W_0N_0 + \int_0^1 Z_0(j)dj - P_0T_0, \quad (3.4)$$

and the budget constraint at time 1 is given by

$$P_1C_1 + \frac{B_1 + X_1}{1 + i_1} + Q_1D_1 = B_0 + X_0 + (1 + Q_1)D_0 + W_1N_1 + \int_0^1 Z_1(j)dj - P_1T_1. \quad (3.5)$$

As at time 2, the household cannot borrow and has no incentive to save, the budget constraint at this time is written as

$$P_2C_2 = B_1 + X_1 + D_1 + W_2N_2 + \int_0^1 Z_2(j)dj - P_2T_2. \quad (3.6)$$

The household maximizes its expected utility (3.1) subject to the budget constraints (3.4), (3.5), and (3.6). The Euler equation is given by

$$\frac{1}{1 + i_t} = \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \Pi_{t+1}^{-1} \frac{\xi_{t+1}}{\xi_t} \right] \text{ for } t = 0,1, \quad (3.7)$$

where $\Pi_t \equiv P_t / P_{t-1}$ is the gross inflation. Optimal decision on labor supply leads to

$$\frac{W_t}{P_t} = C_t \text{ for } t = 0,1,2. \quad (3.8)$$

Given the no-arbitrage condition between one-period and long-term bonds, the price of long-term bonds must satisfy

$$Q_0 = \mathbb{E}_0 \frac{1}{1 + i_0}(1 + Q_1), \quad (3.9)$$

$$Q_1 = \frac{1}{1 + i_1}. \quad (3.10)$$

Here, $Cov_0\left(\beta \frac{\xi_1}{\xi_0} \frac{C_0}{C_1} \frac{P_1}{P_0}, 1 + Q_t\right)$ is assumed to be equal to 0. We can interpret the expectations hypothesis as a linearized version of the no-arbitrage condition between one-period and long-term bonds. Combining (3.9) and (3.10) yields

$$Q_0 = \mathbb{E}_0\left[\frac{1}{1+i_0} + \frac{1}{(1+i_0)(1+i_1)}\right]. \tag{3.11}$$

(3.10) and (3.11) indicate that the price of long-term bonds is determined by the discounted present discount value of coupons.

3.2.2 Firms

There is a continuum of monopolistically competitive firms indexed by $j \in [0,1]$. Firm j produces goods using the production technology:

$$y_t(j) = n_t(j)\,. \tag{3.12}$$

where $n_t(j)$ is the labor hired. Firm j faces a downward-sloping demand curve given by

$$y_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\theta} Y_t. \tag{3.13}$$

where $Y_t \equiv \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}}\,dj\right]^{\frac{\theta}{\theta-1}}$ denotes aggregate output. At times 0 and 1 firms face adjustment costs in changing their prices. Following Rotemberg (1982), firm j faces price adjustment costs:

$$\frac{\gamma}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t \text{ for } t = 0,1. \tag{3.14}$$

The profits of firm j at times 0 and 1 are then expressed as

$$Z_t(j) = \left[(1+s)p_t(j)y_t(j) - W_t y_t(j) - P_t\frac{\gamma}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t\right] \text{ for } t = 0,1. \tag{3.15}$$

where $s \equiv (\theta - 1)^{-1}$ is a production subsidy that offsets distortion from the monopolistic competition. As at time 2 there are no price adjustment costs, profits at this time are

$$Z_2(j) = [(1+s)p_2(j)y_2(j) - W_2 y_2(j)\,]. \tag{3.16}$$

Firm j sets its price at each time $\{p_0(j), p_1(j), p_2(j)\}$ by solving the following problems:

$$\max_{p_0(j)} \left[(1+s)Y_0 p_0(j)^{1-\theta} P_0^{\theta} - W_0 Y_0 p_0(j)^{-\theta} P_0^{\theta} - P_0 \frac{\gamma}{2}\left(\frac{p_0(j)}{p_{-1}(j)} - 1\right)^2 Y_0\right]$$

$$+\beta \mathbb{E}_0\left[\frac{C_0}{C_1}\Pi_1^{-1}\frac{\xi_1}{\xi_0}\right]\left[(1+s)Y_1 p_1(j)^{1-\theta} P_1^{\theta} - W_1 Y_1 p_1(j)^{-\theta} P_1^{\theta} - P_1 \frac{\gamma}{2}\left(\frac{p_1(j)}{p_0(j)} - 1\right)^2 Y_1\right], \tag{3.17}$$

$$\max_{p_1(j)} \left[(1+s)Y_1 p_1(j)^{1-\theta} P_1^{\theta} - W_1 Y_1 p_1(j)^{-\theta} P_1^{\theta} - P_1 \frac{\gamma}{2}\left(\frac{p_1(j)}{p_0(j)} - 1\right)^2 Y_1\right] \tag{3.18}$$

$$\max_{p_2(j)} \left[(1+s)Y_2 p_2(j)^{1-\theta} P_2^{\theta} - W_2 Y_2 p_2(j)^{-\theta} P_2^{\theta}\right]. \tag{3.19}$$

Deriving the first-order conditions at each time and focusing on a symmetric equilibrium where $p_t(j) = P_t$, the following conditions are obtained:

$$\gamma \Pi_0(\Pi_0 - 1) = \theta(C_0 - 1) + \beta\gamma \mathbb{E}_0\left[\frac{\xi_1}{\xi_0}\frac{C_0}{C_1}\frac{Y_1}{Y_0}\Pi_1(\Pi_1 - 1)\right], \tag{3.20}$$

$$\gamma \Pi_1(\Pi_1 - 1) = \theta(C_1 - 1), \tag{3.21}$$

$$C_2 = 1. \tag{3.22}$$

3.2.3 Market clearing

Clearing in the goods market implies that

$$\left[1 - \frac{\gamma}{2}(\Pi_t - 1)^2\right] Y_t = C_t \;\text{ for }\; t = 0,1 \tag{3.23}$$

$$Y_2 = C_2. \tag{3.24}$$

3.2.4 Government

The fiscal authority imposes lump-sum taxes on the household and pays the production subsidy to firms. Fiscal surplus is then given by

$$F_t = T_t - sY_t. \tag{3.25}$$

We abstract from government consumption. At time 0, the fiscal authority pre-commits to a certain sequence of fiscal surpluses as follows:

$$F_t = F \text{ for } t = 0,1,2. \tag{3.26}$$

The fiscal authority does not make fiscal adjustments needed to maintain government solvency unlike in standard models. For simplicity, we assume that the fiscal authority holds fiscal surpluses constant. While in this study we consider a situation in which the fiscal authority does not take responsibility for stabilizing government debt, we assume that $F > 0$. This is necessary to guarantee the existence of an equilibrium. The important point is that the fiscal authority does not adjust primary surpluses with no regard for government solvency.

The central bank is subject to the ZLB constraint on the nominal interest rate:

$$i_t \geq 0, \tag{3.27}$$

and sets the nominal interest rate according to the rule:

$$i_t = \max(0, r_t^n), \tag{3.28}$$

where $r_t^n \equiv (\beta \mathbb{E}_t \xi_{t+1})^{-1} \xi_t - 1$ is the natural interest rate.[6] The central bank tracks the natural interest rate as long as it is positive and sets the short-term nominal interest rate at zero when the natural interest rate is negative. For simplicity, the inflation target is assumed to be zero. The central bank also follows this rule with no regard for government solvency.

The fiscal authority issues one-period and long-term bonds, which are held by the household or the central bank. The central bank issues reserves. Assuming that the one-period

[6] To guarantee the uniqueness of an equilibrium, the central bank is assumed to set the nominal interest rate following a passive policy rule. Further, we consider the simplest case in which the central bank does not completely react to inflation.

bond is in zero net supply (i.e., $B_t = 0$), the flow budget constraint of the consolidated government is then given by[7]

$$(1+Q_t)D_{t-1} + X_{t-1} = Q_t D_t + \frac{X_t}{1+i_t} + P_t F \quad \text{for } t = 0,1,2. \tag{3.29}$$

This can be written in real terms

$$\frac{1}{P_t}[(1+Q_t)D_{t-1} + X_{t-1}] = Q_t d_t + \frac{x_t}{1+i_t} + F \quad \text{for } t = 0,1,2, \tag{3.30}$$

where $d_t \equiv D_t/P_t$ is the real value of long-term bonds, and $x_t \equiv X_t/P_t$ is that of reserves. In the numerical analysis conducted in the next section, we compute the paths of inflation and output for given values of d_{-1} and x_{-1}.[8]

3.2.5 Equilibrium

Since the fiscal authority does not adjust fiscal surpluses, endogenous variables must be determined so as to ensure government solvency. The government solvency condition at each time can be expressed as

$$\frac{1}{P_0}\left[\mathbb{E}_0\left\{1 + \frac{1}{1+i_0} + \frac{1}{(1+i_0)(1+i_1)}\right\}D_{-1} + X_{-1}\right] = \mathbb{E}_0\left[1 + \beta\frac{\xi_1}{\xi_0}\frac{C_0}{C_1} + \beta^2\frac{\xi_2}{\xi_0}\frac{C_0}{C_2}\right]F, \tag{3.31}$$

$$\frac{1}{P_1}\left[\left(1 + \frac{1}{1+i_1}\right)D_0 + X_0\right] = \mathbb{E}_1\left[1 + \beta\frac{\xi_2}{\xi_1}\frac{C_1}{C_2}\right]F, \tag{3.32}$$

$$\frac{1}{P_2}(D_1 + X_1) = F. \tag{3.33}$$

When deriving (3.31) and (3.32), we have used the consumption Euler equation (3.7) that describes the relationship between the real interest rates and the path of consumption. The two conditions show that a change in the future path of nominal interest rates, which changes

[7] As the household is indifferent between one-period bond and reserves, allocation for each security is indeterminate. Without loss of generality, we assume that one-period bond is in net zero supply.
[8] To determine inflation at time 0, we need to choose a value for P_{-1}.

the price of long-term bonds, has equilibrium implications.

The equilibrium in this model is a collection of processes $\{Y_t, C_t, \Pi_t\}_{t=0}^{2}$ that satisfy (3.20)-(3.24) and (3.31)-(3.33), given the predetermined variables $\{D_{-1}, X_{-1}, P_{-1}\}$ and policy variables $F > 0$ and $\{i_0, i_1\}$ that satisfy (3.27) and (3.28). To uniquely determine the equilibrium path of inflation, we need to choose a value for P_{-1}.

In this model, the fiscal authority issues long-term bonds, and prices are rigid. Since the path of the price of long-term bonds is exogenously given, endogenous variables that should adjust to maintain government solvency are inflation and the real interest rate. As C_2 is fixed, an expansion in total government liabilities in period 1 requires a higher level of consumption and higher inflation. Since at time 2 prices are perfectly flexible, the price level is simply determined so as to maintain government solvency; the price level is adjusted to equate the real value of government liabilities and a fixed value of fiscal surpluses.

3.2.6 Assumptions used in numerical analysis

This section explains the assumptions used in the numerical analysis conducted in the next section. The objective of this study is to examine how purchases of long-term bonds influence the economy at the ZLB. In line with the work of Eggertsson and Woodford (2003), the ZLB is binding due to a negative shock to the natural interest rate. It follows a two-state Markov process: when $r_t^n = \beta^{-1} - 1$, the natural interest rate is positive and when $r_t^n = r_{ZLB} < 0$, it takes a negative value. We assume that at time 0, a negative shock initially arises so that the economy falls into the ZLB. At time 1, the natural interest rate remains negative with probability μ, and returns to a steady state with probability $1 - \mu$, where $0 < \mu < 1$. These assumptions regarding the ZLB imply that the price of long-term bonds at time 0 thus is expressed as

$$
\begin{aligned}
Q_0 &= \mathbb{E}_0\left[\frac{1}{1+i_0} + \frac{1}{(1+i_0)(1+i_1)}\right] \\
&= \frac{1}{1+i_0} + \frac{1}{1+i_0}\mathbb{E}_0\left[\frac{1}{1+i_1}\right] \\
&= 1 + \mu + \beta(1-\mu).
\end{aligned}
\tag{3.34}
$$

The price of long-term bonds is unchanged regardless of the amount of long-term bonds the central bank purchases. Therefore, the effectiveness of purchases of long-term bonds discussed in the next section is not due to a change in the price of long-term bonds.

At time 0, the central bank purchases a fraction κ of long-term bonds held by the household with issuing reserves: $d_0 = (1-\kappa)d_{-1}$ and $x_0 = Q_0\kappa d_{-1}$. The interest-rate rule (3.28) implies that the short-term nominal interest rate liftoffs from its ZLB as soon as the natural interest rate returns to positive territory. Because of uncertainty regarding a time of liftoff from ZLB, purchases of long-term bonds expose the central bank to the risk of losses on its balance sheet due to an unexpected decline in the price of long-term bonds.

These losses are pooled in the balance sheet of the consolidated government. We restrict attention to a scenario where the risk materializes at time 1, leading to an expansion in total government liabilities. Total government liabilities at time 1 in this case can be expressed as

$$
\begin{aligned}
\left(1 + \frac{1}{1+i_1}\right)d_0 + x_0 &= \left[\left(1 + \frac{1}{1+i_1}\right)(1-\kappa)d_{-1} + Q_0\kappa d_{-1}\right] \\
&= [(1+\beta)(1-\kappa)d_{-1} + \kappa\{1+\mu+\beta(1-\mu)\}d_{-1}] \\
&= [\mu(1-\beta)\kappa + \beta + 1]d_{-1}.
\end{aligned}
\tag{3.35}
$$

This is increasing in κ, meaning that more aggressive purchases of long-term bonds at time 0 expand total government liabilities at time 1. In the next section, we numerically show that more aggressive purchases of long-term bonds result in higher inflation and output at time 1, which mitigate deflation and a recession at the ZLB.

3.3 Numerical Example

This section presents a numerical example to show how purchases of long-term bonds influence the macroeconomy at the ZLB. In the parameterization, we adopt $\beta = 0.96$ and $\theta = 9$. We choose $\gamma = 90$, implying that the slope of the Phillips curve at times 0 and 1, which is given by θ/γ, is equal to 0.1. It is assumed that the natural interest rate when the economy falls into the ZLB, $r_{ZLB} = -0.04$, and the probability that the economy at the ZLB at time 1, $\mu = 0.8$. Fiscal surplus F is set to 0.2.

Outstanding long-term bonds and reserves at times 0 are given by $d_{-1} = 0.2$ and $x_{-1} = 0$. The reason $d_{-1} = 0.2$ is chosen that, when the central bank does not purchase long-term bonds at time 0, the efficient allocation, which is characterized by $\Pi_1 = 1$ and $Y_1 = 1$, is attained at time 1. It is well-known that achieving an efficient allocation as soon as the natural interest rate returns to positive territory causes deflation and a recession at the ZLB. The numerical example presented below shows that purchases of long-term bonds mitigate the adverse effects of a negative shock to the natural interest rate.

Figure 3.1 displays the dynamics of output and inflation at times 0 and 1 when the nominal interest rate liftoffs from zero at time 1. Three values of κ are considered: $\kappa = 0, 0.25, 0.5$. As previously explained, a more massive purchases of long-term bonds at time 0 leads to an expansion in total government liabilities at time 1. Since the fiscal authority does not adjust fiscal surpluses, this induces positive a wealth effect on the household and then stimulates aggregate demand, resulting in an economic boom and higher inflation at time 1. When the private sector expects higher future inflation at time 0, deflation and a decline in output due to the ZLB are mitigated.

3.4 Concluding Remarks

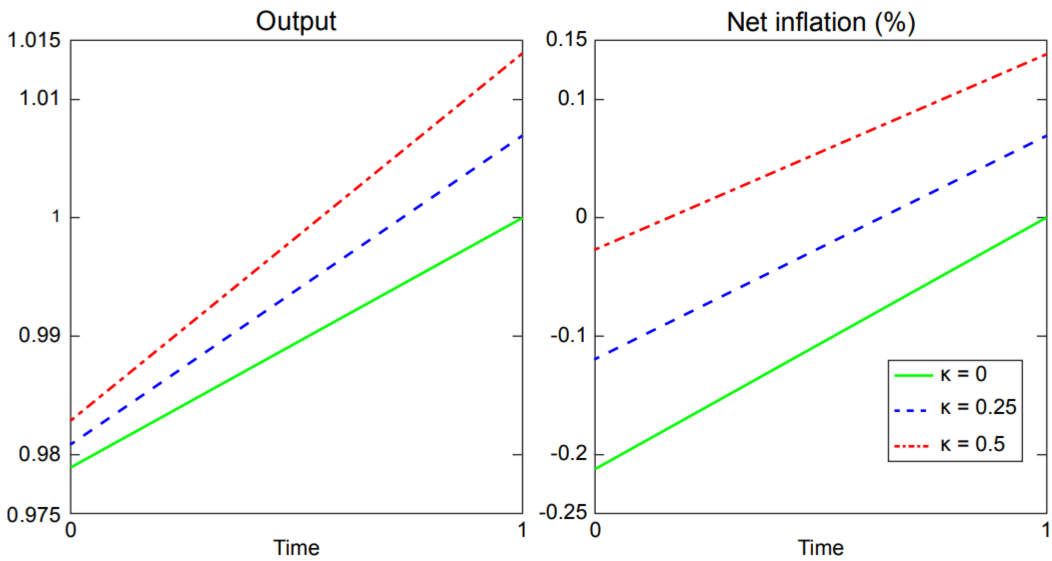

Figure 3.1. Dynamics of output and inflation when the nominal interest rate liftoffs from zero at time 1.

This chapter has demonstrated that central bank purchases of long-term bonds by issuing reserves can increase inflation and output at the ZLB, even without any changes in expectations about the future path of short-term nominal interest rates. We considered a model in which the fiscal authority commits to a certain sequence of fiscal surpluses and the central bank commits to tracking the natural interest rate. In this setup, purchases of long-term bonds at the ZLB lead to an expansion in nominal government liabilities in the future, and then increase expected inflation today. Commitment by the fiscal authority to covering possible future losses on the central bank's balance sheet plays a central role.

This result has an important implication for monetary policy at the ZLB. While the literature on optimal monetary policy at the ZLB has emphasized the importance of commitment, the management of expectations at the ZLB has been a serious challenge for central banks. The results imply that in light of the current fiscal situations in developed countries, purchases of long-term bonds can be an alternative tool by which central banks can influence the macroeconomy during a next deep recession. Central banks must make an appropriate agreement with their fiscal authorities regarding possible future losses before introducing the operations.

Chapter 4

A Fiscal Theory of Central Bank Solvency: Perils of the Quantitative and Qualitative Monetary Easing

This chapter, especially Section 4.1, 4.2, and 4.3, is based on Niwa (forthcoming)

4.1 Introduction

4.1.1 Motivation

The Bank of Japan introduced the Quantitative and Qualitative Monetary Easing (QQE) in April 2013. It has purchased long-term Japanese government bonds by issuing excess reserves. This program has expanded the size of the Bank of Japan's balance sheet and lengthened the duration of assets it holds. In the future, the Bank of Japan could exit from the QQE and increase the policy rate from an effective lower bound. The objective of this chapter is to construct a model to study the Bank of Japan's strategy to exit from the QQE.

We pay particular attention to the following two concerns from a fiscal perspective. First, when the Bank of Japan exits from the QQE, the public would doubt the fiscal authority's willingness to stabilize public debt. For example, Okina (2013, 2015) and Hayakawa (2016) point out that public confidence in fiscal sustainability must be maintained not to constrain the Bank of Japan's strategy to exit from the QQE. Second, this chapter focuses on the financial stability of the Bank of Japan. Some have expressed concern about possible losses on the Bank of Japan's balance sheet after exiting from the QQE (e.g., Iwata et al., 2014, 2018; Okina, 2015). The Bank of Japan's strategy to exit from the QQE may also be constrained by its financial condition.

How does the interaction of these two constrain the Bank of Japan's strategy to exit from the QQE and the ability of monetary policy to stabilize inflation after liftoff from the zero lower bound (ZLB)? To address this question, this chapter develops a theory to consider solvency of the whole government sector and that of the central bank in a unified way. There are three key assumptions.

4.1.2 Key assumption I: Fiscal policy rules

The first assumption is motivated by the concern about fiscal sustainability. To clarify how important it is that the public expects the fiscal authority to take responsibility for stabilizing government debt, we analyze both the case of a passive fiscal policy and an active fiscal policy.

4.1.3 Key assumption II: Intertemporal budget constraint of the central bank

The last two assumptions are motivated by the concern that the Bank of Japan's strategy to exit from the QQE may also be constrained by its financial condition. As discussed in Chapter 1, such an issue is irrelevant if the assumption that the fiscal authority commits to covering losses on the central bank's balance sheet holds. Under this assumption, the central bank cannot be insolvent separately from the solvency of the overall government, as discussed by Reis (2015). In this case the central bank can choose its action with no regard for its financial condition.

In this chapter, we deviate from this assumption and instead separate the balance sheets of the fiscal authority and the central bank. More specifically, we assume that the central bank is subject to a no-Ponzi game condition following Reis (2013). As explained below, the recent theoretical literature on central bank's solvency, which was initiated by Reis (2013), assume that the central bank is subject to an intertemporal budget constraint (Bassetto and Messer, 2013; Reis, 2013, 2015; Del Negro and Sims, 2015; Benigno and Nisticò, 2020; Benigno, 2020):

$$\frac{\text{nominal value of central bank's net worth}}{\text{Price level}} + \text{present value of seigniorage revenues}$$

$$\geq \text{ present value of remittances to the fiscal authority.}$$

This condition implies that the central bank can pursue its policy stance without financial support from the fiscal authority as long as the sum of the real value of its net worth and the present discount value (PDV) of seigniorage revenues is non-negative. A key feature of this equation is that this does not contain information about a future path of long-term bonds held by the central bank, implying whether the central bank holds long-term bonds to maturity is irrelevant from a PDV perspective, as discussed by Del Negro and Sims (2015) and Cavallo et al. (2019). The intuition behind this result is as follows. The price of long-term bonds contains all relevant information about the central bank's future profits (or losses). Accordingly, whether the central bank sells long-term bonds to realize losses at a time of liftoff from the ZLB or keeps long-term bonds in its portfolio by financing it via reserves to smooth losses on its balance sheet over time does not matter from an intertemporal perspective.

This insight is relevant to the debate on the financial stability of the Bank of Japan after it exits from the QQE. According to Okina (2013) and Iwata et al. (2019), after liftoff from the ZLB the Bank of Japan is likely to hold long-term bonds that it has purchased under the QQE to maturity because selling bonds would have substantial impacts on the government bonds market. As the Bank of Japan adopts an amortized cost method for its accounting, it would not incur capital losses after liftoff from the ZLB.[1] In this case, however, the Bank of Japan has to pay interest on excess reserves which have been expanded under the QQE. Based on the discussion by Del Negro and Sims (2015), selling long-term bonds at once to earn seigniorage revenues immediately and holding bonds to maturity makes little difference for the PDV calculations.

It also should be noted that some readers may suspect that it is unrealistic to assume that the Japanese government will not recapitalize the Bank of Japan so that it is meaningless to focus on the financial stability of the Bank of Japan. For example, Meltzer (1999, p.12) states that

> I see no reason to believe there would be any doubt about the government's obligation to stand behind the BOJ. No central bank has ever faced default, and no responsible government would permit that to happen. It is unclear to me what could be meant by failure of a central bank.

However, it is not necessarily guaranteed that the government always recapitalizes the central bank. Against Meltzer (1999), Ueda (2003) points out that

[1] This point is explained by Fujiki and Tomura (2015, 2017) and Iwata et al. (2019) in detail.

> Such arguments [i.e., the above discussion by Meltzer (1999)] seem to be, however, based on a rather naive view of the relationship between the central bank and the government, and the procedures by which the government formulates budgets. First of all, even if the government's recapitalization of a central bank would contribute to the central bank's conduct of monetary policy to maintain price stability, an important question is whether the government would have the incentive to do so. If that were not a significant problem, central bank independence from the government would not be an issue in the first place. The issue of central bank independence has become a subject of discussion in light of the risk that the government, if in charge of monetary policy, is prone to choose a socially undesirable rate of inflation.

Indeed, even in the United States where the aforementioned legislation is put in place, possible losses on the Federal Reserve's balance sheet were politically controversial. As of March 2022, no plan to share possible losses on the Bank of Japan's balance sheet between the government and the Bank of Japan has been announced. It can be argued that possible losses on the Bank of Japan's balance sheet would also be politically controversial as in the United States in 2013. One cannot rule out the possibility that the Bank of Japan's strategy to exit from the QQE will be constrained by its financial condition.

4.1.4 Key assumption III: Remittances rules

In addition, we assume that the central bank is obligated to satisfy its intertemporal budget constraint keeping its commitment to a particular type of remittances rule. As discussed by Reis (2015), when considering the issue of central bank's solvency, the assumption about remittances rule plays an important role. In this study, we assume that the central bank that incurs losses on its balance sheet is permitted to make zero remittances to rebuild its net worth along the lines of Benigno and Nisticò (2020) and Benigno (2020). We call this remittances rule "deferred asset rule," in which a solvency condition of the central bank can be relevant.

The modeling choice of Benigno and Nisticò (2020) and Benigno (2020) is motivated by the case of the Federal Reserve. As explained in Section 1, when incurring losses on its balance sheet, the Federal Reserve is permitted to record deferred assets as negative liabilities on its balance sheet. While the Japanese government and the Bank of Japan have not made an agreement on how to share possible losses on the Bank of Japan's balance sheet, this rule in the United State would be a precedent for a rule according to which the Bank of Japan will make remittances to the government. Indeed, Fujiki and Tomura (2015, 2017) and Iwata et al. (2014, 2018) assume that the Bank of Japan makes zero remittances until its profitability is restored.

We also consider a case in which the fiscal authority commits to covering possible losses on the central bank's balance sheet. We call this "full fiscal support rule." The motivation to study this case is to assess how important it is that the fiscal authority commits not to provide financial support for the central bank that incurs losses on its balance sheet.

4.1.5 Baseline analysis

In this setup, we consider a situation where the central bank that holds long-term bonds raises the nominal interest rates. Due to a decline in the price of long-term bonds, the central bank incurs losses on its balance sheet. In our baseline analysis, we assume that the central bank commits to an entire path of current and future nominal interest rates at a time of liftoff.

The main message of the baseline analysis is that a lack of public confidence in fiscal sustainability, combined with a commitment of the fiscal authority not to provide financial support for the central bank, constrains monetary policy after liftoffs from the zero lower bound (ZLB). To demonstrate this point, we consider the following three types of combination of fiscal policy rule and remittances rule. The first is a combination of an active fiscal policy and the full fiscal support rule. The second is a combination of a passive fiscal policy and the deferred asset rule. The third is a combination of an active fiscal policy and the deferred asset rule. While the third case is the main focus of our study, analyzing the first two cases helps assess how the interaction of an active fiscal policy and the deferred asset rule constraints monetary policy after liftoff from the ZLB.

