No. DP22-1

RCESR Discussion Paper Series

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May 2022

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7 May, 2022

Abstract

We advocate the multilateral Walsh index as a viable alternative to the Gini-Eltetö-Köves-Szulc (GEKS) and Geary-Khamis methods used in the Penn World Table and the International Comparison Program (ICP). We show that it is the only symmetric average fixed basket price index that satisfies transitivity, country symmetry, and invariance to proportional changes in quantities. Simplicity and its superior axiomatic properties including identity and monotonicity, and with associated substitution bias comparable to that of the GEKS_Fisher index, and plausibility and comparability of results based on the 2017 ICP data make the multilateral Walsh method an ideal choice for international price comparisons. (100 words)

JEL Codes: E31, C43

Key words: Price comparisons; Transitivity; Proportionality; Real expenditures

This paper is a revised version, along with a new title, of the RCESR Discussion Paper Series DP22-1, Hitotsubashi University with the title "Towards a simplified approach to international price comparisons: A case for the Multilateral Walsh Index".

Abe's work was supported by JSPS KAKENHI, 19H01467, 16H06322, and 18H00864. Rao acknowledges support from Hitotsubashi Institute for Advanced Study (HIAS) which facilitated visits to HIAS and funding from the Australian Research Council Discovery Project DP170103559. The authors are grateful to Erwin Diewert and Mizobuchi Hideyuki for their comments on earlier versions of this paper. The authors are thankful to the Global Unit of the International Comparison Program at the World Bank for providing detailed basic heading level data from their Researcher database.

The authors declare that they have not received any financial support or have personal relationships with other people or organizations that could inappropriately influence their work.

1. Introduction

Internationally comparable macroeconomic aggregates including gross domestic product (GDP), Household Consumption; Government Expenditure; and Gross Capital Formation are compiled on a regular basis as a part of the International Comparison Program (ICP) at the World Bank. The ICP data provide valuable insights into the structure and composition of the world economy. It is an important source for the analysis of price levels (Deaton and Schreyer, 2021; Heston and Rao, 2021), measurement of global and regional growth and inflation (Balk et al, 2020; Heston and Rao, 2021); regional and global inequality and poverty (Deaton, 2021; Atamanov et al, 2020); and estimates of PPPs and real expenditures are critical inputs into measures like the Human Development Index (United Nations Development Program, 2021) and in the formulation of Sustainable Development Goals (United Nations 2016) and the assessment of performance of nations against these goals. The most recent results (World Bank, 2020) focus on the 2017 benchmark year covering 177 countries. The Penn World Table (PWT) in contrast provides a panel of internationally comparable macro series, PWT Version 10.0 covers 183 countries and the period 1950 to 2019.

The methodology, including the design and conduct of surveys for price collection in participating countries and the index number methods used in aggregating price data leading to estimates of purchasing power parities (PPPs), has evolved over the last fifty years since the inception of the project in 1968[1]. Data related issues have been discussed in Deaton and Heston (2010), Chen and Ravallion (2010), Feenstra et al., (2013) and more recently in Deaton and Aten (2017) and Inklaar and Rao (2017). Aspects of ICP aggregation methodology are discussed in various contributions in Neary (2004), Balk (2009), Rao (2009), Feenstra et al., (2009), Rao (2013a), Diewert (2013).

Aggregation of price and expenditure data in ICP is similar to that used in the compilation of the consumer price index. It is implemented in two stages (Rao, 2013a). First, item level prices are aggregated using the Country-Product-Dummy (CPD) method (Rao, 2013b) leading to PPPs for basic headings[2]. In the second stage, the basic heading PPPs are combined with expenditure data are aggregated using the Gini-Eltetö-Köves-Szulc (GEKS) method (Diewert, 2013). Until the 2005 ICP, the Geary-Khamis (GK) method (Kravis et al, 1982; Diewert, 2013) was the preferred aggregation method[3].

Our focus is on the second stage aggregation and our objective is to propose a simple and easy to understand and explain, and yet analytically superior alternative to the GEKS and GK methods. Though the GEKS and GK methods, discussed in Section 2 below, have intuitive appeal with some desirable properties (Diewert, 1999; Balk, 2008), they both have theoretical shortcomings (Neary, 2004; Diewert 2013) and fail to satisfy basic axiomatic properties like identity test and monotonicity in extreme cases (Rao, 1972). Further, the GK based comparisons are known to suffer from severe substitution bias. Under the GEKS method, price comparisons between pairs of countries are influenced by price data from all the remaining countries. Despite these shortcomings, the GEKS and GK methods continue to play a significant role in international price and expenditure comparisons due to the lack of viable alternatives.

In this paper we advocate a simplified approach anchored on the multilateral Walsh fixed basket index which has superior axiomatic properties than the GEKS and GK methods. We first show that multilateral

[1] The framework including the complex governance structure and various steps involved in the collection of price and national accounts data as well as the steps involved in the ICP are described in Rao (2013a, World Bank, 2020).

[2] The notion of *basic heading* is the spatial counterpart to the concept of *elementary level* in the consumer price index. In concept, the basic heading is the lowest level aggregate at which expenditure data are available. For example, the "rice" basic heading in ICP comprises a range of rice varieties. While no expenditure or quantity data are available for different types of rice, total expenditure or expenditure share of "rice" is available.

[3] Many other methods are available for this purpose. See Hill (1997) for a taxonomy of multilateral index number methods for international price comparisons.

Walsh index is the only fixed basket index that satisfies country symmetry, transitivity and is invariant to proportional changes in the quantity vectors. Our result generalizes the Diewert (2001) result for the bilateral Walsh (1901) index. This method based on the multilateral Walsh index satisfies identity, proportionality and monotonicity tests. In addition, our proof of the main result relaxes the requirement to have some zero prices central to the proof offered in Diewert (2001), and Balk (2008). Our result holds when all the prices are strictly positive.

The use of fixed basket indexes like the Laspeyres, Paasche and the Lowe index in the context of temporal comparisons is often associated with substitution bias (Balk and Diewert, 2010). As the multilateral Walsh belongs to the class of fixed basket indexes, the issue of substitution bias associated with this index of international comparisons is important. Extending the notion of cost of living (COLI) is somewhat challenging when it comes to cross-country comparisons. Issues of non-homothetic and/or heterogeneous preferences along with the requirement of transitivity for price comparisons make the notion of COLI and substitution bias associated with the GEKS, Geary-Khamis and the MW indices difficult to handle. We show, in Section 3, that substitution bias associated with the MW index is of similar order as that of the Fisher index when it comes to bilateral price comparisons, and that the substitution bias for the MW index is conceptually more tractable than the substitution bias associated with the GEKS index currently used in international comparisons. Our analysis shows that the MW and GEKS indexes may induce substitution bias of similar magnitudes and that the bias can be positive or negative. Our empirical findings, exhibiting no systematic differences between MW and GEKS based price comparisons, show that the MW index does not suffer from substitution bias any more than the GEKS index.

The paper is structured as follows. Section 2 establishes the notation and describes the nature of multilateral cross-country comparisons of prices and real expenditures. The Gini-Eltetö-Köves-Szulc (GEKS) and the Geary-Khamis (GK) methods currently used in international comparisons are described. Section 3 presents the main result characterizing the multilateral Walsh index and its axiomatic and economic theoretic properties. The nature and size of substitution bias associated with the multilateral Walsh index relative to that of the Fisher, GEKS and Geary-Khamis indices are also discussed. International price comparisons based on the multilateral Walsh, GEKS and GK methods computed using the ICP price and expenditure data for household consumption for 174 countries are presented in Section 4. Relative differences in estimated purchasing power parities from different methods are compared and evaluated. The paper concludes with Section 5.

2. Notation and Multilateral Cross-Country Comparisons of Prices

We consider the general case with *N* commodities and *M* countries (or spatial entities like regions within a country). Let $\left\{ p_{ij}, q_{ij} : i = 1,2,...,N; \text{and } j = 1,2,...,M \right\}$ represent, respectively, prices and quantity data. All the prices and quantities are assumed to be strictly positive[4]. In vector form, $\left\{ \mathbf{p_j}, \mathbf{q_j} \right\}$ represent price and quantity vectors of order *Nx1* for country *j*; and $\left\{ \mathbf{p}_i, \mathbf{q}_i \right\}$ are (M x 1) vectors of price and quantities of commodity *i* in different countries. International comparisons of prices typically involve comparisons between all pairs of countries and all such comparisons are deemed equally relevant. This means that every country is compared with every other country included in comparisons. For example,

[4] We often encounter in international comparison work zero expenditures, and hence zero quantities, for some items or commodity groups. All the results and work in this paper extends to this more realistic case. However, in this paper we consider only the case where all the prices are strictly positive.

USA is compared with the UK, Germany, India, China and all other countries. Similarly, comparisons of Japan with China, India, Korea, USA and other countries are equally important.

Let $\{PI(s,t): s,t = 1,2,...,M)\}$ represent price index for country *t* (comparison country) with country *k* as the base or reference country. Then, comparisons between all pairs of countries can be presented in a matrix of order $(M \times M)$ with all diagonal elements equal to 1.

$$PI_{M\times M} = \begin{bmatrix} 1 & PI(1,2) & ... & PI(1,M) \\ PI(2,1) & 1 & ... & PI(2,M) \\ . & . & ... & . \\ . & . & ... & . \\ . & . & ... & . \\ PI(M,1) & PI(M,2) & ... & 1 \end{bmatrix}$$

Each element in this matrix, $\{PI(s,t): s,t = 1,2,...,M\}$, is a positive real valued function of the observed price-quantity data, $\{p_s, q_s : s = 1,2,...,M\}$.

A straightforward approach would be to select a known index number formula to compute price indices for all the countries. For example, we may choose the Fisher index as it is known as the ideal index number satisfying many axiomatic properties and is shown to be superlative Diewert (1976). The Fisher index is:

$$PI^F(s,t) = \left[\frac{\sum_{i=1}^{N} p_{it} q_{is}}{\sum_{i=1}^{N} p_{is} q_{is}} \cdot \frac{\sum_{i=1}^{N} p_{it} q_{it}}{\sum_{i=1}^{N} p_{is} q_{it}} \right]^{0.5} \quad for\ s,t = 1,2,...M$$

However, this simple approach is somewhat problematic in that the matrix of Fisher-based price comparisons are not internally consistent. For example, it is easy to check that a comparison of Japan with USA as base would not generally equal the product of comparisons between USA and UK, and between UK and Japan. Thus,

$$PI^F(USA, Japan) \neq PI^F(USA, UK) \cdot PI^F(UK, Japan)$$

We encounter the same problem with the Tornqvist, Edgeworth-Marshal, Sato-Vartia and the bilateral Walsh indexes. Over time, new classes of index numbers (see Hill, 1997) have been developed specifically for measuring cross-country price level differences. These formulae are designed to satisfy three essential requirements stated below in the form of axioms.

Axiom 1: *Country symmetry:* The international price comparisons should treat all countries symmetrically.

For example, if price comparisons between countries are anchored on quantity data from a specific country, say USA, then the price comparisons would be transitive but USA is treated differently from all other countries thus failing this axiom

Axiom 2: *Transitivity:* The matrix of multilateral index numbers $\{PI(s,t): s,t = 1,2,...,M)\}$ satisfies transitivity if for any three countries $s, t\ and\ k$

$$PI(s,t) = PI(s,k) \times PI(k,t) \quad \forall s,t\ and\ k = 1,2,...,M \tag{1}$$

This means that a direct price comparison between, for example, USA and Germany must equal an indirect comparison through a third country, say, Canada, [5](product of index between USA and Canada, and between Canada and Germany).

Axiom 3: *Invariance to proportional changes in quantity vector of a country*: This means that price comparisons are unaffected when the quantities observed in a country are multiplied by a constant.

This means that the price comparisons remain the same whether the total consumption or per capita consumption of a country is used. This axiom is similar to *the invariance to proportional changes in current quantities test* in Diewert (2001, p.207) stated for bilateral temporal comparisons.

There are many other axioms for international price comparisons (Diewert, 1999 and Balk, 2008) but these three axioms are sufficient for us to establish the main result of the paper.

As our main aim is to offer a simple method that is superior to the current practice, we focus on two methods that have occupied the central stage in international comparisons and show that our proposed method has better axiomatic properties. The Gini-Eltetö-Köves-Szulc (Gini, 1924; Eltetö and Köves, 1964); and Szulc, 1964) and the Geary-Khamis (Geary, 1958 and Khamis, 1972) are the two principal methods of aggregation used in international comparisons since its inception in 1968. In the earlier phases of the International Comparison Program (ICP), Kravis et al., (1982) employed the Geary-Khamis method as it provided additively consistent set of comparisons of the gross domestic product and its components. The GK method was replaced by the GEKS method during 2005 ICP and it remains the main aggregation method for the ICP since then. In parallel with the ICP, the OECD and Eurostat have been conducting international price and real income comparisons for member countries since 1980 and the GEKS method has been their preferred method (Eurostat-OECD, 2012). The Penn World Table (PWT) uses both of these methods though the Geary-Khamis method has been the main PWT aggregation method. These two methods are briefly described below[6].

The Gini-Eltetö-Köves-Szulc (GEKS) Method

The GEKS method is an ingenious technique to produce a matrix of *transitive* multilateral comparisons out of a set of bilateral price comparisons that are not transitive. The original GEKS is built on Fisher binary comparisons. As the Fisher index does not satisfy transitivity, the GEKS procedure finds the matrix of price comparisons that is closest (in terms of logarithmic least squares of deviations) to the Fisher binary indexes. Let $\left\{PI^{F}(s,t): s,t=1,2,...,M\right\}$ represent the matrix of bilateral comparisons between all pairs of countries computed using the Fisher index. Then the GEKS price index, denoted by $PI^{GEKS}(s,t)$, is obtained by solving the following problem:

$$Min_{PI^{GEKS}} \sum_{s=1}^{M}\sum_{t=1}^{M}\left[\ln PI^{GEKS}(s,t)-\ln PI^{F}(s,t)\right]^{2}$$
$$subject\ to\ PI^{GEKS}(s,t)=PI^{GEKS}(s,k)\cdot PI^{GEKS}(k,t)\ \forall s,t\ and\ k$$

The solution to this minimization problem (Rao and Banerjee, 1986) is given by:

$$PI_{jk}^{GEKS}=\prod_{l=1}^{M}\left[PI^{F}(j,l)\cdot PI^{F}(l,k)\right]^{1/M}\qquad for\ all\ \ j,k=1,2,...,M \tag{2}$$

[5] This idea is similar to the absence of arbitrage in exchange rates of countries.
[6] Further details can be found in Balk (2008) and Diewert (2013).

Properties of the GEKS index are discussed in Balk (2008), Rao (2009) and Diewert (2013).

The Geary-Khamis (GK) Method

This method was first proposed by Geary (1958) in the context of agricultural output comparisons across countries and later popularized through Khamis (1972) and other related papers. This method differs in its approach to standard index number formulae. The GK method is anchored on the concept of purchasing power parity (PPPs) of currencies and international average prices of commodities. Let $\{PPP_s : s = 1, 2, ..., M\}$ and $\{P_i : i = 1, 2, ..., N\}$ denote, respectively, purchasing power parities of currencies and the international average prices commodities. The GK system consists of the following interrelated set of equations:

$$PPP_s = \frac{\sum_{i=1}^{N} p_{is} q_{is}}{\sum_{i=1}^{N} P_i q_{is}} \quad for\ s = 1, 2, ..., M \tag{3a}$$

and

$$P_i = \frac{\sum_{s=1}^{M} p_{is} q_{is} / PPP_s}{\sum_{j=1}^{M} q_{is}} \quad for\ i = 1, 2, ..., N \tag{3b}$$

The GK system consists of (M+N) linear homogeneous equations in as many unknowns and it has a solution which is positive and unique up to a factor of proportionality. In practice, one of the PPPs is set to 1 and the remaining unknowns are solved. The system can be solved through matrix inversion or using an iterative process which starts with an initial set of values for PPPs (any positive numbers) and then iterated until the solution converges.

Once this system is solved for the unknowns, then the Geary-Khamis price index is given by:

$$PI_{st}^{GK} = \frac{PPP_t}{PPP_s} \quad for\ all\ s, t = 1, 2, ..., M \tag{4}$$

A discussion of the properties and its shortcomings can be found in Kravis et al (1982), Balk (2008) and Diewert (2013), among others.

