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INTERNATIONAL INCOME TRANSFERS UNDER TECHNOLOGICAL UNCERTAINTY*

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Abstract

This paper examines the effects of international income transfers in the presence of technological uncertainty and shows the following results. First, a transfer paradox can occur only if the rates of return from assets are not equalized between the donor and the recipient. Second, the more risk-averse consumers are in both countries, the more likely a transfer paradox is to occur.

Key Words: Transfer; Uncertainty; Attitude toward risk
JEL classification: F10; D80

I. *Introduction*

With the increase in the volume of aid from developed countries to less developed countries, many international economists have attempted to explore the welfare effects of such international transfers. For example, Ohyama (1974), Brecher and Bhagwati (1982), and Kemp and Kojima (1985) examine a transfer with additional constraints imposed (i.e. tied aid). Bhagwati, Brecher and Hatta (1983) and Yano (1983) examine a bilateral transfer in a multilateral world. Bhagwati, Brecher and Hatta (1985) and Yano and Nugent (1999) examine a transfer with a tariff or a tax-cum-subsidy on production and consumption.[1] Many of these works show that the so-called *transfer paradox*, a phenomenon where the welfare of a transfer-receiving (transfer-paying) country paradoxically falls (rises), can occur in various situations.

On the other hand, we also have a vast array of literature which introduces elements of uncertainty into trade models and explores their impacts.[2] Some of these works show that the attitude toward risk of agents (which is one of the most fundamental elements in models with

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[1] See Brakman and van Marrewijk (1998) for supplementary literatures.

[2] Pomery (1984) provides a comprehensive survey.

uncertainty) has crucial impacts on the consequences of trade policies. For example, Newbery and Stiglitz (1984) and Shy (1988) show that opening up trade can be Pareto-inferior to the autarky according to the attitudes toward risk.

The purpose of this paper is to examine the role of the attitudes toward risk in another situation, that is, the situation where international income transfers are executed. Fries (1983) has already dealt with transfers under uncertainty. However he only constructed a numerical example of the transfer paradox. Since the model specified in his paper is an extreme case, we cannot judge from his analysis whether the transfer paradox will occur or not in situations other than the one he considers, and especially we cannot tell what role the attitudes toward risk play in determining the effects of transfers.

In contrast, using the contribution of Helpman and Razin (1978a), this paper constructs a model with uncertainty within which we can explicitly investigate the relation between attitudes toward risk and the effects of transfers. As a result, we get the following conclusions. First, the non-equalization of the rates of return from assets is the necessary condition for a transfer paradox to arise. Second, the more risk averse consumers are, the more likely the transfer paradox is to occur.

Since our model is a general equilibrium one with many goods, utility in each state depends not only on income, but on good prices in each state. This feature introduces important elements because it makes the expected utility of consumers depend on good prices, and therefore their portfolio choices also come to depend on good prices. As will be shown later, this relation between portfolio choice and good prices is crucial in determining the effects of transfers.

This paper is organized as follows. Section II presents the basic model and equilibrium conditions. Section III analyzes the effects of transfers and derives the necessary condition for the transfer paradox to arise. Section IV explores the relation between risk aversion and the effects of transfers. Section V states the cause of the paradox. Finally, Section VI provides the concluding remarks.

II. *The Model*

There are two sectors ($j=1, 2$) and each sector consists of many identical competitive firms. As is usual with the Helpman-Razin model, we view each sector as producing and selling a 'real-equity' competitively.[3] A real-equity is defined as a state-contingent claim to goods such that one unit of real-equity j is entitled to $\theta_j(s)$ units of good j in state s where $\theta_j(s)$ is a random variable that takes a different value according to states. Let Z_j denote the output of real-equity j and $x_j(s)$ denote the output of good j in state s, then the relation between Z_j and $x_j(s)$ is given by $x_j(s)=\theta_j(s)Z_j$. At period 0, consumers choose portfolios to maximize their expected utility. At period 1, after the resolution of uncertainty, they spend income from portfolios and transfers to purchase goods.

There are two countries (home and foreign) that consist of many identical consumers and we consider a representative consumer in each country. Both countries trade goods freely with

[3] For details of the Helpman-Razin model, see Helpman and Razin (1978a, chap. 5) or Helpman and Razin (1978b).

each other, however we assume that they cannot trade real-equities. As will be shown later, this assumption is pivotal in determining the effects of transfers. We employ asterisks to refer to foreign variables.