We demonstrate the following three results. First, under the combination of an active fiscal policy and the full fiscal support rule, the action of the central bank is not constrained by its financial condition. Under the full fiscal support rule, losses on the central bank's balance sheet at a time of liftoff are directly translated into liabilities of the consolidated government. When fiscal policy is active, an equilibrium price level is determined to maintain the solvency of the consolidated government through the mechanism highlighted by the fiscal theory of the price level (FTPL) with long-term bonds. Accordingly, regardless of the amount of losses that the consolidated government incurs at a time of liftoff, its solvency can be maintained solely by a sufficient rise in the price level.

Second, we show that a similar result can be obtained under the combination of a passive fiscal policy and the deferred asset rule. The action of the central bank after liftoff from the ZLB is not constrained by its financial condition. This is because the central bank can maintain its solvency by allowing the price level to increase to an arbitrarily high level to reduce the real value of its liabilities, regardless of the amount of losses on its balance sheet. Note that larger losses on the central bank's balance sheet due to an unexpected decline in the price of long-term bonds at a time of liftoff from the ZLB require a higher price level.

Finally, we study the combination of an active fiscal policy and the deferred asset rule. Under this case, in contrast to the previous two cases, the central bank that holds long-term bonds above a certain threshold cannot freely raise the path of the nominal interest rates at a time of liftoff. In our model, the central bank that incurs losses on its balance sheet must increase the present discounted value (PDV) of seigniorage revenues. This decreases the PDV of lifetime income of households if there is no change in the PDV of primary surpluses and thus imposes an upper bound on the price level through the mechanism highlighted by the FTPL. Therefore, the action of the central bank is constrained by the amount of long-term bonds on its balance sheet.

4.1.6 Extension I: Monetary policy rules

Next, we explore richer implications of our study for policy under the combination of an active fiscal policy and the deferred asset rule. To do this, we extend the baseline model by specifying a rule according to which the central bank controls the short-term nominal interest rate after liftoff from the ZLB. First, we study the case of a passive monetary policy in the sense of Leeper (1991), as fiscal policy is assumed to be active. We then show that under certain conditions, the central bank can achieve its inflation target after liftoff at least in the long run. In this case, however, inflation right after liftoff must undershoot the central bank's target inflation. This means that large-scale purchases of long-term bonds at the ZLB would not lead to an increase in inflation expectations at the ZLB.

Second, we study the case of an active monetary policy; we assume that the Taylor principle is satisfied. The main result is that if the central bank follows the Taylor principle, under certain conditions, it fails to prevent the economy from converging to the deflationary steady state after liftoff. We also provide numerical examples to show that the Bank of Japan cannot stabilize inflation at a positive level after liftoff using recent Japanese data. The result implies that a central bank that engages massive purchases of long-term bonds would not achieve its inflation target after liftoff by actively controlling the short-term nominal interest rate. It is also worth noting that purchases of long-term bonds would not lead to an increase in inflation expectations at the ZLB regardless of whether the Taylor principle is satisfied.

4.1.7 Extension II: Partial default at a time of liftoff

Finally, we incorporate a possibility that the fiscal authority partially defaults on outstanding government bonds at a time of liftoff from the ZLB. We introduce a mechanism through which the equilibrium default rate is determined along the lines of Uribe (2006) and consider a one-off default at a time of liftoff. The possibility of default on Japanese government bonds is also an important issue when considering the Bank of Japan's exit strategy from the QQE. For example, Okina (2015) and Hayakawa (2016) argue that despite the high level of public debt in Japan, the QQE has prevented a fiscal crisis from materializing and contributed to a substantial decline in interest rates. They also express concern that a credit event, such as a sudden rise in interest rates, could occur when the Japanese economy exits from the ZLB.

From a theoretical perspective, however, a relation between purchases of long-term bonds by the central bank and default on government bonds is not clear. In particular, few theoretical studies shed light on such a relation in a setting where the balance sheets of the fiscal authority and the central bank are separated. Our motivation here is to fill this gap, at least to some extent, by extending the above model.

More specifically, we address this issue in terms of price stability. Generally, there are only two consequences when the fiscal authority does not make fiscal adjustments needed to ensure government solvency. The first is the default on government bonds. The second is an increase in inflation, which increases seigniorage revenues and reduces the real value of government bonds outstanding (see, e.g., Shirakawa, 2012). As discussed by Uribe (2006), the experience of Argentina in the 1990s is a clear example, in which when sufficient fiscal adjustments are not implemented, the government must give up on either pursuing price stability or keeping its commitment not to default on government bonds. The Argentine government introduced the currency board arrangement in 1991, which played a central role in suppressing very high inflation during the late-1980s. On the other hand, the public debt was increasing during the 1990s, and "eventually the required primary surplus became implausibly large, particularly in relation to the political system's ability to deliver. […] By 2001, almost no strategy would have succeeded without a sovereign debt restructuring that reduced the present value of Argentina's public debt burden." (IMF 2003, p. 67)

Our analysis here is especially motivated by Kocherlakota (2011), who demonstrates that even under the case of an active fiscal policy, the central bank achieves independent control of inflation if it is willing to allow the fiscal authority to default on its bonds. This result is shown under the assumption that the balance sheets of the fiscal authority and the central bank are consolidated. We address the question: does the result of Kocherlakota (2011) hold even when the balance sheets of the two authorities are separated? The main result is that a central bank exposed to the risk of losses on its balance sheet cannot achieve its inflation target by following the Taylor principle even if it is willing to allow the fiscal authority to default on its bonds. This result implies that even if government bonds are defaultable, large-scale purchases of long-term bonds by the central bank would undermine the ability of monetary policy to stabilize inflation after liftoff.

4.1.8 Related literature

This study is related to the growing literature explained in Chapter 1. Reis (2013, 2015) focus only on central bank solvency, implicitly assuming that the fiscal authority takes responsibility for maintaining the sustainability of public debt. We differ from his works in analyzing a situation where the fiscal authority does not make fiscal adjustments needed to stabilize public debt. Bassetto and Messer (2013) and Del Negro and Sims (2015) assume that both the central bank and the fiscal authority take responsibility for maintaining their solvency. In our model, as explained below, the fiscal authority is allowed to engage in Ponzi games provided that solvency of the whole government and that of the central bank are maintained.

The most closely related to our study is Benigno and Nisticò (2020), which also analyze the effect of losses on the central bank's balance sheet resulting from purchases of long-term bonds under the assumption of an active fiscal policy. However, when analyzing this case, they do not impose a solvency condition on the central bank. The key feature of our setup is that the action of the central bank is subject to both its solvency condition and a solvency condition of the whole government.

4.1.9 Layout

The remainder of this chapter is structured as follows. Section 4.2 presents the model. In Section 4.3, the model is used to investigate what constraints the central bank holding long-term bonds faces at a time of liftoff. We analyze the case of a passive fiscal policy and that of an active fiscal policy, respectively. Section 4.4 extends the model by specifying a rule according to which the central bank controls the short-term nominal interest rate after liftoff from the ZLB. We study a case of a passive monetary policy. Section 4.5 studies a case of an active monetary policy. Section 4.6 provides numerical examples. In Section 4.7, we further incorporate a possibility that the fiscal authority partially defaults on its bonds at a time of liftoff from the ZLB. Finally, Section 4.8 concludes.

4.2 Model

We consider an infinite-horizon model along the lines of Benigno and Nisticò (2020). The economy is populated by a representative household, the fiscal authority, and the central bank.

The household receives a constant endowment of goods each period. The fiscal authority issues one-period and long-term bonds. The central bank issues non-interest-bearing liabilities, money, and interest-bearing liabilities, reserves. The household is indifferent between one-period government bonds and reserves so that prices of the two securities are equalized.

4.2.1 Households

The representative household derives utility from consumption and real money balances. The household's lifetime utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t) + V\left(\frac{M_t}{P_t}\right) \right] \xi_t, \tag{4.1}$$

where$\beta \in (0,1)$ is the discount factor, C_t is consumption, $m_t \equiv M_t/P_t$ is real money balances. M_t is nominal money balances and P_t is the price level. ξ_t is a demand shock. $U(\cdot)$ is increasing, concave and continuously differential. $V(\cdot)$ is concave and increasing in real balances up to a satiation level $\bar{m}$. $\delta_t \in [0,1)$ is the default rate that will be endogenized in Section 4.7. Until Section 4.7, $\delta_t = 0$ in every period.[2]

The household faces the budget constraint

$$\begin{aligned} & P_t C_t + M_t + \frac{B_t + X_t}{1 + i_t} + Q_t D_t \\ & \qquad \le M_{t-1} + B_{t-1} + X_{t-1} + (1 - \delta_t)(1 + \rho Q_t) D_{t-1} + P_t Y - T_t, \end{aligned} \tag{4.2}$$

where B_t is the household's holding of one-period risk-less nominal government bonds at the beginning of period t, D_t is a long-term bond portfolio with price Q_t, and X_t is reserves issued by the central bank, Y is a constant endowment, and T_t is lump-sum taxes. Long-term bonds are assumed to be perpetuities. Specifically, a bond issued at time t pays ρ^k in nominal

[2] In Section 4.7, we explain how we treat partial default in detail.

terms at time $t+k+1$, as in Woodford (2001), for $k \geq 1$ and $\rho \in [0,1]$. For example, when $\rho = 0$, the bond is a short-term bond. $\rho = 1$ corresponds to a classic console bond. The duration of this bond in an environment where the gross inflation is stable at Π is $\Pi/(\Pi - \beta\rho)$, and its *yield to maturity* is given by $Q_t - (1-\rho)$.

The household is also subject to the following constraint that prevents it from engaging in Ponzi games

$$\lim_{\tau \to \infty} \mathbb{E}_t \left[R_{t,\tau} \frac{1}{P_\tau} \left(Q_\tau D_\tau + \frac{B_\tau + X_\tau}{1+i_\tau} + M_\tau \right) \right] \geq 0, \tag{4.3}$$

where $R_{t,\tau} \equiv \beta^{\tau-t} U_C(C_\tau)/U_C(C_t)$ is the stochastic discount factor.

The household chooses consumption and asset allocations to maximize expected utility (4.1) under the constraints (4.2) and (4.3). The optimal choice with respect to consumption requires

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t \left[\frac{U_C(C_{t+1})}{U_C(C_t)} \Pi_t^{-1} \frac{\xi_{t+1}}{\xi_t} \right], \tag{4.4}$$

where $\Pi_t \equiv P_t/P_{t-1}$. is the gross inflation. Optimal substitution between consumption and real money balances leads to the static first-order condition

$$\frac{V_m(M_t/P_t)}{U_C(C_t)} = \frac{i_t}{1+i_t}. \tag{4.5}$$

This implies that a money demand function takes a form

$$\frac{M_t}{P_t} \geq L(i_t, C_t), \tag{4.6}$$

with

$$i_t \geq 0, \tag{4.7}$$

together with the complementary slackness condition which requires that at least one must hold with equality in every period. A money demand function $L(\cdot,\cdot)$ is defined as $L(\cdot,\cdot) \equiv$

$V_m^{-1}[(U_C i_t)/(1+i_t)]$. The absence of opportunities to arbitrage between one-period and long-term bonds requires that

$$Q_t = \mathbb{E}_t\left[\left\{\beta \frac{U_C(C_{t+1})}{U_C(C_t)} \Pi_{t+1}^{-1} \frac{\xi_{t+1}}{\xi_t}\right\}(1-\delta_{t+1})(1+\rho Q_{t+1})\right], \tag{4.8}$$

Finally, the household optimization implies the transversality condition

$$\lim_{\tau\to\infty} \mathbb{E}_t\left[R_{t,\tau}\frac{1}{P_\tau}\left(Q_\tau D_\tau + \frac{B_\tau + X_\tau}{1+i_\tau} + M_\tau\right)\right] = 0. \tag{4.9}$$

4.2.2 Fiscal authority

The fiscal authority issues one-period and long-term bonds. It imposes lump-sum taxes T_t on the household, and receives transfers T_t^C from the central bank. The budget constraint of the fiscal authority is then given by

$$(1-\delta_t)\left(1+\rho Q_t\right)D_{t-1}^F = Q_t D_t^F + T_t + T_t^C, \tag{4.10}$$

where D_{t-1}^F is long-term bonds outstanding at the beginning of period t.

4.2.3 Central bank

The central bank issues non-interest-bearing liabilities, money M_t, and interest-bearing liability, reserves X_t, and holds one-period and long-term bonds B_t^C and D_t^C respectively. The central bank's net worth is given by

$$N_t = \underbrace{Q_t D_t^C + \frac{B_t^C}{1+i_t}}_{assets} - \underbrace{\left(M_t + \frac{X_t}{1+i_t}\right)}_{liabilities}. \tag{4.11}$$

This evolves according to the law of motion

$$N_t = N_{t-1} + \Psi_t^C - T_t^C, \tag{4.12}$$

where Ψ_t^C is the central bank's profits, defined as follows

$$\Psi_t^C \equiv \frac{i_{t-1}}{1+i_{t-1}}(B_{t-1}^C - X_{t-1}^C) + [(1-\delta_t)(1+\rho Q_t) - Q_{t-1}]D_{t-1}^C \tag{4.13}$$

The first term of (4.13) represents net interest receipts on one-period debt. The second term captures excess gains or losses from holding long-term bonds. The central bank holding long-term bonds can incur losses on its balance sheet due to an unexpected decline in the price of long-term bonds. The central bank makes transfers to the fiscal authority according to a particular type of rule, as explained in the next subsection.

Substituting (4.11) and (4.13) into (4.12) gives the central bank's flow budget constraint

$$\begin{aligned} Q_t D_t^C + \frac{B_t^C}{1+i_t} - M_t - \frac{X_t}{1+i_t} \\ = (1-\delta_t)\left(1+\rho Q_t\right)D_{t-1}^C + B_{t-1}^C - X_{t-1} - M_{t-1} - T_t^C. \end{aligned} \tag{4.14}$$

4.2.4 Assumptions about institutional arrangements between the two authorities

In this subsection we make assumptions about institutional arrangements between the fiscal authority and the central bank, which play a central role in our analysis. The central bank is assumed to be subject to a borrowing limit of the form

$$\lim_{\tau\to\infty} \mathbb{E}_t \left[R_{t,\tau}\frac{N_\tau}{P_\tau}\right] \geq 0, \tag{4.15}$$

which prevents it from engaging in Ponzi games. The central bank must honor this no-Ponzi game condition, keeping its commitment to a remittances rule. We consider two types of remittances rules as follows.

Definition 1. *Under **full fiscal support rule**, the central bank makes remittances to the fiscal authority according to the following rule:*

$$T_t^C = \Psi_t^C. \tag{4.16}$$

Definition 2. *Under* ***deferred asset rule****, central bank makes remittances to the fiscal authority according to the following rule:*

$$T_t^C = \begin{cases} \Psi_t^C & if \quad N_t \geq \overline{N} \\ 0 & if \quad otherwise \end{cases} \quad . \tag{4.17}$$

The full fiscal support rule (4.16) has the following two implications. First, whenever the central bank makes positive profits, they are transferred to the fiscal authority. Second, whenever the central bank incurs losses on its balance sheet, the fiscal authority recapitalizes the central bank. Under the full fiscal support rule, therefore, the balance sheets of the two authorities are consolidated.

Under the deferred asset rule (4.17), the central bank must transfer its profits provided that $N_t \geq \overline{N}$, where $\overline{N}$ is an initial level of its net worth. Once its net worth is below an initial level due to balance sheet losses, the central bank makes zero remittances. When an initial level $\overline{N}$ is recovered, the bank returns to make transfers to the fiscal authority.

By iterating the central bank's flow budget constraint (4.14) forward and imposing the no-Ponzi game condition (4.15), we obtain the intertemporal budget constraint of the central bank

$$\frac{N_t}{P_t} + \mathbb{E}_t \sum_{\tau=t}^{\infty} R_{t,\tau} s(i_\tau, C_\tau) \geq \mathbb{E}_t \sum_{\tau=t}^{\infty} R_{t,\tau} \frac{T_\tau^C}{P_\tau}, \tag{4.18}$$

where $S(\cdot,\cdot)$ is seigniorage revenues in period t, which is defined as

$$s(i_t, C_t) \equiv \frac{i_t}{1 + i_t} L(i_t, C_t). \tag{4.19}$$

Moreover, the fact that T_t^C is non-negative in every period implies that the central bank must honor the following solvency condition

$$\frac{N_t}{P_t} + \mathbb{E}_t \sum_{\tau=t}^{\infty} R_{t,\tau} s(i_\tau, C_\tau) \geq 0. \tag{4.20a}$$

This condition implies that to maintain its solvency, the central bank with negative net worth must allow the current price level to increase in order to decrease the real value of its liabilities and/or increase the PDV of seigniorage revenues. As long as the central bank's net worth is non-negative, (4.20a) is irrelevant.

4.2.5 Equilibrium

Equilibrium in the goods market requires

$$Y = C_t. \tag{4.21}$$

Equilibrium in the money market implies that

$$M_t \geq P_t L(i_t, Y), \tag{4.22}$$

while the complementary slackness condition is given by

$$i_t[M_t - P_t L(i_t, Y)] = 0. \tag{4.23}$$

We next consider equilibrium implications of the institutional arrangements between the two authorities. First of all, we consider the implications of the deferred asset rule, which is the main focus of this study. The no-Ponzi game condition of the central bank (4.15), combined with the transversality condition (4.9), implies that in equilibrium the following must hold

$$\lim_{\tau \to \infty} \mathbb{E}_t \left[R_{t,\tau} \frac{Q_\tau D_\tau^F}{P_\tau} \right] = \lim_{\tau \to \infty} \mathbb{E}_t \left[R_{t,\tau} \frac{N_\tau}{P_\tau} \right] \geq 0. \tag{4.24}$$

Roughly speaking, this condition implies that if the central bank is expected to "leave positive assets," then the fiscal authority can engage in Ponzi games. This is allowed, provided that both solvency of the whole government and that of the central bank are maintained.

Bassetto and Messer (2013) and Del Negro and Sims (2015) focus on an equilibrium in which the left-hand side of (4.24) becomes zero; namely, they assume that both fiscal and monetary authorities have responsibility for their own solvency. On the other hand, we are assuming that the fiscal authority does not have such a responsibility. It also should be noted that, when analyzing the case of an active fiscal policy, Benigno and Nisticò (2020) do not impose a solvency condition on the central bank; that is, they do not rule out the possibility that the left-hand side of (4.24) becomes negative.

Iterating the budget constraint of the fiscal authority (4.10) forward and imposing (4.24), we obtain a key condition to consider implications of the institutional arrangement for equilibrium price levels:

$$\frac{(1+\rho Q_t)d_{t-1}^F}{\Pi_t} \geq \mathbb{E}_t \sum_{\tau=t}^{\infty} R_{t,\tau}\left[t_\tau + \frac{T_\tau^C}{P_\tau}\right], \tag{4.25}$$

where $d_{t-1}^F \equiv D_t/P_t$ is the real value of long-term bonds and $t_t \equiv T_t/P_t$ is the real value of lump-sum taxes. To further investigate the implications of this condition, we need to specify a rule according to which the fiscal authority sets the path of taxes. We define two types of fiscal regime: a passive fiscal policy and an active fiscal policy.

Definition 3. *Under a passive fiscal policy, the path of lump-sum taxes* $\{T_\tau/P_\tau\}_{\tau=t}^{\infty}$ *is specified to ensure the transversality condition (4.9), given the predetermined variables* D_{t-1}^F *and any paths of* $\{\Pi_\tau, Q_\tau, T_\tau^C\}_{\tau=t}^{\infty}$.

Definition 4. *Under an active fiscal policy, the fiscal authority pre-commits to the path of lump-sum taxes* $\{\bar{t}_\tau\}_{\tau=t}^{\infty}$ *for all* t.

Under a passive fiscal policy, taxes are adjusted to maintain the solvency condition of the whole government, so that (4.25) is irrelevant. Under an active fiscal policy, the fiscal authority does not make fiscal adjustments needed to stabilize government debt. An equilibrium price level thus must be determined so as to satisfy condition (4.25).

Moreover, under an active fiscal policy, the commitment of the central bank to the remittances rule sets an upper bound on the PDV of fiscal surpluses, and thus imposes an upper bound on an equilibrium price level:

$$\Pi_t \leq \frac{(1+\rho Q_t) d_{t-1}^F}{\mathbb{E}_t \sum_{\tau=t}^{\infty} R_{t,\tau} \bar{t}_\tau}. \tag{4.26}$$

The logic behind this result is as follows. With the above institutional arrangements put in place, the central bank that incurs large losses on its balance sheet must increase the PDV of seigniorage revenues. This decreases the PDV of lifetime income of the household if there is no change in the PDV of primary surpluses, thereby depressing its demand for goods. Through this mechanism, losses on the central bank's balance sheet induce negative wealth effects on consumption and thus put downward pressure on the price level.

There are three determinants of the upper bound on inflation; the following three induce negative wealth effects on the household and thus lower the upper bound on inflation through the mechanism highlighted by the FTPL as explained in Section 2. The first is an increase in the PDV of primary surpluses; the second is a decrease in the outstanding bonds at the beginning of a time of liftoff from the ZLB; and the third is a decline in the price of long-term bonds. In the FTPL with long-term bonds, a change in the price of long-term bonds affects the price level through wealth effects, as well as a change in primary surpluses.

Next, we consider the equilibrium implications of the full fiscal support rule. Under the combination of an active fiscal policy and the full fiscal support rule, since the balance sheets of the two authorities are consolidated, the equilibrium is determined as in the FTPL with long-term bonds, which is explained in Section 2.4. The solvency condition of the consolidated government is given by

$$\frac{1}{P_t}[B_{t-1}^F + (1+\rho Q_t)D_{t-1}^F - \bar{N} - \Psi_t^C] = \sum_{\tau=t}^{\infty} \beta^{t-\tau}[\bar{t}_t + s(i_t, Y)]. \tag{4.27a}$$

Note that under the combination of a passive fiscal policy and the full fiscal support rule, neither the solvency condition of the whole government nor that of the central bank is irrelevant. Purchases of long-term bonds by the central bank then do not affect the economy, as explained in Chapter 1. This combination is not considered in this study.

4.3 Liftoff from the Zero Lower Bound

In this section we investigate under what constraints the central bank holding long-term bonds must raise the path of the nominal interest rates at a time of liftoff from the ZLB. Suppose that the economy is at the ZLB at time $T-1$ due to a demand shock that occurred in the past. We consider a situation in which at time T a negative shock disappears and the central bank sets the path of the nominal interest rates. For the sake of simplicity, we assume that after time T a demand shock does not occur again, so that $R_{T,\tau} = \beta^{\tau-T}$ for all $\tau \geq T$. In a perfect foresight equilibrium, the expectations hypothesis of the term structure holds;

$$Q_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{\rho^{\tau-t}}{\prod_{s=t}^{\tau}(1+i_s)}. \tag{4.28}$$

Substituting $i_{T-1} = 0$ into (4.13), the central bank's profits Ψ_T^C can be written as

$$\Psi_T^C = [(1+\rho Q_T) - Q_{T-1}]D_{T-1}^C. \tag{4.29}$$

This confirms that the central bank holding long-term bonds must incur losses on its balance sheet due to an unexpected decline in the price of long-term bonds. As explained, as long as $N_T > 0$, the solvency condition of the central bank is irrelevant.

We focus our attention on a case in which the resulting value of N_T is negative. Then, since at time T the central bank makes zero remittances, its net worth can be expressed as

$$N_T = \bar{N} + [(1 + \rho Q_T) - Q_{T-1}]D^C_{T-1}, \tag{4.30}$$

and its solvency condition is then given by

$$\frac{1}{P_T}[\bar{N} + \{(1 + \rho Q_T) - Q_{T-1}\}D^C_{T-1}] + \mathbb{E}_T \sum_{\tau=T}^{\infty} \beta^{\tau-T} \frac{i_\tau}{1 + i_\tau} L(i_\tau, Y) \geq 0. \tag{4.20b}$$

Under the above assumptions, we investigate under what constraints the central bank must set the path of the nominal interest rates at time T. We study three combinations of fiscal policy rule and remittances rule: an active fiscal policy and the full fiscal support rule, a passive fiscal policy and the deferred asset rule, and an active fiscal policy and the deferred asset rule. While the main focus of this study is the third case, we first consider the other two cases that help assess how an active fiscal policy and the deferred asset rule are combined to constrain monetary policy after liftoff from the ZLB.

4.3.1 Active fiscal policy and full fiscal support rule

First, we consider the combination of an active fiscal policy and the full fiscal support rule. In this case, the solvency condition of the consolidated government is relevant, and the action of the central bank is subject to only the solvency condition of the consolidated government (4.27a).

Proposition 1. *For any path of the nominal interest rates $\{i_\tau\}_{\tau=T}^{\infty}$, there exists a price level P_T such that:*

$$\frac{1}{P_T}[B_{t-1}^F + (1+\rho Q_t)D_{t-1}^F - \bar{N} - \Psi_t^C] = \sum_{\tau=T}^{\infty} \beta^{T-\tau}[\bar{t}_\tau + s(i_\tau, Y)] \tag{4.27b}$$

given the predetermined variables $\{B_{-1}, D_{T-1}, X_{t-1}, M_{t-1}\}$ *and the path of lump-sum taxes* $\{\bar{t}_\tau\}_{\tau=T}^{\infty}$.

Proposition 1 confirms that under the full fiscal support rule, the central bank can freely set the future path of nominal interest rates. This is the case even when fiscal policy is active. Regardless of the amount of losses that the consolidated government incurs due to a decline in the price of long-term bonds, its solvency can be maintained solely by a sufficient rise in the price level at a time of liftoff from the ZLB.

4.3.2 Passive fiscal policy and deferred asset rule

Next, we consider the combination of a passive fiscal policy and the deferred asset rule. In this case, the solvency condition of the whole government is irrelevant. The action of the central bank is subject to only its solvency condition (4.20b).

Proposition 2. *For any path of the nominal interest rates* $\{i_\tau\}_{\tau=T}^{\infty}$, *there exists an arbitrarily high price level* P_T *such that:*

$$\frac{N_T}{P_T} + \sum_{\tau=T}^{\infty} \beta^{\tau-T} s(i_\tau, Y) = 0, \tag{4.20c}$$

regardless of the value of N_T.

At time T, the central bank can maintain its solvency solely by allowing the current price level to increase to an arbitrarily high level, reducing the real value of its liabilities. It therefore can set any path of the nominal interest rates regardless of the amount of long-term bonds on its balance sheet. Note that Larger losses on the central bank's balance sheet due to an unexpected decline in the price of long-term bonds at a time of liftoff from the ZLB require a higher price level.

4.3.3 Active fiscal policy and deferred asset rule

Finally, we consider the combination of an active fiscal policy and the deferred asset rule, which is the main focus of this study. In this case the action of the central bank is subject to not only its solvency condition (4.20b) but also that of the whole government (4.26).

Proposition 3. *Under the combination of an active fiscal policy and the deferred asset rule, the path of the nominal interest rates* $\{i_\tau\}_{\tau=T}^{\infty}$ *must be consistent with the following condition:*

$$-\frac{\bar{N}/P_{T-1} + [(1+\rho Q_T) - Q_{T-1}]d_{T-1}^C}{\sum_{\tau=T}^{\infty} \beta^{T-\tau} s(i_\tau, Y)} \leq \Pi_T \leq \frac{(1+\rho Q_T) d_{T-1}^F}{\sum_{\tau=T}^{\infty} \beta^{\tau-T}\, \bar{t}_\tau}, \tag{4.31a}$$

given the predetermined variables $\{d_{T-1}^F,\ d_{T-1}^C,\ Q_{T-1},\ P_{T-1}\}$ *and the path of lump-sum taxes* $\{\bar{t}_\tau\}_{\tau=T}^{\infty}$.