Though the GEKS and GK have been the principal methods employed in international comparisons to date, both methods suffer from some fundamental weaknesses. The identity is one of the fundamental axioms in index number theory, but the price index defined in equation (2) for GEKS and in equation (4) for GK fail this test. Both methods satisfy a weaker form of identity test which requires that when two countries have the same price and quantity data then the price index equals 1. These methods also fail to satisfy some basic properties. It is easy to check that both GEKS and GK methods fail the proportionality test. Rao (1972, p. 95) provides a numerical example to show that the GK method fails monotonicity. It is established using an example of three countries with price vectors $p_1 \le p_2 \le p_3$ where the GK indices show values of PI_{12}^{GK} and PI_{13}^{GK} to be less than unity thus violating monotonicity. Rao (1972, pp 97-100) shows that under some extreme cases the GEKS index also fails monotonicity (Rao, 1972). The GEKS and GK methods do not possess any significant economic theoretic properties. Though the GEKS method is anchored on the Fisher index, which is superlative, the GEKS comparisons are not superlative (Neary,

2004) as long as they differ from the respective Fisher binary indexes. The GK price comparisons between countries do not have a direct economic theoretic interpretation though the PPPs defined in (3a) are exact for very restrictive Leontief-type fixed coefficient utility functions. An additional disadvantage with these methods is that price comparisons between a given pair of countries are affected by price data from all the remaining countries. Our objective is to propose a simpler alternative that is also free from these deficiencies.

3. Symmetric average fixed basket approach and the Multilateral Walsh Index

In this paper we purse the fixed basket approach to price index numbers discussed in ILO-IMF-ECE (2021) Manual, *Consumer Price Index Theory*. This approach has a long history dating back to Lowe (1823) and Walsh (1901, 1921). The fixed basket approach compares costs of buying a representative basket of goods and services at the prices prevailing in the comparison and reference countries. The Manual explains the main virtue of this approach, "Price statisticians tend to be very comfortable with a concept of the price index that is based on pricing out a constant "representative" basket of commodities, … The main reason why price statisticians might prefer a member of the family of Lowe or fixed basket price indices defined by (15) is *that the fixed basket concept is easy to explain to the public."* (ILO-ECE, Chapter 1, pp. 9-10).

Let $\left\{q_{if} : i = 1, 2, ..., N\right\}$ denote a fixed basket of quantities. Then the fixed basket price index between countries *s* and *t* is defined as:

$$PI_{st}^{f} \equiv \frac{\sum_{i=1}^{N} p_{it} q_{if}}{\sum_{i=1}^{N} p_{is} q_{if}} \qquad \forall s, t = 1, 2, ...M \tag{5}$$

The Laspeyres, Paasche, Edgeworth and Walsh index numbers are common forms of the fixed basket approach. The Walsh index (Walsh, 1901, 1921) for bilateral comparisons is:

$$PI_{st}^{W} = \frac{\sum_{i=1}^{N} p_{it} \left(q_{is} \cdot q_{it}\right)^{1/2}}{\sum_{i=1}^{N} p_{is} \left(q_{is} \cdot q_{it}\right)^{1/2}} \tag{6}$$

While Walsh (1921) provided an intuitive justification of the use of average of quantities in the two periods as weights in measuring price changes leading to (6), it is Diewert (2001) who provided an analytical framework and showed that the bilateral Walsh index in (6) is the only index that is based on symmetric averages of the quantities in the two periods and is invariant to proportional changes in observed quantity vectors. Diewert (2001) has also shown that the Walsh price index in (6) is a superlative index as it is exact for a generalized Leontief utility function.

The fixed-basket approach is intuitive, and it can be helpful when it comes to explaining the methodology to users of ICP data. Can such a simple approach form the basis for international comparisons? Can it be on par or better than the GEKS and GK methods discussed in Section 2? As we demonstrate in this paper, it is possible to generalize this fixed-basket concept for multilateral comparisons, and as shown in section 3.3 this generalized method is superior to the GEKS and GK methods as it satisfies the *identity test, monotonicity, and also unaffected by price data from other countries.*

3.1 Multilateralisation of the Walsh index

We turn to multilateral comparisons and the Walsh index. The index in (6) is clearly not transitive and therefore violates one of the fundamental internal consistency requirements. How do we generalize the Walsh index for use in cross-country comparisons? We have two options.

The first is to simply apply the GEKS procedure on bilateral Walsh indexes[7] leading to:

$$PI_{st}^{MW-GEKS} = \prod_{l=1}^{M}\left[PI_{sl}^{W} \cdot PI_{lt}^{W}\right]^{1/M} \quad for\ all\ s,t = 1,2,...,M \tag{7}$$

This approach is similar to the use of the Fisher index in the GEKS formula. The index in (7), therefore, has the same deficiencies and issues that are observed for the Fisher-based GEKS index.

The second option is to generalize the result in Diewert (2001) for multilateral comparisons anchoring on the notion of a fixed basket based on symmetric averages of the observed quantities, $\left\{\mathbf{q_j} : j = 1,2,...,M\right\}$. We start with the most general specification where we allow the fixed basket to take a different functional form for each pair of countries, *s* and *t*, denoted by $\mathbf{q_f^{st}} = \left\{q_{if}^{st} : i = 1,2,...,N\right\}$; and also for different commodities. Thus, the *i-th* element of $\mathbf{q_f^{st}}$ is defined as:

$$q_{if}^{st} \equiv m_i^{st}\left(q_{1i}, q_{2i}, .., q_{Mi}\right)\ for\ \ i = 1,...,N \tag{8}$$

where $m_i^{st} : \mathbf{R}_{++}^{M} \rightarrow \mathbf{R}_{++}; and\ \forall\, i, m_i^{st}\left(a,a,...,a\right) = a\ \ where\ a \in \mathbf{R}_{++}$.

Given this specification of the fixed basket, the fixed basket price index number is given by:

$$PI(s,t;\mathbf{q}_f^{\mathbf{st}}) = \frac{\sum_{i=1}^{N} p_{it}\, m_i^{st}(q_{i1}, q_{i2},..., q_{iM})}{\sum_{i=1}^{N} p_{is}\, m_i^{st}(q_{i1}, q_{i2},..., q_{iM})} \quad for\ all\ s,t = 1,2,...,M \tag{9}$$

We narrow the choice of the fixed basket and uniquely determine the functional form in (8) by invoking the three fundamental Axioms 1 to 3 – country symmetry, transitivity and invariance to proportional changes in quantity vectors of countries - discussed in section 2. The country-symmetry axiom implies that the price index in (9) should be invariant to the order in which countries are considered. This means that the index value should be the same for all permutations of the countries. This property implies that the averaging function in (9) should be a symmetric function.

Axiom 3 implies that when the quantity vector of country *k* is multiplied by λ, then the price index (9) remains unchanged. Thus

$$PI(s,t;\mathbf{q}_f^{\mathbf{st}}) = \frac{\sum_{i=1}^{N} p_{it}\, m_i^{st}(q_{i1}, q_{i2},..., q_{iM})}{\sum_{i=1}^{N} p_{is}\, m_i^{st}(q_{i1}, q_{i2},..., q_{iM})} = \frac{\sum_{i=1}^{N} p_{it}\, m_i^{st}(q_{i1}, q_{i2}, \lambda q_{ik}..., q_{iM,})}{\sum_{i=1}^{N} p_{is}\, m_i^{st}(q_{i1}, q_{i2}, \lambda q_{ik}..., q_{iM,})}\ \ for\ k = 1,2,...,M \tag{10}$$

The main result of the paper is stated below:

Proposition 1: Suppose the multilateral price index in (9) is defined for all strictly positive price and quantity vectors. Then the price index in (9) satisfies the axioms of *country symmetry, transitivity, and*

[7] Rao and Banerjee (1986) observed that GEKS is a simple procedure to generate transitive comparisons from a matrix of non-transitive bilateral comparisons that satisfy country reversal test. Consequently, GEKS can be applied to indexes other than the Fisher index.

invariance to proportional changes in the quantity vector of any country if and only if the averaging function is of the form:

$$m_i^{st}\left(q_{1i},q_{2i},..,q_{Mi}\right) = \prod_{j=1}^{M}\left(q_{ij}\right)^{1/M} \quad \forall i=1,2,...,N; and\ s,t=1,2,...,M \tag{11}$$

The *if* part is straightforward to check. Proof of the *only if* part of the proposition is given in the Appendix A.

In addition to providing a generalization of Proposition 6 in Diewert (2001), another feature of our proof is that it holds in a more restricted domain when prices and quantities are strictly positive. Our result strengthens the result in Diewert (2001, p.243), Proposition 6, and its proof, which relies on the assumption of some zero prices. Balk (2008, pp 101-102) in presenting the proof of Proposition 6 of Diewert (2001), states: "For this proof the domain definition of $P^{LIN}(.)$ must be extended to $p, p' \in \Re_{+}^{N}$ "[8]. Diewert's proof makes use of the presence of zero prices. Existence of zero prices is unrealistic[9] in the context of price index numbers. Our proof holds in the case where observed prices are in the strictly positive domain $\Re_{++}^{N}$.

We also note that the averaging function in (11) takes the same form for all the commodities even though we started with a general specification in (8) that potentially allowed for the use of different types of averages for different commodities.

Proposition 1 provides a generalization of the Walsh index for multilateral price comparisons, we refer to this as the *multilateral* Walsh (MW) index. The symmetric averaging of quantity vectors (11) leads to the following form for the multilateral Walsh index:

$$PI^{MW}(s,t;\mathbf{q_f^{st}}) = \frac{\sum_{i=1}^{N} p_{it}\prod_{j=1}^{M}\left(q_{ij}\right)^{1/M}}{\sum_{i=1}^{N} p_{is}\prod_{j=1}^{M}\left(q_{ij}\right)^{1/M}} \quad for\ all\ s,t=1,2,...,M \tag{12}$$

This is the multilateral Wash index we implement in our empirical section and provide a comparative assessment of results from different methods.

The fixed basket from Proposition 1 has important properties. First, the basket is derived using the same weight for quantity vectors from different countries irrespective of their size and level of development. This property, reflecting symmetric treatment of countries, is also a virtue attributed to the GEKS system where all the countries are treated the same in the construction of the GEKS index in (2). Second property of this basket is that its effect on the index is size neutral. This means that only the structure of the consumption basket from each country is important but not its absolute size. Consequently, the fixed basket may be interpreted as an average global structure.

[8] This means that the price vector is non-negative allowing the possibility of zero prices.

[9] In practice, price statisticians encounter the problem of disappearing goods in which case there are no observed prices and quantities. In such cases, reservation prices (which are positive) are recommended (Fox and Diewert, 2021). It is common to encounter cases where prices are positive but with zero consumption or quantities. We encounter this problem in the empirical work we report in Section 4.

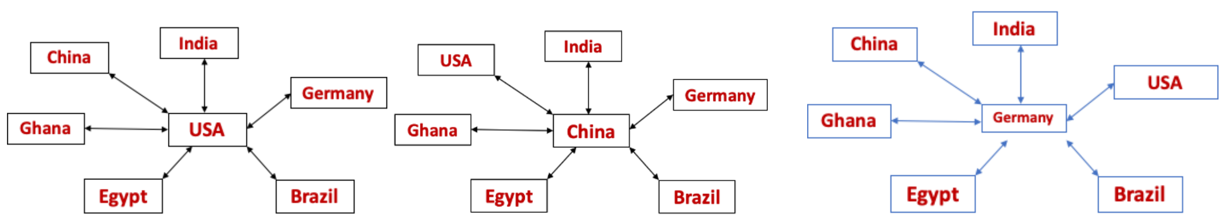
3.2 Structural similarity of international price comparisons based on GEKS, Geary-Khamis and the multilateral Walsh indices

The formulae and the approaches that underpin these three methods for international price comparisons are quite different. These methods were developed using different strands of reasoning and are algebraically quite distinct. However, there are structural similarities between these methods. All of these methods share a common feature that the price comparisons are built using a "*star-system*".

Let us start with the Fisher-based GEKS used in the ICP. Consider a star-system of comparisons where comparisons between countries are through a pre-selected star country and using the Fisher price index number formula. For example, if USA is selected as the star, the star system of comparisons is shown in Panel A of Figure 1 below. Panel B shows star-system comparisons through China, and Panel C through Germany.

Figure 1: Start-system price comparisons using different countries and the GEKS Method

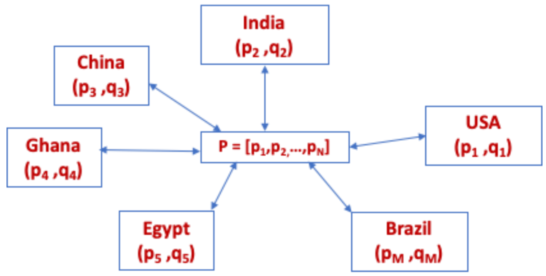
Panel A: USA | Panel B: China | Panel c: Germany



In Panel A, comparison between China and India is the product of China-USA and USA-India bilateral comparisons each computed using the Fisher index. In Panel B, this comparison is the direct China-India comparison, and in Panel C this is the product of China-Germany and Germany-India comparisons. Since the Fisher index is not transitive, each star-sytem with different star countries produce a different comparison. The GEKS method suggests that comparison between two countries, say China and India, is the geometric average of comparisons between these two countries using each of the countries as the star countries. If there are 177 countries in the comparison, as is the case with the ICP in 2017, comparison between China and India is the geometric average of 177 such comparisons. Transitivity of this method is straightforward to check.

Now, let us turn to the Geary-Khamis (GK) method. The GK method computes international average prices of all the commodities represented by the vector $\mathbf{P} = (P_1, P_2, ..., P_N)$. Then prices in each country are compared with this average price vector, as shown in the schematic diagram below.

Figure 2: Schematic diagram for price comparisons with the Geary-Khamis Method



Price level for each country is measured relative to the international average prices as the ratio of costs of buying the country's bundle at its own prices and at the international prices. For example, for India this measure would be $\sum_{i=1}^{N} p_{i2}q_{i2} \Big/ \sum_{i=1}^{N} P_i q_{i2}$, denoted as PPP_2. Then the price index for a country, say Ghana, with USA as the refence country is given by the ratio of PPPs of the two countries, in this case the ratio is $PI_{14}^{GK} = PPP_4 / PPP_1$. These comparisons are transitive by construction.

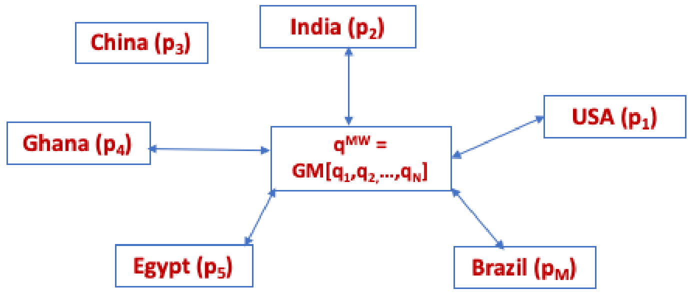
There is some similarity between the GK price comparisons and the GEKS_Tornqvist index advocated in Caves, Christensen and Diewert (CCD) (1982) where they make use of unweighted geometric average of observed prices[10]. Under the CCD approach, each country compares its prices with the geometric average of prices from all countries, and in the price comparison country specific expenditure shares and the geometric average of shares of all the countries are used[11]. In contrast, the GK index uses a quantity weighted average of prices and a Paasche-type index for price comparisons. Except for this difference, price comparisons between countries have the same structure as that of the GK price comparisons.

Finally, we turn to the multilateral Walsh (MW) index. This index is somewhat similar to the star-system used in the Geary-Khamis method. In Figure 2 for the GK system, all the price comparisons are anchored on the vector of international average prices. In the case of MW index, comparisons are anchored on a symmetric average of quantity vectors of all the countries. The quantity vector that underpins the MW comparisons is the geometric average of quantity vectors of all the countries, $q^{MW} = \left[q_1 \cdot q_2 \cdots q_M \right]^{1/M}$. The star-system of comparisons for the MW-system are shown below.

[10] Though the index in CCD (1982) is in the context of quantity comparisons, the same approach can be used for price comparisons.

[11] We note that this interpretation holds only for GEKS based on Tornqvist index and does not extend to GEKS_Fisher comparisons.

Figure 3: Star-system for Multilateral Walsh Price Comparisons



Under the multilateral Walsh method, price comparisons between countries are all based on the common quantity vector. For example, comparison between India and USA is given by $\sum_{i=1}^{N} p_{i2}\left(q_{i1}q_{i2}...q_{iM}\right)^{1/M} \Big/ \sum_{i=1}^{N} p_{i1}\left(q_{i1}q_{i2}...q_{iM}\right)^{1/M}$.

The three methods share a star-like structure for price comparisons. However, there are some important differences. The GEKS method averages several star-country comparisons, as a result, price comparisons between any given pair of countries are affected by price data from all the countries. Given the complex nature of the process, the resulting comparisons do not satisfy important properties like the identity test and monotonicity. The GK and MW methods in Figures 2 and 3 are very similar in structure, the GK method uses international average prices whereas the MW is based on international average of quantities but have very different properties. Since the GK international prices are averages of prices in all the countries which are simultaneously determined with PPPs, the resulting price comparisons between countries are affected by prices in all the countries. Further, the international average prices are affected by proportional changes in quantity vectors. The GK price comparisons therefore are affected whether total quantities or per capita quantities are used in comparisons. The GK comparisons, like GEKS, fail to satisfy identity and monotonicity tests. In comparison, price comparisons with MW are not affected by price data from other countries and at the same time satisfy these test properties.