For simplicity, we make the following three assumptions. First, (A-1) for $\forall\ s, j=1, 2$

$$\theta_j(s)=\begin{cases}1 & \text{if } j=s\\ 0 & \text{if } j\neq s\end{cases} \qquad \theta_j^*(s)=\begin{cases}0 & \text{if } j=s\\ 1 & \text{if } j\neq s\end{cases} \tag{1}$$

That is, each real-equity is supposed to bring about return only in one state like an Arrow-Debreu security, and moreover real-equity j of the home (foreign) country is profitable only in state $j(j'\neq j)$. Second, (A-2) the production possibility frontier (PPF) of real-equities is a Ricardian type. Finally, (A-3) consumers in both countries exhibit constant relative risk aversion (CRRA) with respect to income variations. From (A-1), the home (foreign) country specializes its production to good s (s') in state s (s and s' denote the opposite numbers, i.e. if s is equal to 1, s' is equal to 2, etc.). From (A-2) and the assumption of no international trade in real-equities, the equilibrium domestic relative price of real-equities is equal to its constant marginal rate of transformation.

Throughout the paper, we use real variables in terms of export goods. For example, $y(s)$ ($y^*(s)$) is the home (foreign) country's real income in state s in terms of good s (s').[4] Let $p(s)$ ($p^*(s)$) be the relative price of import good of the home (foreign) country in state s (thus, $p(s)=1/p^*(s)$). Additionally, we assume that, at an initial equilibrium, transfers are not executed in both states. In deriving an equilibrium of the economy, we take the following procedures. First, we take good prices as given constant and derive an ex-ante equilibrium of real-equity markets in each country. Then, considering the ex-post good markets clearing condition, we determine equilibrium good prices.

Consider the home country at period 1. Suppose that state s happens at period 1 and that the values of prices and income $(p(s), y(s))$ are determined. The consumer's problem in the home country is given by

$$\max_{\{c_j(s)\}} u(c_1(s), c_2(s))$$
$$\text{s.t. } p(s)c_{s'}(s)+c_s(s)\leq y(s) \qquad s, s'=1, 2\ s\neq s',$$

where $u(\cdot)$ is the utility function and $c_j(s)$ is the consumption of good j in state s. The utility level which he/she can achieve in state s is given by the indirect utility function $v(p(s), y(s))$. Let $e(p(s), u(s))$ be the expenditure function. Using $v(p(s), y(s))$, we can express the expected utility function at period 0 as $\sum_{s=1}^{2}\pi(s)v(p(s), y(s))$ where $\pi(s)$ is the subjective probability for state s to occur. Note that this expected utility depends not only on incomes in both states, but on good prices in both states.

The home country's income in state s is given by

$$y(s)=p(s)x_{s'}(s)+x_s(s)+t(s) \qquad s, s'=1, 2\ s\neq s', \tag{2}$$

where $x_j(s)$ is the output of good j in state s and $t(s)$ is the receipt of a transfer in state s. (2) means that the income is the sum of portfolio income (which is equal to producer income) and transfer income. From (A-1), we have $x_s(s)=Z_s$ and $x_{s'}(s)=0$, thus (2) can be rewritten as

[4] From (A-1), the home (foreign) country exports good s (s') in state s.

$$y(s) = Z_s + t(s) \qquad s = 1,\ 2. \tag{3}$$

On the other hand, from (A-2), real-equity's PPF is given by $q_1Z_1 + q_2Z_2 = Z$ where Z is a given constant and q_1/q_2 is the constant MRT between real-equities. Combining this and (3), we get

$$\sum_{s=1,2} q_s y(s) = \sum_{s=1,2} q_s t(s) + Z. \tag{4}$$

This gives the income combinations feasible to the home country, given the volumes of transfers $t \equiv (t(1),\ t(2))$. We write this equation as $y(2) = F(y(1);\ t) \equiv -qy(1) + qt(1) + t(2) + Z/q_2$ where $q \equiv q_1/q_2$.

Maximizing the expected utility under the income constraint (4), we can get the equilibrium incomes given t and $p \equiv (p(1),\ p(2))$:[5]

$$V(p,\ t) \equiv \max_{\{y(s)\}} \{\sum \pi(s) v(p(s),\ y(s)) \parallel y(2) = F(y(1);\ t)\}. \tag{5}$$

Let $y_s(p,\ t)$ denote the solution to this problem. The utility level and compensated import demand of the home country in state s are respectively given by $u_s(p,\ t) = v(p(s),\ y_s(p,\ t))$ and $e_p(p(s),\ u_s(p,\ t)) \equiv \partial e(p(s),\ u_s(p,\ t))/\partial p(s)$.[6] Throughout the paper, subscripts attached to functions (except for subscripts s and s') denote the (partial) differentiation with respect to a particular variable.