When N_T turns to negative, the current price level must increase to ensure the central bank solvency. Larger losses on the central bank's balance sheet due to an unexpected decline in the price of purchases long-term bonds require a higher price level. Therefore, when the price level is determined by the fiscal factors, the central bank that holds long-term bonds above a certain threshold cannot freely raise the path of the nominal interest rates at a time of liftoff from the ZLB. Note that as long as $N_T > 0$, the central bank can set any path of nominal interest rate after liftoff as its solvency condition is irrelevant.

The three results in this chapter demonstrate that the combination of an active fiscal policy and the deferred asset rule imposes an upper bound on the amount of losses that the central bank is allowed to incur, and thus constrains monetary policy after liftoff from the ZLB. In the rest of this chapter, we further investigate the implications of this type of combination for monetary policy.

4.4 Monetary Policy Rules: Passive Monetary Policy

In the previous section, we considered a situation in which the central bank sets the path of the nominal interest rates at a time of liftoff. In the rest of this chapter, we explore richer implications of our analysis for policy by extending the model in some directions. First, we specify a rule according to which the central bank controls the short-term nominal interest rate after liftoff. Second, in Section 4.7, we further incorporate the possibility that the fiscal authority partially defaults on its bonds.

To address the issue of liftoff from the ZLB in a way consistently the existing literature, we specify a rule according to which the central bank controls the short-term nominal interest rate after liftoff. We assume that monetary policy follows a Taylor rule:

$$1 + i_t = \max\left[\frac{\Pi^*}{\beta}\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi}, 1\right], \tag{4.32}$$

where Π^* is the central bank's target inflation.

This section studies the case of a passive monetary policy in the sense of Leeper (1991); the central bank cannot increase the short-term nominal interest rate more than one-to-one in response to an increase in inflation. Leeper (1991) demonstrates that this is needed to stabilize inflation under an active fiscal policy. This case is worth considering in light of the current fiscal situation in Japan.

The Taylor rule, combined with the Euler equation (4.4) with the condition that $\xi_\tau = 1$ for all $\tau \geq T$, implies that equilibrium paths of inflation must satisfy the following difference equation:

$$\Pi_{t+1} = \max\left[\Pi^* \left(\frac{\Pi_t}{\Pi^*}\right)^\phi, \beta\right], \tag{4.33}$$

for all $\tau \geq T$. When $0 < \phi < 1$, this differential equation has the globally unique and stable steady state, as seen in Figure 4.1.

(4.33) implies that an equilibrium path of inflation, $\{\Pi_\tau\}_{\tau=T}^{\infty}$, can be indexed by the value of Π_T, and so does the path of the nominal interest rates. This enables us to express the price of long-term bonds at a time of liftoff and the PDV of seigniorage revenues as a function of Π_T:

$$Q_T \equiv Q(\Pi_T), \tag{4.34}$$

and

$$\sum_{\tau=T}^{\infty} \beta^{T-\tau} s(i_T, Y) \equiv S(\Pi_T, Y). \tag{4.35}$$

Note that $Q(\cdot)$ is decreasing in Π_T.

The following proposition characterizes the condition under which the central bank cannot overshoot inflation after liftoff from the ZLB.

Proposition 4. (Impossibility of overshooting inflation) *If the following two conditions are satisfied at time* T*, under the combination of an active fiscal policy and a passive monetary policy, any path of inflation that that starts from* $\Pi_T \in [\Pi^*, \infty)$ *disallowed as equilibria*

$$\Psi_T^C = [\{1 + \rho Q(\Pi^*)\} - Q_{T-1}] D_{T-1}^C < 0, \tag{4.36}$$

and

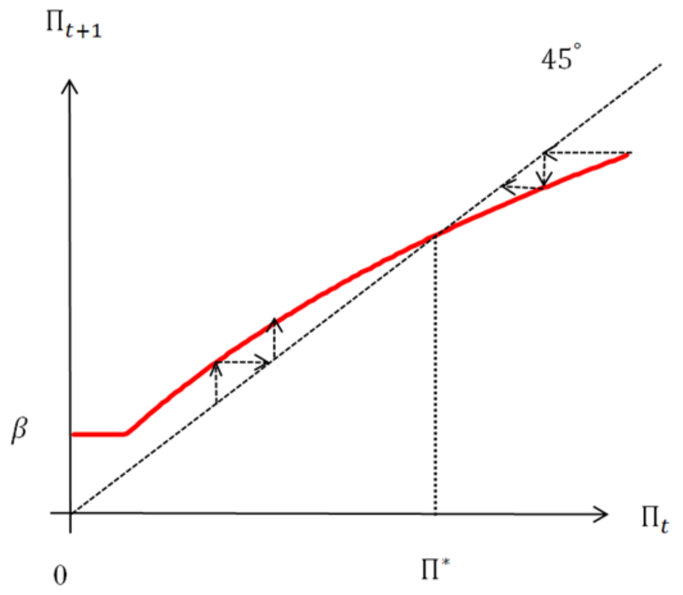

Figure 4.1. Globally unique and stable solution to the difference equation (4.33) under a passive monetary policy.

$$\Pi^* > \frac{[1+\rho Q(\Pi^*)]d^F_{T-1}}{\sum_{\tau=T}^{\infty}\beta^{T-\tau}\bar{t}_\tau}, \tag{4.37a}$$

given the predetermined variables $\{d^F_{T-1}, D^C_{T-1}, Q_{T-1}\}$, *and the path of lump-sum taxes* $\{\bar{t}_\tau\}_{\tau=T}^{\infty}$.

Condition (4.39) holds when the central bank purchases long-term bonds at a sufficiently high price when the economy is at the ZLB; $Q_{T-1} > 1 + \rho Q(\Pi^*)$. Inflation is bounded above whenever the central bank incurs losses on its balance sheet.

Right-hand side of (4.37a) is the upper bound on inflation at time T when the central bank achieves its inflation target. This upper bound is decreasing in Π^* since raising the central bank's target inflation decreases the price of long-term bonds, which induces negative wealth effects on the household as explained in Section 4.2.5. If Π^* exceeds this upper bound, as depicted in Figure 4.2, any paths of inflation that starts from $\Pi_T > \Pi^*$ are not equilibria.

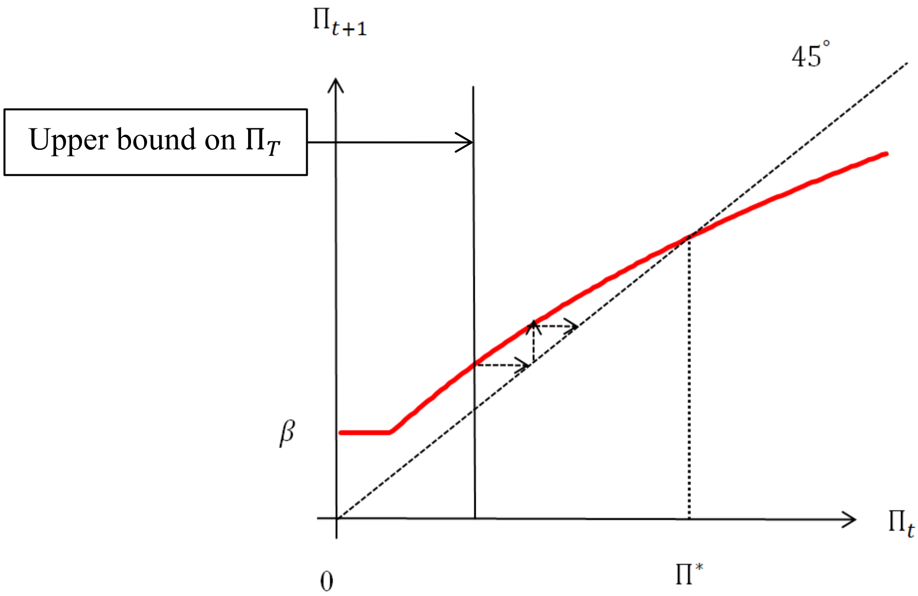

Figure 4.2. Dynamics of inflation after liftoff from the ZLB. Suppose that the conditions of Proposition 4 are satisfied.

Proposition 4 implies that by following a passive monetary policy the central bank may be able to achieve its inflation target at least in the long run after liftoff. In this case, however, inflation right after liftoff from the ZLB must undershoot the central bank's inflation target. Only paths on which the two solvency conditions are maintained are supported as equilibria. In other words, large-scale purchases of long-term bonds at the ZLB would not lead to an increase in inflation expectations at the ZLB.

4.5 Monetary Policy Rules: Active Monetary Policy

Next, we consider the case of an active monetary policy in the sense of Leeper (1991), we assume that the Taylor principle is satisfied. This assumption is more standard in the monetary policy literature. As seen in Figure 4.3, in the case of an active monetary policy the difference equation (4.33) has two stationary solutions, implying the possibility of the

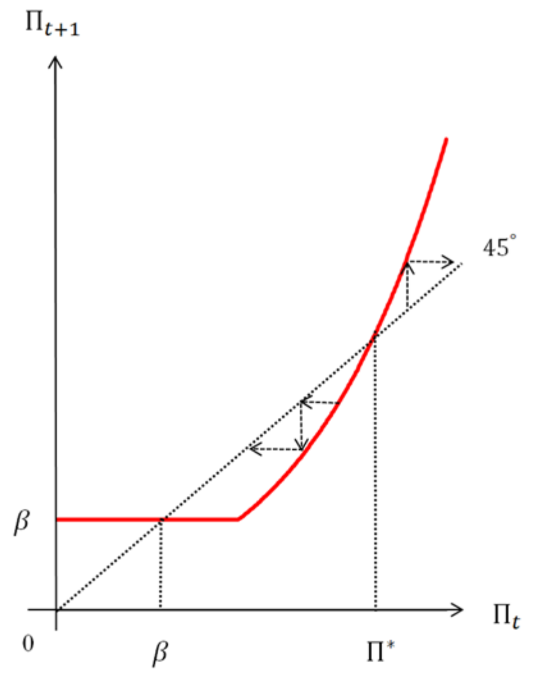

Figure 4.3. Multiple solutions to the difference equation (4.33) under an active monetary policy.

existence of two steady states. One is the intended steady state in which the central bank can achieve its inflation target. The other is the deflationary steady state characterized by deflation and the ZLB (see Benhabib et al., 2001, 2002). If the public expects that inflation is above target at time T, hyperinflation paths develop. If instead inflation at time T is expected to be below target, the economy converges to the deflationary steady state. The central bank achieves its inflation target if and only if the public expects that $\Pi_\tau = \Pi^*$ for all $\tau \geq T$.

The following proposition characterizes the condition under which the central bank cannot achieve its inflation target.

Proposition 5. (Impossibility of achieving inflation target) *If (4.36) and (4.37a) conditions are satisfied at time* T*, under active fiscal and monetary policies, the stationary and the inflationary solutions are disallowed as equilibria given the predetermined variables* $\{d_{T-1}^F,\ D_{T-1}^C,\ Q_{T-1}\}$*, and the path of lump-sum taxes* $\{\bar{t}_\tau\}_{\tau=T}^{\infty}$.

As explained right-hand side of (4.37a) is the upper bound on inflation at time T when the central bank achieves its inflation target. If Π^* exceeds this upper bound, as depicted in Figure 4.4, the intended steady state is ruled out as equilibrium. Under the two conditions, any paths starting from $\Pi_T > \Pi^*$ are also excluded since on these paths the price of long-term bonds is below $Q(\Pi^*)$. Only deflationary paths on which both solvency conditions are maintained are therefore supported as equilibria.

Note that there is a possibility that condition (4.37a) holds for any $\Pi_T \in [\beta, \infty)$. But this does not mean that there exists no equilibrium in the model. The reason is that there exists some $\widehat{\Pi}^* \in [\beta, \infty)$ such that the central bank can keep its profits non-negative by setting its inflation target $\widehat{\Pi}^*$; in this case, condition (4.36) does not hold, and the equilibrium price level is uniquely determined through the mechanism highlighted by the FTPL. For example, in the extreme case in which the central bank's inflation target is set to $\Pi^* = \beta$, Ψ_T^C is non-negative for any Q_{T-1}.[3]

This result has two implications for policy options on which a central bank may rely at the ZLB. First, our result immediately implies that a central bank that is exposed to the risk of losses on its balance sheet would not be able to achieve its inflation target after liftoff from the ZLB by actively controlling the short-term nominal interest rate. The second option, which is recurrent in policy debate, is raising the central bank's target inflation (Krugman, 1998). This has been thought to be effective in stimulating economic activities at the ZLB because an increase in inflation expectations at the ZLB leads to a decline in the real interest rate today. With inappropriate institutional arrangements between the fiscal authority and the central bank put in place, however, raising the central bank's inflation target would backfire because this decreases the upper bound on inflation at a time of liftoff.

[3] See Appendix for detail.

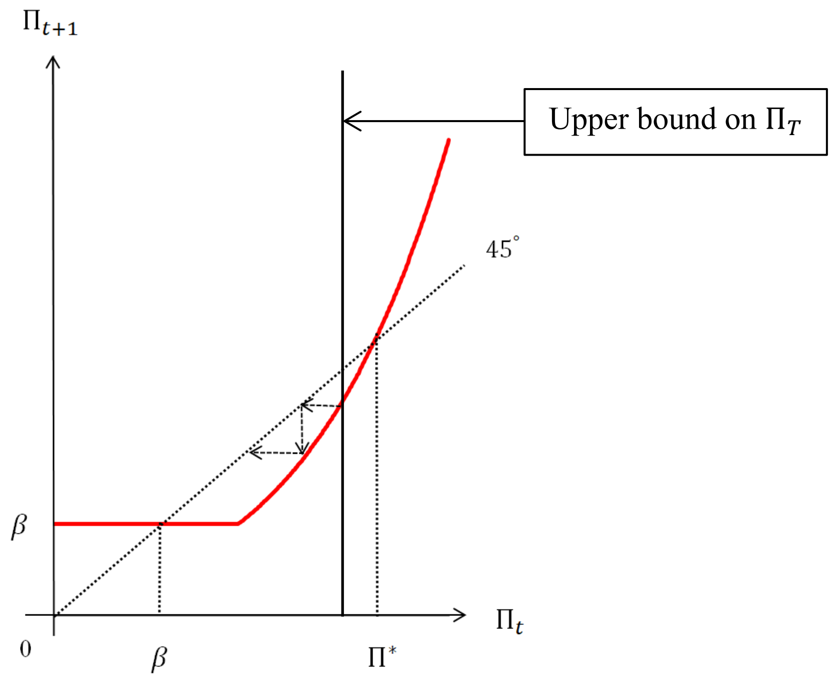

Figure 4.4. Dynamics of inflation after liftoff from the ZLB. Suppose that the conditions of Proposition 5 are satisfied.

4.6 Numerical Examples

In this section, we present numerical examples to investigate the quantitative importance of Proposition 3 using Japanese data. First, suppose that the Bank of Japan exits from the QQE at the end of fiscal year 2020. We calculate the central bank's profits and the upper bound on inflation at a time of liftoff. Second, we conduct a similar analysis, supporting that the Bank of Japan exits from the QQE at the end of fiscal year 2014 as originally planned in April 2013. A comparison between the two cases makes clear how the effects of the QQE have changed over time.

We take the model's frequency to be annual and adopt $\beta = 0.99^4$. The duration of long-term bonds is assumed to be 10 years, implying that $\rho \cong 0.955$.[4] A constant endowment Y,

[4] When calculating this value, we assume that inflation is stable at 2%.

which is regarded as real GDP, is normalized to 1. The value of lump-sum taxes is set such that in a steady state with two percent inflation the debt-to-GDP ratio remains at 2.4.

4.6.1 Exit from the QQE in fiscal year 2020

First, we consider a case in which the Bank of Japan exits from the QQE at the end of fiscal year 2020. The outstanding debt-to-GDP ratio at the beginning of a time of liftoff is set to 2.4. The share of outstanding government bonds held by the central bank is set to 0.45. Theprice of long-term bonds in a period right before liftoff Q_{T-1} is calculated by using the average of daily data on 10-year Japanese government bond yield in fiscal year 2020, which is approximately 0.037%.

Figure 4.5 displays a numerical example of the central bank's profits and the upper bound on inflation at a time of liftoff from the ZLB. We report the results for alternative values of Π^*. The left panel shows that a slight increase in interest rates is enough to turn the central bank's profits negative. This is due to a substantially high value of Q_{T-1}. The green line in the right panel confirms that the upper bound decreases as the target inflation rate increases. This line crosses the 45-degree line at the target inflation rate of approximately $-1\,\%$, implying that the central bank cannot stabilize inflation at a positive level as long as it follows the Taylor principle.

4.6.2 Exit from the QQE in fiscal year 2014

Second, we suppose that the Bank of Japan exits from the QQE at the end of fiscal year 2014. The outstanding debt-to-GDP ratio at the beginning of a time of liftoff is set to 2.3. The share of outstanding government bonds held by the central bank is set to 0.2. The average of daily data on 10-year Japanese government bond yield in fiscal year 2014 is approximately 0.48%.

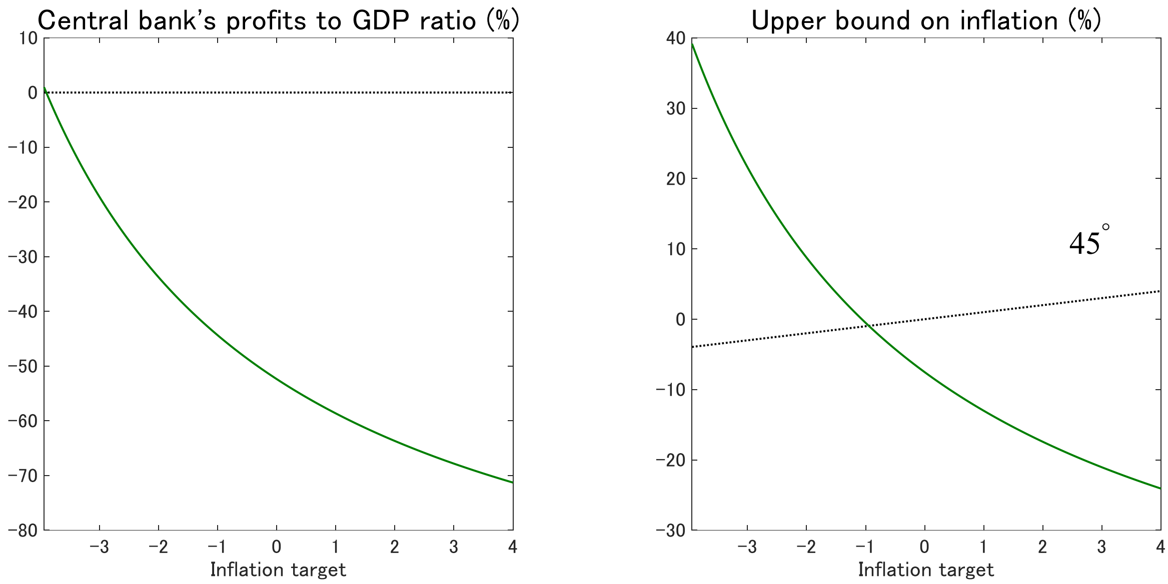

Figure 4.5. A numerical example of central bank's profits and the upper bound of inflation at alternative values of Π^*. Exit from the QQE at the end of fiscal year 2020.

Figure 4.6 depicts numerical examples of the central bank's profits and the upper bound on inflation at a time of liftoff from the ZLB. We report results for both cases, and red lines represent the results for the case in which the Bank of Japan exits the QQE at the end of fiscal year 2014. Comparing the two cases, we can show the following two results. First, losses on the Bank of Japan's balance sheet becomes larger at the end of fiscal year 2020. This is because of the increase in the amount of long-term bonds held by the Bank of Japan and the price of long-term bonds at the ZLB. Second, while the upper bound on inflation at a time of liftoff becomes lower for any value of $\Pi^* \in [0.99^4, 1.04]$ at the end of fiscal year 2020, there is little change in the upper bound on inflation. A key to understanding this is that in our model whenever the central bank incurs losses on its balance sheet, inflation is bounded above by the right-hand side of (4.37a), which depends only on fiscal factors. Therefore, if there is no change to fiscal factors, the upper bound on inflation does not change regardless of the amount of losses on the central bank's balance sheet.

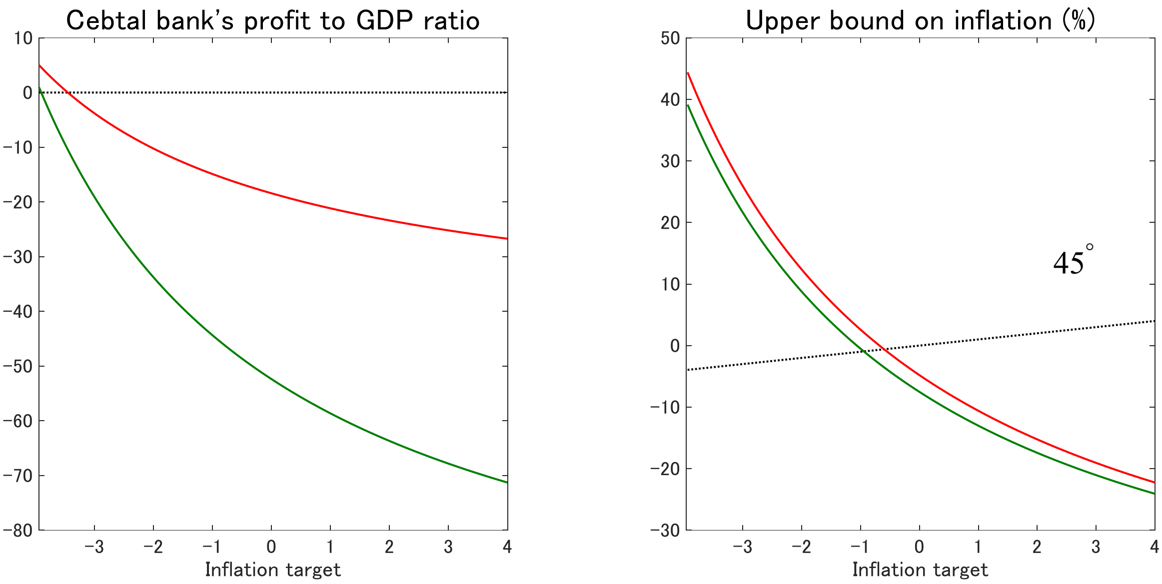

Figure 4.6. Numerical examples of central bank's profits and the upper bound of inflation at alternative values of Π^*. Green lines: Exit from the QQE at the end of fiscal year 2020. Red lines: Exit from the QQE at the end of fiscal year 2014.

The decrease in the upper bound at the end of fiscal year can be attributed to the increase in the price of long-term bonds, which reduces the amount of bonds outstanding at the end of fiscal year 2020. However, this effect on the upper bound on inflation does not significantly differ between the results in the two cases. The reason is that the 10-year Japanese government bond yield was already relatively low in fiscal year 2014.

The results in this section suggest that the mechanism described in Proposition 5 would be quantitatively important in considering the Bank of Japan's strategy to exit from the QQE.

4.7 Partial Default at a Time of Liftoff

A way that the central bank may be able to take to stabilize inflation after liftoff from the ZLB is forcing the fiscal authority to default on its bonds. As discussed by Kocherlakota (2011), the central bank achieves independent control of inflation even under the case of an active fiscal policy if it is willing to allow the fiscal authority to default on its bonds.[5] In this section we ask the question: does this result hold in our situation of interest? To do so, we assume that both fiscal and monetary policies are active.

Specifically, in this section, we consider one-off default at a time of liftoff. It is assumed that from period $T+1$ onward, the fiscal authority never defaults on its bonds again; $\delta_\tau = 0$ for all $\tau \geq T+1$. Under this assumption, we study how the equilibrium default rate δ_T is determined.[6] We first analyze the case in which the central bank does not purchase long-term bonds at the ZLB and then turn to the case with operations.

4.7.1 No long-term bonds on the central bank's balance sheet

The assumption that the central bank does not hold long-term bonds (i.e., $D^C_{T-1} = 0$) implies that $\Psi^C_\tau = T^C_\tau \geq 0$ for all $\tau \geq T$. The equilibrium price level at a time of liftoff is thus determined to maintain a solvency condition of the consolidated government. The government solvency condition at time T is written as:

$$\frac{1}{P_T}[X_{T-1} + M_{T-1} + B_{T-1} + (1-\delta_T)\{1+\rho Q(\Pi_T)\}D_{T-1}] \\ = S(\Pi_T, Y) + \sum_{\tau=T}^{\infty} \beta^{T-\tau}\bar{t}_\tau. \tag{4.27c}$$

Let δ^* denote a default rate at time T when the central bank achieves its inflation target after liftoff, so that δ^* must satisfy the following:

$$\delta^* \equiv 1 - \frac{\Pi^*[S(\Pi^*) + \sum_{\tau=T}^{\infty} \beta^{T-\tau}\bar{t}_\tau] - (X_{T-1} + M_{T-1} + B_{T-1})/P_{T-1}}{\{1+\rho Q(\Pi^*)\}D_{T-1}/P_{T-1}}. \tag{4.38}$$

5 Uribe (2006) provides a more formal analysis of a relation between price stability and default on government bonds.
6 Here we do not examine how anticipation of partial default at a time of liftoff from the ZLB influences the economy at the ZLB. We leave this for future research.

Proposition 6. *If $\delta^* \in (0,1)$ then the central bank that follows the Taylor principle can achieve its inflation target (the only intended steady state is supported as equilibrium) by allowing the fiscal authority to default on its bonds.*

This result confirms that the discussion by Kocherlakota (2011) also holds in a setting with long-term bonds. Even if both fiscal and monetary policies are active, the government can achieve price stability by imposing explicit taxation on government bondholders, which depresses their demand for goods. As long as $\delta^* \in (0,1)$, inflation is completely stabilized.