3.3 Axiomatic Properties of the Multilateral Walsh Index

The desirability of index number formulae is assessed using the axiomatic or test approach whereby each formula is judged by the properties it satisfies. Chapter 2 in the latest version of the ILO-IMF-ECE (2021), *Consumer Price Index Theory* has a comprehensive list of axioms or test properties[12]. Based on the descriptions of these tests in that chapter, Proposition 2 lists a selected set of properties of the multilateral Walsh index.

Proposition 2: The multilateral Walsh index in (9) satisfies the following properties: (1) positivity; (2) continuity; (3) identity or constant prices test; (4) proportionality in current prices; (5) inverse proportionality in base country prices; (6) invariance to proportional changes in quantity vectors of countries; (7) invariance to changes in the order of commodities; (8) commensurability or invariance to changes in the units of measurement; (9) country reversal test; (10) transitivity; (11) mean value test for

[12] Balk (2008) offers an excellent review of the test properties for index numbers.

prices, i.e., the index lies within the bounds of the minimum and maximum of price ratios; and (12) monotonicity in comparison and reference/base country prices.

The proof of the position is straightforward as each of these tests can be verified using the MW index in (11). Of these properties, we single out commensurability and invariance to proportional changes to quantity vectors for further comment. As will be seen in the empirical section, commensurability of the index is critical in the case of where the method is applied at the basic heading level. This property ensures that the results are invariant to the choice of the reference country for the basic heading PPPs. The invariance to proportional changes to quantity vector is even more important. This means that price comparisons will remain the same when the quantity vector of a country is proportionally increased ten-fold! This means that comparisons are essentially size neutral and only the structures of quantity vectors really matter for international comparisons.

3.4 The multilateral Walsh index and the Funke, Hacker and Voeller (1979) Theorem

After listing properties satisfied by the multilateral Walsh price index, we consider it necessary to discuss Proposition 2 in relation to the often-quoted important theorem by Funke, Hacker and Voeller (FHV) (1979, Theorem 3.13) which is stated below in a slightly rephrased form.

Funke, Hacker and Voeller (FHV) Theorem: A price index which maps prices and quantities observed in two periods, $P(p_1, q_1, p_0, q_0): \Re_{++}^{4N} \to \Re_{++}$ satisfies the axioms: (i) monotonicity; (ii) linear homogeneity; (iii) identity test; (IV) commensurability and (V) the circular test if and only if the price index P is the "Cobb-Douglas" index

$$P(p_1, q_1, p_0, q_0) = \prod_{i=1}^{N} \left(\frac{p_{i1}}{p_{i0}} \right)^{\alpha_i} \quad where\ \alpha_i > 0\ for\ all\ i;\ and \sum_{i=1}^{N} \alpha_i = 1.$$

In Proposition 2 we proved that the multilateral Walsh index satisfies all the five axioms stated in FHV theorem and yet the MW index formula is not a Cobb-Douglas index. How can this be?

This is an interesting puzzle which requires some explanation as to how both results could be true at the same time. After careful reading of the proof by FHV theorem it is clear that this discrepancy arises due to the technical definition of price index for bilateral used in their work. Their definition restricts the price index to be a function of *only* prices and quantities observed in the two periods or two countries. However, we are working in a multilateral comparison framework where the price index is a mapping from all observed price and quantity data to positive real numbers. This means that indices like the MW index, the focus of our work, are not considered as bilateral indexes and, therefore, are technically outsidethe framework of FHV (1979).

3.5 Substitution Bias GEKS, GK and multilateral Walsh price comparisons and

The Fisher binary index is a superlative index which is exact, and therefore without any substitution bias, when preferences are quadratic. Similarly, the bilateral Walsh index is superlative index that is exact when for generalized Leontief preferences (Diewert, 1976, 2001. These indexes are likely to have some substitution bias if the true preferences differ, respectively, from the quadratic and generalized Leontief preferences. However, these properties of binary indices do not necessarily translate to their multilateral counterparts, the GEKS_Fisher and the multilateral Walsh index. It is useful to examine the possible nature of substitution bias associated with these multilateral indics.

To facilitate exposition, we simplify our discussion by assuming identical but not necessarily homothetic preferences across all the countries. Let $U(q)$ represent a continuous increasing function and representative consumer's cost function representing the minimum expenditure required to achieve the utility level u at a price vector p is defined as:

$$C(p,u) = \min_q \left\{ \sum_{i=1}^{N} p_i q_i \;\middle|\; U(q)=u \right\}$$

Substitution bias associated with a price index number formula is discussed relative to the Konus cost of living index or price index. The Konus index comparing price vectors p_k *and* p_j at utility level u is given by:

$$PI_{jk}^{Konus} = \frac{C(p_k,u)}{C(p_j,u)}$$

Then the substitution bias associated with a price index number formula PI relative to the Konus index is defined as:

$$Sub\,Bias_{jk}^{PI} = \frac{PI_{jk}}{PI_{jk}^{Konus}} - 1 = \frac{PI_{jk}}{C(p_k,u)/C(p_j,u)} - 1 \tag{13}$$

If the bias is positive, then the price index formula under consideration overstates the change in the cost of living, and vice versa. Let us consider the substitution bias associated with the Fisher, MW and GK indices.

The Fisher Index

As the Fisher index is the geometric average of the Laspeyres and Paasche indices, the substitution bias associated with these indices, under the assumption that the observed quantities are optimal or cost-minimising, are given by:

$$Sub\;Bias_{jk}^{L} = \frac{\sum_{i=1}^{N} p_{ik}q_{ij}}{\sum_{i=1}^{N} p_{ij}q_{ij}} \Bigg/ \frac{C(p_k,U(q_j))}{C(p_j,U(q_j))} - 1 = \frac{\sum_{i=1}^{N} p_{ik}q_{ij}}{\sum_{i=1}^{N} p_{ij}q_{ij}} \Bigg/ \frac{C(p_k,U(q_j))}{\sum_{i=1}^{N} p_{ij}q_{ij}} - 1 = \lambda_k > 0 \tag{14}$$

$$Sub\;Bias_{jk}^{P} = \frac{\sum_{i=1}^{N} p_{ik}q_{ik}}{\sum_{i=1}^{N} p_{ij}q_{ik}} \Bigg/ \frac{C(p_k,U(q_k))}{C(p_j,U(q_k))} - 1 = \frac{\sum_{i=1}^{N} p_{ik}q_{ik}}{\sum_{i=1}^{N} p_{ij}q_{ik}} \Bigg/ \frac{\sum_{i=1}^{N} p_{ik}q_{ik}}{C(p_j,U(q_k))} - 1 = \frac{1}{1+\mu_j} - 1 = \beta_j < 0 \tag{15}$$

Substitution bias for the Fisher index, under homothetic preferences, can then be shown to be:

$$Sub\,Bias_{jk}^{Fisher} = \frac{PI_{jk}^{Fisher}}{C(p_k,u)/C(p_j,u)} - 1 = \left[\frac{1+\lambda_k}{1+\mu_j}\right]^{0.5} - 1 = \eta_{jk} \tag{16}$$

If λ *and* μ are of similar magnitude, then the ratio $(1+\lambda_j)/(1+\mu_j)$ is close to 1. Diewert (1976) shows that Fisher index is exact when the utility function is quadratic in which case the bias reduces to zero.

The Geary-Khamis PPPs

We may recall from Section 2 that the GK PPP for the currency of country *j* is defined as:

$$PPP_j = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}}$$

Then the substitution bias associated with this PPP is given by:

$$Sub\ Bias_j^{GK-PPP} = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ik}} \bigg/ \frac{C(p_j, U(q_j))}{C(P, U(q_j))} - 1 = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}} \bigg/ \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{C(P, U(q_j))} - 1 = \frac{1}{1+\delta_j} - 1 = \phi_j < 0. \tag{17}$$

This negative bias is what is observed in practice and the result is that the real expenditure of country *j* is likely to be overstated. The extent of this bias varies from country to country and the magnitude depends on the degree of similarity in the domestic and international price structures.

The multilateral Walsh index

The multilateral Walsh index is defined using the fixed basket quantity vector which is shown to be the geometric average of quantity vectors in all the countries, denoted by q_f. Then the bias in the index is given by:

$$Sub\ Bias_{jk}^{MW} = \frac{\sum_{i=1}^{N} p_{ik} q_{if}}{\sum_{i=1}^{N} p_{ij} q_{if}} \bigg/ \frac{C(p_k, U(q_f))}{C(p_j, U(q_f))} - 1 = \frac{\sum_{i=1}^{N} p_{ik} q_{if} \big/ C(p_k, U(q_f))}{\sum_{i=1}^{N} p_{ij} q_{if} \big/ C(p_j, U(q_f))} - 1 = \frac{1+\alpha_k}{1+\beta_j} - 1 = \psi_{jk} \tag{18}$$

This expression (18) is somewhat similar to the expression associated with the Fisher index, (16), except that here the same reference quantity vector, geometric mean of the observed quantities, is used. If α_k *and* β_j are similar in magnitude, then the substitution bias is close to zero. Further, when the comparison is bilateral, i.e., the reference quantity vector is the geometric average of the quantities in countries *j* and *k*, Diewert (1976) has shown that the bilateral Walsh index is exact for the generalized Leontief utility function.

We also note that the substitution biases associated with the Fisher and MW indices tend to be similar in absolute magnitudes but may differ in their signs. But a proper comparison of biases would be between those associated with GEKS and the MW indexes.

The GEKS Index

The GEKS index for comparing prices between countries *j* and *k* based on the Fisher bilateral formula are given by:

$$PI_{jk}^{GEKS} = \prod_{l=1}^{M} \left[PI_{jl}^{F} \; P_{lk}^{F} \right]^{1/M}$$

In this case, substitution bias associated with the GEKS index under homothetic preferences is:

$$Sub\,Bias_{jk}^{GEKS} = \frac{PI_{jk}^{GEKS}}{PI_{jk}^{Konus}} - 1 = \frac{PI_{jk}^{GEKS}}{C(p_k, u)/C(p_j, u)} - 1 \qquad (19)$$

The substitution bias associated with GEKS is complex and it involves biases associated with all the other countries in the multilateral comparisons. Consequently, it is difficult to assess or get a handle on the bias associated with GEKS. As the GEKS comparison is designed to be close to the binary Fisher index (according to a logarithmic least squares criterion), it is expected but not necessarily guaranteed that the bias associated with GEKS is similar to that associated with the Fisher index.

In summary, we have shown that while the actual magnitudes of the substitution bias are not tractable or measurable the bias associated with MW index for price comparisons would be of similar magnitude as that of the GEKS_Fisher index though the direction of bias could be different. Computation of substitution bias requires the specification of a utility or cost function which can be used as a basis for comparison of a given price index with the cost of living index derived using the specific utility function. Balk and Diewert (2010) make use of second-order approximation of the unknown cost function, and the assumption of existence of long run trends in prices, and on elasticity assumptions to provide an indication of the substitution bias associated with the Lowe index for temporal comparisons. As these assumptions do not automatically hold in the case of cross-country comparisons, it is difficult to speculate on the nature of substitution bias associated with these indices. Melser and Webster (2021) report the substitution bias and chain drift associated with several multilateral methods, including the GEKS index, for temporal comparisons. Estimates of substitution bias are based on a simulation experiment where price and quantity data are generated using CES preferences with different values for elasticity of substitution. Their findings suggest that the substitution bias associated with Fisher_GEKS can be significant for large values of the elasticity of substitution. Given the complexities in measuring substitution bias associated with GEKS_Fisher, MW and the GK indices, we have restricted our discussion to illustrate that substitution bias exists even in the case of GEKS_Fisher index.

We may emphasize here that in the discussion here, we have abstracted from reality and based this discussion on identical and homothetic preferences across all the countries. The international comparison data tends to point towards non-homothetic preferences (Neary, 2004; Feenstra et al, 2013) and heterogeneous preferences. The apparatus used in Balk and Diewert (2010) or the exposition here is not sufficiently powerful to handle this general case scenario and as such would be subject matter for future research.

3.5 Indirect quantity or real expenditure comparisons

The MW index does not satisfy the factor reversal test which states that the product of the price and quantity indexes computed using the same formula but by simply changing the roles of prices and quantities must equal the value ratio. The Fisher index satisfies this test, but most other formulae fail this test. Instead, the MW index can be combined with the factor test which requires that the product of the price and quantity indexes, computed using different formulae, equals the value ratio.

As the multilateral Walsh index does not satisfy the factor reversal test, we recommend the use of the indirect or implicit Walsh quantity index defined as:

$$QI^{ID-MW}(s,t:p_i^{st}) = \frac{\sum_{i=1}^{N} p_{it}q_{it}}{\sum_{i=1}^{N} p_{is}q_{is}} \Big/ PI^{MW}(s,t;\mathbf{q}_\mathbf{f}^{\mathbf{st}}) = \frac{\sum_{i=1}^{N} p_{it}q_{it} \Big/ PI^{MW}(s,t;\mathbf{q}_\mathbf{f}^{\mathbf{st}})}{\sum_{i=1}^{N} p_{is}q_{is}} \quad \forall s,t = 1,2...,M \qquad (20)$$

This indirect quantity index satisfies various tests including transitivity. We prefer the indirect quantity index to the direct Walsh quantity index, $\frac{\sum_{i=1}^{N} q_{it}\left(p_{is}\cdot p_{it}\right)^{1/2}}{\sum_{i=1}^{N} q_{is}\left(p_{is}\cdot p_{it}\right)^{1/2}}$, which is a constant price quantity index that is also is known to be exact for a Generalized Linear utility function (Diewert, 2001, p. 209).

4 Cross-country comparisons of prices and real expenditures using multilateral Walsh index and the 2017 ICP Data

As the objective of this paper is to provide a simpler alternative to the existing methods used in cross-country comparisons of prices, we use data from the ICP. We use the most recent 2017 ICP data covering 177 countries of the world. Details of the methods used in the compilation of data and the subsequent compilation of PPPs and real expenditures at the regional and global level can be found in the 2017 ICP reports from the World Bank (2020) and the Asian Development Bank (ADB, 2020). Our empirical application uses unpublished ICP data consisting of PPPs and expenditures in national currencies for 155 basic headings or commodity groups[13] covering Household Consumption, Government Expenditure, Gross Capital Formation, and Net Exports[14].

4.1 ICP 2017 Data

Though the ICP 2017 covers 177 countries, data for two countries which served as bridge countries for the purpose of linking regional comparisons[15] are duplicated. These are Egypt and Sudan. In our work, we eliminate duplication by placing Egypt in Africa, and Sudan in Western Asia. In World Bank (2020), Russian Federation is under CIS-EUO. We opted to place it under CIS. Finally, Gautemala does not have quantity information and thus dropped. consequently, our computations are based on data for 174 countries. The ICP data covers all the major aggregates of the national accounts. In this paper we focus on data for household consumption which consists of 109 basic headings. The full list of these basic headings is provided in Appendix B of the paper. We have opted to focus on Consumption instead of the whole of GDP as our focus has been on methods with useful axiomatic and economic theoretic properties.

We use price data available in the form of PPPs at the basic heading level for each country, expressed using USA as the reference or base country. Thus, we have PPP and expenditure matrices of order 109 x

[13] Basic headings are similar to commodity groups for elementary indices used in the compilation of the consumer price index.

[14] We thank the Global ICP Unit for providing the detailed data. Users can obtain these data upon application, to the ICP unit at the World Bank, for release of data for research purposes.

[15] For details of the ICP methodology for linking regional comparisons, see Rao (2013a).

174[16]. For our index computations, we treat the PPPs as price data and quantities are obtained by converting expenditures using PPPs. Thus, we have:

$$p_{ij} = PPP_{ij} \quad and \;\; q_{ij} = \frac{e_{ij}}{PPP_{ij}} \;\; for \; i = 1, 2, ..., 109; \; and \;\; j = 1, 2, ..., 174$$

As USA is the base country, PPP_{ij} equals 1 for all commodities when *j* = USA. Following procedures established in the early phases of the ICP by Kravis et al (1982), quantities of the basic headings which are like composite commodities are indeed real expenditures or expenditures converted into US dollars using PPPs. In the case of USA, the observed expenditures in US dollars also serve as real expenditures.

What happens if PPPs at the basic heading are expressed using currency of another country, say South Africa, as the reference currency? As the multilateral Walsh index satisfies commensurability and is independent of units of measurement, the overall price and real expenditure comparisons are invariant to the choice of the reference currency. The GK and GEKS methods also satisfy commensurability.

4.2 Commodities with zero expenditures/quantities

The ICP data we are working with is at the basic heading level where prices of items as well as expenditures on items are already aggregated. Further, as the ICP uses annual average prices and annual expenditures one would expect limited occurrence of zero expenditures. In our data set with 174 countries and 109 composite commodities (basic headings), we have 18966 country-by-item observations of which we found 4.3 percent recorded zero expenditures. Sixty three out of 109 basic headings record positive expenditures for all the countries. The highest percentage of zero expenditures are recorded for narcotics (54.5%), combined passenger transport (57.5%), animal drawn vehicles (67.2%) and prostitution (67.8%). Low levels of expenditures are recorded for these basic headings for countries where positive expenditures were reported.