As for the foreign country, the structures are basically the same as the home country except that it specializes in the opposite good. Thus the equation corresponding to (4) becomes[7]

$$\sum q_{s'}^* y^*(s) = \sum q_{s'}^* t^*(s) + Z^*.$$

Note that the numeraire in the foreign country is the opposite good to the one in the home country. $y_s^*(p^*,\ t^*)$ and $u_s^*(p^*,\ t^*)$ can be derived in the same way as before.

For later references, it is convenient to see how incomes respond to changes in the volume of transfers and good prices. For this purpose, we use the first-order condition for the expected utility maximization (5). Taking into account $F_y(y(1);\ t) = -q$, it is given by

$$\pi(1) v_y(p(1),\ y(1)) - \pi(2) v_y(p(2),\ F(y(1);\ t)) q = 0, \tag{6}$$

or $\pi(1)v_y(1)/\pi(2)v_y(2) = q$.[8] That is, the MRS between $(y(1),\ y(2))$ is equal to their MRT. This determines $y(1)$ as the function of $(p,\ t)$ and $y(2)$ is determined by the relation $y(2) = F(y(1);\ t)$.

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5 This procedure for deriving equilibrium quantities is valid because we consider a perfectly competitive economy with no externalities.

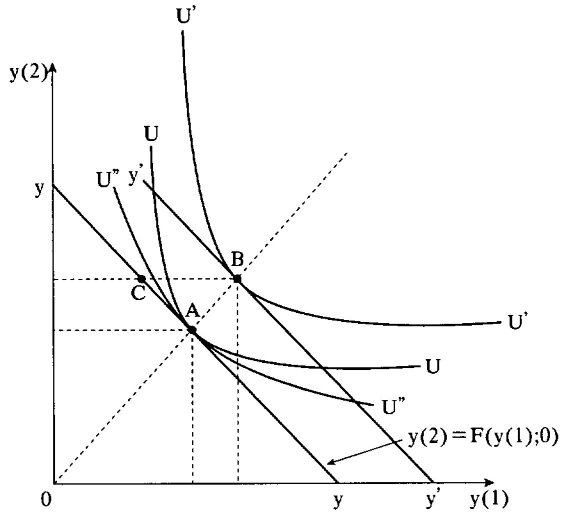
6 Recall that neither country produces its import goods, so demands for import goods are equal to import demands.

7 The equations corresponding to (2) and (3) are respectively

$$y^*(s) = p^*(s) x_s^*(s) + x_{s'}^*(s) + t^*(s) \qquad s,\ s' = 1,\ 2\ \ s \neq s',$$
$$y^*(s) = Z_{s'}^* + t^*(s) \qquad s,\ s' = 1,\ 2\ \ s \neq s'.$$

8 The arguments of $v_y(s)$ (i.e. $p(s)$ and $y(s)$) are omitted for notational simplification. Similarly, in the remainder of the paper, we often omit the arguments of functions.

FIG. 1



Before presenting the results, we define some notations. Let $m(s) \equiv e_p(s)$ be the home country's compensated import demand function in state s. Let $c(s) \equiv c(p(s), y(s))$ be the Marshallian demand function of import goods in state s, and $\sigma(s) \equiv c_y(s)y(s)/c(s)$ the income elasticity of demand of import goods. Let ρ denote the constant degree of relative risk aversion, i.e. $\rho \equiv -v_{yy}(s)y(s)/v_y(s)$, and finally $\omega(s) \equiv q_s Z_s/Z$(by definition, $\omega(1)+\omega(2)=1$). Foreign variables are defined similarly (asterisks are attached to them). We only present the results (see appendix 1 for the derivation).

$$\frac{\partial y_s}{\partial t(s)} = \omega(s), \qquad \frac{\partial y_s}{\partial t(s')} = \frac{q_{s'}}{q_s}\omega(s), \tag{7}$$

$$\frac{\partial y_s}{\partial p(s)} = m(s)\left(1 - \frac{\sigma(s)}{\rho}\right)\omega(s'), \qquad \frac{\partial y_s}{\partial p(s')} = -m(s')\left(1 - \frac{\sigma(s')}{\rho}\right)\frac{q_{s'}}{q_s}\omega(s). \tag{8}$$