4.7.2 Implication of purchases of long-term bonds

As discussed above, the central bank holding long-term bonds can incur losses on its balance sheet. We focus our attention on a case in which the resulting value of Ψ_T^C is negative. In addition, suppose the target inflation rate Π^* satisfies the following condition:

$$-\frac{\bar{N}/P_{T-1} + [\{1+\rho Q_T(\Pi^*)\} - Q_{T-1}]d_{T-1}^C}{S(\Pi^*)} \leq \Pi^* \leq \frac{[1+\rho Q(\Pi^*)]d_{T-1}^F}{\sum_{\tau=T}^{\infty}\beta^{T-\tau}\bar{t}_\tau}. \quad (4.31b)$$

In the case of no default the intended steady state is supported as equilibrium, but other paths can also be equilibria. As shown in the above subsection, if the central bank does not hold long-term bonds, default on bonds can be used to exclude the deflationary and inflationary paths. This is not the case if the central bank holds long-term bonds on its balance sheet:[7]

Proposition 7. *If the default rate at time T is sufficiently high such that*

[7] In the opposite case $\left(\Pi^* \leq \frac{(1-\delta_T)[1+\rho Q_T(\Pi^*)]d_{T-1}^F}{\sum_{\tau=T}^{\infty}\beta^{T-\tau}\bar{t}_\tau}\right)$, we cannot trim deflationary and inflationary paths, as long as

$$-\frac{\bar{N}/P_{T-1} + [\{1+\rho Q_T(\Pi^*)\} - Q_{T-1}]d_{T-1}^C}{S(\Pi^*)} \leq \Pi^*$$

$$\Pi^* > \frac{(1-\delta_T)[1+\rho Q_T(\Pi^*)]d_{T-1}^F}{\sum_{\tau=T}^{\infty}\beta^{T-\tau}\bar{t}_\tau}, \tag{4.37b}$$

given the predetermined variables d_{T-1}^F and the path of lump-sum taxes $\{\bar{t}_\tau\}_{\tau=T}^{\infty}$, the intended steady state and inflationary paths are disallowed as equilibria.

The upper bound on inflation at a time of liftoff from the ZLB, which is given by the right-hand side of (4.37b), is decreasing in the default rate δ_T. A large haircut is thus followed by deflation. This is because default, which is explicit taxation on government bondholders, induces negative wealth effects on the household, which puts downward pressure on the price level further. The result suggests that, even if government bonds are defaultable, purchases of long-term bonds by the central bank would undermine the ability of monetary policy to stabilize inflation.

4.8 Concluding Remarks

In this chapter, we have developed a framework for thinking about the solvency of the whole government and that of the central bank. Using the model, we investigated how a lack of public confidence in fiscal sustainability constrains the Bank of Japan's strategy to exit from the QQE. The main result was that the central bank that holds long-term bonds above a certain threshold cannot raise the path of the nominal interest rates at a time of liftoff from the ZLB. By analyzing the extended models, we also showed that massive purchases of long-term bonds at the ZLB would undermine the abilities of monetary policy to stabilize inflation.

The main message of this chapter is that a lack of public confidence in fiscal sustainability, combined with a commitment of the fiscal authority not to provide financial support for the central bank, would undermine the ability of the central bank that engages in large-scale purchases of long-term bonds at the ZLB to stabilize inflation after liftoff. A similar result can be obtained even when introducing the possibility of partial default on government bonds at a time of liftoff. When considering the effects of purchases of long-term bonds, we should focus on the relation between the fiscal authority and the central bank.

While the perspective of this chapter was on looking into the future, the results of our study could also help understand how the QQE has influenced the Japanese economy. Since April 2013, the Bank of Japan has intended to bring Japanese inflation up to 2 % mainly by reducing long-term interest rates. Despite a substantial decline in long-term interest rates, inflation has been below target. Our study offers a possible explanation for these observations. In our model, massive purchases of long-term bonds at the ZLB would contribute to a decline in long-term interest rates, which is consistent with the standard view explained in Chapter 1. However, unlike in the standard story, in our model this does not necessarily lead to an increase in inflation expectations at the ZLB. Rather, the central bank holding long-term bonds is forced to lower interest rates because inflation is bounded above. Future research can explore this possibility further by constructing a richer model augmented to include the mechanism highlighted in this chapter.

4.A Appendix

In this appendix, we conform that when the central bank's inflation target is set to $\Pi^* = \beta$, Ψ_T is non-negative for any Q_{T-1}. To this end, we show that the following is satisfied: $1 + \rho Q(\beta) - Q_{T-1} \geq 0$ for any Q_{T-1}.

Because of the presence of the ZLB, Q_t is bounded above by $(1-\rho)^{-1}$:

$$Q_t = \sum_{\tau=t}^{\infty} \frac{\rho^{\tau-t}}{\prod_{s=t}^{\tau}(1+i_s)}$$

$$\leq \sum_{\tau=t}^{\infty} \rho^{\tau-t}$$

$$= \frac{1}{1-\rho}.$$

Since $Q(\beta)$ also takes $(1-\rho)^{-1}$, it can be shown that Ψ_T^C is non-negative:

$$\Psi_T \geq 1 + \frac{\rho}{1-\rho} - \frac{1}{1-\rho}$$

$$= 0.$$

Chapter 5

Fragmented Fiscal Policymaking in a New Keynesian Model

5.1 Introduction

5.1.1 Motivation

As explained in Chapter 1, the existing literature has emphasized that fiscal and monetary policies are linked through the consolidated government budget constraint and public expectations about how current and future policies interact play an important role in shaping the macroeconomy. In particular, one of the most important policy implications of the works of Leeper (1991) and Woodford (2001) is that when the fiscal authority takes no responsibility for stabilizing debt, the central bank must lower its sensitivity to inflation to stabilize the macroeconomy.[1]

In this chapter, we develop a dynamic general equilibrium model augmented to include a particular type of political economic aspect of fiscal policymaking. In the model used in the literature, fiscal policy is decided by a single policy maker in a centralized manner. However, this assumption is not necessarily realistic. This is because, in reality, several interest groups

[1] Basically, to support the standard story that the central bank can achieve price stability by choosing a sufficiently high sensitivity to inflation as an equilibrium, we must assume that the fiscal authority accepts responsibility for stabilizing debt.

are involved in a process of fiscal policymaking, and thus they would not be able to coordinate to achieve a certain conduct of fiscal policy. Specifically, our study is motivated by the possibility that when monetary policy is used to stabilize debt, it would induce competition among interest groups for fiscal resources, delaying debt stabilization. Moreover, due to such political constraints, the central bank in turn has to conduct policy considering how it influences interest groups' incentives; consequently, this would distort socially optimal conduct of fiscal and monetary policy. In standard dynamic stochastic general equilibrium (DSGE) models, fiscal policy is assumed to be decided by a benevolent planner, so that these political issues do not arise.

5.1.2 Setup

Thus motivated, we construct a New Keynesian model in which fiscal policymaking is influenced by two fragmented interest groups and interactions among interest groups and the central bank have implications for equilibrium dynamics. This government structure induces a common pool problem when interest groups expect the central bank to accommodate their free-riding behaviors. The main objective of this chapter is to study how coordination failure between interest groups distorts the optimal conduct of fiscal and monetary policy. Particular attention is paid to the roles the central bank is forced to play in a stabilization process.

To this end, we formalize a linear-quadratic dynamic game relying on the technique developed in the literature on optimal monetary policy (Clarida et al., 1999; Woodford, 2003). When investigating optimal monetary policy in a New Keynesian model, it is common to make the following three assumptions. The first assumption is the availability of lump-sum taxes; the fiscal authority has access to non-distortionary tax instruments. The second assumption is the presence of a benevolent fiscal policy maker; the fiscal policy decisions are made by a benevolent planner in a centralized manner. The third assumption is a passive fiscal policy; a planner adjusts lump-sum taxes to ensure government solvency given any current and future monetary policy and resulting macroeconomic variables. Based on these assumptions, we can characterize optimal monetary policy with no regard for fiscal consequences of alternative monetary policies, as we know that their budgetary impacts are neutralized by an appropriate adjustment of lump-sum taxes. However, to study the macroeconomic outcome of political dynamics as mentioned above, we have to deviate from the three common assumptions regarding fiscal policy. Each of the three deviations explained below characterizes a key feature of the model presented in this study.

First, we deviate from the assumption of the availability of lump-sum taxes. Instead, we adopt an assumption of fixed lump-sum taxes. Specifically, the fiscal authority cannot adjust lump-sum transfer payments due to political reasons and can only choose labor income taxes, which are distortionary. An increase in the labor income tax increases marginal costs and thus produces inflationary pressure.

This assumption implies that the Ricardian equivalence does not hold in our model. To stabilize accumulated debt, the government has to raise distortionary taxes and/or increase inflation, reducing the real value of outstanding debt; both involve welfare costs. In this environment, the government aiming to minimize welfare losses due to debt stabilization has to consider how fiscal and monetary policies should be coordinated.

This modeling is in line with the literature that studies optimal fiscal-monetary policy in a New Keynesian model in which lump-sum taxes are unavailable. Benigno and Woodford (2003, 2007) develop a technique to use a linear-quadratic method to characterize optimal fiscal and monetary policy in this type of New Keynesian model. In addition, Leeper and Zhou (2021) extend Benigno and Woodford's (2003) model to a case with long-term government bonds to investigate the roles played by debt maturity in optimal policy. Specifically, the model presented in this study builds on that of Leeper and Zhou (2021). In their model, the central bank and the fiscal authority are coordinated to maximize social welfare. The important difference between our model and their model is that our main objective is to study fiscal and monetary policies from a positive perspective.[2] In our model, fiscal policy decisions are made by interest groups.

Specifically, the second key feature of our model is an assumption of fragmented fiscal policymaking. We assume that tax policy decisions are made in a decentralized manner, allowing each interest group to determine tax rates. We also assume that the government is weak and has to meet the requirements from the interest groups at face value. Our model is populated by identical households, which are divided into two symmetric groups. We embed such a structure into our model following the literature on a monetary union DSGE model. The baseline model in the literature is developed by Benigno (2004).[3]

In the politico-economic literature, a common pool problem that arises when fiscal policymaking is influenced by fragmented interest groups is regarded as one of main causes of socially excessive public spending or public debt stock (see, e.g., Weingast et al.,1981; Chari and Cole, 1993; von Hagen and Harden, 1995; and Velasco, 2000). Velasco (2000) develops an infinite-horizon model in which this type of "common pool" problem causes excessive debt accumulation. We differ from Velasco (2000) in studying how "common pool" problem distorts optimal stabilization policies. This study assumes that in a steady state a benevolent planner would choose optimal allocation, regardless of whether the interest groups coordinate. Our setup is along the lines of Leeper (1991). In his model, fiscal and

[2] There are also two differences from them. First, while their focus is on a distorted steady state, we assume that steady state distortions from the monopolistic competition are eliminated by an appropriate tax policy following the literature on the optimal monetary policy in a New Keynesian model. This is due to the difference in the objective of research. Benigno and Woodford (2003) and Leeper and Zhou (2021) study optimal policy when lump-sum taxes not available. In contrast, we aim to analyze the consequences of coordination failure between interest groups. Second, while they characterize optimal policy from a timeless perspective, we focus on a Markov-perfect equilibrium as explained later.

[3] See Beetsma and Jensen (2005), Gali and Monacelli (2008), Ferrero (2009), and Farhi and Werning (2017) for studies that extend Benigno's (2004) model to explicitly incorporate the presence of fiscal authorities in member countries. Nakamura and Steinsson (2014) also use a similar framework to examine a closed economy with two regions.

monetary policies are coordinated to stabilize the macroeconomy around a steady state. He shows that the response of the economy to debt shocks differs depending on whether inflation is used to reduce the real value of government bonds to its steady-state level.

Finally, we assume a fiscal leadership, rather than a passive fiscal policy, in order to consider an environment in which coordination failure between the interest groups has consequences for an equilibrium path on which accumulated debt is stabilized. To do this, we analyze a game in which the interest groups are the leader and the central bank is the follower. Each period is divided into two stages. At the first stage, the two interest groups simultaneously choose tax rates imposed on their own households. At the second stage, after observing the interest groups' decisions, the central bank sets the short-term nominal interest rate. At the first stage of this game each interest group aim to maximize welfare of their own households, whereas the central bank conducts monetary policy to maximize social welfare. As prices are rigid, loss function of a benevolent planner depends on variations of economy-wide inflation and output gap. Conversely, loss function of the interest groups depends on group-specific inflation and output gap.

An important feature of our model is that the interest groups and the central bank interact to shape the macroeconomy. The interest groups' decisions on taxes affect fiscal surpluses and thus impose a restriction on possible choices for the central bank in the second stage. The action of the central bank in turn affects economy-wide inflation and output. In the model, monetary policy has effects on the current state of the economy through two channels. The first effect is due to a change in the real interest rate as is standard in the New Keynesian literature, and the second one is via a change in the price of long-term bonds. By changing the price of long-term bonds, a change in the future path of the nominal interest rate affects the current state of the economy. In this setting, the decision of each interest group has externality through its effect on monetary policy. Accordingly, whether they internalize this externality plays a central role in examining fiscal and monetary policy in our model.

In this setup, we analyze a non-cooperative game between the two interest groups. We focus on a Markov-perfect equilibrium assuming that the players cannot credibly commit to their future actions. In each period, the players re-optimize their behaviors given observed state variables recognizing a game with similar structure will be repeated in the future. We compare paths of variables that a benevolent planner would choose and those resulting from a non-cooperative game.

5.1.3 Baseline analysis

Using the above model, we first examine how the economy in which the debt stock is initially above a steady-state level is stabilized over time. The results of the numerical analysis are summarized as follows. First, it is socially optimal to increase the labor income tax and inflation to decrease government debt. A benevolent planner would choose the timing of the tax collection and inflation to maximize social welfare. Because of the presence of long-term bonds, variations in future interest rates can be used to optimally choose the timing of tax collection and inflation.

Second, in a non-cooperative game, a resulting response of the tax rate becomes negative. Against the inflationary pressure from accumulated debt, interest groups find it optimal to lower the tax rate. They aim to attenuate upward pressure on group-specific inflation by decreasing marginal costs. The interest groups do not fully internalize that their actions affect economy-wide inflation through their budgetary effects. This free-riding activity puts socially excessive downward pressure on marginal costs and slows down debt stabilization excessively. The former in turn puts downward pressure on current inflation, whereas the later delays the timing of inflation. Overall, the free-riding activity of the interest groups causes a positive response of inflation excessively gradual. Then, the central bank is forced to make a response of the nominal interest rate excessively gradual. This is required to smooth the real interest rate and thus output gap given a gradual response of inflation.

We also examine how a change in debt maturity affects the results. In the model, debt maturity is a key determinant of how the government can spread the distortions due to debt stabilization over time. Specifically, shortening debt maturity decreases the sensitivity of the price of long-term bonds to changes in future interest rates, and therefore weakens the government's ability to spread distortions over time. We present a numerical example to demonstrate that when debt maturity is decreased, a benevolent planner would make a positive response of economy-wide inflation sharper to stabilize debt rapidly. In addition, we numerically show that shortening debt maturity provides the interest groups a stronger incentive to free-ride. Against the upward pressure on economy-wide inflation coming from shortening debt maturity, the interest groups find it optimal to cut tax rates further to attenuate upward pressure on group-specific inflation.

5.1.4 Negative demand shock

In the rest of this chapter, we introduce a large negative shock to the natural interest rate that causes the zero lower bound (ZLB) to bind. As in the baseline case, the initial stock of debt is above a steady-state level. The main difference is that the economy is hit by a temporary negative demand shock. The goal is to examine how a negative demand shock, along with the presence of the ZLB, changes the results in the baseline case in which the natural interest rate remains at a steady-state level.

The model presented in this study is useful for addressing issues of unconventional monetary policy measures. Fiscal-monetary interactions have received attention especially during the recession after the onset of the global financial crisis, which is characterized by zero interest rates and sharp increases in the debt-to-GDP ratio in many developed countries.

Indeed, then central bankers expressed concern about fiscal sustainability (e.g., Bernanke, 2009; Shirakawa, 2012). Moreover, it seems that some have considered several fiscal consequences which purchases of long-term bonds may have as one of the most harmful possible side effects of unconventional monetary policy measures (see, e.g, Plosser, 2012; Bank for International Settlements, 2014; Okina, 2015; Hayakawa, 2016).

First, we study how the economy responds to a negative demand shock assuming that the nominal interest rate can be lowered below zero. The government then can use not only fiscal policy but also monetary policy to respond to a negative demand shock. The objective of studying a case without the ZLB is to assess how important the presence of the ZLB is in our model separately from the effects of a negative shock on the macroeconomy.

In the standard New Keynesian model in which the fiscal authority adjusts lump-sum taxes to neutralize budgetary effects of monetary policy, the central bank that is allowed to lower the nominal interest rate below zero can achieve complete stabilization of output gap and inflation even when the economy is hit by a negative demand shock. Monetary policy can mimic the natural interest rate regardless of whether this rate is positive. We numerically show that a similar result can be obtained in our model; lowering the nominal interest rate below zero is effective in mitigating a contractionary effect of a negative demand shock.

Next, we study the case with the ZLB in which the nominal interest rate cannot be lowered below zero. While the nominal interest rate is temporarily stuck at zero, the economy eventually converges to a steady state. When analyzing how this type of negative demand shock affects the macroeconomy, public expectations about future conduct of fiscal and monetary policy play an important role. In particular, as discussed in Chapter 1, the literature has emphasized the effectiveness of forward guidance. In our model, however, the players cannot credibly commit to their future actions, and therefore a time inconsistent commitment to future conduct of monetary policy has no effect on the economy.

Meanwhile, fiscal policy at the ZLB involves a change in public expectations about future macroeconomic conditions and fiscal and monetary policy because it affects the stock of government debt at a time of liftoff from the ZLB. Accordingly, to examine its effects, it is important to consider how fiscal policy responds to a negative demand shock. First, we analyze the socially optimal conduct of fiscal policy at the ZLB and then study how this is distorted by coordination failure between the interest groups. Subsequently, we numerically show that coordination failure causes a recession at the ZLB more severe.

Finally, we analyze the effects of purchases of long-term bonds on the economy at the ZLB. This type of operation shortens the maturity structure of government debts held by the public and thus affects the macroeconomy in our model. The main results of this analysis are twofold. First, if the interest groups coordinate, then the operation has an expansionary effect on the economy at the ZLB. Second, coordination failure between the interest groups weakens this expansionary effect of purchases of long-term bonds.

5.1.5 Related literature

This work is related to discussion on free-riding problems that would arise in the European Monetary Union; that is, member countries delegate their monetary policy to the European Central Bank but remain authorized to decide fiscal policy. It has been pointed out that this structure would induce a free-riding behavior of the member countries and thus prevent the European Central Bank from achieving its objective.[4] In models developed to address these issues, the fiscal authority in each member country honors its budget constraint.[5] The key feature of our setup is that the interest groups and the central bank share the consolidated government constraint. They interact with each other given that their decisions have

[4] See, for example, Dixit and Lambertini (2001), Uhlig (2002), Cooper and Kempf (2004), Chari and Kehoe (2007), and Kirsanova et al. (2018).
[5] Such a setup is motivated by the presence of the Pact of Stability and Growth, which requires the member countries to maintain fiscal discipline.

budgetary implications.

This study is closely related to Bhattarai et al. (2015).[6] They analyze a New Keynesian model similar to ours in which the central bank acts in a discretionary manner. They demonstrate that purchases of long-term bonds can prevent deflation and a recession caused by a negative demand shock by mitigating the time-inconsistency problem of forward guidance. Our result that if the interest groups coordinate, then purchases of long-term bonds have an expansionary effect on the economy at the ZLB is consistent with theirs. However, there are two main differences from them. First, while they examine optimal fiscal-monetary policy, our main focus is on a non-cooperative game between the interest groups. Second, the government in their model optimally chooses their policy instruments only after liftoff from the ZLB. At the ZLB, it adjusts taxes to keep the stock of government debt constant given current inflation that is implied by inflation and output gap at a time of liftoff in the future. Conversely, in our study, the interest groups optimally choose their actions, regardless of whether they coordinate, not only after liftoff but also while the economy is at the ZLB.

5.1.6 Layout

The remainder of this chapter is structured as follows. Section 5.2 introduces the model. In Section 5.3, we log-linearize equilibrium conditions and derive a quadratic welfare loss function. Section 5.4 presents a formal definition of a Markov-perfect equilibrium and then outlines the optimal policy problems. In Section 5.5, a baseline case is numerically analyzed. Section 5.6 introduces a negative demand shock and explains the setup of a game in a case without the ZLB. In Section 5.7, we numerically analyze a game in the case without the ZLB. Section 5.8 analyzes the case with the ZLB. In Section 5.9, we examine the effects of purchases of long-term bonds on the economy at the ZLB. Finally, Section 5.10 concludes.

[6] Their model builds on the model of Eggertsson (2006).

5.2 Structure of the Model

There is a continuum of economic agents. Each agent is a monopolist in producing a single differentiated good. The economy is divided into two symmetric interest groups G_i for $i = 1$ or 2. The population of segment $[0.0.5) \equiv G_1$ belongs to group1, while segment $[0.5,1] \equiv G_2$ belongs to group 2. The agents demand goods produced in both the groups. Each interest group chooses labor tax rate imposed on agents in its own group. The central bank controls the short-term nominal interest rate.

5.2.1 Households

Household j has the following utility function

$$\mathbb{E}_0 U^j = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t^j - \frac{y_t(j)^{1+\eta}}{1+\eta}\right] \xi_t, \tag{5.1}$$

where C_t^j is consumption, $y_t(j)$ is the production of the differentiated good produced by agent j, $\mathbb{E}_t$ is the expectation operator conditional upon the information set available at time t, $\beta \in (0,1)$ is the discount factor, and ξ_t is a demand shock common across the interest groups. Until Section 5.6, $\xi = 1$ in every period.

The consumption index C^j is defined as

$$C^j \equiv \frac{1}{2}\left(C_1^j \cdot C_2^j\right)^{\frac{1}{2}}, \tag{5.2}$$

where C_i^j is the consumption index across the continuum of goods produced by interest group i. C_i^j is in turn defined as

$$C_i^j \equiv \left[2^{\frac{1}{\varepsilon}} \int_{l \in G_i} c^j(l)^{\frac{\varepsilon-1}{\varepsilon}} dl\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{5.3}$$

where $\varepsilon > 1$ parameterizes the elasticity of substitution across goods produced in interest group j.

The consumption price index, P_t, associated with (5.3) is defined as

$$P \equiv (P_1 \cdot P_2)^{\frac{1}{2}}. \tag{5.4}$$

Here, P_i is the producer price index in interest group i, defined as

$$P_i \equiv \left[2 \int_{l \in G_i} p_i(l)^{1-\varepsilon} dl\right]^{\frac{1}{1-\varepsilon}}, \tag{5.5}$$

where $p_i(l)$ is the price of goods produced by firm l. As the households in the economy are homogeneous and purchasing power parity holds, they face same consumption price index regardless of their group affiliation. The structure of preference implies that the demand for goods produced by household j in interest group i is given by

$$y_i(j) = \left[\frac{p(j)}{P_i}\right]^{-\varepsilon} \left(\frac{P_i}{P}\right)^{-1} C^W, \tag{5.6}$$

where $C^W = \int_0^1 C^l \, dl$ is the economy-wide consumption. Given this demand function, we can express aggregate demand in interest group i as

$$Y_i = \left(\frac{P_i}{P}\right)^{-1} C^W. \tag{5.7}$$

Household j in interest group i is subject to a sequence of flow budget constraints

$$\begin{aligned} &P_t C_t^j + D_t^j + Q_t B_t^j \\ &\quad \leq (1 + i_{t-1}) D_{t-1}^j + (1 + \rho Q_t) B_{t-1}^j + \left(1 - \tau_{i,t}\right) p_t(j) y_t(j), + P_t Z_t, \end{aligned} \tag{5.8}$$

where P_t is the price level, i_t is the risk-free nominal interest rate, $\tau_{i,t}$ is the rate of production tax imposed on households in interest group i, and Z_t is lump-sum transfers payments. D_t^j is a one-period bond, and B_t^j is a long-term bond with price Q_t. In particular, long-term bonds are assumed to be perpetuities. A bond issued at time t pays ρ^k in nominal terms at time $t+k+1$, as in Woodford (2001), for each $k \geq 1$ and $0 < \rho < 1$.

The household maximizes its expected utility (5.1) subject to (5.8), yielding the following first-order conditions:

$$\left(1-\tau_{i,t}\right)\frac{1}{C_t^j} = \frac{\varepsilon}{\varepsilon-1}[y_t(j)]^{\eta}, \tag{5.9}$$

$$\frac{1}{1+i_t} = \beta \mathbb{E}_t\left[\frac{C_{t+1}^j}{C_t^j}\frac{\xi_t}{\xi_{t+1}}\Pi_{t+1}^{-1}\right], \tag{5.10}$$

$$Q_t = \beta \mathbb{E}_t\left[\frac{C_{t+1}^j}{C_t^j}\frac{\xi_t}{\xi_{t+1}}\Pi_{t+1}^{-1}\right](1+\rho Q_{t+1}). \tag{5.11}$$

(5.9) is the intratemporal optimality condition for labor supply, (5.10) is the consumption Euler equation, and (5.11) is implied by the absence of opportunities to arbitrage between one-period and long-term bonds.

5.2.2 Firms

Monopolistically competitive firms produce differentiated goods. We introduce nominal rigidities following Calvo (1983). Each period, a fraction $1-\alpha$ of firms are allowed to re-optimize their price, while the remaining fraction α leave their price unchanged. Calvo probability α is assumed to be common across the interest groups. Firm j in interest group i that revises its price at time t chooses its new price $p_t^*(j)$ to maximize the present discounted value of profits:

$$\mathbb{E}_t\sum_{k=0}^{\infty}(\alpha\beta)^k\left[\frac{1}{C_{t+k}^j P_{t+k}}\left(1-\tau_{i,t+k}\right)p_t^*(j)y_{t,t+k}^*(j) - \frac{\left[y_{t,t+k}^*(j)\right]^{1+\eta}}{1+\eta}\right], \tag{5.12}$$

subject to the downward-sloping demand function:

$$y_{t,t+k}^{*}(j) = \left(\frac{p_t^{*}(j)}{P_{i,t+k}}\right)^{-\varepsilon} Y_{i,t+k}. \tag{5.13}$$

The first-order condition for this problem is given by

$$p_t^{*}(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[y_{t,t+k}^{*}(j)\right]^{1+\eta}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left(1 - \tau_{i,t+k}\right) \frac{C_{t+k}}{P_{t+k}} y_{t,t+k}^{*}(j)}. \tag{5.14}$$

5.2.3 Government

Assuming that one-period bond D_t is in zero-net supply, the government budget constraint is given by

$$(1 + \rho Q_t) \frac{b_{t-1}}{\Pi_t} = Q_t b_t + \frac{1}{2} \sum_{i=1}^{2} \tau_{i,t} Y_{i,t} - Z_t, \tag{5.15}$$

where $b_t \equiv B_t / P_t$ is the real value of long-term bonds. Here for simplicity we abstract from government consumption.

The government cannot adjust a path of lump-sum transfers payments $\{Z_\tau\}_{\tau=t}^{\infty}$ due to political reasons. To stabilize debt, it must collect production tax and/or increase inflation, reducing the real value of outstanding debt; both involve welfare costs. This assumption means that monetary policy influences inflation and the economic activities through its fiscal consequences. Specifically, the expected path of nominal interest rates determines the price of long-term bond Q_t, thereby affecting the dynamics of bond, inflation, and output. This causes a coordination failure between the interest groups, as discussed below.

Debt in a steady state with zero inflation can be expressed as

$$b = \frac{1 - \beta\rho}{1 - \beta} (\tau Y - Z). \tag{5.16}$$

In the numerical analyses below, we choose a steady-state value of lump-sum transfer payments so that the debt-to-output ratio in a steady state remains unchanged for alternative values of ρ.