How do we treat these basic headings? How will zero expenditures impact on the estimated PPPs? We considered three options. First option is to simply ignore the problem and compute PPPs using different methods. Most methods can be computed in the presence of zero expenditures and hence zero quantities. In the case of multilateral Walsh index, which relies on geometric averages of observed quantities, use of data with zero expenditures essentially means that all those basic headings with at least one country reporting zero expenditure would automatically be excluded from index number computation. The estimated PPPs would then rely only on data for the 63 basic headings with positive expenditures for all the commodities. Such an approach means that a significant proportion of data would be discarded, not a statistically sound procedure.

The second option is to aggregate data to a higher level with fewer composite commodities but without any zero expenditures at that level. In our preliminary analysis, we mapped the 109 basic headings into 37 higher level aggregates ensuring positive expenditure at that level. For example, the basic headings *rice; other cereals, flour and others; bread; other bakery products; and pasta products and couscous* are combined to form the aggregate, *Bread and Cereals.* The critical part here is to construct price data for higher level aggregates which in turn requires handling basic headings with zero expenditures. Therefore, the problem of zero expenditure would still be intrinsically present.

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[16] These are not published but are available for research purposes upon request from the World Bank.

As a third option, which is our preferred option, we have devised a procedure based on the notion of a generalized mean that makes it possible to utilize all the data available for all the 109 basic headings. We recall that the multilateral Walsh method uses the following quantity vector shown in equation (11):

$$m_i^{st}\left(q_{1i}, q_{2i}, .., q_{Mi}\right) = \prod_{j=1}^{M}\left(q_{ij}\right)^{1/M} \quad \forall i = 1, 2, ..., N; and\ s, t = 1, 2, ..., M$$

Unlike the simple geometric mean which equals zero when some quantities are zero, we use the generalized mean which is strictly positive as long as there is at least one positive quantity[17].

The generalized mean is computed in two steps. First, for each commodity *i*, we compute the proportion of countries where the basic heading *i* records a zero expenditure, denoted by z_i as:

$$z_i = \frac{1}{M}\sum_{j=1}^{M} I_{ij}\left(q_{ij} = 0\right) \qquad for\ \ i = 1, 2, ..., N$$

where $I_{ij}\left(q_{ij} = 0\right)$ is an indicator function which takes value 1 if the basic heading has zero expenditure in country *j*, and zero otherwise. In our data set, there are 63 basic headings with positive expenditures recorded in all the countries. For these basic headings, z_i equals zero.

The generalized mean function is defined as:

$$m_i(q_i, z_i) = \left(\sum_{j=1}^{M} \frac{1}{M}\left(q_{ij}\right)^{z_i}\right)^{1/z_i}, \ z_i > 0 \tag{21}$$

This mean function converges to the unweighted geometric mean of quantities as z_i, the proportion of countries reporting zero quantities, goes to zero. Thus, we have

$$\lim_{z_i \to 0} m_i(q_i, z_i) = \prod_{j=1}^{M}\left(q_{ij}\right)^{1/M}$$

The use of the generalized mean helps us fully utilise all available price and quantity data. Empirical results reported below are based on the use of this generalized mean to compute multilateral Walsh indices.

4.3 Purchasing Power Parities and Real Expenditures from different methods

We have computed PPPs using the 2017 ICP data for 174 countries and 109 basic headings. To gain an understanding of the performance of various methods, we have computed PPPs and real expenditures using a range of bilateral methods: Laspeyres, Paasche, Fisher, Sato-Vartia and bilateral Walsh; and multilateral methods including Fisher-based GEKS, Walsh-based GEKs indices, multilateral Walsh, and the Geary-Khamis methods. The full set of PPPs from all the methods are shown in Appendix C. Table 1 presents purchasing power parities (PPP) of currencies for selected countries and selected methods with US dollar as the reference currency. A PPP of Rs. 19.779 for India based on GEKS_Fisher means that what

[17] This is always guaranteed. If there is a basic heading which has zero quantities in all the countries, that basic heading can simply be dropped.

can be purchased in USA with one dollar can be purchased in India using 19.779 rupees. This PPP is lower at 19.505 rupees when multilateral Walsh method is used.

Table 1: Price comparisons using selected aggregation methods
(Reference currency: US dollar)

Country Name	Fisher_bi	Walsh_bi	Walsh_Multi	GEKS_Fisher	GK_percapita	GK_Original
Ghana	1.489	1.641	1.761	1.644	1.373	1.414
Kenya	42.323	40.501	44.139	43.185	39.098	38.976
South Africa	6.438	6.946	6.746	6.483	6.273	6.224
China	4.034	4.019	4.203	4.205	4.016	4.062
India	19.176	18.820	19.505	19.779	17.882	17.760
Switzerland	1.271	1.266	1.342	1.326	1.273	1.267
Germany	0.798	0.797	0.806	0.798	0.777	0.768
United Kingdom	0.825	0.819	0.825	0.775	0.761	0.757
Brazil	2.435	2.470	2.310	2.354	2.301	2.274
Kuwait	0.181	0.209	0.239	0.180	0.160	0.159
Sudan (WAS)	5.328	5.241	5.182	5.478	5.226	5.107

Source: Authors' computations using data from 2017 ICP Research Database

The PPPs from multilateral Walsh index (column 3) are numerically close but generally higher than PPPs from to GEKS_Fisher method. In the case of China and India PPPs are marginally lower. These results demonstrate that the multilateral Walsh index provides similar estimates of PPPs to those from GEKS_Fisher and at the same time the index has superior axiomatic properties. Differences in PPPs from these methods are systematically analysed below. These results indicate that the likely effect of using the multilateral Walsh index is that the real size of global consumption would be marginally lower. As there are no systematic patterns, we expect that global inequality in consumption to be similar in magnitude to that obtained from GEKS_Fisher method.

Results from the GK method warrant further comment. First, PPPs from GK_original are systematically lower than the GEKS_Fisher, differences significant for some countries. This result is consistent with the Gerchenkron effect induced by the GK method. The last two columns serve as a demonstration that the GK method is not invariant to proportional changes in quantity vectors. The GK_original uses total quantities in each country whereas GK_percapita uses scaled down per capita quantities.

These systematic differences using the ratio of PPPs from the GEKS_Fisher and multilateral Walsh methods are shown in Figure 1 below.

Figure 1: Ratio of Multilateral Walsh to GEKS_Fisher

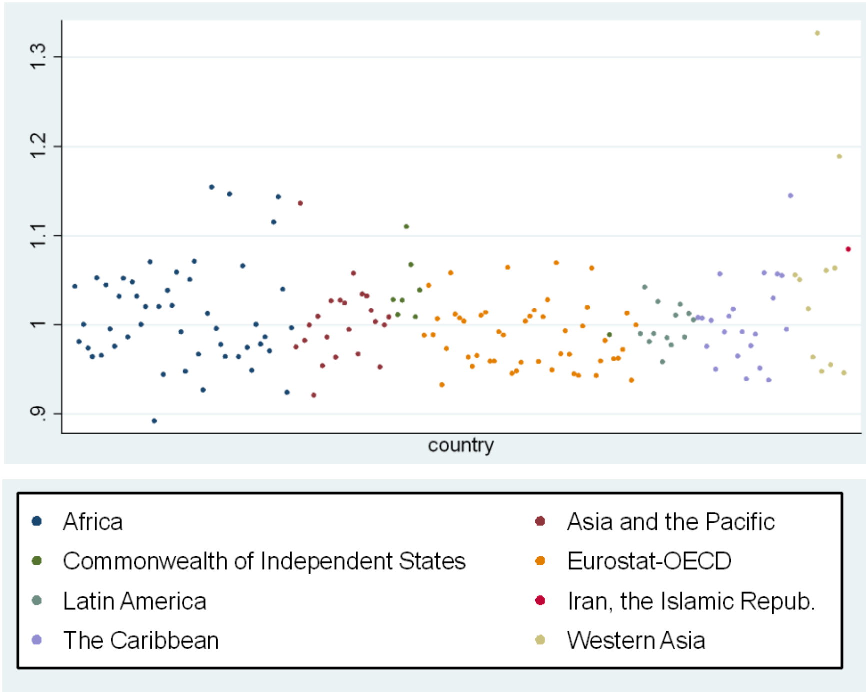

Figure 1 shows dispersion of these ratios by regions. Africa and Western Asia show biggest variation in the ratios. The following table presents descriptive statistics for this ratio by regions.

Table 2: Ratios of Multilateral Walsh and GK relative to GEKS_Fisher PPPs

Multilateral Walsh/GEKS_Fisher		
Area	Mean	Std
Africa	1.011	0.057
Asia and the Pacific	1.004	0.044
Commonwealth of Independent States	1.036	0.038
Eurostat-OECD	0.988	0.036
Latin America	0.999	0.023
Special Participation	1.085	.
The Caribbean	1.005	0.049
Western Asia	1.053	0.116
Total	1.006	0.054

GK/GEKS_Fisher		
Area	Mean	Std
Africa	0.915	0.059
Asia and the Pacific	0.923	0.034
Commonwealth of Independent States	0.929	0.036
Eurostat-OECD	0.985	0.022
Latin America	0.955	0.024
Special Participation	0.847	.
The Caribbean	0.947	0.041
Western Asia	0.950	0.035
Total	0.945	0.049

Note: Speical Participation contains Iran, Islainc Republic only.

The left panel shows that the average difference between the Multilateral Walsh and the GEKS_ PPPs is a negligible 0.6 of a percentage point. However, significant differences of 3.6 and 5.3 percent, on average, are present for the CIS and Western Asian regions. The right panel shows that GK PPPs are significantly lower for lower income countries which means that the use of GK based PPPs to convert household consumption would show a more equally distributed world compared to that shown by GEKS or multilateral Walsh method. method. The GK and GEKS_Fisher PPP ratios are consistent with the existence of Gerchenkron effect associated with the GK method (Dowrick and Akmal, 2004; Deaton and Heston, 2010; Almas, 2012).

Results presented in Figure 1 and Table 2 are consistent with our discussion on the nature of substitution bias associated with the Fisher, multilateral Walsh, GK and GEKS price index numbers. The absence of any systematic pattern in differences between MW and GEKS indices may be reflecting the differing nature of the substitution biases associated with these indices.

To further examine the nature of the deviations of PPPs from GEKS and multilateral Walsh methods, we plot these ratios against the relative difference between the Laspeyres and Paasche indices measured using (Laspeyres – Paasche)/Laspeyeres = 1 – (Paasche/Laspeyres). As Laspeyres index is usually greater than Paasche index, this relative difference is generally positive.

Figure 2: Ratio of MW and GEKS_Fisher PPPs against Laspeyres-Paasche Spread

The first interesting feature in the figure is that there are 15 countries (two countries from Africa and 13 from OCED-EU region, see Appendix C with full list of PPPs) where the Laspeyres index (with USA as base) is smaller than the Paasche index. Explanation for this observation may lie in the fact that Laspeyres index would be greater than the Paasche index when preferences are homothetic and identical across countries. This means that one or both of these assumptions may have been violated in the 2017 ICP data. Similarity in price structures could play a role. The fact that this has been observed for high income OECD-EU countries is particularly surprising. In the presence of these results, we think that it is not relevant to discuss economic theoretic aspects of these indices and whether some indices are superlative or not becomes irrelevant. In view of this, our focus is on the axiomatic or test properties discussed in section 3.3. On this score, the multilateral Walsh has better credentials than the GEKS and GK methods

The figure also shows that as the gap between Laspeyres and Paasche indices increases, the range for the ratio of multilateral Walsh and GEKS_Fisher PPPs tend to increase. When the gap is in the range of 20 to 40 percent, the ratio ranges from around 0.95 to 1.3. Even when the spread is close to zero, the ratio seems to range from 0.95 to 1.05.

As the multilateral Walsh index relies on the geometric mean of the quantities of all the participating countries, we have conducted robustness checks to examine the effects on PPPs when the world average quantity vector is replaced by the average quantity vector of different regions. Results from robustness checks are available in Appendix E. As expected, some differences in PPPs are observed. However, features reported in Table 2 and Figure 2 still remain. Use of average quantity vector from any particular region violates the axiom of country symmetry and hence the use of world average quantity vector is the preferred option for multilateral Walsh index.

We now turn to the levels and distribution of real consumption expenditure implied by PPPs from different methods as well as from the use of market exchange rates.

Table 3: The Real Consumptions by Multilateral Walsh, GEKS_Fisher, GK and Market Exchange Rates (in US dollars)

	Multilateral Walsh		GEKS_Fisher		GK		EXR	
Area	Mean	GINI (weighted)	Mean	GINI (weighted)	Mean	GINI (weighted)	Mean	GINI (weighted)
Africa	3511	0.433	3534	0.435	3838	0.434	1613	0.395
Asia and the Pacific	8847	0.164	8870	0.163	9552	0.154	4934	0.303
Commonwealth of Independent States	7731	0.102	7987	0.092	8535	0.089	2829	0.128
Eurostat-OECD	19071	0.239	18937	0.243	19224	0.237	16831	0.322
Latin America	8044	0.162	8050	0.159	8412	0.154	4787	0.190
Special Participation	6138	0.000	6661	0.000	7864	0.000	2927	0.000
The Caribbean	13504	0.236	13733	0.254	14608	0.261	13601	0.374
Western Asia	10017	0.331	10841	0.369	11442	0.378	6573	0.462
Total	10701	0.465	10769	0.467	11228	0.453	8361	0.631

Note: Multilateral Walsh, GEKS_Fisher and GK PPPs are all computed per capita quantities. EXR is the market exchange rate in 2017 provided by the ICP 2017.

Table 3 reports differences in the real household expenditures per capita across different PPPs in each region. As expected, the last row of the table shows negligible differences between GEKS_Fisher and multilateral Walsh based results. Both levels of per capita consumption and Gini measure of inequality are roughly the same for both methods. In contrast, the per capita real consumption from Geary-Khamis is greater than the real consumption derived using multilateral Walsh and the GEKS_Fisher methods. The last two columns confirm the stylized facts concerning the use of market exchange rates which tend to show significantly lower consumption levels and higher level of inequality.

The results presented in this Table augur well for the multilateral Walsh method for compiling PPPs. The price levels, real expenditures and inequality from the method are similar to those from GEKS_Fisher method and there are no significant systematic differences. From the perspective of the results, there is little to choose between these two methods. However, the analytical properties of the multilateral Walsh method and its simplicity anchored on the notion of fixed basket comparisons are a distinct advantage.

3 Conclusions

In this paper we advocate the use of the multilateral Walsh index which belongs to the class of fixed basket indexes for making international price and real income comparisons. A fixed basket index is easy to explain and easily understood by the end-users. The method conforms to the intuitive notion that the International Comparison Program compares relative costs of a fixed basket. Added to its conceptual simplicity, the method possesses a whole range of axiomatic properties expected of price index numbers. In this paper we have proved that the multilateral Walsh index is the only fixed-basket index which uses symmetric averages of quantities as weights, transitive, and invariant to proportional changes in quantity vectors. The invariance property is particularly important as it implies that it is not the absolute size of the quantity vector of a country but its structure that is more important for price comparisons. This method gives the same PPPs and price levels irrespective of whether we use the quantity vector of USA (or any other country) or a vector that is 10 or 100 times that of USA as long as the structure is maintained.

The multilateral Walsh index is shown to possess superior axiomatic properties compared to the Gini-Eltetö-Köves-Szulc and the Geary-Khamis methods currently used in ICP and the Penn World Table. In addition to satisfying identity, proportionality and monotonicity, the index has the advantage that comparisons between any two countries are not affected by price data, and the associated measurement errors in data, from the remaining countries. The empirical results from the application of the multilateral Walsh method are similar to those obtained using GEKS and there are no systematic differences in results from these two methods. An analysis of the substitution biases associated with the Fisher, GEKS, GK and multilateral Walsh indices support the empirical findings reported here. Simplicity of the multilateral Walsh method, its superior axiomatic properties compared to the current ICP and PWT methods, and the plausibility and comparability of results make it an ideal choice and a strong alternative to the Gini-Eltetö-Köves and the Geary-Khamis methods for international price and real expenditure comparisons

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Appendix A

Proof of Proposition 1

In this Appendix we consider the problem of choice in the case of fixed basket index numbers. The notation used here is the same as that used in the main body of the paper. We consider the following fixed-basket index number formula for comparison country *k* with respect to a reference or base country *j*.

$$PI^{st}(q_f) = \frac{\sum_{i=1}^{N} p_{it} q_{if}}{\sum_{i=1}^{N} p_{is} q_{if}} \quad \text{(A1)}$$

If q_f is constant for all $s,t,$ while mathematically simple, to determine the reference vector, we need to use information other than observed price and quantity vectors. We consider the case when the quantity vector, q_f , is a function of observed quantity vectors. That is, the reference vector is not an exogenous vector. For the price index between s and *t*, $PI^{st}\left(q_f\right)$, the reference vector q_f might be different for comparisons between s and *t*, and the index between *k* and *m*. Thus, we add superscript, q_f^{st} for the price index between s and t.