We can get the results corresponding to the foreign country by (i) attaching asterisks and (ii) exchanging q_s for $q^*_{s'}$ and $\omega(s)$ for $\omega^*(s')$.[9]

There are two things worth pointing out. First, (7) does not directly depend on the degree of risk aversion ρ. This means that the effects of transfers on the equilibrium incomes do not depend on the degree of risk aversion as long as good prices are given constant. The reason for this can be explained as follows. Consider an initial equilibrium with $t=0$. This is depicted in

[9] The results are as follows.

$$\frac{\partial y^*_s}{\partial t^*(s)} = \omega^*(s'), \qquad \frac{\partial y^*_s}{\partial t^*(s')} = \frac{q^*_s}{q^*_{s'}}\omega^*(s'),$$

$$\frac{\partial y^*_s}{\partial p^*(s)} = m^*(s)\left(1 - \frac{\sigma^*(s)}{\rho^*}\right)\omega^*(s), \qquad \frac{\partial y^*_s}{\partial p^*(s')} = -m^*(s')\left(1 - \frac{\sigma^*(s')}{\rho^*}\right)\frac{q^*_s}{q^*_{s'}}\omega^*(s').$$

Note that the roles played by q_s and q'_s or $\omega(s)$ and $\omega(s')$ are changed in the foreign country. It is because the real-equity which is profitable in each state is opposite in each country by the assumption (A-1).

Figure 1. Since $t=0$, the income constraint is represented by $y(2)=F(y(1);0)$ (yy line). The expected utility function is represented by an indifference curve UU. Since the consumer is risk averse, the expected utility indifference curve is convex to the origin. In addition, since the consumer has CRRA, the expected utility function is homothetic in $(y(1), y(2))$. Given (p, t), the equilibrium incomes are represented by a point A where MRS$=q$. Since $t=0$, the portfolio incomes are equal to the total incomes.

Now suppose that the transfer is executed in state 1, i.e., $t(1)$ rises. In this case, $F(\cdot)$ shifts rightward to $y'y'$, and since the expected utility indifference curve is homothetic, the new equilibrium income combination shifts to a point B. At the same time, the portfolio income shifts to a point C (the length BC represents $t(1)$). Note that the changes in income, whether total or portfolio, are independent of the degree of risk aversion (ρ) as long as ρ is constant (CRRA) giving us a homothetic indifference curve.[10] For this reason, (7) is independent of ρ. This means that the change in good supply against the change in transfer is also independent of ρ because good supply has the one-to-one relation with portfolio incomes.

Second, in contrast to (7), (8) depends on the degree of risk aversion. Especially, (8) indicates that as the degree of relative risk aversion approaches zero, the response of income to the change in good prices become large.[11] The change in good prices makes the indifference curve rotate because the expected utility depends on good prices. As consumers are more risk-neutral, the expected utility indifference curve becomes flatter (like $U''U''$ in Figure 1), therefore the change in income by rotation of indifference curve which is caused by the good prices changes becomes large.

Now consider an equilibrium in this economy. Equilibrium conditions are given by

$$p(s)e_p(p(s), u_s(p, t))=e^*_p(p^*(s), u^*_s(p^*, t^*))+t(s) \qquad s=1, 2, \tag{9}$$

$$p^*(s)=1/p(s) \qquad s=1, 2, \tag{10}$$

$$t^*(s)=-t(s)/p(s) \qquad s=1, 2, \tag{11}$$

(9) is the equilibrium condition of the balance of payments.[12] By definition, $p^*(s)$ is the reciprocal number of $p(s)$. Since the home country's receipts of transfers are equal to the foreign country's payments of transfers, (11) must hold. Given the volume of transfers t, equilibrium prices p can be determined from (9) to (11) (however, recall that transfers are not executed initially).

III. *The Effects of Transfers*

In this section, we examine the effects of transfers on the terms of trade and welfare of both countries. Before doing this, we first examine the cause of the change in welfare. In the remainder of the paper, we only consider the case of a transfer in state 1 and assume that the transfer is executed from the home country to the foreign country, but the same kind of the

[10] In the case of constant absolute risk aversion, the same argument holds.