5.3 Linear-Quadratic Approach

We employ a linear-quadratic method to analyze the policy problems considered in subsequent sections, which allow us to solve the game between the interest groups that we formalize in the next section analytically. We log-linearize the equilibrium conditions and compute a quadratic approximation of the utility function. The approximation is taken around the steady state in which the following relationships hold:[7]

$$\frac{p(j)}{P_i} = 1, \qquad \frac{P_i}{P} = 1, \quad \tau_i = 1 - \frac{1}{2}\left(\frac{\varepsilon}{\varepsilon - 1}\right), \quad i = \beta^{-1} - 1, \qquad Q = \frac{\beta}{1 - \beta\rho}. \quad (5.17)$$

From now on, for a generic variable X_t, which takes $\bar{X}$ in the steady state, $\hat{x}$ is defined as $\hat{x}_t \equiv \ln(X_t/\bar{X})$, and for a generic interest group-i specific variable v_i, we define $v^W \equiv \frac{1}{2}(v_1 + v_2)$.

5.3.1 Log-linearized equilibrium conditions

We log-linearize the equilibrium conditions around the steady state. First, (5.14) is linearized to give the Phillips curve in interest group i:

$$\pi_{i,t} = \beta \mathbb{E}_t \pi_{i,t+1} + \kappa\left(\hat{y}_{i,t} + \psi \hat{\tau}_{i,t}\right), \quad (5.18)$$

where κ and ψ are functions of structural parameters. As production tax increases the marginal costs, it puts an upward pressure on inflation in interest group i. Second, we linearize the consumption Euler equation in interest group i (5.10) to derive the condition that

[7] The steady state level of labor tax rate is chosen such that we can obtain a quadratic loss function. Benigno and Benigno (2003) drive conditions under which one can apply a liner-quadratic approach to a two-country open-economy DSGE model.

links group-specific variables to the short-term nominal interest rate:

$$\hat{y}_{i,t} = \mathbb{E}_t\hat{y}_{i,t+1} - \left(\hat{\imath}_t - \mathbb{E}_t\pi_{i,t+1} + \hat{r}_t^N \right), \tag{5.19}$$

where $\hat{r}_t^N \equiv \xi_t - \mathbb{E}_t\xi_{t+1}$ is the natural interest rate. In the analysis conducted in Sections 4.6 and 4.7, we assume that the ZLB becomes binding due to a large negative shock to the natural interest rate. In the baseline case such a shock does not occur; $\hat{r}_t^N = 0$ in every period.

Third, the linearized government budget constraint is written as

$$\hat{b}_{t-1}^W = \pi_t^W + \beta\hat{b}_t^W + \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) + \beta(1-\rho)\hat{Q}_t, \tag{5.20}$$

where $\gamma \equiv \left(\frac{\tau Y}{\tau Y - Z}\right)$. This equation shows that, as $0 < \rho < 1$, an increase in the price of long-term bonds puts downward pressure on inflation. Precisely, it has both inflationary and deflationary effects. Inflationary effect arises because the price of outstanding bonds is increased, which imposes the fiscal burden. An increase in the price of long-term bonds is deflationary because it enables the government to sell new bonds at a higher price, which reduces the fiscal burden. Under the assumption of decaying coupons, the price of newly issued bonds is more sensitive to changes in nominal interest rates than that of outstanding bonds. Therefore, the deflationary effect dominates the inflationary effect.

Finally, the asset-pricing condition (5.15) is linearized to yield

$$\hat{Q}_t = -\hat{\imath}_t + \beta\rho\mathbb{E}_t\hat{Q}_{t+1}. \tag{5.21}$$

5.3.2 Quadratic welfare loss function

Here, we derive a second-order approximation to the utility function (5.1) around the steady state to obtain a quadratic welfare loss function of interest group $L_{i,0}$, which can be expressed as

$$L_{i,0} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_{i,t}^2 + \lambda \hat{y}_{i,t}^2\right), \tag{5.22}$$

where λ is function of structural parameters. This is the payoff function that interest group i aims to maximize in the game formalized in the next section.

Meanwhile, the central bank is assumed to be benevolent so that its objective is to maximize the joint welfare of the two interest groups L_0^W defined as

$$L_0^W = \frac{1}{2}\sum_{i=1}^{2} L_{i,0} = -\frac{1}{4}\mathbb{E}_0 \sum_{i=1}^{2}\sum_{t=0}^{\infty} \beta^t \left(\pi_{i,t}^2 + \lambda \hat{y}_{i,t}^2\right). \tag{5.23}$$

While some readers may suspect that one should not use a linear-quadratic method to characterize optimal policy correctly, this is a standard treatment in the literature on optimal monetary policy. Woodford (2003) explains two major advantages of using this method. The first is that it is "simply mathematically convenience" (p.383). The second is the "comparability of the results with those of the traditional literature on monetary policy evaluation, which almost always assumes a quadratic loss function of one sort or another" (p.384). In particular, some recent studies that examine optimal policy at the ZLB also use a linear-quadratic approach (see, e.g., Burgert and Schmidt, 2014; Nakata et al., 2019; Bilbiie, 2019).

5.4 Markov-Perfect Equilibrium

We analyze a game among the interest groups and the central bank by using the conditions derived in Section 4.3. We focus on a Markov-perfect equilibrium assuming that the players cannot credibly commit to their future actions. Therefore, their strategies depend only on the welfare-relevant state variable, which is the stock of government debt $\hat{b}_{t-1}$. In addition, we restrict our attention to an equilibrium in which the two interest groups are symmetric, following Velasco (2000).

5.4.1 Markov-perfect equilibrium

In this subsection, we provide a formal definition for a Markov-perfect equilibrium, in which expectations are required to be a function of the stock of debt.

Definition. *A Markov-perfect equilibrium is a set of* V_t, $\{\bar{\pi}_{i,t}, \bar{y}_{i,t}, \bar{\tau}_{i,t}, \bar{b}_{i,t}\}_{i=1}^{2}$, *and* $\{\bar{\imath}_t, \bar{Q}_t\}$ *such that:*

(i) for all $\hat{b}_{t-1}$

$$\pi_t^W = \frac{1}{2}\sum_{i=1}^{2} \bar{\pi}_{i,t}\big(\hat{b}_{t-1}^W\big), \qquad \hat{y}_t^W = \frac{1}{2}\sum_{i=1}^{2} \bar{y}_{i,t}\big(\hat{b}_{t-1}^W\big),$$

$$\hat{\tau}_t^W = \frac{1}{2}\sum_{i=1}^{2} \bar{\tau}_{i,t}\big(\hat{b}_{t-1}^W\big), \qquad \hat{b}_t^W = \frac{1}{2}\sum_{i=1}^{2} \bar{b}_{i,t}\big(\hat{b}_{t-1}^W\big)$$

(ii) for all $\hat{b}_{t-1}$

$$\big\{\bar{\pi}_{i,t}\big(\hat{b}_{t-1}^W\big), \bar{y}_{i,t}\big(\hat{b}_{t-1}^W\big), \bar{\tau}_{i,t}\big(\hat{b}_{t-1}^W\big), \bar{b}_{i,t}\big(\hat{b}_{t-1}^W\big), \bar{\imath}_t\big(\hat{b}_{t-1}^W\big), \bar{Q}_t\big(\hat{b}_{t-1}^W\big)\big\}$$

$$= \underset{\pi_{i,t}, \hat{y}_{i,t}, \hat{\tau}_{i,t}, \hat{b}_{i,t}, \hat{\imath}_t, \hat{Q}_t}{\operatorname{argmin}} \left[\frac{1}{2}\big(\pi_{i,t}^2 + \lambda \hat{y}_{i,t}^2\big) + \beta V_t\big(\hat{b}_t^W\big)\right]$$

subject to

$$\hat{y}_{i,t} = \bar{y}_{i,t+1}\big(\hat{b}_t^W\big) - \big[\hat{\imath}_t - \bar{\pi}_{i,t+1}\big(\hat{b}_t^W\big) - \hat{r}_t^N\big]$$

$$\pi_{i,t} = \beta \bar{\pi}_{i,t+1}\big(\hat{b}_t^W\big) + \kappa\big(\hat{y}_{i,t} + \psi \hat{\tau}_{i,t}\big)$$

$$\hat{b}_{t-1}^W = \pi_t^W + \beta \hat{b}_t^W + \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) + \beta(1-\rho)\hat{Q}_t$$

$$\hat{Q}_t = -\hat{\imath}_t + \beta\rho \bar{Q}_{t+1}\big(\hat{b}_t^W\big)$$

given

$$\big\{\bar{\pi}_{g,t}\big(\hat{b}_{t-1}^W\big), \bar{y}_{g,t}\big(\hat{b}_{t-1}^W\big), \bar{\tau}_{g,t}\big(\hat{b}_{t-1}^W\big), \bar{b}_{g,t}\big(\hat{b}_{t-1}^W\big)\big\}_{g\neq i} \text{ and } \big\{\bar{\imath}_t\big(\hat{b}_{t-1}^W\big), \bar{Q}_t\big(\hat{b}_{t-1}^W\big)\big\}.$$

Generally, the strategies of players are not necessarily time-invariant, but in the baseline analysis we assume this because the game is stationary.

5.4.2 Benevolent planner's problem

In this subsection, we characterize optimal fiscal and monetary policy that a benevolent planner would choose. This policy aims to maximize social welfare given by (5.22). While the main focus of our study is on a non-cooperative game between the two interest groups, it is instructive to analyze a planner's solution to interpret the result of the main analysis.

Before characterizing optimal stabilization paths, a planner would take into account the fact that in an equilibrium the interest groups choose same actions. This implies that, for all $i \in \{1,2\}$, $\pi_{i,t} = \pi_t^W$, $\hat{y}_{i,t} = \hat{y}_t^W$, $\hat{\tau}_{i,t} = \hat{\tau}_t^W$, $\hat{b}_{i,t} = \hat{b}_t^W$, $\bar{\pi}_{i,t}(\cdot) = \bar{\pi}_t(\cdot)$, $\bar{y}_{i,t}(\cdot) = \bar{y}_t(\cdot)$, $\bar{\tau}_{i,t}(\cdot) = \bar{\tau}_t(\cdot)$, and $\bar{b}_{i,t}(\cdot) = \bar{b}_t(\cdot)$. Given this symmetricity condition, a benevolent planner would choose $\{\pi_t^W, \hat{y}_t^W, \hat{\tau}_t^W, \hat{b}_t^W, \hat{\imath}_t, \hat{Q}_t\}$ to maximize social welfare. Its optimization problem is then written recursively as

$$V_t\left(\hat{b}_{t-1}^W\right) = \min\left[\frac{1}{2}\{(\pi_t^W)^2 + \lambda(\hat{y}_t^W)^2\} + V_{t+1}\left(\hat{b}_t^W\right)\right]$$

subject to

$$\hat{y}_{i,t} = \bar{y}_{i,t+1}\left(\hat{b}_t^W\right) - \left[\hat{\imath}_t - \bar{\pi}_{i,t+1}\left(\hat{b}_t^W\right) - \hat{r}_t^N\right]$$

$$\pi_{i,t} = \beta\bar{\pi}_{i,t+1}\left(\hat{b}_t^W\right) + \kappa\left(\hat{y}_{i,t} + \psi\hat{\tau}_{i,t}\right)$$

$$\hat{b}_{t-1}^W = \pi_t^W + \beta\hat{b}_t^W + \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) + \beta(1-\rho)\hat{Q}_t$$

$$\hat{Q}_t = -\hat{\imath}_t + \beta\rho\bar{Q}_{t+1}\left(\hat{b}_t^W\right)$$

5.4.3 Non-cooperative game

We next formalize a non-cooperative game. Each interest group maximizes its own welfare, taking the strategy of the other group as given. At the first stage of the game, the two interest groups recognize that, in the second stage, the central bank follows them to set the short-term nominal interest rate after observing their decisions on the tax rate. Accordingly, interest group i can choose the tax rate taking into account how its decision changes the central bank' optimal response. The optimization problem of interest group i is then written as

$$V_t(\hat{b}_{t-1}^W) = \min\left[\frac{1}{2}(\pi_{i,t}^2 + \lambda\hat{y}_{i,t}^2) + \beta V_{t+1}(\hat{b}_t^W)\right]$$

subject to

$$\hat{y}_{i,t} = \bar{y}_{i,t+1}(\hat{b}_t^W) - [\hat{\imath}_t - \bar{\pi}_{i,t+1}(\hat{b}_t^W) - \hat{r}_t^N]$$

$$\pi_{i,t} = \beta\bar{\pi}_{i,t+1}(\hat{b}_t^W) + \kappa(\hat{y}_{i,t} + \psi\hat{\tau}_{i,t})$$

$$\hat{b}_{t-1}^W = \pi_t^W + \beta\hat{b}_t^W + \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) + \beta(1-\rho)\hat{Q}_t$$

$$\hat{Q}_t = -\hat{\imath}_t + \beta\rho\bar{Q}_{t+1}(\hat{b}_t^W)$$

given

$$\{\bar{\pi}_{g,t}(\cdot), \bar{y}_{g,t}(\cdot), \bar{\tau}_{g,t}(\cdot), \bar{b}_{g,t}(\cdot)\}_{g\neq i} \quad \text{and} \quad \{\bar{\imath}_t(\cdot), \bar{Q}_t(\cdot)\}.$$

Solving this optimization problem provides the best-response function according to which interest group i chooses the tax rate $\hat{\tau}_{i,t}$.

Note that, in this setup, the decision on the tax rate by each interest group has an economy-wide externality. The government budget constraint (5.23) shows that an increase in taxes attenuates the inflation pressure, and therefore contributes to stabilizing economy-wide inflation. However, in the non-cooperative game, each interest group does not fully internalize the positive externality since its objective is to stabilize group-specific output and inflation. Consequently, the central bank is forced to choose a sub-optimal policy in the second stage of the game.

5.4.4 Solving the optimization problems

In this subsection, we explain the method to solve the above optimization problems following Bhattarai et al. (2015). Since we use the linear-quadratic approach, we can conjecture that the solution of the non-cooperative game takes the linear form:

$$\bar{\pi}_{i,t}\left(\hat{b}^{W}_{t-1}\right) = \bar{\pi}_{i,BL}\hat{b}^{W}_{t-1}, \;\; \bar{y}_{i,t}\left(\hat{b}^{W}_{t-1}\right) = \bar{y}_{i,BL}\hat{b}^{W}_{t-1}, \;\; \bar{\tau}_{i,t}\left(\hat{b}^{W}_{t-1}\right) = \bar{\tau}_{i,BL}\hat{b}^{W}_{t-1},$$
$$\bar{b}_{i,t}\left(\hat{b}^{W}_{t-1}\right) = \bar{b}^{BL}_{i}\hat{b}^{W}_{t-1}, \;\; \bar{\imath}_{t}\left(\hat{b}^{W}_{t-1}\right) = \bar{\imath}^{BL}\hat{b}^{W}_{t-1}, \text{ and } \;\; \bar{Q}_{t}\left(\hat{b}^{W}_{t-1}\right) = \bar{Q}^{BL}\hat{b}^{W}_{t-1}, \tag{5.24}$$

where $\bar{\pi}^{BL}_{i}$, $\bar{y}^{BL}_{i}$, $\bar{\tau}^{BL}_{i}$, $\bar{b}^{BL}_{i}$, $\bar{\imath}^{BL}$, and $\bar{Q}^{BL}$ are unknown coefficients that are determined in an equilibrium. The interest groups are assumed to play stationary strategies given that the game in the baseline case has the stationary structure.

When choosing policy instruments, a benevolent planner would take in account that the two interest groups are symmetric so that the following relationships hold:

$$\pi_{i,t} = \pi^{W}_{t}, \;\; \hat{y}_{i,t} = \hat{y}^{W}_{t}, \;\; \hat{\tau}_{i,t} = \hat{\tau}^{W}_{t}, \;\; \hat{b}_{i,t} = \hat{b}^{W}_{t},$$
$$\bar{\pi}^{BL}_{i} = \bar{\pi}^{BL}, \quad \bar{y}^{BL}_{i} = \bar{y}^{BL}, \quad \bar{\tau}^{BL}_{i} = \bar{\tau}^{BL}, \text{ and } \quad \bar{b}^{BL}_{i} = \bar{b}^{BL}. \tag{5.25}$$

When solving the non-cooperative game, we focus on a symmetric equilibrium in which the two interest groups choose the same action. Generally, cooperative and non-cooperative solutions are different because of the externality that the interest groups' decisions has through monetary policy. The details of the procedure for determining the coefficients are in the Appendix.

5.5 Numerical Results: Baseline Case

In this subsection, we present numerical examples to study how the coordination failure between the two interest groups distorts socially optimal conduct of fiscal and monetary policy. Suppose that, at the beginning of period 0, government debt is accumulated for exogenous reasons and from period 0 onward this is started to be stabilized. We take the period of the model to be a quarter, and adopt $\beta = 0.99$, $\varepsilon = 6$, and $\alpha = 0.66$. We choose $\eta = 2$, implying that the Frisch elasticity of labor supply is 0.5. The Frisch elasticity is a key parameter that is the determinant of how a change in the tax rate affects group-specific inflation and output gap. In Appendix D, we study the sensitivity of our results to alternative values of the Frisch elasticity. The steady-state value of the debt-to-GDP ratio is set to 0.49, which is the average of the U.S. data from 1948 to 2019. Regarding the duration of long-term government bonds, which is determined by ρ, we consider two cases, 14 years and 10 years, which are the longer duration case and shorter duration case, respectively.

5.5.1 Longer duration case

We first numerically analyze the outcome of the game in the longer duration case. The initial stock of debt is set to $\hat{b}_{-1}^{W} = 0.2$.[8] We compare the benevolent planner's solution and the non-cooperative solution.

Figure 5.1 illustrates how the accumulated debt and the macroeconomy are stabilized over time. Solid blue lines indicate the paths that a benevolent planner would choose, while the dotted blue lines indicate the paths resulting from the non-cooperative game. As shown in the figure, it is socially optimal to increase both the labor income tax and inflation to decrease the accumulated debt. A planner would choose the policy instruments to smooth the distortions due to debt stabilization over time. The presence of long-term bonds enables a planner to use variations in future nominal interest rates in order to optimally choose the timing of the tax collection and inflation.

In the non-cooperative game, the resulting response of the tax rate becomes negative. Against the inflationary pressure from the accumulated debt, the interest groups find it

[8] The choice of the initial value does not qualitatively affect the results below.

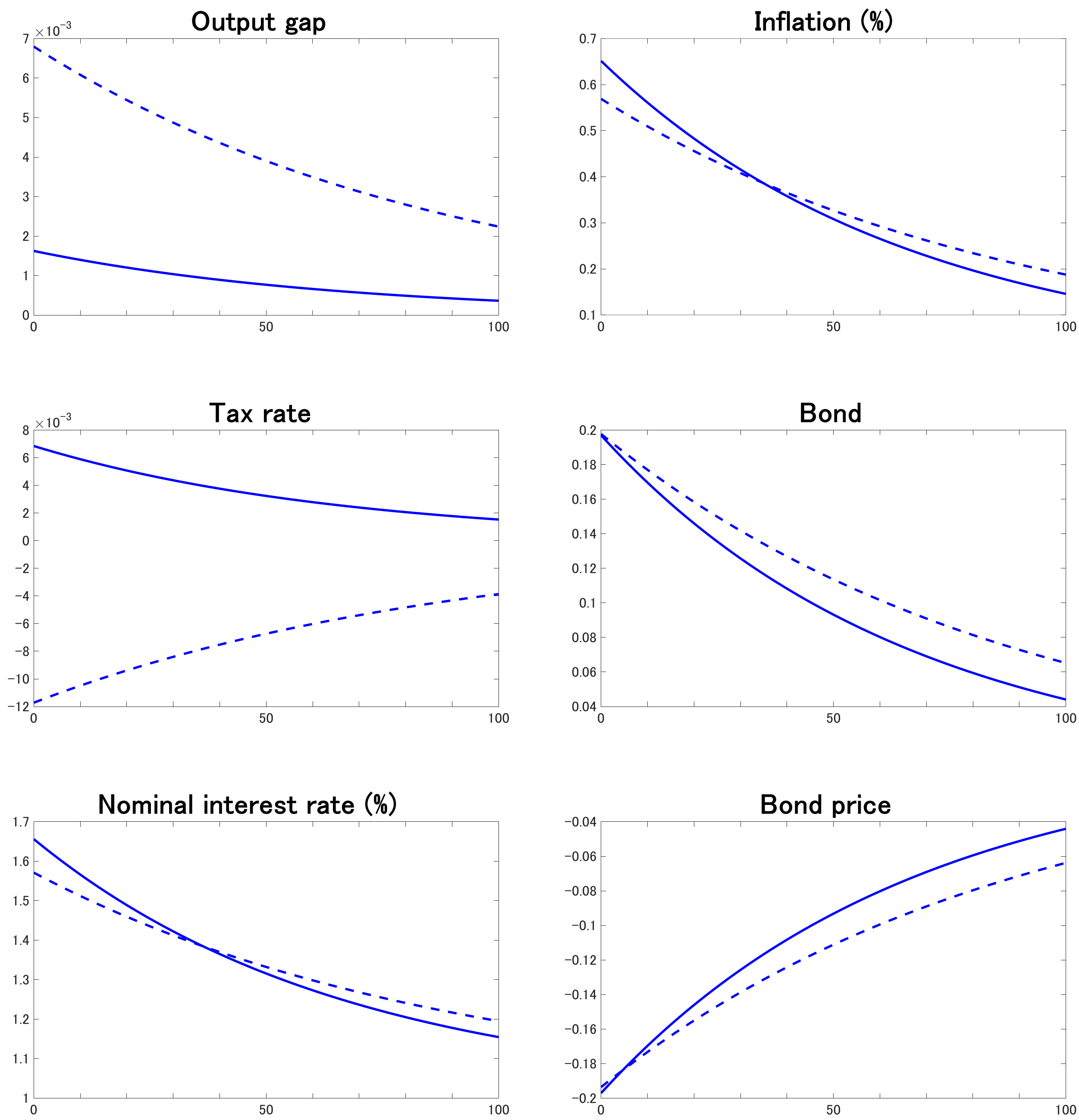

Figure 5.1. Baseline case: longer duration. Solid blue lines: responses that a benevolent planner would choose. Dotted blue lines: responses resulting from the non-cooperative game.

optimal to lower the tax rate to attenuate upward pressure on group-specific inflation. They do not fully internalize that the tax cut influences inflation through its budgetary impacts.

This free-riding behavior of the interest groups puts pressure on the response of economy-wide inflation to be excessively gradual due to the following two reasons. First, the free-riding behavior puts socially excessive downward pressure on the marginal costs and thus downward pressure on inflation. Second, the free-riding behavior slows the speed of debt stabilization excessively and then delays the timing of inflation. Further, it should be noted that a negative response of the tax rate puts pressure on a positive response of output gap to be socially excessive.

Given this free-riding behavior of the interest groups, the central bank finds it optimal to attach a larger weight on output-gap smoothing than in the case of cooperation. This is because the loss function of the central bank depends on variations of economy-wide inflation and output gap. The pressure on economy-wide inflation to be excessively gradual forces the central bank to make a positive response of the nominal interest rates excessively gradual as well to smooth the real interest rate and thus output gap. Consequently, the negative response of the price of long-term bonds is also forced to be excessively gradual.

5.5.2 Shorter duration case

We next conduct a similar exercise under the shorter duration case. The results are reported in Figure 5.2. Solid red lines represent the paths that a benevolent planner would choose in the shorter duration case, whereas the dotted red lines represent the paths resulting from the non-cooperative game in the shorter duration case.

This numerical result shows that in the shorter duration case it is socially optimal that economy-wide output gap and inflation are less smoothed than in the longer duration case. The logic behind this is that shortening debt maturity makes the price of long-term bonds less

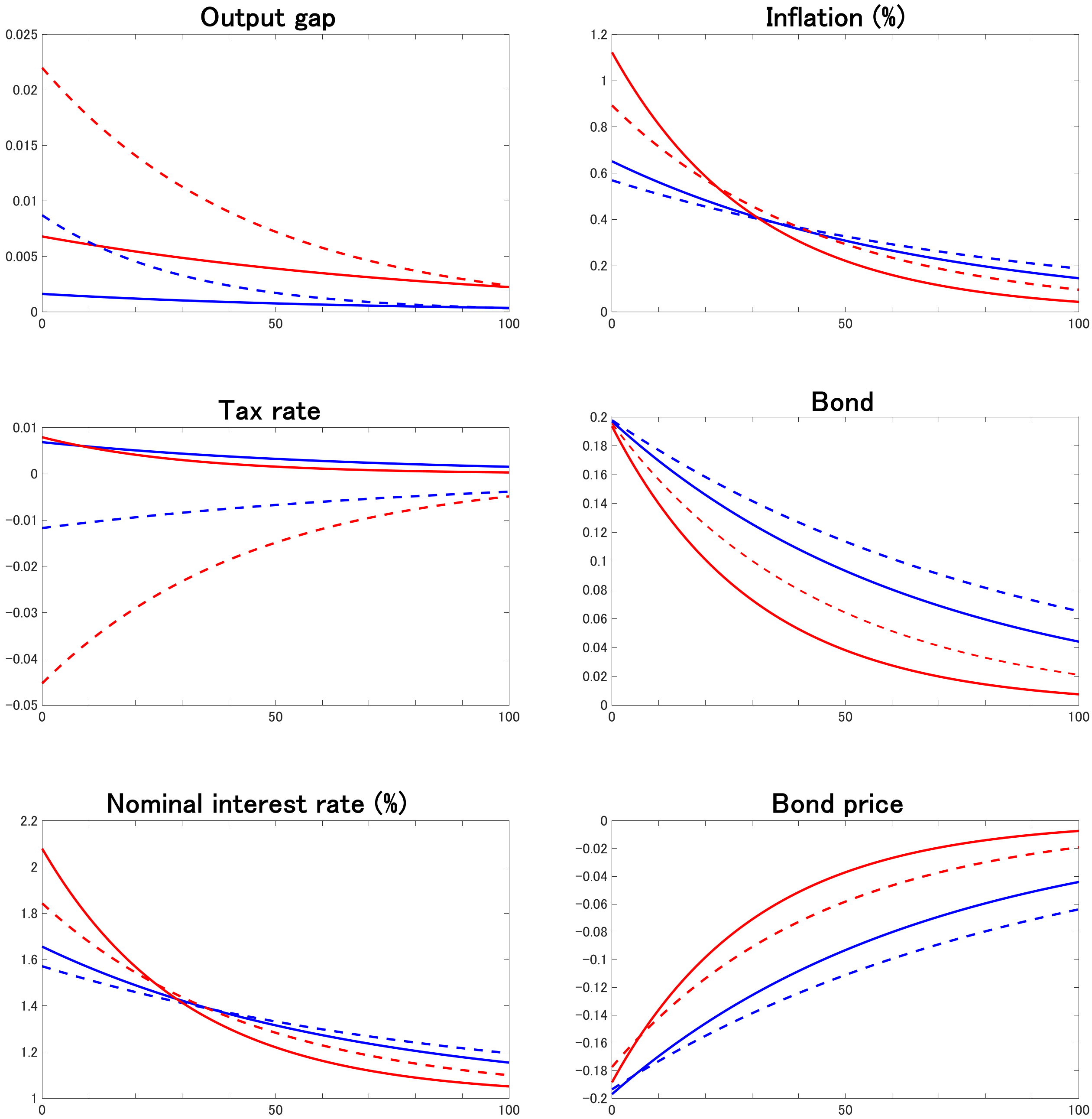

Figure 5.2. Baseline case: longer duration vs. shorter duration. Blue solid lines: responses that a benevolent planner would choose in the longer duration case. Blue dotted lines: responses resulting from the non-cooperative game in the longer duration case. Red solid lines: responses that a benevolent planner would choose in the shorter duration case. Red dotted lines: responses resulting from the non-cooperative game in the shorter duration case.

sensitive to changes in current and future nominal interest rates, thereby reducing the government's ability to spread the distortions due to debt stabilization over time. Hence, a benevolent planner would give up attaining the optimal path in the longer duration case, but find it optimal to make the responses of the tax rate and inflation stronger to reduce debt rapidly.