Consider the following functional form for q_f^{st}

$$q_f^{st} = m_i^{st}\left(q_{1i}, q_{2i}, .., q_{Mi}\right) \text{ for } i = 1,..., N \quad \text{where } m_i^{st} : \mathbf{R}_{++}^{M} \rightarrow \mathbf{R}_{++}$$

$$m_i^{st}\left(a, a, ..., a\right) = a \text{ for any } a \in \mathbf{R}_{++} \text{ for } i = 1,..., N$$

where M is the number of locations or times. q_{ji} is quantity of commodity i at location *j*. Note that we do not restrict the domain of the reference function, m_i^{st}, to quantity vectors at the two states, s and t. Rather, we extend the domain to the all states[18] (regions or times). We introduce the notions of symmetric averaging function as well as country reversal and transitivity properties.

[18] In the context of this paper, states refer to different countries. However, we wish to make this Appendix as general as possible.

Denote the quantity vector in which q_{is} and q_{it} are exchanged as $q_{i\_st} = \left(q_{i1},.,q_{is},...,q_{it},..,q_{iM}\right)$. Then, by construction, the following equation must hold for all $i = 1,2,...,N$ and $s,t = 1,2,...,M$,

$$m_i^{ts}\left(q_i\right) = m_i^{st}\left(q_{i\_st}\right).$$

Definition 1: *Symmetric function : A function* $m_i : \mathbf{R}_{++}^M \to \mathbf{R}_{++}$ *is symmetric if it is invariant to changes in order of the variables, that is, for any* $s \neq t$, *we have*

$$m_i\left(q_{i1},.,q_{is},...,q_{it},..,q_{iM}\right) = m_i\left(q_{i1},.,q_{it},...,q_{is},..,q_{iM}\right)$$

Definition 2: *State Reversal: for any* $s,t,$ $p_t, p_s, q_s,$ *and* q_t, *we get* $PI^{st}\left(q_f^{st}\right) \times PI^{ts}\left(q_f^{ts}\right) = 1$

Definition 3: *Transitivity:* $PI^{st}\left(q_f^{st}\right)$ *is transitive if for any* $s,t,k,$ *we always have*

$$PI^{st}\left(q_f^{st}\right) \times PI^{tk}\left(q_f^{st}\right) = PI^{sk}\left(q_f^{st}\right)$$

We note that if $PI^{st}\left(q_f^{st}\right)$ is transitive, it passes the state reversal test.

We state and prove the following theorem.

Proposition 1: Suppose the multilateral price index, $PI^{st}(q_f)$, in (A1) is defined for all positive price and quantity vectors and that the averaging function $m_i^{st}\left(q_{1i}, q_{2i},.., q_{Mi}\right)$ is symmetric. Then, the price index in (A1) which is invariant to proportional changes in the quantity vector of any country satisfies transitivity if and only if the averaging function is of the form:

$$m_i^{st}\left(q_{1i}, q_{2i},.., q_{Mi}\right) = \prod_{j=1}^{M}\left(q_{ij}\right)^{1/M} \quad \forall i = 1,2,...,N; and\ s,t = 1,2,...,M$$

Proof: The *if* part of the result is straightforward to check. The only if part of the theorem is proved using a series of lemmas which are stated and proved below.

Lemma 1: *Suppose* $PI^{st}\left(q_f^{st}\right)$ *is transitive. Then, for all* $s,t = 1,2,...,M$,

$$m_i^{st}\left(q_i\right) = m_i\left(q_i\right), \text{ and } m_i\left(q_i\right) = m_i\left(q_{i\_st}\right).$$

that is, all the reference vectors have the identical symmetric functional form across countries.

Proof: If the index satisfies state reversal test, then

$$PI^{st}(q_f^{st}) \cdot PI^{ts}(q_f^{ts}) = 1$$

$$\Rightarrow \frac{\sum_{i=1}^{N} p_{it} m_i^{st}\left(q_i\right)}{\sum_{i=1}^{N} p_{is} m_i^{st}\left(q_i\right)} \times \frac{\sum_{i=1}^{N} p_{is} m_i^{ts}\left(q_i\right)}{\sum_{i=1}^{N} p_{it} m_i^{ts}\left(q_i\right)} = 1$$

Using $m_i^{ts}\left(q_i\right) = m_i^{st}\left(q_i\_st\right)$, we get

$$\frac{\left(\sum_{i=1}^{N} p_{is} m_i^{ts}(q_i)\right)}{\left(\sum_{i=1}^{N} p_{it} m_i^{ts}(q_i)\right)} = \frac{\left(\sum_{i=1}^{N} p_{is} m_i^{st}(q_{i\_st})\right)}{\left(\sum_{i=1}^{N} p_{it} m_i^{st}(q_{i\_st})\right)}$$

$$\Rightarrow \frac{\sum_{i=1}^{N} p_{it} m_i^{st}(q_i)}{\sum_{i=1}^{N} p_{is} m_i^{st}(q_i)} \times \frac{\sum_{i=1}^{N} p_{is} m_i^{st}(q_{i\_st})}{\sum_{i=1}^{N} p_{it} m_i^{st}(q_{i\_st})} = 1$$

$$\Rightarrow \frac{\sum_{i=1}^{N} p_{it} m_i^{st}(q_i)}{\sum_{i=1}^{N} p_{it} m_i^{st}(q_{i\_st})} = \frac{\sum_{i=1}^{N} p_{is} m_i^{st}(q_{i\_st})}{\sum_{i=1}^{N} p_{is} m_i^{st}(q_i)}$$

Suppose we have

$$m_i^{st}(q_i) \neq m_i^{st}(q_i\_st).$$

Then, there exists $X \neq 1$ such that,

$$\frac{\sum_{i=1}^{N} p_{it} m_i^{st}(q_i)}{\sum_{i=1}^{N} p_{it} m_i^{st}(q_{i\_st})} = X \neq 1$$

Further

$$\frac{\sum_{i=1}^{N} p_{it} m_i^{st}(q_i)}{\sum_{i=1}^{N} p_{it} m_i^{st}(q_{i\_st})} = \frac{\sum_{i=1}^{N} p_{is} m_i^{st}(q_{i\_st})}{\sum_{i=1}^{N} p_{is} m_i^{st}(q_i)} \Rightarrow \frac{\sum_{i=1}^{N} p_{is} m_i^{st}(q_{i\_st})}{\sum_{i=1}^{N} p_{is} m_i^{st}(q_i)} = X .$$

Since $m_i^{st}(q_i) \neq m_i^{st}(q_i\_st).$ we can choose p_{is} such that

$$\frac{\sum_{i=1}^{N} p_{is} m_i^{st}(q_{i\_st})}{\sum_{i=1}^{N} p_{is} m_i^{st}(q_i)} \neq X \ .$$

This is a contradiction. Therefore, the following equation must always hold.

$$m_i^{st}(q_i) = m_i^{st}(q_{i-st}).$$

Since the index is transitive, we have

$$.\,PI^{st}(q_i) \times PI^{tk}(q_i) = \frac{\sum_{i=1}^{N} p_{it} m_i^{st}(q_i)}{\sum_{i=1}^{N} p_{is} m_i^{st}(q_i)} \times \frac{\sum_{i=1}^{N} p_{ik} m_i^{tk}(q_i)}{\sum_{i=1}^{N} p_{it} m_i^{tk}(q_i)} = \frac{\sum_{i=1}^{N} p_{ik} m_i^{sk}(q_i)}{\sum_{i=1}^{N} p_{is} m_i^{sk}(q_i)} = PI^{sk}(q_i)$$

Rearranging the terms in the equation, we have

$$\frac{\sum_{i=1}^{N} p_{it} m_i^{st}\left(q_i\right)}{\sum_{i=1}^{N} p_{it} m_i^{tk}\left(q_i\right)} \times \frac{\sum_{i=1}^{N} p_{ik} m_i^{tk}\left(q_i\right)}{\sum_{i=1}^{N} p_{is} m_i^{st}\left(q_i\right)} = \frac{\sum_{i=1}^{N} p_{ik} m_i^{sk}\left(q_i\right)}{\sum_{i=1}^{N} p_{is} m_i^{sk}\left(q_i\right)} .$$

We note that the right hand side of the eqation does not depend on prices in country *t*. This means that the first term on the left hand side does not depend on *t*. This can happen only if for any observed p_{it} the following must hold:

$$\frac{\sum_{i=1}^{N} p_{it} m_i^{st}\left(q_i\right)}{\sum_{i=1}^{N} p_{it} m_i^{tk}\left(q_i\right)} = n \in \mathbf{R}_{++} \qquad \textit{for any choice of } \; p_{it}$$

This can happen only if

$$m_i^{st}\left(q_i\right) = n \times m_i^{tk}\left(q_i\right).$$

Because we have $m_i^{st}\left(a, a, ..., a\right) = a$ for any $a \in \mathbf{R}_{++}$ for $i = 1, ..., N$, n shoud be unity. Thus, we have

$$m_i^{st}\left(q_i\right) = m_i^{tk}\left(q_i\right) = m_i\left(q_i\right).$$

Proof of Lemma 1 is complete.

If the price index, $PI^{st}\left(q_f^{st}\right)$, is transitive, the above proposition shows that for any $s, t,$ the reference vector must take the following form,

$$q_f^{st} = \left(m_1\left(q_1\right), m_2\left(q_2\right), .., m_N\left(q_N\right)\right) \text{for all } s, t \;\; \text{where } q_i = \left(q_{i1}, q_{i2}, ..., q_{iM}\right).$$

This means that when transitivity holds, the reference quantity vector used in all bilateral comparisons is the same. We note here that this result still allows for the functional form for each commodity, each element of the reference quantity vector, to be different.

The following Lemma is useful in proving the main result.

Lemma 2: *Let* $\mathbf{q_1} = \left(q_{11}, q_{21} ..., q_{N1}\right), \mathbf{q_2} = \left(q_{12}, q_{22} ..., q_{N2}\right) \in \mathbf{R}_{++}^{N}$. *Suppose for all* $\lambda > 0$ *and quantity vectors* $\mathbf{q_1}, \mathbf{q_2}$ *the following equation holds:*

$$\frac{m_1\left(\lambda q_{11}, q_{21,} ..\right)}{m_1\left(q_{11}, q_{21,} ...\right)} = \frac{m_2\left(\lambda q_{12}, q_{22,} ..\right)}{m_2\left(q_{12} . q_{22,} ..\right)}.$$

Then, there exists a function, $f\left(\lambda\right)$, that satisfies

$$\frac{m_1\left(\lambda q_{11}, q_{21,} ..\right)}{m_1\left(q_{11}, q_{21,} ...\right)} = \frac{m_2\left(\lambda q_{12}, q_{22,} ..\right)}{m_2\left(q_{12} . q_{22,} ..\right)} = f\left(\lambda\right).$$

Proof. Set $\left(q_{11}, q_{21} ..., q_{N1}\right) = \left(1, 1 ..., 1\right)$, then, for any $q_2 \in \mathbf{R}_{++}^{N}$, the following equation holds:

$$\frac{m_1\left(\lambda, 1 ..\right)}{m_1\left(1, 1 ...\right)} = \frac{m_2\left(\lambda q_{12}, q_{22,} ..\right)}{m_2\left(q_{12} . q_{22,} ..\right)}.$$

Similarly, set $\left(q_{12},q_{22}...,q_{N2}\right)=(1,1...,1)$. Then, for any $q_1 \in \mathbf{R}_{++}^N$, the following equation holds:

$$\frac{m_1\left(\lambda q_{11},q_{21,}..\right)}{m_1\left(q_{11},q_{21,}...\right)}=\frac{m_2\left(\lambda,1,1,..\right)}{m_2\left(1.1..\right)}.$$

Then we get

$$\frac{m_1\left(\lambda q_{11},q_{21,}..\right)}{m_1\left(q_{11},q_{21,}...\right)}=\frac{m_2\left(\lambda q_{12},q_{22,}..\right)}{m_2\left(q_{12}.q_{22,}..\right)}=\frac{m_1\left(\lambda,1..\right)}{m_1\left(1,1...\right)}==\frac{m_2\left(\lambda,1,1,..\right)}{m_2\left(1.1..\right)}=f(\lambda).$$

The next Lemma completes the proof of the theorem.

Definition 4 *Invariant to proportional changes of a country*

Suppose all the quantities in country *j are* multiplied by $\lambda>0$, that is, the new quantity vector for country j becomes,

$$\mathbf{q_j}=\boldsymbol{\lambda}\mathbf{q_j}=\lambda\left(q_{1j},q_{2j},...,q_{Nj}\right)=\left(\lambda q_{1j},\lambda q_{2j},...,\lambda q_{Nj}\right)$$

with the matrix of quantities of all commodities in all countries denoted as

$$\mathbf{q}=\left(\mathbf{q_1},...,\mathbf{q_{j-1}},\mathbf{q_j},\mathbf{q_{j+1}},..,\mathbf{q_M}\right),$$

then, $PI^{st}\left(\mathbf{p_s},\mathbf{p_t},\mathbf{q}\right)$ is unchanged, that is,

$$PI^{st}\left(\mathbf{p_s},\mathbf{p_t},\mathbf{q}\right)=PI^{st}\left(\mathbf{p_s},\mathbf{p_t},\mathbf{q}\right).$$

Lemma 3: *If* $PI^{st}\left(p_s,p_t,q\right)$ *in (A1) satisfies country symmetry and is Invariant to proportional changes of any country and passes the transitivity test,* m_i *should have the following functional form,*

$$m_i=\prod_{m=1}^{M}\left(q_{im}\right)^{1/M}\quad for\ all\ i=1,2,...,N$$

Proof: Without loss of generality, let us change quantities of country a multiple λ. Then invariance of the fixed basket index implies

$$\frac{\sum_{i=1}^N p_{it}m_i(q_{i1},q_{i2},...,q_{iM})}{\sum_{i=1}^N p_{is}m_i(q_{i1},q_{i2},...,q_{iM})}=\frac{\sum_{i=1}^N p_{it}m_i(\lambda q_{i1},q_{i2},...,q_{iM})}{\sum_{i=1}^N p_{is}m_i(\lambda q_{i1},q_{i2},...,q_{iM})}. \tag{A2}$$

We first prove the result for the case of two countries, denoted by *s* and *t*. Then we have

$$\frac{\sum_{i=1}^N p_{it}m_i(q_{is},q_{it})}{\sum_{i=1}^N p_{is}m_i(q_{is},q_{it})}=\frac{\sum_{i=1}^N p_{it}m_i(\lambda q_{is},q_{it})}{\sum_{i=1}^N p_{is}m_i(\lambda q_{is},q_{it})}.$$

Rearranging this equation we have

$$\frac{\sum_{i=1}^{N} p_{it} m_i(q_{is}, q_{it})}{\sum_{i=1}^{N} p_{it} m_i(\lambda q_{is}, q_{it})} = \frac{\sum_{i=1}^{N} p_{is} m_i(q_{is}, q_{it})}{\sum_{i=1}^{N} p_{is} m_i(\lambda q_{is}, q_{it})} .$$

Here the RHS does not depend on p_{it} whereas the LHS does not depend on p_{is}. Then, there exists a function $f(\lambda, q_s, q_t)$ such that

$$\frac{\sum_{i=1}^{N} p_{it} m_i(\lambda q_{is}, q_{it})}{\sum_{i=1}^{N} p_{it} m_i(q_{is}, q_{it})} = \frac{\sum_{i=1}^{N} p_{is} m_i(\lambda q_{is}, q_{it})}{\sum_{i=1}^{N} p_{is} m_i(q_{is}, q_{it})} = f(\lambda, q_s, q_t) ,$$

which leads to the following equation:

$$\sum_{i=1}^{N} p_{it} m_i(\lambda q_{is}, q_{it}) = f(\lambda, q_s, q_t) \sum_{i=1}^{N} p_{it} m_i(q_{is}, q_{it}) ,$$

$$\sum_{i=1}^{N} p_{is} m_i(\lambda q_{is}, q_{it}) = f(\lambda, q_s, q_t) \sum_{i=1}^{N} p_{is} m_i(q_{is}, q_{it}) .$$

Since the above is the identity for prices, for each *i*, we have

$$m_i(\lambda q_{is}, q_{it}) = f(\lambda, q_s, q_t) m_i(q_{is}, q_{it}) ,$$

$$m_j(\lambda q_{js}, q_{jt}) = f(\lambda, q_s, q_t) m_j(q_{js}, q_{jt}) .$$

These two equations can be arranged as

$$f(\lambda, q_s, q_t) = \frac{m_j(\lambda q_{js}, q_{jt})}{m_j(q_{is}, q_{it})} = \frac{m_i(\lambda q_{is}, q_{it})}{m_j(q_{is}, q_{it})}$$