[11] If ρ rises beyond income elasticity, the change in income becomes larger, too. But this change is limited by some upper bound. On the other hand, as ρ approaches zero, the size of income change is not limited.

[12] At first glance, it seems that (9) only means the equilibrium of ex-post goods markets. However, since $u_s(p, t)$ and $u^*_s(p^*, t^*)$ are constructed so that ex-ante equilibrium is realized, it follows that (9) means the whole equilibrium.

arguments can be applied to the other cases.

Using the envelope theorem and the first order condition of the expected utility maximization, we have $\partial V/\partial p(s)=\pi(s)v_p(s)$ and $\partial V/\partial t(1)=\pi(1)v_y(1)$. Thus we can decompose the change in the expected utility of the home country as follows

$$dV=-\sum_{s=1}^{2}\pi(s)v_y(s)m(s)dp(s)+\pi(1)v_y(1)dt(1). \tag{12}$$

This shows that the change in the home country's expected utility is the sum of the direct effect of a transfer and the effects of changes in the terms of trade. Since the direct effect of a transfer always raises welfare, it follows that if a transfer paradox occurs in the model, it is because the deterioration of the terms of trade outweighs the positive direct effects of a transfer.

Furthermore, from the first-order condition of the expected utility maximization, (12) can be written as

$$\frac{dV}{\pi(1)v_y(1)}=-m(1)dp(1)-\frac{m(2)}{q}dp(2)+dt(1). \tag{13}$$

Similarly, with regard to the foreign country,

$$\frac{dV^*}{\pi^*(1)v_y^*(1)}=-m^*(1)dp^*(1)-\frac{m^*(2)}{q^*}dp^*(2)+dt^*(1), \tag{14}$$

where $q^*\equiv q_2^*/q_1^*$.[13] From (9), (10) and (11),

$$m^*(s)=p(s)m(s), \qquad dp^*(s)=-dp(s)/p(s)^2, \qquad dt^*(1)=-dt(1)/p(1), \tag{15}$$

hold at an initial equilibrium. Combining (13), (14) and (15), we get the following relation

$$\frac{dV}{\pi(1)v_y(1)}+\frac{dV^*}{p^*(1)\pi^*(1)v_y^*(1)}=\frac{1}{qq^*}\left(\frac{p(1)}{p(2)}q-q^*\right)m(2)dp(2).$$

This gives the relation of the changes in welfare caused by the transfer, in both countries. Since the transfer we consider here is a non-distortionary lump-sum type, if there is no distortion in the economy, the transfer only makes the economy move from one Pareto efficient allocation to another. Therefore sign $dV=-$sign dV^*must hold. But, here if $p(1)q/p(2)\neq q^*$, the possibility that sign $dV=$sign dV^* arises. $p(1)q/p(2)\neq q^*$ means that the rates of return from real-equities are not equalized between the two countries.[14] This means that if the rates of return from real-equities are not equalized, the Pareto-efficiency is not satisfied ex-ante. Bhagwati et al. (1983, 1985) pointed out that there usually exist some kinds of distortions in the economy when the transfer paradox happens. From this, we can conjecture that the

[13] Note that while we define $q\equiv q_1/q_2$, here we define $q^*\equiv q_2^*/q_1^*$ conversely. See footnote 9.

[14] Suppose that you have one unit of income. If you spend it to buy real-equity 1 in the home country, you will get $p_1(1)/q_1$ of income in state 1 ($p_i(s)$ is the price of good i in state s in the home country) and if you spend it to buy real-equity 2 in the foreign country, you get $p_2^*(1)/q_2^*$ of income in state 1 (both real-equities bring no return in state 2). If both real-equities have the same rate of return in state 1, $p(1)=q_2^*/q_1$. Similarly, if home's real-equity 2 and foreign's real-equity 1 have the same rate of return in state 2, $p(2)=q_1^*/q_2$. So, if real-equities in both countries have the same rate of return, $p(1)q/p(2)=q^*$.

non-equalization of rates of return has something to do with the effects of transfers. Indeed, as will be shown later, this non-equalization of rates of return (abbreviated as non-ERR) is the necessary condition for the transfer paradox to arise.[15]

Using (9), (10) and (11), we can execute comparative statics. However, the model presented above is too complicated to analyze. Moreover, our main theme is to explore the relation between the attitudes toward risk (ρ, ρ^*) and the effects of transfers. Therefore, to exclude the other irrelevant elements in the model, we specify utility functions and consider a symmetric equilibrium as follows.