Next, we consider the non-cooperative game. Given that shorting debt maturity puts additional upward pressure on inflation, the interest groups find it optimal to decrease the tax rate further in order to attenuate upward pressure on group-specific inflation. Accordingly, in the shorter duration case, the consequence of the free-riding activities of the interest groups becomes more severe; the negative response of the tax rate becomes larger than in the longer duration case.

5.6 Negative Demand Shock: Case without the Zero Lower Bound

In the rest of this chapter, we introduce a large negative shock to the natural interest rate that causes the ZLB to bind. As in the baseline case, the initial stock of debt is above the steady-state level. The main difference is that the economy is hit by a temporary shock to the natural interest rate. The goal is to examine how a demand negative shock, along with the presence of the ZLB, changes the results in the baseline case in which the natural interest rate remains at the steady-state level.

Before proceeding, this section studies how the economy responds to a negative demand shock assuming that the central bank can lower the nominal interest rate below zero. The government in this case can use not only fiscal policy but also monetary policy to respond to a negative demand shock. The reason for studying the case without the ZLB (or the "negative rates case" for short) is to assess how important the presence of the ZLB is in our model, separately from the effects of a negative demand shock. In section 5.8 we study the case in which the nominal interest rate is subject to the ZLB.

A shock is assumed to be deterministic as in Jung et al. (2005).[9] Specifically, a negative demand shock hits the economy in period 0 and disappears in period K. From period 0 to $K-1$, the natural interest rate remains at a negative level, and from period K onward it stays at the steady-state level; $\hat{r}_t^N = \hat{r}_L < -(1-\beta)$ for $t = 0, 1, \cdots, K-1$ and $\hat{r}_t^N = 0$ for all $t \geq K$.

As the natural interest rate changes over time, the game we will analyze is no longer stationary. Specifically, from period 0 to $K-1$ the economy gradually approaches a time of liftoff from the ZLB. Conversely, the subgame starting period K is stationary as in the baseline case. This fact enables us to easily analyze the game presented in this section by regarding the game during periods of a demand shock as a K-period repeated game, followed by the infinite-horizon game.

As in the baseline case, the players use Markovian strategies. The following two are common knowledge among them: (i) a negative shock disappears in period K, (ii) after a shock disappears, the players use the same stationary strategies they use in the baseline case. In period K, the players choose their actions given the level of government debt accumulated during periods of a demand shock, $\hat{b}_{K-1}^W$. Foreseeing this, the interest groups and the central bank choose their actions from period 0 to $K-1$. We allow strategies that the interest groups and the central bank play from period 0 to $K-1$ to be time-variant. From period K onward, the players use the stationary strategies.

5.7 Negative Rates Case: Numerical Results

[9] See, for example, Werning (2012), Correia et al. (2013), and Gabaix (2020) for other studies that address the issues of policy at the ZLB under this assumption. Eggertsson and Woodford (2003) assume that a shock to the natural interest rate follows a two-state Markov process.

In this section, we numerically analyze the negative rates case. At the beginning of period 0 debt is accumulated and from period 0 onward the players optimally choose their actions. For the structural parameters, we use the same values as in the baseline case. In addition, K is set to 6, implying that a negative shock lasts for a year and a half. The value of $\hat{r}_L$ is chosen such that the natural interest rate drop to $-2\,\%$ from period 0 to 5. The initial debt stock is set to$\hat{b}_{-1}^{W} = 0.2$ again.

Figure 5.3 shows the responses of variables to a negative demand shock in the negative rates case. The solid green lines with crosses display the responses that a planner would choose, while the dotted black lines with diamonds are the responses resulting from the non-cooperative game. The numerical result shows that, during the periods of the negative natural interest rate, the central bank has to lead the nominal interest rate into negative territory and the fiscal authority has to increase the tax rate. In particular, large responses are required in the first period. A planner would use monetary policy mainly for output stabilization and fiscal policy mainly for inflation stabilization. A planner would lower the nominal interest rate to increase output gap, producing inflationary pressure. The spike in the tax rate in the first period is required to attenuate the inflationary pressure coming from the negative interest rate, along with the accumulated government debt.

Next, we consider the non-cooperative game. In this case, the positive response of the tax rate at the first period is socially insufficient. This is because the interest groups do not fully internalize the aforementioned social benefits of the tax hike during the periods of the negative natural interest rate. Thus, the central bank is forced to lower the nominal interest rate further in order to stimulate the economy, causing excessive responses of output gap.

Figure 5.4 displays the responses of output gap and inflation in the negative rates case from period 1 onward. The numerical results in this section show that if the central bank can lower

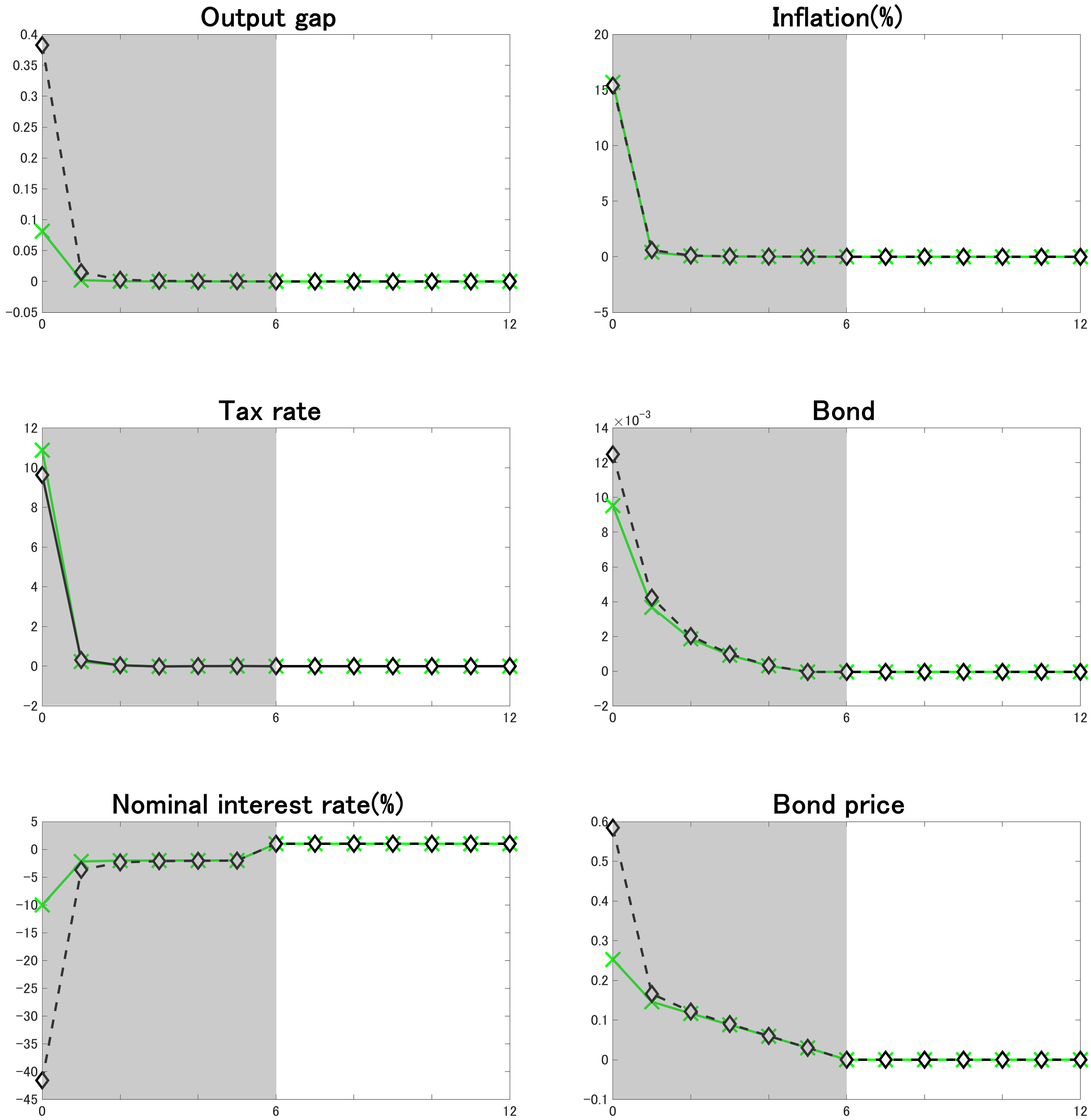

Figure 5.3. Negative rates case: longer duration. Solid green lines with crosses: responses that a benevolent planner would choose in the negative rates case. Dotted black lines with diamonds: responses resulting from the non-cooperative game in the negative rates case. A demand shock occurs in period 0 and disappears in period 6.

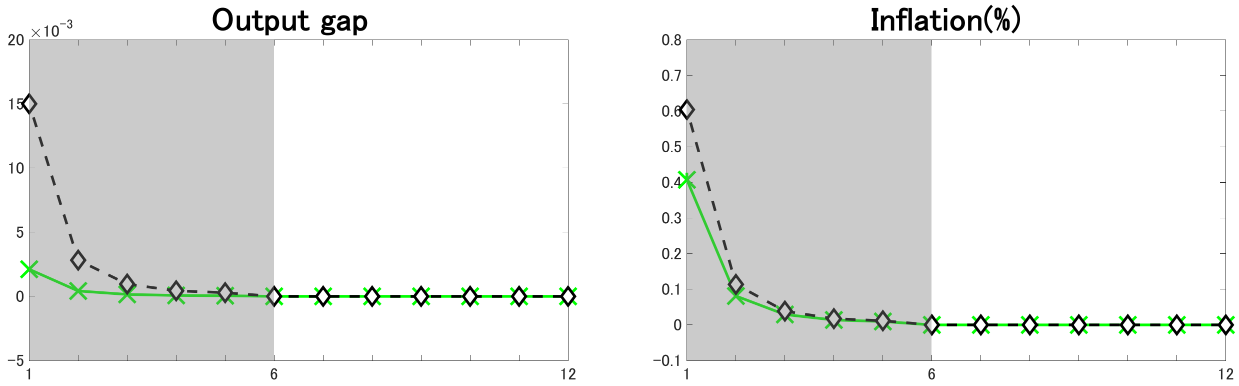

Figure 5.4. Responses of output gap and inflation in the negative rates case from period 1 onward. Solid green lines with crosses: responses of output and inflation that a benevolent planner would choose in the negative rates case. Dotted black lines with diamonds: responses of output and inflation resulting from the non-cooperative game in the negative rates case. A demand shock occurs in period 0 and disappears in period 6.

the nominal interest rate below zero, a negative shock to the natural interest rate cannot be contractionary.

5.8 Case with the Zero Lower Bound

Next, this chapter studies the case with the ZLB (or the "ZLB case" for short). We compute the responses of the economy to a negative demand shock under the same assumptions regarding the initial stock of government debt and the dynamics of the natural interest rate used in the previous section. The only difference is that the central bank cannot lower the nominal interest rate below zero. As in the baseline case, the players use Markovian strategies.

A large negative demand shock, along with the presence of the ZLB, produces downward pressure on inflation at the ZLB due to the following two reasons. First, due to the presence of the ZLB, a negative demand shock puts upward pressure on the real interest rate. Second, a decline in the nominal interest rate in response to a demand shock puts upward pressure on the price of long-term bonds. Note that an increase in future nominal interest rates directly translates into a decline in the price of long-term bonds at the ZLB. As it is assumed that the players cannot credibly commit to their future actions, the central bank cannot influence the state of the economy at the ZLB by committing to the future conduct of monetary policy.

During the ZLB periods, fiscal policy plays an essential role in changing public expectations about the macroeconomic conditions and fiscal and monetary policies after liftoff. While the tax cut puts downward pressure on inflation as explained, it also has inflationary effects. The key point is that the tax cut at the ZLB leads to an increase in the stock of government debt at the time of liftoff. This puts upward pressure on inflation at the ZLB due to the following two reasons. First, debt accumulation at the ZLB involves an increase in inflation at the time of liftoff. This prospect leads the private sector to expect higher future inflation. Second, debt accumulation at the ZLB leads to a decline in the price of long-term bonds at the time of liftoff. This translates into a decline in the price of long-term bonds during the ZLB periods.

Figure 5.5 depicts the responses of variables to a negative shock to the natural interest rate in the ZLB case. The solid blue lines with circles display the responses a planner would choose, while the dotted blue lines with circles are the responses resulting from the non-cooperative game. We first consider the case of coordination. When the nominal interest rate cannot be lowered below zero, it is socially optimal to slow down the speed of debt stabilization, which leads to larger responses of inflation and output gap after liftoff from the ZLB, as shown in Figure 5.6. This mitigates low inflation and a recession during the ZLB periods. A planner would find that the inflationary effects of the tax cut dominate the deflationary effect.

Next, we consider the case of non-cooperation. During the ZLB periods, the interest groups foresee that the free-riding problem arises after liftoff from the ZLB. The results in the baseline case show that the free-riding behavior of interest groups reinforces the deflationary

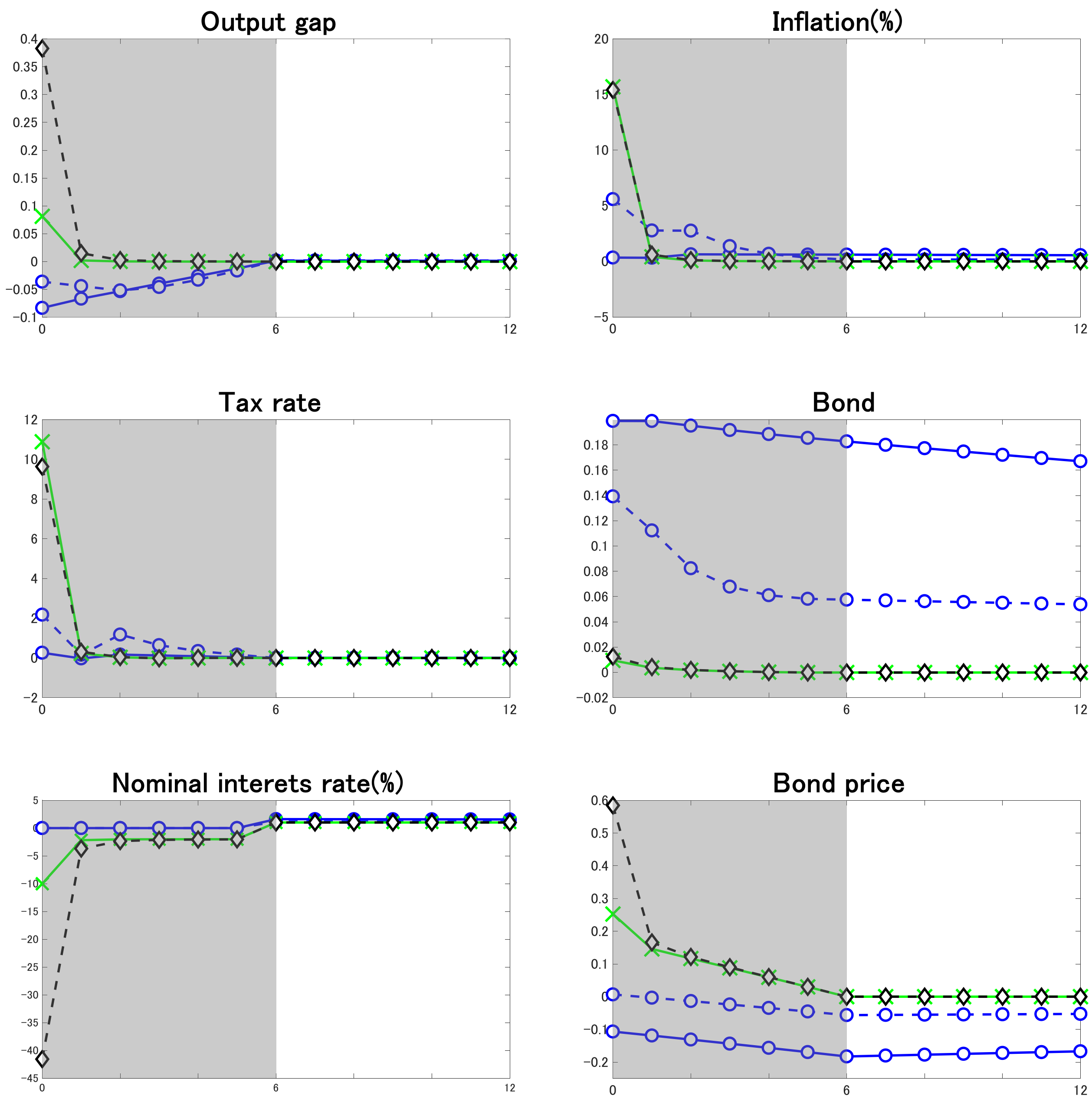

Figure 5.5. Comparison between the ZLB case and the negative rates case: longer duration. Solid blue lines with circles: responses that a planner would choose in the ZLB case. Dotted blue lines with circles: responses resulting from the non-cooperative game in the ZLB case. Solid green lines with crosses: responses that a benevolent planner would choose in the negative rates case. Dotted black lines with diamonds: responses resulting from the non-cooperative game in the negative rates case. A demand shock occurs in period 0 and disappears in period 6.

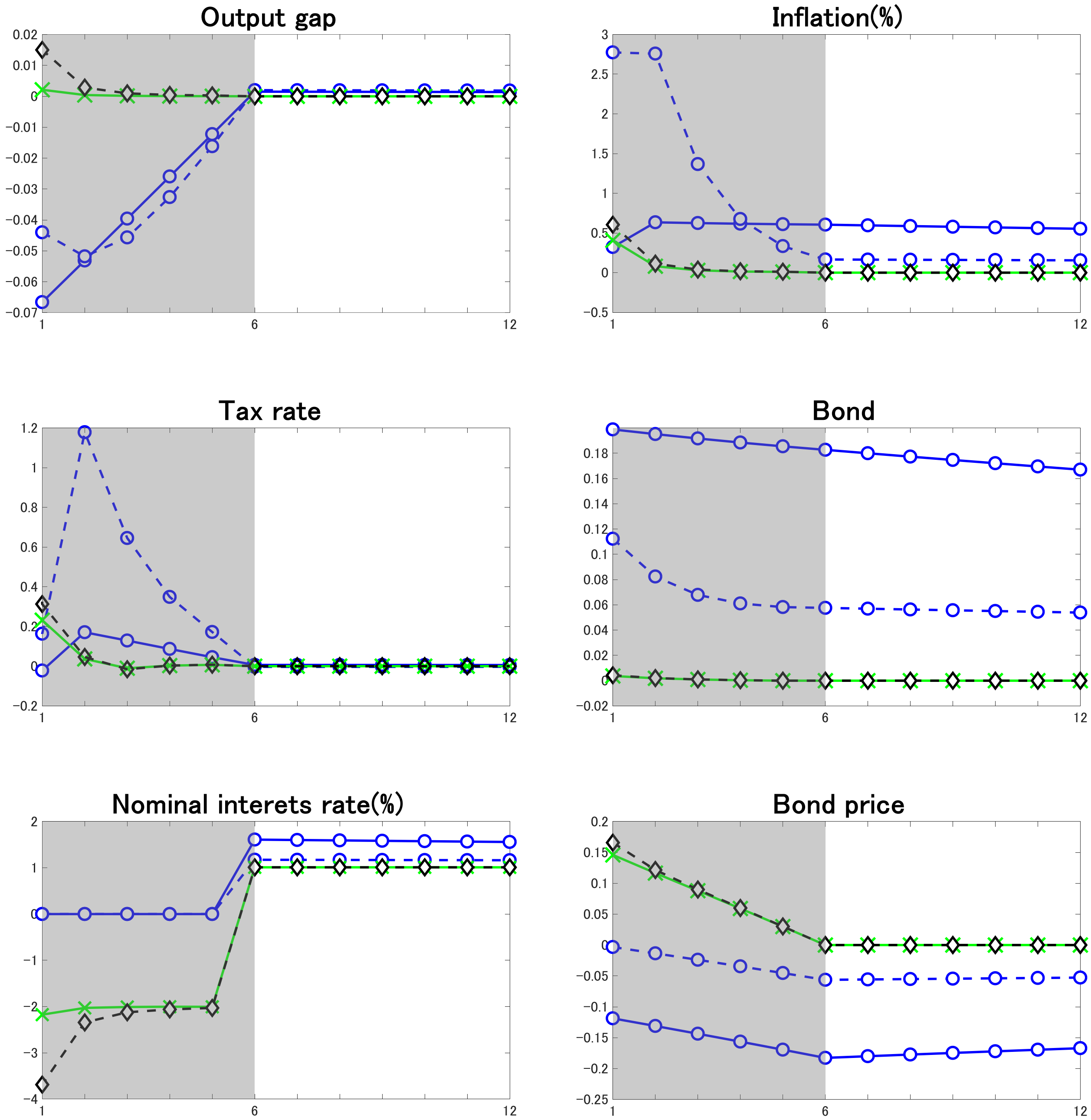

Figure 5.6. Comparison between the ZLB case and the negative rates case: longer duration. Responses of variables from period 1 onward. Solid blue lines with circles: responses that a planner would choose in the ZLB case. Dotted blue lines with circles: responses resulting from the non-cooperative game in the ZLB case. Solid green lines with crosses: responses that a benevolent planner would choose in the negative rates case. Dotted black lines with diamonds: responses resulting from the non-cooperative game in the negative rates case. A demand shock occurs in period 0 and disappears in period 6.

effects of a negative demand shock. The reasons for this are twofold. First, the free-riding behavior puts downward pressure on inflation at the time of liftoff and thus on inflation during the ZLB periods. Second, the free-riding behavior put upward pressure on the price of long-term bonds at the time of liftoff. Therefore, the interest groups choose the tax rate during the ZLB periods.

It should also be noted that the interest groups do not fully internalize that their actions influence the macroeconomy through their budgetary effects. Against the deflationary pressures coming from the free-riding problem that arises after liftoff, during the ZLB periods the interest groups find it optimal to increase the tax rate in order to increase group-specific inflation. Therefore, in the case of non-cooperation, the response of the tax rate becomes positive. This speeds up the debt stabilization excessively, leading to low inflation at the time of liftoff from the ZLB. This, along with the tax hike, causes a recession at the ZLB more severe, as shown in Figure 5.6.

The results in this section imply that our model shares a similar property with the standard New Keynesian model; when the nominal interest rate is subject to the ZLB constraint, a large negative shock to the natural interest rate causes a recession at the ZLB. This is the case even when the interest groups coordinate. Coordination failure makes a recession at the ZLB more severe. In the next section, we address the question: do purchases of long-term bonds help mitigate the contractionary effects of a negative demand shock on the economy at the ZLB?

5.9 Effects of Purchases of Long-Term Bonds

In this section, we examine how shortening debt maturity affects the results. In our model, the operation has both inflationary and deflationary effects on the economy at the ZLB. Shortening debt maturity is inflationary because it puts upward pressure on inflation at the time of liftoff, as shown in the baseline case. Deflationary effect arises because it puts upward pressure on the price of long-term bonds during the ZLB periods due to the following two reasons. First, as shown in the baseline case, shortening debt maturity puts upward pressure on the price long-term bonds at the time of liftoff. Second, shortening debt maturity lowers the sensitivity of long-term bonds at the ZLB to change in future nominal interest rates. Overall, how fiscal policy responds to a negative demand shock is one of the key determinants of the macroeconomic effect purchases of long-term bonds.

Figure 5.7 displays the responses of the variables to a negative demand shock in the ZLB case. The solid red lines with circles represent the responses that a planner would choose in the shorter duration case, while the dotted red lines with circles represent the resulting responses in the shorter duration case. During the ZLB periods, a planner would cut tax further. A planer would intend to amplify the inflationary effect and attenuate the deflationary effect. The tax cut mitigates deflation and a recession during the ZLB periods.

Next, we consider the non-cooperative game. In the case of non-cooperation, shortening debt maturity also has both inflationary and deflationary effects, as in the case of cooperation. The numerical result implies that the inflationary effect dominates the deflationary effect, and shortening debt maturity is expansionary. However, the expansionary effect is smaller than in the case of cooperation.