Since the above is the identity with respect to quantity vectors, $f(\lambda, q_s, q_t)$ should not depend on the quantity vectors. Therefore, we get

$$f(\lambda, q_s, q_t) = f(\lambda)$$

Then, $f(\lambda)$ can be written as

$$\frac{\sum_{i=1}^{N} p_{it} m_i(\lambda q_{is}, q_{it})}{\sum_{i=1}^{N} p_{it} m_i(q_{is}, q_{it})} = \frac{\sum_{i=1}^{N} p_{is} m_i(\lambda q_{is}, q_{it})}{\sum_{i=1}^{N} p_{is} m_i(q_{is}, q_{it})} = f(\lambda) ,$$

which leads to the following equation:

$$\sum_{i=1}^{N} p_{it} m_i(\lambda q_{is}, q_{it}) = f(\lambda) \sum_{i=1}^{N} p_{it} m_i(q_{is}, q_{it}) .$$

Since this equation holds for all p_{it} and observed quantities q_{is} and q_{it}, this equation implies that for all *i*, we have

$$m_i(\lambda q_{is}, q_{it}) = f(\lambda) m_i(q_{is}, q_{it}) . \qquad \text{(A3)}$$

Denote $q_{is} = a, q_{it} = b,$ then,

$$m_i\left(\lambda a, b\right) = f\left(\lambda\right) \times m_i\left(a, b\right) .$$

Set $b = 1$,

$$m_i(\lambda a,1) = f(\lambda) \times m_i(a,1). \quad \text{(A4)}$$

Set $a = 1$. Because $m_i(1,1) = 1$, we get

$$m_i(\lambda,1) = f(\lambda).$$

This also means,

$$m_i(\lambda b,1) = f(\lambda b),$$

$$m_i(b,1) = f(b).$$

Set $a = b$ in (A4)

$$m_i(\lambda b,1) = f(\lambda) \times m_i(b,1)$$

$$= f(\lambda b) \times f(b)$$

$$= f(\lambda b).$$

Therefore, we get

$$f(\lambda b) = f(\lambda) \times f(b).$$

This is one of the Cauchy's functional equations whose general solution is

$$f(\lambda) = \lambda^c, c \neq 0.$$

Note that we have

$$m_i(\lambda a,b) = f(\lambda) \times m_i(a,b)$$

Set $a = 1$,

$$m_i(\lambda,b) = f(\lambda) \times m_i(b,1)$$

$$= f(\lambda) \times f(b)$$

$$= \lambda^c b^c.$$

Because $m_i(a,a) = a$,

$$m_i(a,a) = a^{2c}$$

$$= a,$$

thus, we get

$$c = 1/2.$$

Therefore, if $M = 2$, for all ,

$$m_i\left(q_{is}, q_{it}\right) = q_{is}^{1/2} q_{it}^{1/2}.$$

This completes the proof for *M* = 2.

Let us turn to the general case where *M* > 2. We start with equation (A2) and rearranging as:

$$\frac{\sum_{i=1}^{N} p_{it} m_i(\lambda q_{i1}, q_{i2}, ..., q_{iM})}{\sum_{i=1}^{N} p_{it} m_i(q_{i1}, q_{i2}, ..., q_{iM})} = \frac{\sum_{i=1}^{N} p_{is} m_i(\lambda q_{i1}, q_{i2}, ..., q_{iM})}{\sum_{i=1}^{N} p_{is} m_i(q_{i1}, q_{i2}, ..., q_{iM})} .$$

In this equation the LHS does not depend on p_{is} whereas the RHS does not depend on p_{it}. Further this must hold for all values of p_{it} and p_{is}, and for all possible values for quantities. Following the same procedure when *M* = 2., we can show that there exists a function $f(\lambda)$ such that

$$\sum_{i=1}^{N} p_{it} m_i(\lambda q_{i1}, q_{i2}, ..., q_{iM}) = f(\lambda) \sum_{i=1}^{N} p_{it} m_i(q_{i1}, q_{i2}, ..., q_{iM}) \qquad \text{(A4)}$$

Set all factors, prices and quantities other than q_{i1} to unity. Then we have

$$m_i\left(\lambda q_{i1}, 1, 1, ..., 1\right) = f(\lambda) m_i\left(q_{i1}, 1, 1, ..., 1\right) . \qquad \text{(A5)}$$

If we further set $q_{i1} = 1$, then we have

$$m_i\left(\lambda, 1, 1, ..., 1\right) = f(\lambda) m_i\left(1, 1, 1, ..., 1\right) .$$

Since $m_i(1,1,...,1) = 1$, we get

$$m_i\left(\lambda, 1, 1, ..., 1\right) = f(\lambda) .$$

Now set all factors other than q_{i2} to unity. Then from (A4) we have

$$m_i\left(\lambda, q_{i2}, 1, ..., 1\right) = f(\lambda) m_i\left(1, q_{i2}, 1, ..., 1\right) .$$

Since $m_i(...)$ is symmetric, we have

$$m_i\left(\lambda, q_{i2}, 1, ..., 1\right) = f(\lambda) m_i\left(1, q_{i2}, 1, ..., 1\right) = f(\lambda) m_i\left(q_{i2}, 1, .1, .., 1\right) = f(\lambda) f\left(q_{i2}\right) .$$

By iterating this process *M* times, we get

$$m_i(q_{i1}, q_{i2}, ..., q_{iM}) = \prod_{m=1}^{M} f(q_{im}) .$$

Further we have

$$f(\lambda b) = f(\lambda) m_i(b, 1, 1, ..., 1) = f(\lambda) f(b) . \qquad \text{(A6)}$$

Equation (A6) is one of the classical Cauchy equations whose general solution is:

$$f(\lambda) = \lambda^c, c \neq 0 .$$

Since we know that

$$m_i(\lambda, \lambda, ..., \lambda) = f(\lambda)^M = \lambda^{cM} = \lambda .$$

Since we know that

$$m_i(\lambda,\lambda,...,\lambda) = f(\lambda)^M = \lambda^{cM} = \lambda$$

We have

$$cM = 1 \Rightarrow c = \frac{1}{M}$$

Therefore, for all *i* we obtain

$$m_i(q_{i1}, q_{i2}, ..., q_{iM}) = \prod_{m=1}^{M} \left(q_{iM}\right)^{1/M} \quad .$$

This completes the proof of Proposition 1 in the paper.

Appendix B

Table A1: Basic Headings in Individual Consumption Expenditure by Households(ICP 2017)

Item Name	zero ratio	Item Name	zero ratio
Rice	0.000	Major tools and equipment (BH)	0.017
Other cereals, flour and other cereal products	0.000	Small tools and miscellaneous accessories (BH)	0.000
Bread	0.000	Non-durable household goods (BH)	0.000
Other bakery products	0.000	Domestic services	0.000
Pasta products and couscous	0.000	Household services	0.052
Beef and veal	0.000	Pharmaceutical products (BH)	0.000
Pork	0.092	Other medical products (BH)	0.034
Lamb, mutton and goat	0.017	Therapeutic appliances and equipment (BH)	0.011
Poultry	0.000	Medical services (BH)	0.000
Other meats and meat preparations	0.000	Dental services (BH)	0.017
Fresh, chilled or frozen fish and seafood	0.000	Paramedical services (BH)	0.017
Preserved or processed fish and seafood	0.000	Hospital services (BH)	0.023
Fresh milk	0.000	Motor cars (BH)	0.000
Preserved milk and other milk products	0.000	Motor cycles (BH)	0.011
Cheese and curd	0.000	Bicycles (BH)	0.006
Eggs and egg-based products	0.000	Animal drawn vehicles (BH)	0.672
Butter and margarine	0.000	Fuels and lubricants for personal transport equipment (BH)	0.006
Other edible oils and fats	0.000	Maintenance and repair of personal transport equipment (BH)	0.000
Fresh or chilled fruit	0.000	Other services in respect of personal transport equipment (BH)	0.034
Frozen, preserved or processed fruit and fruit-based products	0.000	Passenger transport by railway (BH)	0.259
Fresh or chilled vegetables, other than potatoes and other tuber vegetables	0.000	Passenger transport by road (BH)	0.000
Fresh or chilled potatoes and other tuber vegetables	0.000	Passenger transport by air (BH)	0.000
Frozen, preserved or processed vegetables and vegetable-based products	0.006	Passenger transport by sea and inland waterway (BH)	0.103
Sugar	0.000	Combined passenger transport (BH)	0.575
Jams, marmalades and honey	0.000	Other purchased transport services (BH)	0.161
Confectionery, chocolate and ice cream	0.000	Postal services (BH)	0.000
Food products n.e.c. (BH)	0.000	Telephone and telefax equipment (BH)	0.000
Coffee, tea and cocoa (BH)	0.000	Telephone and telefax services (BH)	0.000
Mineral waters, soft drinks, fruit and vegetable juices (BH)	0.000	Audio-visual, photographic and information processing equipment	0.000
Spirits (BH)	0.046	Recording media (BH)	0.000
Wine (BH)	0.046	Repair of audio-visual, photographic and information processing e	0.040
Beer (BH)	0.046	Major durables for outdoor and indoor recreation (BH)	0.052
Tobacco (BH)	0.000	Maintenance and repair of other major durables for recreation and	0.201
Narcotics (BH)	0.546	Other recreational items and equipment (BH)	0.000
Clothing materials, other articles of clothing and clothing accessories (BH)	0.000	Garden and pets (BH)	0.011
Garments (BH)	0.000	Veterinary and other services for pets (BH)	0.046
Cleaning, repair and hire of clothing (BH)	0.006	Recreational and sporting services (BH)	0.000
Shoes and other footwear (BH)	0.000	Cultural services (BH)	0.000
Repair and hire of footwear (BH)	0.011	Games of chance (BH)	0.190
Actual rentals for housing (BH)	0.132	Newspapers, books and stationery (BH)	0.000
Imputed rentals for housing (BH)	0.000	Package holidays (BH)	0.086
Maintenance and repair of the dwelling (BH)	0.006	Education - HHC (BH)	0.000
Water supply (BH)	0.006	Catering services (BH)	0.000
Miscellaneous services relating to the dwelling (BH)	0.040	Accommodation services (BH)	0.000
Electricity (BH)	0.000	Hairdressing salons and personal grooming establishments (BH)	0.000
Gas (BH)	0.006	Appliances, articles and products for personal care (BH)	0.000
Other fuels (BH)	0.011	Prostitution (BH)	0.678
Furniture and furnishings (BH)	0.000	Jewellery, clocks and watches (BH)	0.000
Carpets and other floor coverings (BH)	0.000	Other personal effects (BH)	0.000
Repair of furniture, furnishings and floor coverings (BH)	0.046	Social protection - HHC (BH)	0.092
Household textiles (BH)	0.000	Insurance (BH)	0.006
Major household appliances whether electric or not (BH)	0.000	Financial Intermediation Services Indirectly Measured (FISIM) (BH	0.057
Small electric household appliances (BH)	0.000	Other financial services n.e.c. (BH)	0.063
Repair of household appliances (BH)	0.034	Other services n.e.c. (BH)	0.029
Glassware, tableware and household utensils (BH)	0.000		