We assume that the home country has a symmetric Cobb-Douglas-CRRA type utility function like

$$u(c_1(s), c_2(s)) \equiv \begin{cases} \dfrac{1}{1-\rho}\left[\left(c_1(s)^{1/2}c_2(s)^{1/2}\right)^{1-\rho}-1\right] & \text{if } \rho \neq 1, \\ \ln(c_1(s)^{1/2}c_2(s)^{1/2}) & \text{if } \rho = 1, \end{cases} \tag{16}$$

Then ρ becomes the constant degree of relative risk aversion. Suppose that the foreign country has the same type (asterisks are attached to them).[16] To consider a symmetric equilibrium, we assume that (B-1) $q=\pi(1)/\pi(2)$, $q^*=\pi^*(1)/\pi^*(2)$. (B-2) $Z=Z^*$, $q_1+q_2=q_1^*+q_2^*$. (B-1) means that the MRT of real-equities is equal to the ratio of the subjective probabilities.[17] From this, if $p(s)=1$, each country chooses the same level of incomes in both states (i.e., $y(1)=y(2)$ and $y^*(1)=y^*(2)$). (B-2) means that both countries have the same scales. Under (B-1) and (B-2), the following relations hold at the initial equilibrium (see appendix 2):

$$p(s)=1, \qquad y(s)=y^*(s'), \qquad m(s)=m^*(s'), \qquad \forall\ s, s'=1, 2. \tag{17}$$

For convenience, we define notations as follows. First, $\tau \equiv q^*/q$ so that $\tau \neq 1$ denotes the existence of non-ERR.[18, 19] Let $\varepsilon \equiv \tau(1+q)/(1+\tau q)$. Since ε is an increasing function of τ, and $\varepsilon=1$ when $\tau=1$, $\varepsilon \neq 1$ denotes non-ERR, as well. From the definitions of τ and ε, the more q and q^* are separated from each other, the more ε is separated from one. It follows that the

[15] Note that it is meaningless to compare q and q^* directly (that is, to compare the relative prices of real-equities) because the real-equities in both countries are not the same assets by the assumption of $\theta_i(s)$ and $\theta_i^*(s)$. Only if $p(1)=p(2)$ holds, is the comparison of the relative prices equivalent to that of the rates of return.

[16] With this utility function, the expenditure function and the Marshallian demand function of import good of the home country are respectively given by

$$e(p(s), u(s)) = 2p(s)^{1/2}[u(s)(1-\rho)+1]^{\frac{1}{1-\rho}},$$
$$c(p(s), y(s)) = y(s)/2p(s).$$

[17] Under (B-1), if $q \neq q^*$ as we will consider later, $\pi(1)/\pi(2) \neq \pi^*(1)/\pi^*(2)$. That is, both countries have different subjective probabilities.

[18] Originally, $p(1)q/p(2) \neq q^*$ is non-ERR. However since $p(s)=1$ holds at a symmetric equilibrium, $q \neq q^*$ means non-ERR.

[19] Let $r(s)$ ($r^*(s)$) be the rate of return from home (foreign) country's real-equity which brings return in state s. Then, $r(1)=p_1(1)/q_1$, $r(2)=p_2(2)/q_2$, $r^*(1)=p_2(1)/q_2^*$, $r^*(2)=p_1(2)/q_1^*$ (see footnote 14). Using this notations, $p(1)=p_2(1)/p_1(1)=1$ and $p(2)=p_1(2)/p_2(2)=1$, we can express τ in terms of the rates of return.

$$\tau = \frac{r(1)}{r^*(1)}\frac{r^*(2)}{r(2)}$$

From this, it is shown that the rise in $r(1)$ and $r^*(2)$ or the fall in $r^*(1)$ and $r(2)$ lead to the rise in τ.

more ε is separated from one, the larger the distortion is.