This is due coordination failure that arises during the ZLB periods. The positive response of the tax rate in the shorter duration case is larger than that of the longer duration. This is because during the ZLB periods the interest groups foresee that after liftoff from the ZLB, they have a stronger incentive to free-ride, which reinforces the deflationary pressures coming from the free-riding problem. Against the additional deflationary pressures, during the ZLB periods, the interest groups find it optimal to increase the tax rate further in order to

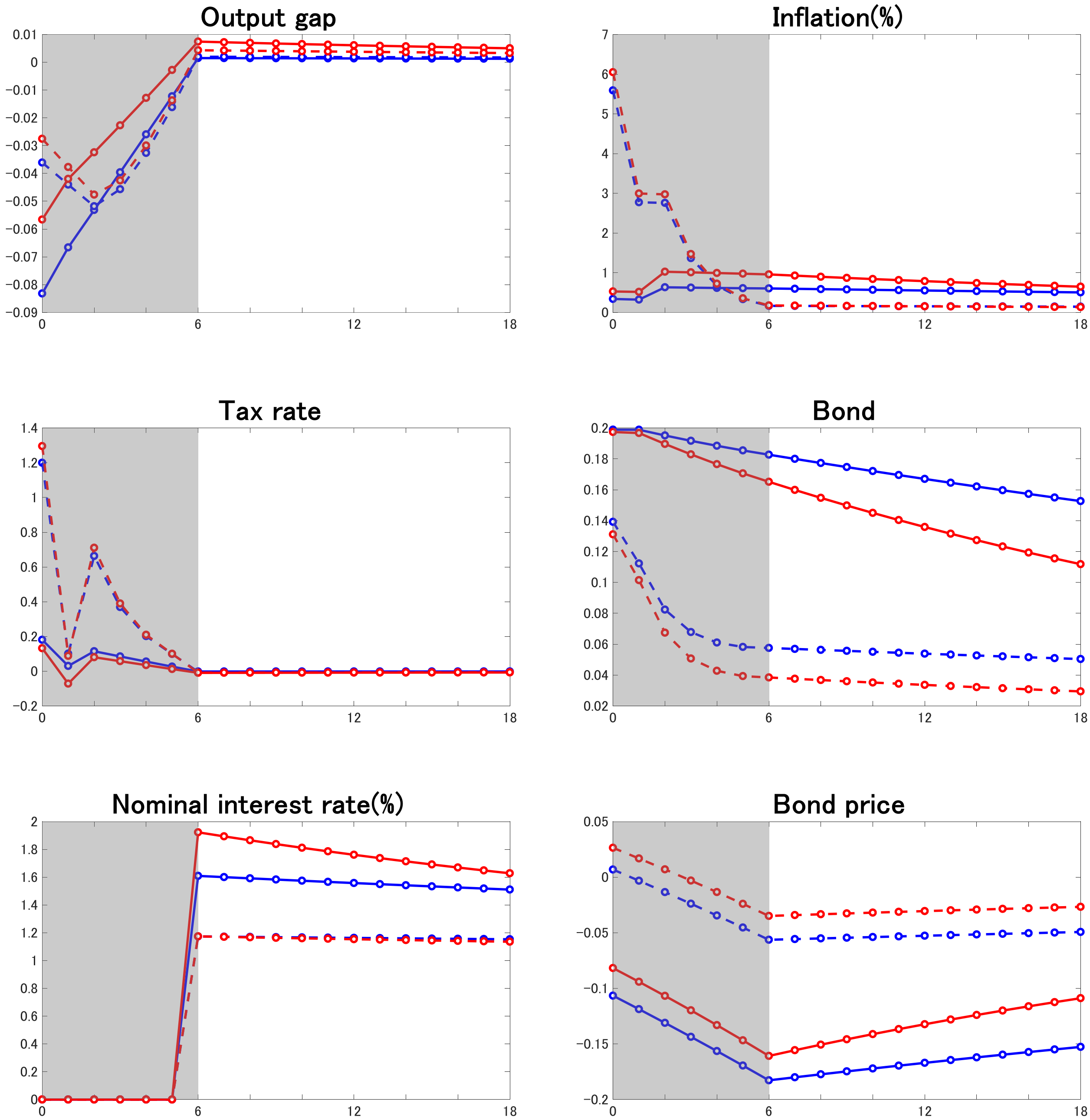

Figure 5.7. ZLB case: longer duration vs. shorter duration. Solid blue lines with circles: responses that a benevolent planner would choose in the longer duration case. Dotted blue lines with circles: responses resulting from the non-cooperative game in the longer duration case. Solid red lines with circles: responses that a benevolent planner would choose in the shorter duration case. Dotted red lines with circles: responses resulting from the non-cooperative game in the shorter duration case. A demand shock occurs in period 0 and disappears in period 6.

increase group-specific inflation. The resulting tax hike weakens the expansionary effect of shortening debt maturity.

5.10 Concluding Remarks

In this chapter, we have presented a dynamic general equilibrium model augmented to include a particular type of political economic aspect of fiscal policymaking. We modeled a situation where tax policy is decided by decentralized interest groups and embedded it in a New Keynesian model developed to investigate optimal fiscal and monetary policy. The main focus was on how a common pool problem distorts the optimal conduct of policy.

First, we studied how the economy in which the debt stock is initially above a steady-state level is stabilized over time. The main result was that coordination failure delays debt stabilization and causes a positive response of inflation excessively gradual. In addition, we numerically showed that shortening debt maturity reinforces this mechanism by giving the groups a stronger incentive to free-ride.

Second, we investigated how a negative demand shock that causes the ZLB to bind changes the above results. The numerical results imply that if the groups coordinate, then purchases of long-term bonds, which shorten the maturity structure of government debts held by the public, have an expansionary effect on the economy at the ZLB. We also numerically showed that coordination failure between the interest groups weakens this expansionary effect.

5.A Appendix

In Appendix 5.A.1, we explain how to solve policy problems in the baseline case. In Appendix 5.A.2, we explain how to solve policy problems in the negative rates case. In Appendix 5.A.3, we explain how to solve policy problems in the ZLB case. Appendix 5.A.4 provides sensitivity analyses.

5.A.1 Baseline Case

5.A.1.1 Benevolent planner's solution

The period Lagrangian for the planner's problem is given by

$$\mathcal{L}_t = \frac{1}{2}[(\pi_t^W)^2 + \lambda(\hat{y}_t^W)^2] + \beta \mathbb{E}_t V\big(\hat{b}_t^W\big)$$

$$+ \phi_t^{PC}\big[\pi_t^W - \beta\bar{\pi}^{BL}\hat{b}_t^W - \kappa(\hat{y}_t^W + \psi\hat{\tau}_t^W)\big]$$

$$+\phi_t^{IS}\big[\hat{y}_t^W - \bar{y}^{BL}\hat{b}_{t-1}^W + \big(\hat{\imath}_t - \bar{\pi}^{BL}\hat{b}_t^W\big)\big]$$

$$+ \phi_t^{Gov}\big[\hat{b}_{t-1}^W - \pi_t^W - \beta\hat{b}_t^W - \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) - \beta(1-\rho)\hat{Q}_t\big]$$

$$+\phi_t^{Q}\big[\hat{Q}_t + \hat{\imath}_t - \beta\rho\bar{Q}^{BL}\hat{b}_t^W\big],$$

where ϕ_t^{PC}, ϕ_t^{IS}, ϕ_t^{Gov}, and ϕ_t^{Q} are Lagrange multipliers corresponding to (5.18), (5.19), (5.20), and (5.21). We then obtain the first-order conditions:

$$\frac{\partial L_t}{\partial \pi_t^W} = \pi_t^W + \phi_t^{PC} - \phi_t^{Gov} = 0,$$

$$\frac{\partial L_t}{\partial \hat{y}_t^W} = \lambda\hat{y}_t^W + \phi_t^{IS} - \kappa\phi_t^{PC} - \gamma(1-\beta)\phi_t^{Gov} = 0,$$

$$\frac{\partial L_t}{\partial \hat{\tau}_t^W} = -\kappa\psi\phi_t^{PC} - \gamma(1-\beta)\phi_t^{Gov} = 0,$$

$$\frac{\partial L_t}{\partial \hat{\imath}_t} = \phi_t^{IS} + \phi_t^{Q} = 0,$$

$$\frac{\partial L_t}{\partial \hat{Q}_t} = -\beta(1-\rho)\phi_t^{Gov} + \phi_t^{Q} = 0,$$

$$\frac{\partial L_t}{\partial \hat{b}_t^W} = V'\left(\hat{b}_t^W\right) - (\bar{y}^{BL} + \bar{\pi}^{BL})\phi_t^{IS} - \beta\bar{\pi}^{BL}\phi_t^{PC} - \beta\phi_t^{Gov} - \beta\rho\bar{Q}^{BL}\phi_t^{Q} = 0,$$

and the envelop condition:

$$V'\left(\hat{b}_{t-1}^W\right) = \phi_t^{Gov}.$$

The conditions characterizing the dynamics of the aggregate variables are summarized to obtain the system of equations:

$$\hat{y}_t^W - \frac{1}{\lambda}\left[\beta(1-\rho) + \gamma(1-\beta)\left(1-\frac{1}{\psi}\right)\right]\left[1+\frac{\gamma(1-\beta)}{\kappa\psi}\right]^{-1}\pi_t^w = 0,$$

$$\left[(1-\rho)(\bar{y}^{BL} + \bar{\pi}^{BL}) + \left\{\frac{\gamma(1-\beta)}{\kappa\psi}\right\}\bar{\pi}^{BL} - \beta\rho(1-\rho)\bar{Q}_{BL} - 1\right]\pi_t^w + \mathbb{E}_t\pi_{t+1}^W = 0,$$

$$\hat{y}_t^W - \mathbb{E}_T\hat{y}_{t+1}^W + (\hat{\imath}_t - \mathbb{E}_t\pi_{t+1}^W) = 0, \tag{5.A.1}$$

$$\pi_t^W - \beta\mathbb{E}_t\pi_{t+1}^W - \kappa\hat{y}_t^W - \kappa\psi\hat{\tau}_t^W = 0, \tag{5.A.2}$$

$$\hat{b}_{t-1}^W - \hat{\pi}_t^W - \beta\hat{b}_t^W - \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) - \beta(1-\rho)\hat{Q}_t = 0, \tag{5.A.3}$$

$$\hat{Q}_t + \hat{\imath}_t - \beta\rho\mathbb{E}_t\hat{Q}_{t+1} = 0 \tag{5.A.4}$$

Substituting the conjectured solutions provides the conditions to determine the coefficients:

$$\bar{y}^{BL} - \frac{1}{\lambda}\left[\beta(1-\rho) + \gamma(1-\beta)\left(1-\frac{1}{\psi}\right)\right]\left[1+\frac{\gamma(1-\beta)}{\kappa\psi}\right]^{-1}\bar{\pi}^{BL} = 0,$$

$$\left[(1-\rho)(\bar{y}^{BL} + \bar{\pi}^{BL}) + \left\{\frac{\gamma(1-\beta)}{\kappa\psi}\right\}\bar{\pi}^{BL} - \beta\rho(1-\rho)\bar{Q}^{BL} - 1\right] + \beta\bar{b}^{BL} = 0,$$

$$\bar{y}^{BL} - \bar{y}^{BL}\bar{b}_{BL} + \left(\bar{\iota}^{BL} - \bar{\pi}^{BL}\bar{b}^{BL}\right) = 0,$$

$$\bar{\pi}^{BL} - \beta\bar{\pi}^{BL}\,\bar{b}^{BL} - \kappa\bar{y}^{BL} - \kappa\psi\bar{\tau}^{BL} = 0,$$

$$1 - \bar{\pi}^{BL} - \beta\bar{b}^{BL} - \gamma(1-\beta)(\bar{y}^{BL} + \bar{\tau}^{BL}) - \beta(1-\rho)\bar{Q}^{BL} = 0,$$

$$\bar{Q}^{BL} + \bar{\iota}^{BL} - \beta\rho\bar{Q}^{BL}\bar{b}^{BL} = 0.$$

5.A.1.2 Non-cooperative solution

The period Lagrangian for the interest group i's problem is given by

$$\begin{aligned}
\mathcal{L}_t = {} & \frac{1}{2}\left(\pi_{i,t}^2 + \lambda\hat{y}_{i,t}^2\right) + \beta\mathbb{E}_t V\left(\hat{b}_t^W\right) \\
& + \phi_{i,t}^{PC}\left[\pi_t^W - \beta\bar{\pi}_i^{BL}\hat{b}_t^W - \kappa\left(\hat{y}_{i,t} + \psi\hat{\tau}_{i.t}\right)\right] \\
& + \phi_{i,t}^{IS}\left[\hat{y}_{i,t} - \bar{y}_i^{BL}\hat{b}_{t-1}^W + \left(\hat{\iota}_t - \bar{\pi}_i^{BL}\hat{b}_t^W\right)\right] \\
& + \phi_{i,t}^{Gov}\left[\hat{b}_{t-1}^W - \pi_t^W - \beta\hat{b}_t^W - \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) - \beta(1-\rho)\hat{Q}_t\right] \\
& + \phi_{i,t}^{Q}\left[\hat{Q}_t + \hat{\iota}_t - \beta\rho\bar{Q}_i^{BL}\hat{b}_t^W\right],
\end{aligned}$$

where $\phi_{i,t}^{PC}$, $\phi_{i,t}^{IS}$, $\phi_{i,t}^{Gov}$, and $\phi_{i,t}^{Q}$ are Lagrange multipliers corresponding to (5.18), (5.19), (5.20), and (5.21).

Focusing on a symmetric equilibrium in which $\pi_{i,t} = \pi_t^W$, $\hat{y}_{i,t} = \hat{y}_t^W$, $\hat{\tau}_{i,t} = \hat{\tau}_t^W$, and $\hat{b}_{i,t} = \hat{b}_t^W$ hold, the conditions that characterize the dynamics of the aggregate variables are obtained:

$$\hat{y}_t^W - \frac{2}{\lambda}\left[\beta(1-\rho) + \frac{\gamma(1-\beta)}{2}\left(1 - \frac{1}{\psi}\right)\right]\left[1 + \frac{\gamma(1-\beta)}{\kappa\psi}\right]^{-1} \pi_t^W = 0,$$

$$\left[(1-\rho)(\bar{y}_i^{BL} + \bar{\pi}_i^{BL}) + \left\{\frac{\gamma(1-\beta)}{2\kappa\psi}\right\}\bar{\pi}_i^{BL} - \beta\rho(1-\rho)\bar{Q}_i^{BL} - 1\right]\pi_t^W + \mathbb{E}_t\pi_{t+1}^W = 0,$$

and (5.A.1), (5.A.2), (5.A.3), and (5.A.4).

Substituting the conjectured solutions, on which the symmetricity conditions are imposed, yields the conditions to determine the coefficients:

$$\bar{y}_i^{BL} - \frac{2}{\lambda}\left[\beta(1-\rho) + \frac{\gamma(1-\beta)}{2}\left(1 - \frac{1}{\psi}\right)\right]\left[1 + \frac{\gamma(1-\beta)}{\kappa\psi}\right]^{-1} \bar{\pi}_i^{BL} = 0,$$

$$\left[(1-\rho)(\bar{y}_i^{BL} + \bar{\pi}_i^{BL}) + \left\{\frac{\gamma(1-\beta)}{2\kappa\psi}\right\}\bar{\pi}_i^{BL} - \beta\rho(1-\rho)\bar{Q}_i^{BL} - 1\right] + \beta\bar{b}_i^{BL} = 0,$$

$$\bar{y}_i^{BL} - \bar{y}_i^{BL}\bar{b}_i^{BL} + \left(\bar{\iota}_i^{BL} - \bar{\pi}_i^{BL}\bar{b}_i^{BL}\right) = 0,$$

$$\bar{\pi}_i^{BL} - \beta\bar{\pi}_i^{BL}\bar{b}_i^{BL} - \kappa\bar{y}_i^{BL} - \kappa\psi\bar{\tau}_i^{BL} = 0,$$

$$1 - \bar{\pi}_i^{BL} - \beta\bar{b}_i^{BL} - \gamma(1-\beta)(\bar{y}_i^{BL} + \bar{\tau}_i^{BL}) - \beta(1-\rho)\bar{Q}_i^{BL} = 0,$$

$$\bar{Q}_i^{BL} + \bar{\iota}_i^{BL} - \beta\rho\bar{Q}_i^{BL}\bar{b}_i^{BL} = 0.$$

5.A.2 Case without the Zero Lower Bound

5.A.2.1 Benevolent planner's solution

The period Lagrangian for the planner's problem in the periods of a negative demand shock is given by

$$\mathcal{L}_t = \frac{1}{2}[(\pi_t^W)^2 + \lambda(\hat{y}_t^W)^2] + \beta \mathbb{E}_t V_{t+1}(\hat{b}_t^W)$$

$$+\xi_t^{PC}\left[\pi_t^W - \beta\bar{\pi}_t^{NR}\hat{b}_t^W - \kappa(\hat{y}_t^W + \psi\hat{\tau}_t^W)\right]$$

$$+\xi_t^{IS}\left[\hat{y}_t^W - \bar{y}_t^{NR}\hat{b}_{t-1}^W + \left(\hat{\imath}_t - \bar{\pi}_t\hat{b}_t^W + \hat{r}_L\right)\right]$$

$$+\xi_t^{Gov}\left[\hat{b}_{t-1}^W - \pi_t^W - \beta\hat{b}_t^W - \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) - \beta(1-\rho)\hat{Q}_t\right]$$

$$+\xi_t^Q\left[\hat{Q}_t - \hat{\imath}_t - \beta\rho\bar{Q}_t^{NR}\hat{b}_t^W\right],$$

where ξ_t^{PC}, ξ_t^{IS}, ξ_t^{Gov}, and ξ_t^Q are Lagrange multipliers corresponding to (5.18), (5.19), (5.20), and (5.21). We then obtain the first-order conditions:

$$\frac{\partial L_t}{\partial \pi_t^W} = \pi_t^W + \xi_t^{PC} - \xi_t^{Gov} = 0,$$

$$\frac{\partial L_t}{\partial \hat{y}_t^W} = \lambda\hat{y}_t^W + \xi_t^{IS} - \kappa\xi_t^{PC} - \gamma(1-\beta)\xi_t^{Gov} = 0,$$

$$\frac{\partial L_t}{\partial \hat{\tau}_t^W} = -\kappa\psi\xi_t^{PC} - \gamma(1-\beta)\xi_t^{Gov} = 0,$$

$$\frac{\partial L_t}{\partial \hat{\imath}_t} = \xi_t^{IS} + \xi_t^Q = 0,$$

$$\frac{\partial L_t}{\partial \hat{Q}_t} = -\beta(1-\rho)\xi_t^{Gov} + \xi_t^Q = 0,$$

$$\frac{\partial L_t}{\partial \hat{b}_t^W} = V_{t+1}'(\hat{b}_t^W) - (\bar{y}^{NR} + \bar{\pi}^{NR})\xi_t^{IS} - \beta\bar{\pi}^{NR}\xi_t^{PC} - \beta\xi_t^{Gov} - \beta\rho\bar{Q}^{NR}\xi_t^{Q} = 0,$$

and the envelop condition:

$$V'(\hat{b}_{t-1}^W) = \xi_t^{Gov}.$$

The conditions characterizing the dynamics of the aggregate variables are summarized to obtain the system of equations:

$$\hat{y}_t^W - \frac{1}{\lambda}\left[\beta(1-\rho) + \gamma(1-\beta)\left(1-\frac{1}{\psi}\right)\right]\left[1+\frac{\gamma(1-\beta)}{\kappa\psi}\right]^{-1}\pi_t^w = 0,$$

$$\left[(1-\rho)(\bar{y}^{NR} + \bar{\pi}^{NR}) + \left\{\frac{\gamma(1-\beta)}{\kappa\psi}\right\}\bar{\pi}^{NR} - \beta\rho(1-\rho)\bar{Q}^{NR} - 1\right]\pi_t^w + \mathbb{E}_t\pi_{t+1}^W = 0,$$

and (5.A.2), (5.A.3), and (5.A.4) for $t = 0, \cdots, K-1$.

Substituting the conjectured solutions yields the conditions to determine the coefficients

$$\bar{y}_t^{NR} - \frac{1}{\lambda}\left[\beta(1-\rho) + \gamma(1-\beta)\left(1-\frac{1}{\psi}\right)\right]\left[1+\frac{\gamma(1-\beta)}{\psi\kappa}\right]^{-1}\bar{\pi}_t^{NR} = 0,$$

$$\left[(\bar{y}_{t+1}^{NR} + \bar{\pi}_{t+1}^{NR})(1-\rho) + \left\{\frac{\gamma(1-\beta)}{\psi\kappa}\right\}\bar{\pi}_{t+1}^{NR} - \beta\rho(1-\rho)\bar{Q}_{t+1}^{NR}\right]\bar{\pi}_t + \bar{\pi}_{t+1}^{NR}\bar{b}_t^{NR} = 0,$$

$$[\bar{y}_t^{NR} + \bar{\iota}_t^{NR} + \bar{y}_{t+1}^{NR}\bar{b}_t^{NR} - \bar{\pi}_{t+1}^{NR}\bar{b}_t^{NR}]\hat{b}_{t-1}^W + (1-\beta+\hat{r}_L) = 0,$$

$$\bar{\pi}_t^{NR} - \beta\bar{\pi}_{t+1}^{NR}\bar{b}_t^{NR} - \kappa\bar{y}_t^{NR} - \kappa\psi\bar{\tau}_t^{NR} = 0,$$

$$1 - \bar{\pi}_t^{NR} - \beta\hat{b}_t^W - \gamma(1-\beta)(\bar{y}_t^{NR} + \bar{\tau}_t^{NR}) - \beta(1-\rho)\bar{Q}_t^{NR} = 0,$$

given the initial condition $\hat{b}_{-1}^W$ and the fact that $\bar{y}_6^{NR} = \bar{y}^{BL}$, $\bar{\pi}_6^{NR} = \bar{\pi}^{BL}$, and $\hat{Q}_6^{NR} = \bar{Q}^{BL}$.

5.A.2.2 Non-cooperative solution

The period Lagrangian for group i's problem in the periods of a negative demand shock is given by

$$\mathcal{L}_t = \frac{1}{2}\left(\pi_{i,t}^2 + \lambda \hat{y}_{i,t}^2\right) + \beta \mathbb{E}_t V_{t+1}\left(\hat{b}_t^W\right)$$

$$+ \xi_{i,t}^{PC}\left[\pi_t^W - \beta \bar{\pi}_{i,t}^{NR} \hat{b}_t^W - \kappa\left(\hat{y}_{i,t} + \psi \hat{\tau}_{i.t}\right)\right]$$

$$+ \xi_{i,t}^{IS}\left[\hat{y}_{i,t} - \bar{y}_{i,t}^{NR} \hat{b}_{t-1}^W + \left(\hat{\imath}_t - \bar{\pi}_{i,t}^{ZLB} \hat{b}_t^W + \hat{r}_L\right)\right]$$

$$+ \xi_{i,t}^{Gov}\left[\hat{b}_{t-1}^W - \pi_t^W - \beta \hat{b}_t^W - \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) - \beta(1-\rho)\hat{Q}_t\right]$$

$$+ \xi_{i,t}^{Q}\left[\hat{Q}_t - \hat{\imath}_t - \beta\rho \bar{Q}_{i,t}^{ZLB} \hat{b}_t^W\right],$$

for $t = 0, \cdots, K-1$, where$\xi_{i,t}^{PC}$, $\xi_{i,t}^{IS}$, $\xi_{i,t}^{Gov}$, and $\xi_{i,t}^{Q}$ are Lagrange multipliers corresponding to (5.18), (5.19), (5.20), and (5.21).

The conditions characterizing the dynamics of the aggregate variables from period 0 to K are summarized to obtain the system of equations:

$$\bar{y}_{i,t}^{NR} - \frac{2}{\lambda}\left[\beta(1-\rho) + \frac{\gamma(1-\beta)}{2}\left(1 - \frac{1}{\psi}\right)\right]\left[1 + \frac{\gamma(1-\beta)}{\psi\kappa}\right]^{-1} \bar{\pi}_{i,t}^{NR} = 0,$$

$$\left[\left(\bar{y}_{i,t+1}^{NR} + \bar{\pi}_{i,t+1}^{NR}\right)(1-\rho) + \left\{\frac{\gamma(1-\beta)}{2\psi\kappa}\right\}\bar{\pi}_{i,t+1}^{NR} - \beta\rho(1-\rho)\bar{Q}_{i,t+1}^{NR}\right]\bar{\pi}_{i,t} + \bar{\pi}_{i,t+1}^{NR}\bar{b}_t^{NR} = 0,$$

$$\left[\bar{y}_{i,t}^{NR} + \bar{\imath}_{i,t}^{NR} + \bar{y}_{i,t+1}^{NR}\bar{b}_t^{NR} - \bar{\pi}_{i,t+1}^{NR}\bar{b}_t^{NR}\right]\hat{b}_{t-1}^W + (1 - \beta + \hat{r}_L) = 0,$$

$$\bar{\pi}_{i,t}^{NR} - \beta\bar{\pi}_{i,t+1}^{NR}\bar{b}_t^{NR} - \kappa\bar{y}_{i,t}^{NR} - \kappa\psi\bar{\tau}_{i,t}^{NR} = 0,$$

$$1 - \bar{\pi}_{i,t}^{NR} - \beta\hat{b}_t^W - \gamma(1-\beta)\left(\bar{y}_{i,t}^{NR} + \bar{\tau}_{i,t}^{NR}\right) - \beta(1-\rho)\bar{Q}_{i,t}^{NR} = 0,$$

$$\bar{Q}_{i,t}^{NR} + \bar{\imath}_{i,t}^{NR} - \beta\rho\bar{Q}_{i,t+1}^{NR}\bar{b}_t^{NR} = 0,$$

given the initial condition $\hat{b}_{-1}^{W}$ and the fact that $\bar{y}_{i,6}^{NR} = \bar{y}_{i}^{BL}$, $\bar{\pi}_{i,6}^{NR} = \bar{\pi}_{i}^{BL}$, and $\hat{Q}_{6}^{NR} = \bar{Q}_{i}^{BL}$.

5.A.3 Case with the Zero Lower Bound

5.A.3.1 Benevolent planner's solution

The period Lagrangian for the planner's problem during the ZLB periods is given by

$$\mathcal{L}_t = \frac{1}{2}[(\pi_t^W)^2 + \lambda(\hat{y}_t^W)^2] + \beta\mathbb{E}_t V_{t+1}\left(\hat{b}_t^W\right)$$

$$+ \mu_t^{PC}\left[\pi_t^W - \beta\bar{\pi}_t^{ZLB}\hat{b}_t^W - \kappa(\hat{y}_t^W + \psi\hat{\tau}_t^W)\right]$$

$$+\mu_t^{IS}\left[\hat{y}_t^W - \bar{y}_t^{ZLB}\hat{b}_{t-1}^W + \left\{-(1-\beta) - \bar{\pi}_t\hat{b}_t^W + \hat{r}_L\right\}\right]$$

$$+ \mu_t^{Gov}\left[\hat{b}_{t-1}^W - \pi_t^W - \beta\hat{b}_t^W - \gamma(1-\beta)(\hat{y}_t^W + \hat{\tau}_t^W) - \beta(1-\rho)\hat{Q}_t\right]$$

$$+\mu_t^Q\left[\hat{Q}_t - (1-\beta) - \beta\rho\bar{Q}_t^{ZLB}\hat{b}_t^W\right],$$

for $t = 0, \cdots, K-1$, where μ_t^{IS}, μ_t^{PC}, μ_t^{Gov}, and μ_t^Q are Lagrange multipliers corresponding to (5.18), (5.19), (5.20), and (5.21). We then obtain the first-order conditions:

$$\frac{\partial\mathcal{L}}{\partial\pi_t^W} = \pi_t^W + \phi_t^{PC} - \phi_t^{Gov} = 0,$$

$$\frac{\partial\mathcal{L}}{\partial\hat{y}_t^W} = \lambda\hat{y}_t^W + \phi_t^{IS} - \kappa\phi_t^{PC} - \gamma(1-\beta)\phi_t^{Gov} = 0,$$

$$\frac{\partial\mathcal{L}}{\partial\hat{\tau}_t^W} = -\kappa\psi\phi_t^{PC} - \gamma(1-\beta)\phi_t^{Gov} = 0,$$

$$\frac{\partial\mathcal{L}}{\partial\hat{Q}_t} = -\beta(1-\rho)\phi_t^{Gov} + \phi_t^Q = 0,$$

$$\frac{\partial \mathcal{L}}{\partial \hat{b}_t^W} = \beta V'_{t+1}\left(\hat{b}_t^W\right) - \phi_t^{IS}(\bar{y}_t^{ZLB} + \bar{\pi}_t^{ZLB}) - \beta\bar{\pi}_{t+1}\phi_t^{PC} - \beta\phi_t^{Gov} - \beta\rho\bar{Q}_{t+1}^{ZLB}\,\phi_t^{Q} = 0,$$

and the envelop condition:

$$V'_t\left(\hat{b}_{t-1}^W\right) = \phi_t^{Gov},$$

for $t = 0, \cdots, K-1$.