Appendix C

Table A2: PPPs for all countries, Different Methods and ICP 2017 Data

countryname	Fisher_bi	Jevons_bi	Laspeyres_bi	Paashe_bi	SV_bi	Walsh_bi	Walsh_m	GEKS_SV	GEKS_Fisher	GEKS_bi Walsh	GK_percapita	GK_original
Angola	98.53	122.22	118.00	82.27	105.23	104.00	108.93	110.22	104.36	109.21	95.39	94.21
Burundi	634.22	811.01	764.42	526.20	595.65	566.03	647.17	637.60	659.60	616.50	581.89	577.95
Benin	232.31	267.91	262.57	205.53	222.43	218.85	228.98	224.99	228.87	223.38	208.53	205.86
Burkina Faso	232.84	252.22	256.21	211.60	220.45	217.74	207.55	208.75	213.14	207.58	199.88	193.56
Botswana	6.07	6.21	5.83	6.31	6.02	6.03	5.18	5.26	5.38	5.21	5.36	5.33
Central African Republic	319.49	355.52	374.01	272.92	347.59	347.58	332.05	323.06	315.26	319.64	283.91	278.82
Côte d'Ivoire	283.27	301.46	294.80	272.19	286.95	286.44	250.45	257.15	259.28	255.30	257.49	256.70
Cameroon	239.77	290.19	300.48	191.33	243.41	242.36	251.78	244.00	240.91	242.54	213.56	214.26
Congo, Dem. Rep.	707.10	748.79	738.42	677.10	661.02	634.78	657.14	648.92	660.17	637.87	614.07	604.62
Congo, Rep.	281.16	335.32	313.63	252.06	276.19	274.49	279.29	284.88	286.19	282.68	268.88	265.85
Comoros	203.62	269.58	263.49	157.36	187.08	181.98	221.92	204.78	214.96	200.87	180.40	174.73
Cabo Verde	49.19	59.27	55.76	43.40	50.20	50.87	50.90	50.57	48.35	50.25	46.28	45.51
Djibouti	100.05	117.07	116.27	86.09	104.54	105.34	101.48	104.56	102.88	104.46	94.80	92.29
Algeria	39.07	44.24	43.52	35.08	41.46	42.41	41.22	39.76	39.31	39.46	37.49	37.95
Egypt, Arab Rep. (AFR)	3.40	4.49	4.29	2.70	3.22	3.15	3.64	3.51	3.52	3.44	3.09	3.08
Ethiopia	9.12	10.46	10.01	8.31	8.63	8.56	8.65	8.55	8.64	8.47	7.74	7.68
Gabon	323.70	353.09	359.31	291.62	330.11	329.70	334.36	332.99	327.49	332.44	317.51	315.01
Ghana	1.49	1.97	1.95	1.14	1.66	1.64	1.76	1.71	1.64	1.71	1.37	1.41
Guinea	3381.92	3641.51	3711.86	3081.30	3232.58	3080.01	2917.62	3204.65	3269.05	3127.75	3160.99	3125.41
Gambia, The	15.69	17.51	19.06	12.92	14.20	13.82	16.01	15.31	15.68	15.13	13.82	13.72
Guinea-Bissau	272.80	275.74	271.13	274.47	253.04	253.88	234.21	244.41	248.00	248.20	246.94	241.22
Equatorial Guinea	275.78	346.52	335.17	226.91	282.66	278.25	308.63	302.66	297.01	300.46	268.60	263.68
Kenya	42.32	49.24	48.07	37.26	41.46	40.50	44.14	42.40	43.18	41.97	39.10	38.98
Liberia	37.95	57.83	55.17	26.11	38.10	36.34	47.22	45.17	44.56	44.25	34.53	32.98
Lesotho	5.75	6.31	5.86	5.65	5.77	5.75	5.34	5.39	5.39	5.39	5.04	5.00
Morocco (AFR)	4.48	5.23	4.79	4.19	4.44	4.41	4.17	4.33	4.40	4.31	4.17	4.05
Madagascar	979.36	1232.95	1285.20	746.29	931.99	910.52	1031.38	977.12	980.90	965.51	832.16	836.35
Mali	219.37	263.89	274.85	175.08	216.74	216.31	229.55	210.66	214.21	210.03	181.32	172.13
Mozambique	25.40	28.58	27.60	23.37	26.64	26.44	22.68	23.46	23.44	23.22	22.59	21.94
Mauritania	135.55	142.15	143.49	128.05	124.83	121.87	120.88	126.65	130.40	125.15	128.70	121.13
Mauritius	16.50	21.68	20.42	13.34	17.43	17.28	17.89	18.19	17.67	18.11	15.62	14.62
Malawi	233.21	324.77	349.70	155.53	270.68	268.46	278.44	261.16	241.14	261.36	194.68	192.97
Namibia	6.53	8.37	7.59	5.63	6.63	6.57	6.78	6.79	6.81	6.77	6.23	5.97
Niger	260.37	291.95	287.23	236.03	260.27	260.42	244.51	249.26	250.01	247.38	236.78	231.78
Nigeria	120.41	124.80	120.98	119.85	115.48	115.30	113.79	116.11	117.96	115.51	117.54	117.00
Rwanda	306.46	373.89	392.11	239.53	316.24	325.14	346.71	308.52	302.30	308.97	244.06	253.93
Senegal	247.48	272.72	273.63	223.83	237.56	235.94	238.55	247.35	247.43	245.45	238.95	235.53
Sierra Leone	2068.76	2630.25	2649.38	1615.38	2153.21	2135.62	2305.61	2184.42	2161.83	2164.66	1842.19	1869.50
São Tomé and Principe	11.33	12.91	12.04	10.66	10.63	10.35	10.92	11.14	11.20	11.02	10.76	10.55
Eswatini	5.72	7.29	6.70	4.89	5.86	5.78	5.78	6.11	6.09	6.06	5.83	5.69
Seychelles	8.52	10.02	9.92	7.32	8.83	8.89	9.03	9.09	9.02	9.04	8.57	8.51
Chad	260.34	277.58	266.51	254.32	252.51	245.20	242.76	246.64	248.16	244.47	241.11	232.95
Togo	246.23	282.59	274.68	220.72	239.97	233.19	245.41	245.56	248.76	242.06	230.24	226.91
Tunisia	0.77	0.95	0.89	0.67	0.78	0.77	0.74	0.75	0.76	0.75	0.71	0.69
Tanzania	875.78	883.05	996.36	769.79	827.62	822.37	913.18	785.32	818.56	775.93	707.49	695.12
Uganda	1062.95	1589.22	1556.94	725.69	1022.25	998.79	1378.41	1205.16	1204.93	1185.44	910.38	902.10
South Africa	6.44	8.06	7.57	5.47	6.97	6.95	6.75	6.81	6.48	6.77	6.27	6.22
Zambia	4.40	4.97	4.60	4.21	4.21	4.17	4.04	4.32	4.38	4.27	4.28	4.17
Zimbabwe	0.48	0.58	0.57	0.41	0.48	0.46	0.49	0.49	0.49	0.48	0.44	0.42
Bangladesh	30.16	34.83	33.58	27.08	28.07	27.84	28.92	29.00	29.66	28.79	27.13	26.95
Brunei Darussalam	0.71	0.89	0.88	0.57	0.79	0.79	0.79	0.76	0.70	0.74	0.64	0.64
Bhutan	21.15	24.40	22.65	19.76	21.30	21.30	20.59	21.42	20.96	21.29	20.44	20.40
China	4.03	4.89	4.37	3.73	3.98	4.02	4.20	4.24	4.21	4.23	4.02	4.06
Fiji	1.03	1.20	1.06	0.99	1.03	1.03	0.94	1.01	1.02	1.01	1.02	1.03
Hong Kong SAR, China	6.18	6.80	6.76	5.65	6.20	6.21	6.24	6.22	6.19	6.25	5.79	5.80
Indonesia	5165.07	6186.49	5657.00	4715.93	5168.58	5192.77	5015.05	5242.01	5255.48	5208.68	4952.89	4961.71
India	19.18	23.59	23.53	15.63	18.60	18.82	19.51	19.66	19.78	19.54	17.88	17.76
Cambodia	1505.65	1810.18	1795.13	1262.85	1513.30	1528.45	1561.27	1543.16	1519.43	1544.01	1344.95	1346.94
Lao PDR	3232.19	3774.13	3544.86	2947.09	3181.06	3158.64	3122.97	3225.66	3240.67	3179.19	3049.37	3073.40
Sri Lanka	51.41	66.54	67.79	38.99	50.17	49.29	57.97	55.38	56.38	54.31	47.68	46.61
Maldives	9.86	10.55	11.23	8.66	9.65	9.80	10.21	9.86	9.96	9.91	9.03	9.06
Myanmar	400.22	482.86	454.15	352.69	358.02	342.73	395.05	388.62	397.09	380.67	361.53	360.84
Mongolia	851.47	1087.97	1003.53	722.45	877.17	894.92	955.04	914.20	902.32	910.65	795.21	773.40
Malaysia	1.77	2.03	1.87	1.67	1.75	1.77	1.69	1.73	1.75	1.73	1.68	1.67
Nepal	32.22	35.05	40.47	25.66	29.93	29.09	32.49	31.40	31.40	30.81	27.83	27.54
Pakistan	33.36	41.09	39.41	28.24	31.63	30.50	34.31	33.53	33.22	32.94	30.21	29.97
Philippines	18.66	22.98	21.97	15.85	18.44	18.50	19.50	19.26	19.19	19.21	17.51	17.54
Singapore	1.02	1.21	1.22	0.85	1.03	1.04	1.07	1.10	1.07	1.11	0.97	0.94
Thailand	13.82	16.43	14.90	12.81	13.68	13.82	12.93	13.46	13.58	13.39	12.87	12.96
Taiwan, China	15.69	18.88	16.42	14.99	15.68	15.85	16.69	16.65	16.69	16.62	15.77	15.86
Vietnam	7758.78	9261.17	9009.86	6681.43	7739.89	7808.35	8094.83	8118.26	8024.54	8126.64	7464.90	7480.72
Armenia	166.42	218.18	197.57	140.18	162.29	157.52	172.18	165.24	167.36	163.26	148.89	147.00
Azerbaijan	0.50	0.63	0.58	0.43	0.51	0.51	0.50	0.50	0.50	0.50	0.47	0.47
Belarus	0.63	0.78	0.71	0.56	0.64	0.64	0.63	0.61	0.61	0.61	0.58	0.58
Kazakhstan	122.70	143.56	145.59	103.41	129.75	130.70	140.16	129.45	126.20	129.03	115.26	114.55
Kyrgyz Republic	20.44	26.51	25.09	16.66	20.89	20.23	20.96	19.84	19.63	19.47	17.58	17.50
Moldova	6.56	8.11	7.46	5.77	6.85	6.73	6.37	6.37	6.31	6.33	6.00	6.00
Tajikistan	2.69	3.47	3.17	2.28	2.69	2.56	2.68	2.59	2.58	2.52	2.33	2.30
Albania	55.31	62.76	61.15	50.02	55.07	54.66	52.90	53.55	53.52	53.37	50.91	51.46
Australia	1.51	1.53	1.52	1.50	1.52	1.52	1.58	1.52	1.51	1.53	1.48	1.48
Austria	0.83	0.93	0.82	0.84	0.83	0.83	0.83	0.82	0.84	0.82	0.82	0.81
Belgium	0.85	0.91	0.85	0.85	0.85	0.85	0.86	0.85	0.85	0.85	0.84	0.83
Bulgaria	0.80	0.96	0.86	0.74	0.80	0.80	0.75	0.80	0.81	0.80	0.78	0.77
Bosnia and Herzegovina	0.86	1.00	0.91	0.80	0.85	0.84	0.81	0.83	0.83	0.82	0.80	0.79
Canada	1.29	1.37	1.27	1.31	1.29	1.30	1.34	1.28	1.26	1.29	1.25	1.24
Switzerland	1.27	1.36	1.30	1.24	1.27	1.27	1.34	1.31	1.33	1.30	1.27	1.27
Chile	497.06	545.74	528.95	467.09	514.65	514.17	490.85	493.43	487.09	494.37	487.94	488.15

countryname	Fisher_bi	Jevons_bi	Laspeyres_bi	Paashe_bi	SV_bi	Walsh_bi	Walsh_m	GEKS_SV	GEKS_Fisher	GEKS_bi Walsh	GK_percapita	GK_original
Colombia	1441.23	1804.15	1712.19	1213.15	1501.98	1495.04	1529.75	1531.52	1523.53	1529.55	1398.07	1387.64
Costa Rica	380.98	445.96	429.14	338.21	386.33	385.13	370.25	385.91	384.15	385.65	377.50	374.51
Cyprus	0.70	0.76	0.70	0.69	0.70	0.70	0.65	0.68	0.68	0.68	0.69	0.69
Czech Republic	14.12	15.86	14.01	14.24	14.03	14.17	13.62	14.10	14.11	14.14	14.10	13.96
Germany	0.80	0.86	0.80	0.80	0.79	0.80	0.81	0.79	0.80	0.79	0.78	0.77
Denmark	8.07	8.44	8.05	8.09	8.12	8.17	7.99	7.79	7.88	7.84	7.75	7.62
Spain	0.72	0.78	0.73	0.72	0.72	0.73	0.69	0.71	0.72	0.72	0.71	0.70
Estonia	0.64	0.71	0.63	0.64	0.64	0.64	0.58	0.60	0.60	0.60	0.61	0.61
Finland	0.99	1.01	1.00	0.97	0.99	0.99	0.94	0.94	0.95	0.94	0.94	0.93
France	0.83	0.89	0.82	0.85	0.84	0.84	0.82	0.83	0.83	0.84	0.83	0.82
United Kingdom	0.82	0.75	0.85	0.80	0.82	0.82	0.83	0.78	0.77	0.79	0.76	0.76
Greece	0.64	0.75	0.66	0.62	0.64	0.64	0.63	0.66	0.67	0.66	0.65	0.65
Croatia	4.15	4.86	4.25	4.06	4.14	4.13	3.89	4.06	4.11	4.06	4.05	4.06
Hungary	159.32	185.13	164.27	154.52	159.49	159.62	148.97	154.78	155.50	154.61	154.20	153.46
Ireland	1.00	0.99	1.01	0.98	1.00	1.00	0.97	0.98	0.97	0.99	0.97	0.97
Iceland	152.80	159.10	150.60	155.03	153.67	154.47	148.52	144.93	147.09	145.06	145.22	144.27
Israel	4.31	4.37	4.46	4.17	4.37	4.38	4.17	4.20	4.10	4.22	4.07	4.06
Italy	0.81	0.85	0.82	0.79	0.81	0.81	0.75	0.78	0.78	0.78	0.79	0.77
Japan	108.75	128.76	107.92	109.58	108.35	108.79	119.24	116.23	118.18	116.13	111.66	110.22
Korea, Rep.	915.69	1079.11	960.14	873.30	916.47	921.53	1044.90	1018.32	1015.94	1019.71	952.21	939.30
Lithuania	0.54	0.60	0.53	0.55	0.54	0.54	0.47	0.49	0.50	0.49	0.51	0.50
Luxembourg	0.96	0.94	0.96	0.97	0.97	0.98	1.01	0.94	0.94	0.95	0.90	0.90
Latvia	0.59	0.66	0.60	0.58	0.59	0.59	0.56	0.58	0.57	0.58	0.58	0.58
Mexico	10.60	11.16	10.94	10.27	10.62	10.65	9.83	9.97	9.89	10.00	9.99	9.94
North Macedonia	23.35	25.76	24.99	21.83	23.09	23.29	21.95	22.71	22.70	22.69	22.16	22.01
Malta	0.66	0.75	0.67	0.66	0.66	0.66	0.62	0.65	0.66	0.65	0.65	0.65
Montenegro	0.48	0.52	0.48	0.48	0.49	0.49	0.42	0.45	0.45	0.45	0.46	0.46
Netherlands	0.87	0.89	0.87	0.88	0.88	0.88	0.85	0.84	0.85	0.85	0.84	0.83
Norway	11.03	12.01	10.87	11.18	11.07	11.14	11.01	10.67	10.79	10.72	10.52	10.48
New Zealand	1.61	1.62	1.58	1.64	1.63	1.64	1.67	1.58	1.57	1.58	1.53	1.52
Poland	1.95	2.24	2.01	1.90	1.93	1.92	1.78	1.86	1.89	1.86	1.87	1.85
Portugal	0.69	0.77	0.71	0.67	0.69	0.69	0.65	0.67	0.67	0.68	0.67	0.66
Romania	1.97	2.24	2.10	1.85	1.97	1.97	1.87	1.92	1.90	1.92	1.84	1.83
Russian Federation	26.70	30.43	28.11	25.35	27.11	27.04	25.15	25.62	25.43	25.55	25.40	25.39
Serbia	51.64	61.14	55.49	48.06	51.60	51.50	49.05	50.74	50.99	50.53	49.67	49.25
Slovak Republic	0.60	0.66	0.60	0.61	0.60	0.61	0.57	0.60	0.60	0.60	0.60	0.60
Slovenia	0.68	0.75	0.67	0.69	0.68	0.69	0.63	0.65	0.65	0.65	0.66	0.66
Sweden	9.38	10.27	9.51	9.26	9.41	9.43	9.29	9.03	9.17	9.04	8.93	8.79
Turkey	1.78	2.13	1.96	1.61	1.79	1.78	1.69	1.79	1.81	1.78	1.71	1.67
United States	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Argentina	11.17	13.02	11.67	10.68	11.28	11.31	11.03	11.28	11.15	11.29	11.13	11.14
Bolivia	2.57	3.20	2.90	2.28	2.64	2.66	2.72	2.60	2.61	2.61	2.38	2.36
Brazil	2.43	2.73	2.67	2.22	2.47	2.47	2.31	2.35	2.35	2.36	2.30	2.27
Dominican Republic	23.57	29.49	27.98	19.85	23.38	23.01	24.38	24.57	24.62	24.48	22.96	22.45
Ecuador	0.59	0.69	0.66	0.53	0.60	0.61	0.60	0.59	0.58	0.59	0.56	0.56
Honduras	11.78	13.64	12.91	10.75	11.59	11.54	10.69	11.04	11.16	11.03	10.78	10.78
Haiti	33.20	35.43	34.92	31.56	31.91	30.75	31.90	32.26	32.37	32.03	30.85	29.91
Nicaragua	12.02	14.05	14.09	10.26	11.91	12.02	11.81	11.91	12.08	11.91	11.13	11.04
Panama	0.53	0.62	0.61	0.45	0.55	0.55	0.53	0.52	0.52	0.52	0.49	0.48
Peru	1.85	2.15	2.03	1.68	1.94	1.98	2.00	1.97	1.95	1.97	1.88	1.90
Paraguay	2553.94	3162.10	2888.10	2258.45	2555.95	2575.50	2541.87	2594.85	2577.87	2593.62	2509.63	2470.04
El Salvador	0.54	0.63	0.59	0.50	0.55	0.55	0.54	0.53	0.53	0.53	0.51	0.50
Uruguay	24.65	28.01	27.59	22.03	24.54	24.62	25.23	25.24	25.09	25.28	23.94	23.22
Aruba	1.36	1.78	1.51	1.22	1.41	1.41	1.53	1.49	1.52	1.48	1.39	1.33
Anguilla	2.60	2.92	2.83	2.39	2.61	2.61	2.59	2.56	2.58	2.57	2.46	2.38
Antigua and Barbuda	2.49	2.81	2.63	2.35	2.51	2.50	2.43	2.46	2.49	2.46	2.41	2.35
Bahamas, The	1.10	1.20	1.27	0.95	1.10	1.10	1.13	1.11	1.12	1.11	1.05	1.01
Belize	1.36	1.65	1.54	1.21	1.41	1.42	1.39	1.46	1.47	1.46	1.39	1.33
Bermuda	1.54	1.41	1.67	1.41	1.52	1.51	1.57	1.49	1.49	1.50	1.42	1.40
Barbados	2.03	2.37	2.25	1.83	2.01	2.03	2.23	2.22	2.25	2.22	2.08	1.97
Curaçao	1.48	1.71	1.48	1.47	1.53	1.53	1.52	1.50	1.51	1.51	1.47	1.43
Cayman Islands	1.17	1.19	1.27	1.09	1.17	1.17	1.18	1.16	1.16	1.17	1.11	1.09
Dominica	1.81	2.26	2.00	1.64	1.83	1.82	1.83	1.89	1.89	1.88	1.78	1.70
Grenada	1.78	2.29	1.96	1.61	1.80	1.79	1.84	1.85	1.85	1.84	1.76	1.72
Guyana	118.45	143.43	125.29	111.99	119.11	118.63	111.39	118.75	118.56	118.39	116.79	114.17
Jamaica	75.50	87.70	79.92	71.32	75.88	76.48	74.30	75.39	76.09	75.46	72.89	72.91
St. Kitts and Nevis	2.45	2.65	2.57	2.33	2.45	2.47	2.49	2.54	2.51	2.54	2.46	2.37
St. Lucia	2.08	2.49	2.32	1.86	2.15	2.13	2.01	2.12	2.11	2.11	2.10	2.04
Montserrat	2.03	2.70	2.46	1.68	2.09	2.06	2.35	2.20	2.22	2.20	1.91	1.82
Suriname	3.50	3.82	3.51	3.48	3.52	3.55	3.03	3.22	3.23	3.22	3.37	3.38
Sint Maarten	1.33	1.70	1.57	1.14	1.37	1.37	1.55	1.47	1.51	1.47	1.31	1.22
Turks and Caicos Islands	1.29	1.28	1.36	1.23	1.31	1.30	1.28	1.22	1.21	1.23	1.17	1.17
Trinidad and Tobago	4.40	5.16	5.05	3.83	4.46	4.47	4.56	4.46	4.32	4.43	4.15	4.09
St. Vincent and the Grenadines	1.74	2.38	2.12	1.42	1.79	1.77	1.86	1.87	1.86	1.86	1.68	1.62
British Virgin Islands	1.13	1.19	1.24	1.04	1.16	1.17	1.26	1.09	1.10	1.10	0.99	1.00
United Arab Emirates	2.82	2.89	3.08	2.59	2.93	2.92	3.08	3.02	2.91	2.98	2.91	2.86
Bahrain	0.22	0.25	0.25	0.20	0.22	0.22	0.23	0.22	0.22	0.22	0.20	0.20
Iraq	593.59	712.55	641.56	549.22	581.19	573.66	579.24	576.56	568.69	568.21	540.56	532.21
Jordan	0.33	0.41	0.37	0.29	0.32	0.32	0.33	0.34	0.34	0.34	0.32	0.31
Kuwait	0.18	0.21	0.23	0.14	0.22	0.21	0.24	0.22	0.18	0.21	0.16	0.16
Morocco (WAS)	4.48	5.23	4.79	4.19	4.44	4.41	4.17	4.33	4.40	4.31	4.17	4.05
Oman	0.22	0.24	0.22	0.21	0.22	0.22	0.23	0.22	0.22	0.22	0.21	0.21
West Bank and Gaza	2.13	2.44	2.30	1.97	2.18	2.19	2.06	2.14	2.16	2.13	2.11	2.12
Qatar	3.06	2.91	3.19	2.95	3.09	3.06	3.17	2.98	2.98	2.97	2.94	2.86
Saudi Arabia	1.65	2.27	2.04	1.33	1.79	1.76	2.04	1.86	1.71	1.81	1.54	1.51
Sudan (WAS)	5.33	6.45	6.02	4.71	5.23	5.24	5.18	5.43	5.48	5.38	5.23	5.11
Iran, Islamic Rep.	14045.41	18403.68	17965.80	10980.51	12898.53	12625.65	15847.33	14325.98	14602.73	13986.28	12368.29	12173.22

19Appendix D

Table A3: Real Per Capita Expenditure Index (USA = 100)

country name	GEKS Fisher	Walsh M	GK	EXR	country name	GEKS Fisher	Walsh M	GK	EXR
Angola	9.82	9.40	10.74	6.17	Colombia	20.80	20.71	22.66	10.73
Burundi	1.56	1.59	1.77	0.60	Costa Rica	29.11	30.20	29.62	19.70
Benin	3.88	3.88	4.26	1.53	Cyprus	62.71	65.78	61.78	48.32
Burkina Faso	2.54	2.61	2.71	0.93	Czech Republic	41.20	42.68	41.22	24.95
Botswana	18.23	18.90	18.28	9.47	Germany	61.64	61.00	63.30	55.54
Central African Republic	1.90	1.81	2.11	1.03	Denmark	54.81	54.05	55.74	65.60
Côte d'Ivoire	5.76	5.96	5.80	2.56	Spain	53.32	55.58	53.62	43.08
Cameroon	5.83	5.58	6.58	2.41	Estonia	38.11	39.72	37.49	25.96
Congo, Dem. Rep.	2.17	2.18	2.34	0.98	Finland	55.12	55.54	55.59	58.94
Congo, Rep.	5.31	5.44	5.65	2.61	France	54.69	55.33	54.72	51.20
Comoros	6.69	6.48	7.97	3.29	United Kingdom	63.22	59.36	64.35	63.12
Cabo Verde	13.83	13.13	14.45	6.84	Greece	46.26	48.93	47.04	34.75
Djibouti	9.88	10.02	10.72	5.72	Croatia	39.30	41.43	39.82	24.42
Algeria	12.48	11.90	13.09	4.42	Hungary	32.28	33.70	32.55	18.34
Egypt, Arab Rep. (AFR)	27.51	26.64	31.35	5.43	Ireland	50.21	50.02	50.16	55.07
Ethiopia	2.88	2.88	3.21	1.04	Iceland	68.66	68.00	69.54	94.65
Gabon	11.07	10.84	11.42	6.23	Israel	47.99	47.20	48.35	54.72
Ghana	5.69	5.31	6.81	2.15	Italy	56.41	58.82	55.85	49.57
Guinea	5.24	5.87	5.42	1.88	Japan	49.76	49.32	52.67	52.43
Gambia, The	4.39	4.30	4.98	1.48	Korea, Rep.	39.35	38.26	41.99	35.37
Guinea-Bissau	3.94	4.17	3.95	1.68	Lithuania	46.87	49.36	46.11	26.34
Equatorial Guinea	22.12	21.29	24.46	11.29	Luxembourg	83.97	78.46	87.92	89.45
Kenya	7.77	7.60	8.58	3.25	Latvia	35.94	37.15	35.49	23.34
Liberia	2.33	2.20	3.01	0.92	Mexico	29.34	29.54	29.06	15.34
Lesotho	6.63	6.68	7.08	2.68	North Macedonia	21.18	21.90	21.69	8.82
Morocco (AFR)	10.99	11.60	11.58	4.99	Malta	49.34	52.20	49.59	36.59
Madagascar	2.87	2.73	3.39	0.90	Montenegro	36.77	38.97	35.92	18.61
Mali	4.32	4.03	5.11	1.59	Netherlands	55.39	55.46	56.08	53.22
Mozambique	2.22	2.29	2.30	0.82	Norway	58.32	57.17	59.81	76.23
Mauritania	4.49	4.85	4.55	1.64	New Zealand	53.04	49.85	54.27	59.04
Mauritius	44.57	44.01	50.41	22.84	Poland	39.91	42.32	40.43	20.03
Malawi	2.39	2.07	2.96	0.79	Portugal	48.61	50.66	48.78	36.96
Namibia	18.93	19.01	20.68	9.68	Romania	35.54	36.18	36.63	16.71
Niger	1.44	1.47	1.52	0.62	Russian Federation	31.53	31.89	31.57	13.74
Nigeria	8.83	9.15	8.86	3.41	Serbia	23.22	24.14	23.84	11.02
Rwanda	3.64	3.17	4.51	1.32	Slovak Republic	36.29	37.70	36.07	24.48
Senegal	5.75	5.97	5.96	2.45	Slovenia	44.61	45.88	44.17	32.73
Sierra Leone	4.15	3.89	4.87	1.21	Sweden	54.67	53.97	56.15	58.80
São Tomé and Principe	7.22	7.41	7.52	3.72	Turkey	29.65	31.61	31.25	14.67
Eswatini	16.47	17.35	17.20	7.52	United States	100.00	100.00	100.00	100.00
Seychelles	35.91	35.90	37.82	23.75	Argentina	36.26	36.63	36.32	24.40
Chad	2.80	2.86	2.88	1.19	Bolivia	14.53	13.94	15.93	5.45
Togo	2.54	2.58	2.74	1.09	Brazil	21.35	21.76	21.84	15.75
Tunisia	20.52	21.13	22.08	6.46	Dominican Republic	25.84	26.09	27.71	13.38
Tanzania	4.79	4.29	5.54	1.76	Ecuador	16.17	15.75	16.68	9.40
Uganda	4.05	3.54	5.36	1.35	Honduras	9.84	10.26	10.18	4.65
South Africa	18.87	18.13	19.50	9.17	Haiti	3.96	4.02	4.16	2.01
Zambia	3.61	3.91	3.69	1.66	Nicaragua	9.42	9.63	10.22	3.78
Zimbabwe	4.60	4.61	5.05	2.24	Panama	40.17	39.74	42.67	21.07
Bangladesh	7.60	7.79	8.30	2.80	Peru	17.92	17.50	18.65	10.73
Brunei Darussalam	25.56	22.48	27.92	12.90	Paraguay	19.90	20.18	20.44	9.22
Bhutan	14.33	14.59	14.70	4.61	El Salvador	15.14	14.95	15.95	8.06
China	13.35	13.36	13.98	8.31	Uruguay	32.65	32.47	34.22	28.57
Fiji	20.15	21.87	20.22	9.96	Aruba	50.04	49.64	54.64	42.42
Hong Kong SAR, China	93.56	92.69	99.87	74.26	Anguilla	39.16	38.88	40.94	37.35
Indonesia	13.92	14.59	14.77	5.47	Antigua and Barbuda	24.73	25.34	25.60	22.82
India	9.55	9.69	10.57	2.90	Bahamas, The	43.28	43.07	46.25	48.50
Cambodia	7.14	6.94	8.06	2.68	Belize	11.82	12.43	12.46	8.66

(Table A3 continued)

country name	GEKS Fisher	Walsh M	GK	EXR	country name	GEKS Fisher	Walsh M	GK	EXR
Cambodia	7.14	6.94	8.06	2.68	Belize	11.82	12.43	12.46	8.66
Lao PDR	9.16	9.50	9.73	3.55	Bermuda	91.65	86.65	95.88	136.37
Sri Lanka	17.91	17.42	21.18	6.62	Barbados	32.91	33.18	35.67	37.02
Maldives	15.32	14.95	16.90	9.92	Curaçao	38.39	38.02	39.40	32.37
Myanmar	5.83	5.86	6.41	1.70	Cayman Islands	78.11	76.72	81.88	108.62
Mongolia	12.88	12.17	14.61	4.76	Dominica	22.22	23.02	23.55	15.57
Malaysia	35.15	36.34	36.58	14.27	Grenada	32.96	33.22	34.63	22.62
Nepal	5.46	5.27	6.15	1.64	Guyana	12.34	13.13	12.52	6.89
Pakistan	10.35	10.02	11.38	3.26	Jamaica	20.66	21.16	21.57	12.29
Philippines	14.51	14.28	15.90	5.52	St. Kitts and Nevis	33.46	33.81	34.27	31.17
Singapore	69.87	69.63	76.74	53.91	St. Lucia	10.50	11.03	10.58	8.21
Thailand	19.78	20.76	20.86	7.91	Montserrat	30.50	28.80	35.45	25.03
Taiwan, China	58.46	58.47	61.89	32.06	Suriname	16.97	18.08	16.30	7.26
Vietnam	10.81	10.72	11.62	3.88	Sint Maarten	52.65	51.09	60.49	44.38
Armenia	21.90	21.29	24.62	7.59	Turks and Caicos Islands	22.28	21.07	23.00	27.00
Azerbaijan	21.12	20.88	22.60	6.13	Trinidad and Tobago	40.24	38.11	41.91	25.64
Belarus	24.06	23.40	25.39	7.58	St. Vincent and the Grenadines	19.98	20.08	22.24	13.80
Kazakhstan	30.75	27.69	33.67	11.90	British Virgin Islands	34.86	30.44	38.85	38.34
Kyrgyz Republic	9.47	8.87	10.57	2.70	United Arab Emirates	46.48	43.99	46.59	36.86
Moldova	16.83	16.68	17.73	5.75	Bahrain	37.70	35.87	39.92	21.75
Tajikistan	5.00	4.81	5.53	1.50	Iraq	13.58	13.33	14.29	6.15
Albania	20.05	20.29	21.08	9.04	Jordan	16.89	17.52	18.18	8.19
Australia	69.33	66.36	70.57	80.15	Kuwait	35.47	26.73	39.98	21.04
Austria	65.96	66.71	67.49	62.42	Morocco (WAS)	10.99	11.60	11.58	4.99
Belgium	56.91	56.52	58.02	54.77	Oman	28.30	26.66	28.99	15.85
Bulgaria	28.61	30.67	29.52	13.35	West Bank and Gaza	10.86	11.37	11.11	6.51
Bosnia and Herzegovina	23.18	23.81	24.04	11.11	Qatar	45.66	42.91	46.26	37.32
Canada	64.10	60.54	65.03	62.42	Saudi Arabia	45.96	38.65	51.24	20.98
Switzerland	78.11	77.20	81.40	105.27	Sudan (WAS)	7.95	8.41	8.34	2.16
Chile	31.56	31.32	31.51	23.69	Iran, Islamic Rep.	16.75	15.44	19.78	7.36

Appendix E

Robustness Checks

In this appendix, we report results of our robustness checks.

(1) Sensitivity to the proportional changes in quantities
First, we check the numerical impacts of the proportional changes in quantities. Our approximation of the reference vector by (11) makes the index number independent on the changes in the size of quantities. Walsh 1 is the ratio between the Multilateral Walsh index based on country level total quantities and the MW based on per capita quantities. Walsh 2 reports the ratio of the MW based on per capita quantity and the MW when the quantities in the US are multiplied by 100. Appendix Table 1 reports that both Walsh 1 and Walsh 2 are almost unity, implying that the changes in the proportional changes in the quantities hardly affect the index number. GK1 in the table reports the ratio between the GK based on the total quantities and the GK based on per capita quantities, which is very different from unity.

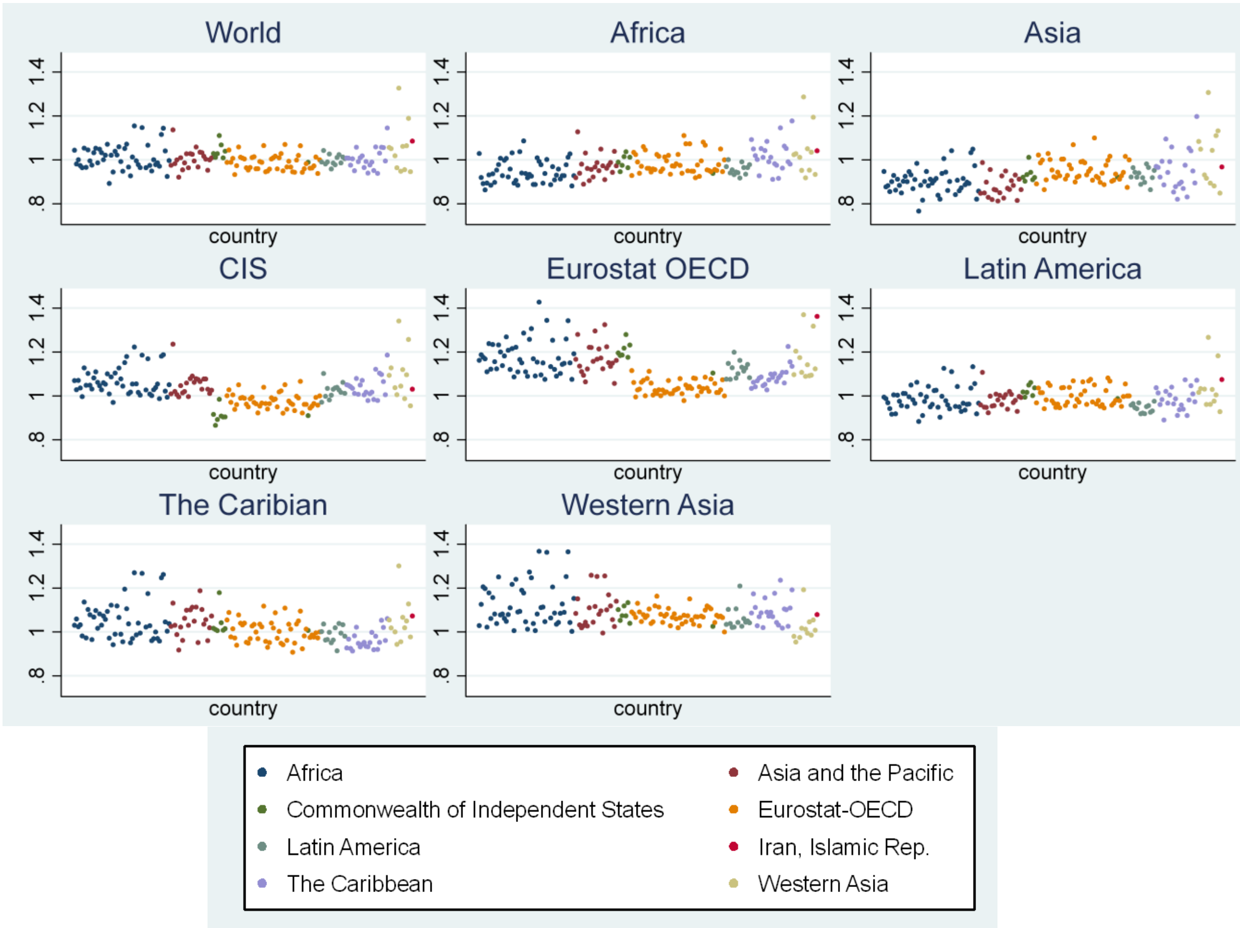
Appendix Table A4: Robustness Check 1

Stats	Walsh_M/GEKS_Fisher	GK/GEKS_Fisher	Walsh Check1	Walsh Check2	GK Check
Mean	1.006	0.945	0.997	1.000	1.013
p50	1.000	0.954	0.997	1.000	1.010
Min	0.892	0.756	0.992	0.999	0.961
Max	1.327	1.041	1.003	1.000	1.076
SD	0.054	0.049	0.002	0.000	0.016

Walsh Check1: Walsh_m based on country level quantity / Walsh_m based on per capita quantity
Walsh Check 2: Walsh_m when the size of the US is multiplied by 100/ Walsh_m
GK Check: Geary_Khamis on country level quantity / Geary_Khamis based on per capita quantity

(2) Sensitivity to the choice of the reference quantity vector
Figure 1 in the main text reports that the differences between the MW and the GEKS_Fisher are heterogeneous across regions. In Figure 1, we use the geometric mean of quantities all over the world. In Appendix Figure A1, we report the same figures as Figure 1, the distribution of the ratio between the Multilateral Walsh and the GEKS_Fisher, but with different reference vector. For example, in the subfigure entitled “Africa” use the mean function, (11) in the main text, in the African regions for the reference vector. As is clear from the figures, the patterns of the heterogeneity in the ratio between the Multilateral Walsh index and the GEKS_Fisher are very similar even if we use different reference vector.

Appendix Figure A1: Different Reference Vector



Note: The ratio of the Multilateral Walsh/GEKS_Fisher. The reference vector is the average defined in equation (11) in the main text. The title of each subfigure shows the region used in obtaining the reference vector.