The conditions characterizing the dynamics of the aggregate variables from period 0 to 5 are summarized to obtain the system of equations:

$$\beta\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\pi_{t+1}^W + \beta\left(\left[\frac{\gamma(1-\beta)}{\kappa}\right]\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\right)\bar{\pi}_{t+1}^{ZLB}\pi_t^W$$

$$-\beta\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\pi_t^W + (\bar{y}_{t+1}^{ZLB} + \bar{\pi}_{t+1}^{ZLB})\left(1 - \frac{1}{\psi}\right)\gamma(1-\beta)\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\pi_t^W$$

$$-\lambda(\bar{y}_{t+1}^{ZLB} + \bar{\pi}_{t+1}^{ZLB})\hat{y}_T^W - \beta^2\rho(1-\rho)\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\bar{Q}_{t+1}\pi_t^W = 0,$$

$$\hat{y}_t^W - \mathbb{E}_t\hat{y}_{t+1}^W - \mathbb{E}_t\pi_{t+1} - (1 - \beta - \hat{r}_{ZLB}) = 0,$$

and (5.A.2), (5.A.3), and (5.A.4) or $t = 0, \cdots, K-1$.

Substituting the conjectured solutions yields the conditions to determine the coefficients

$$\beta\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\bar{\pi}_{t+1}^{ZLB}\bar{b}_t^{ZLB} + \beta\left(\left[\frac{\gamma(1-\beta)}{\kappa}\right]\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\right)\bar{\pi}_t^{ZLB}\bar{\pi}_{t+1}^{ZLB}$$

$$-\beta\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\bar{\pi}_t^{ZLB} + (\bar{y}_{t+1}^{ZLB} + \bar{\pi}_{t+1}^{ZLB})\left(1 - \frac{1}{\psi}\right)\gamma(1-\beta)\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\bar{\pi}_t^{ZLB}$$

$$-\lambda(\bar{y}_{t+1}^{ZLB} + \bar{\pi}_{t+1}^{ZLB})\bar{y}_t - \beta^2\rho(1-\rho)\left[\frac{\gamma(1-\beta)}{\psi\kappa} - 1\right]^{-1}\bar{Q}_{t+1}^{ZLB}\bar{\pi}_t^{ZLB} = 0,$$

$$\bar{y}_t^{ZLB}\hat{b}_{t-1}^W - \left(\bar{y}_{t+1}^{ZLB}\bar{b}_t^{ZLB}\right)\hat{b}_{t-1}^W - \left(\bar{\pi}_{t+1}^{ZLB}\bar{b}_t^{ZLB}\right)\hat{b}_{t-1}^W + (1 - \beta + \hat{r}_L) = 0,$$

$$\bar{\pi}_t^{ZLB} - \beta\bar{\pi}_t^{ZLB}\bar{b}_t^{ZLB} - \kappa\bar{y}_t^{ZLB} - \kappa\psi\bar{\tau}_t^{ZLB} = 0,$$

$$1-\bar{\pi}_t^{ZLB}-\beta\bar{b}_t^{ZLB}-\gamma(1-\beta)(\bar{y}_t^{ZLB}+\bar{\tau}_t^{ZLB})-\beta(1-\rho)\bar{Q}_t^{ZLB}=0,$$

$$\bar{Q}_t^{ZLB}\hat{b}_{t-1}^W-(1-\beta)-\beta\rho\bar{Q}_t^{ZLB}\bar{b}_t^{ZLB}\hat{b}_{t-1}^W=0,$$

for $t=0,\cdots,K-1$, given the initial condition $\hat{b}_{-1}^W$ and the fact that $\hat{y}_6^W=\bar{y}^{BL}\hat{b}_5^W$, $\hat{\pi}_6^W=\bar{\pi}^{BL}\hat{b}_5^W$, $\hat{Q}_6^W=\bar{Q}^{BL}\hat{b}_5^W$.

5.A.3.2 Non-cooperative solution

The period Lagrangian for the planner's problem during the ZLB periods is given by

$$\begin{aligned}
\mathcal{L}_t =& \frac{1}{2}\left(\pi_{i,t}^2+\lambda\hat{y}_{i,t}^2\right)+\beta\mathbb{E}_t V_t\left(\hat{b}_t^W\right) \\
&+\mu_{i,t}^{PC}\left[\pi_t^W-\beta\bar{\pi}_{i,t}^{ZLB}\hat{b}_t^W-\kappa\left(\hat{y}_{i,t}+\psi\hat{\tau}_{i.t}\right)\right] \\
&+\mu_{i,t}^{IS}\left[\hat{y}_{i,t}-\bar{y}_{i,t}^{ZLB}\hat{b}_{t-1}^W+\left\{-(1-\beta)-\bar{\pi}_t\hat{b}_t^W+\hat{r}_L\right\}\right] \\
&+\mu_{i,t}^{Gov}\left[\hat{b}_{t-1}^W-\pi_t^W-\beta\hat{b}_t^W-\gamma(1-\beta)(\hat{y}_t^W+\hat{\tau}_t^W)-\beta(1-\rho)\hat{Q}_t\right] \\
&+\mu_t^Q\left[\hat{Q}_t-(1-\beta)-\beta\rho\bar{Q}_{i,t}^{ZLB}\hat{b}_t^W\right],
\end{aligned}$$

for $t=0,\cdots,K-1$, where $\mu_{i,t}^{PC}$, $\mu_{i,t}^{IS}$, $\mu_{i,t}^{Gov}$, and $\mu_{i,t}^{Q}$ are Lagrange multipliers corresponding to (5.18), (5.19), (5.20), and (5.21).

The conditions characterizing the dynamics of the aggregate variables from period 0 to K are summarized to obtain the system of equations:

$$2\beta\left[\frac{\gamma(1-\beta)}{\psi\kappa}-1\right]^{-1}\bar{\pi}_{i,t+1}^{ZLB}\bar{b}_t^{ZLB}+\beta\left(\left[\frac{\gamma(1-\beta)}{2\kappa}\right]\left[\frac{\gamma(1-\beta)}{\psi\kappa}-1\right]^{-1}\right)\bar{\pi}_{i,t}^{ZLB}\bar{\pi}_{i,t+1}^{ZLB}$$

$$-\beta\left[\frac{\gamma(1-\beta)}{\psi\kappa}-1\right]^{-1}\bar{\pi}_{i,t}^{ZLB}+\frac{1}{2}\left(\bar{y}_{i,t+1}^{ZLB}+\bar{\pi}_{i,t+1}^{ZLB}\right)\left(1-\frac{1}{\psi}\right)\gamma(1-\beta)\left[\frac{\gamma(1-\beta)}{\psi\kappa}-1\right]^{-1}\bar{\pi}_{i,t}^{ZLB}$$

$$-\frac{\lambda}{2}\left(\bar{y}_{i,t+1}^{ZLB}+\bar{\pi}_{i,t+1}^{ZLB}\right)\bar{y}_t-\beta^2\rho(1-\rho)\left[\frac{\gamma(1-\beta)}{\psi\kappa}-1\right]^{-1}\bar{Q}_{i,t+1}^{ZLB}\bar{\pi}_{i,t}^{ZLB}=0,$$

$$\left(\bar{y}_{i,t}^{ZLB}-\bar{y}_{i,t+1}^{ZLB}\bar{b}_{i,t}^{ZLB}-\bar{\pi}_{i,t+1}^{ZLB}\bar{b}_{i,t}^{ZLB}\right)\hat{b}_{t-1}^{W}+(1-\beta+\hat{r}_L)=0,$$

$$\bar{\pi}_{i,t}^{ZLB}-\beta\bar{\pi}_{i,t}^{ZLB}\bar{b}_{i,t}^{ZLB}-\kappa\bar{y}_{i,t}^{ZLB}-\kappa\psi\bar{\tau}_{i,t}^{ZLB}=0,$$

$$1-\bar{\pi}_{i,t}^{ZLB}-\beta\bar{b}_{i,t}^{ZLB}-\gamma(1-\beta)\left(\bar{y}_{i,t}^{ZLB}+\bar{\tau}_{i,t}^{ZLB}\right)-\beta(1-\rho)\bar{Q}_{i,t}^{ZLB}=0,$$

$$\bar{Q}_{i,t}^{ZLB}\hat{b}_{t-1}^{W}-(1-\beta)-\beta\rho\bar{Q}_{i,t}^{ZLB}\bar{b}_{i,t}^{ZLB}\hat{b}_{t-1}^{W}=0,$$

for $t=0,\cdots,K-1$, given the initial condition $\hat{b}_{-1}^{W}$ and the fact that $\hat{y}_6^W=\bar{y}_i^{BL}\hat{b}_5^W$, $\hat{\pi}_6^W=\bar{\pi}_i^{BL}\hat{b}_5^W$, $\hat{Q}_6^W=\bar{Q}_i^{BL}\hat{b}_5^W$.

5.A.4 Sensitivity Analysis

In this section, we assess the sensitivity of the main results presented in this chapter to some alternative values of the Frisch elasticity of labor supply. We report the results for the two alternative values of η (1 and 3). A change in the Frisch elasticity affects the sensitivity of group-specific inflation to a change in the tax rate; thus, the robustness of our results to these changes is less evident. For other structural parameters, we use the same values. The initial debt is set to $\hat{b}_{-1}^{W}=0.2$.

5.A.4.1 Higher value of the Frisch elasticity

First, we consider a higher value of the Frisch elasticity: $\eta^{-1} = 1$. Figure 5.8 report the results for the baseline case. It is notable that a benevolent planner would choose a negative response of the tax rate because, when $\eta^{-1} = 1$. The reason for this is that the inflationary effect of the tax hike dominates its deflationary effect. Nonetheless, the responses of the other variables are similar to these when $\eta^{-1} = 2$. It should also be noted that in the non-cooperative game a negative response of the tax rate becomes larger.

Second, we consider the ZLB case. As shown in Figure 5.9 the three main results of our analysis are robust to this change in the Frisch elasticity. First, the free-riding activity of the interest groups causes deflation at the time of liftoff from the ZLB and thus a severe recession during the ZLB periods. Second, if the interest groups coordinate, shortening debt maturity has an expansionary effect on the economy at the ZLB. Third, coordination failure between interest groups weakens the expansionary effect of purchases of long-term bonds on the economy at the ZLB.

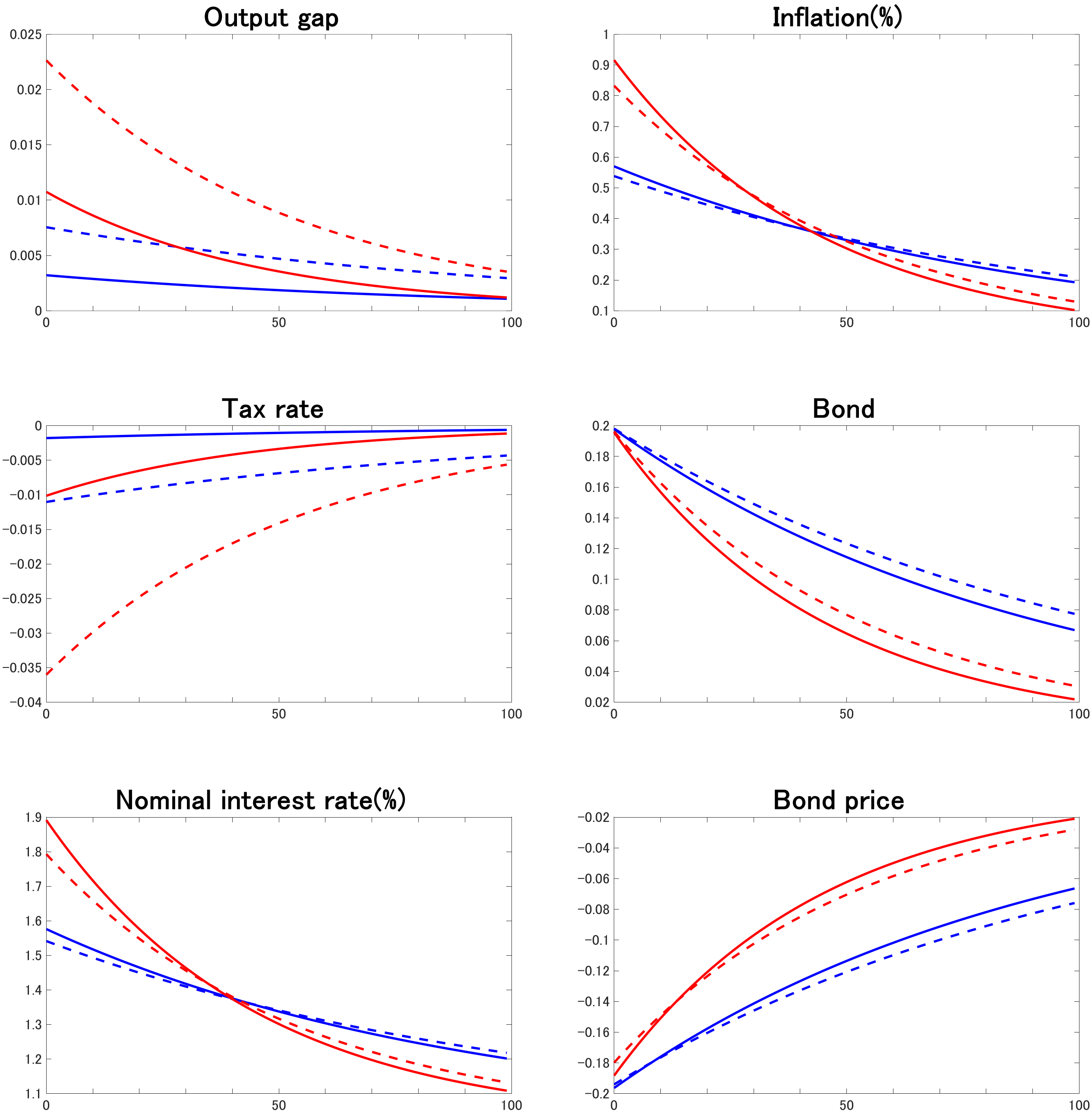

Figure 5.8. Baseline case when $\eta^{-1} = 1$: longer duration vs. shorter duration. Blue solid lines: responses that a benevolent planner would choose in the longer duration case. Blue dotted lines: responses resulting from the non-cooperative game in the longer duration case. Red solid lines: responses that a benevolent planner would choose in the shorter duration case. Red dotted lines: responses resulting from the non-cooperative game in the shorter duration case.

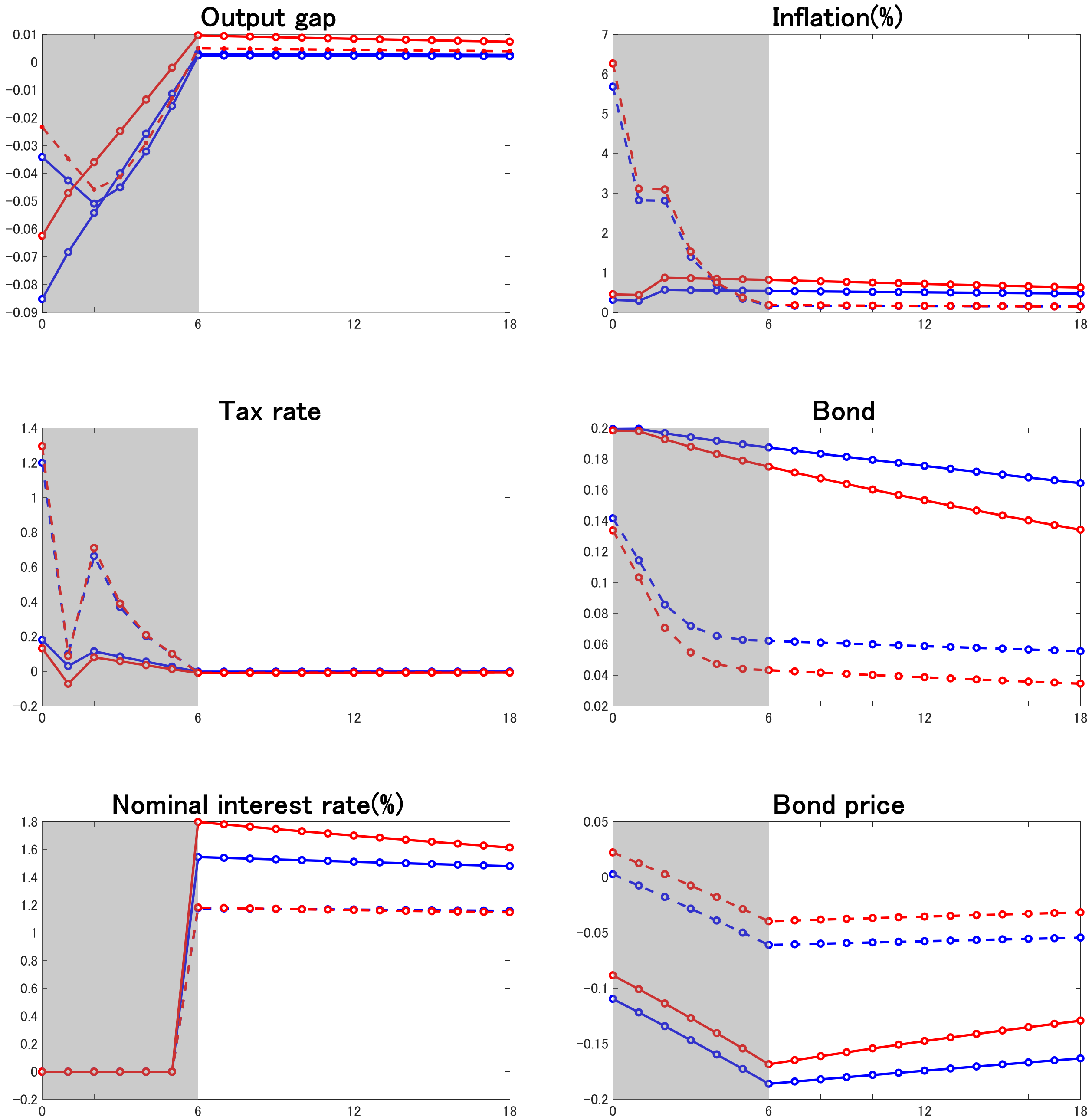

Figure 5.9. ZLB case when $\eta^{-1} = 1$: longer duration vs. shorter duration. Solid blue lines with circles: responses that a benevolent planner would choose in the longer duration case. Dotted blue lines with circles: responses resulting from the non-cooperative game in the longer duration case. Solid red lines with circles: responses that a benevolent planner would choose in the shorter duration case. Dotted red lines with circles: responses resulting from the non-cooperative game in the shorter duration case. A demand shock occurs in period 0 and disappears in period 6.

5.A.4.2 Lower value of the Frisch elasticity

Second, we consider a lower value of the Frisch elasticity: $\eta^{-1} = 1/3$. As shown in Figure 10 and 11, the main results of this chapter are robust to this change.

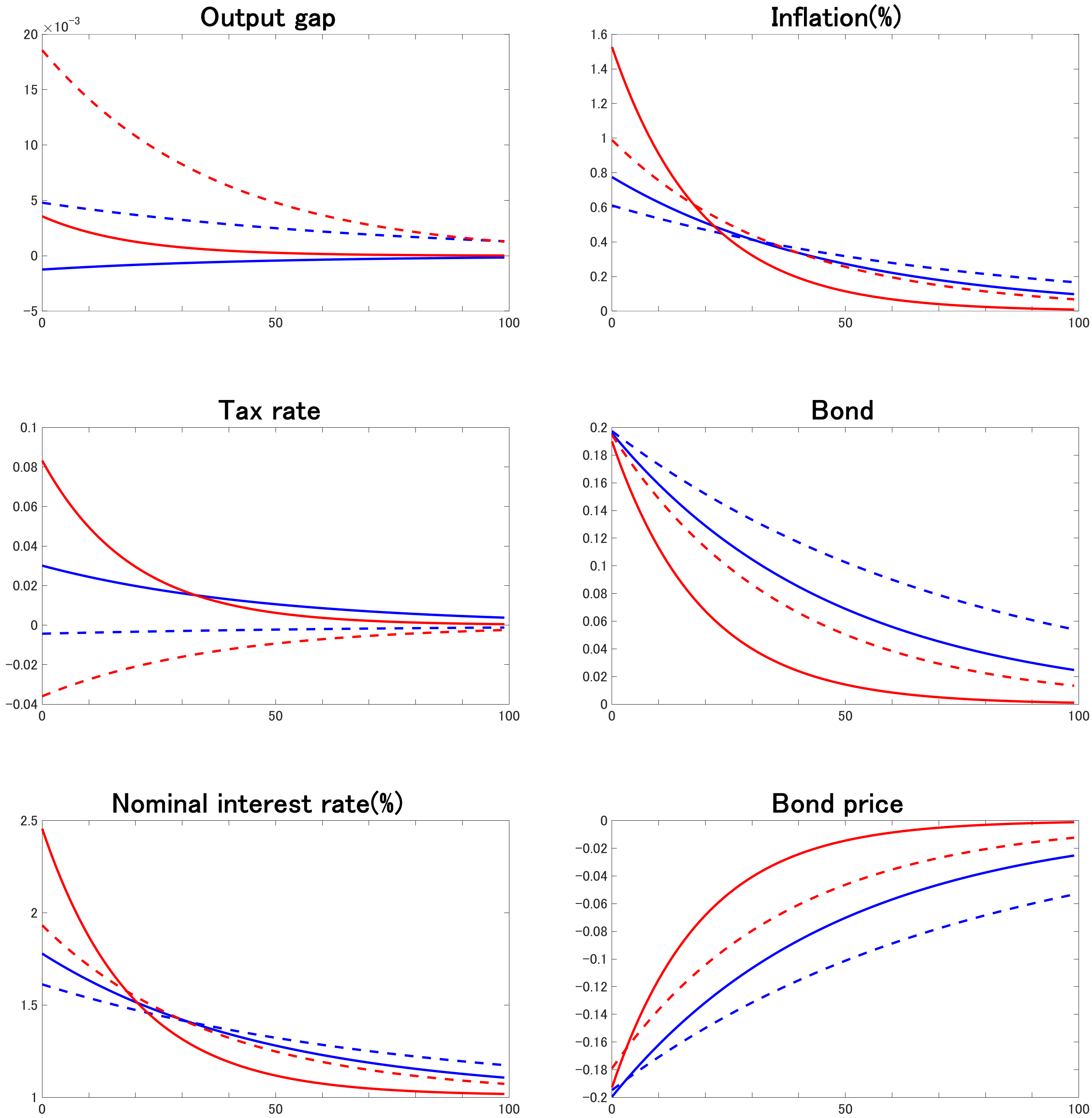

Figure 5.10. Baseline case when $\eta^{-1} = 1/3$: longer duration vs. shorter duration. Blue solid lines: responses that a benevolent planner would choose in the longer duration case. Blue dotted lines: responses resulting from the non-cooperative game in the longer duration case. Red solid lines: responses that a benevolent planner would choose in the shorter duration case. Red dotted lines: responses resulting from the non-cooperative game in the shorter duration case.

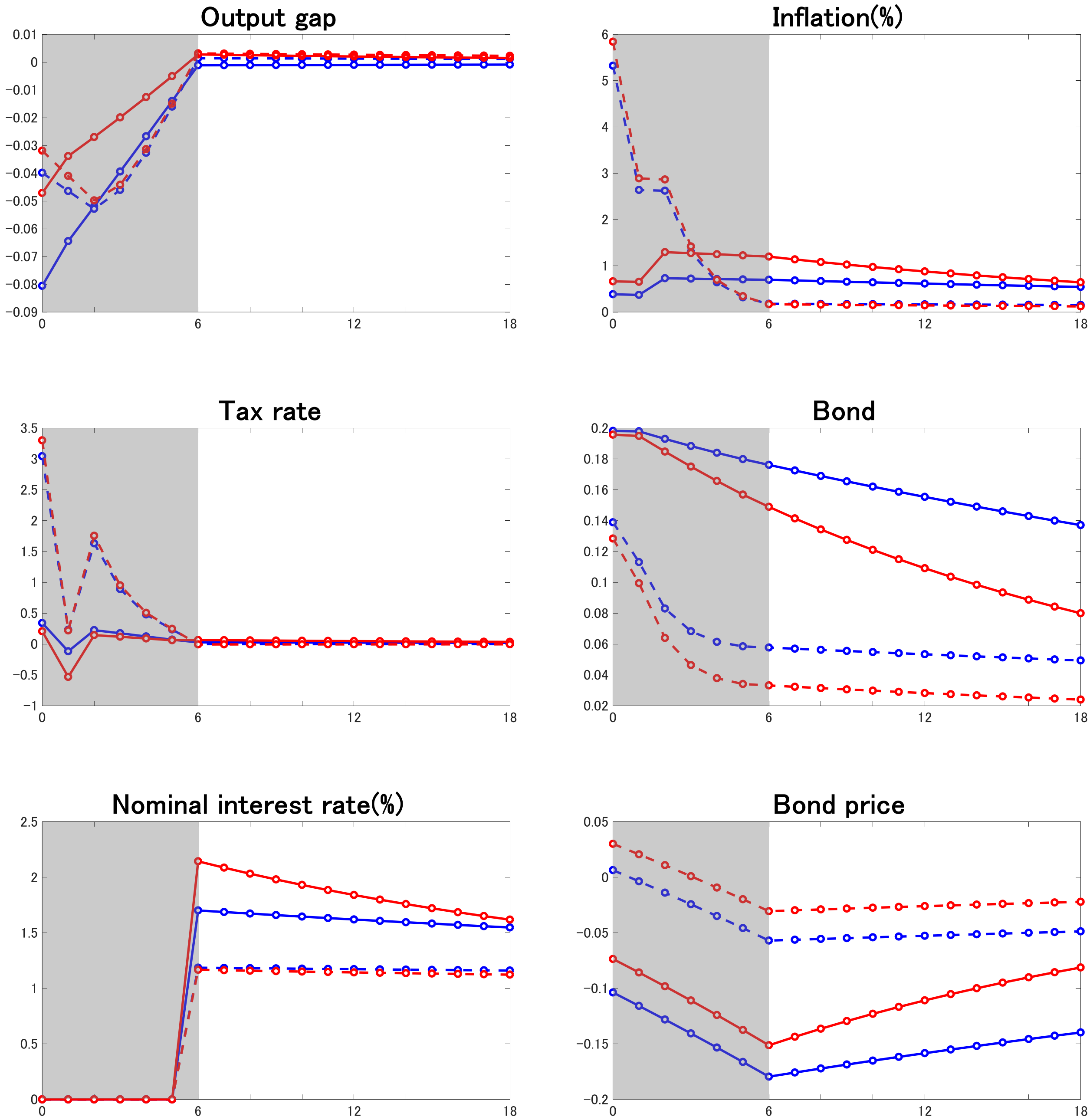

Figure 5.11. ZLB case when $\eta^{-1} = 1/3$: longer duration vs. shorter duration. Solid blue lines with circles: responses that a benevolent planner would choose in the longer duration case. Dotted blue lines with circles: responses resulting from the non-cooperative game in the longer duration case. Solid red lines with circles: responses that a benevolent planner would choose in the shorter duration case. Dotted red lines with circles: responses resulting from the non-cooperative game in the shorter duration case. A demand shock occurs in period 0 and disappears in period 6.

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