Below, we only present the results of comparative statics. With regard to derivations, see appendix 3. First, the effects of a transfer on the terms of trade are

$$\frac{\partial p(1)}{\partial t(1)}=\frac{m(2)\omega(2)}{2\Delta}\left[-2-2\beta-\left(\frac{1}{\rho}-\frac{1}{\rho^*}\right)\beta\right], \tag{18}$$

$$\frac{\partial p(2)}{\partial t(1)}=\frac{m(1)\omega(2)}{2\Delta}\left[2q-2\beta-\left(\frac{1}{\rho}-\frac{1}{\rho^*}\right)\beta\right], \tag{19}$$

where $\Delta\equiv\dfrac{m(1)m(2)}{2}\left(\dfrac{1}{\rho}+\dfrac{1}{\rho^*}\right)>0,$

$\beta\equiv q(1-\varepsilon)/2.$

Δ must be positive for the stability condition to be satisfied. Indeed the condition is satisfied in this case.[20]

Next, we see the effects of a transfer on welfare. From (13), (14), (18) and (19), the changes in the expected utility of each country are given by

$$\frac{\partial V}{\partial t(1)}=\frac{\pi(1)v_y(1)\Omega}{\Delta}Q, \qquad \frac{\partial V^*}{\partial t(1)}=\frac{\pi^*(1)v_y^*(1)\Omega}{\varepsilon\Delta}Q^*. \tag{20}$$

where

$$\Omega\equiv m(1)m(2)/4>0,$$
$$Q\equiv 2(1-\varepsilon)+(3-\varepsilon)/\rho+(1+\varepsilon)/\rho^*,$$
$$Q^*\equiv 2(1-\varepsilon)-(1+\varepsilon)/\rho+(1-3\varepsilon)/\rho^*.$$

This yields the following proposition.

Proposition 1 (*i*) *A transfer paradox occurs in the home country* (*the foreign country*) *if and only if* $Q<0$ ($Q^*>0$). (*ii*) *If the rates of return from real-equities are equalized between the donor and the recipient, the transfer paradox cannot occur.*

Proof. (*i*) *It is clear from* (20). (*ii*) *If* $\varepsilon=1$, $Q=2(\rho^{-1}+\rho^{*-1})>0$ *and* $Q^*=-2(\rho^{-1}+\rho^{*-1})<0$. *Q.E.D.*

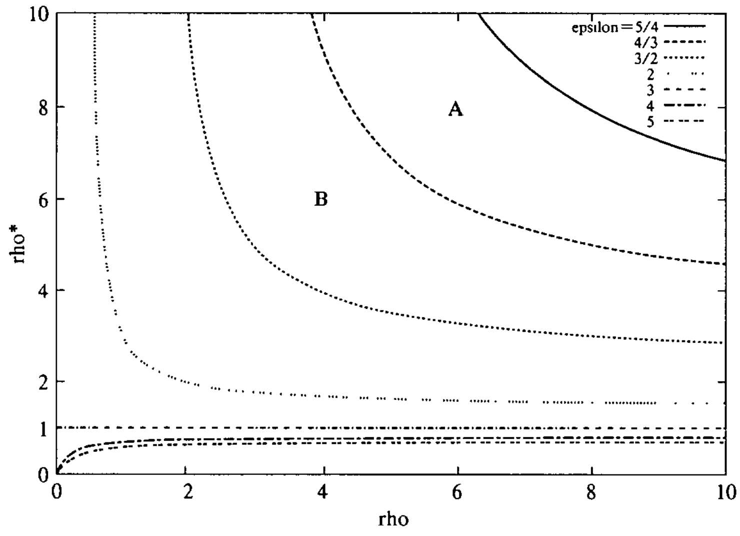
Moreover, from the definition of Q and Q^*, if $\varepsilon>1$, $Q^*<0$, and if $\varepsilon<1$, $Q>0$. So it follows that the simultaneous transfer paradox in both countries (i.e. the strong transfer paradox) cannot occur in this model.

IV. *The Relations between the Degree of Risk Aversion and the Effects of Transfers*

In the previous section, we derived the necessary and sufficient condition for the transfer paradox and showed that the transfer paradox is impossible if ERR holds. Here, we examine in detail the case where ERR does not hold (i.e. the case of $\varepsilon\neq 1$) and show that in what

[20] The stability mentioned here means the stability in the whole equilibrium, and not the one in the ex-post goods markets. But it can easily be shown that the stability in the ex-post goods markets are also satisfied at the symmetric equilibrium.

FIG. 2



situations the transfer paradox will occur (i.e. $Q<0$, or $Q^*>0$).

First, consider the case of $\varepsilon>1$. Since $Q^*<0$ always holds in this case, we only see the sign of Q. The combinations of (ρ, ρ^*) which make Q equal zero are given by

$$\rho^* = \frac{1+\varepsilon}{2(\varepsilon-1)+(\varepsilon-3)/p}. \tag{21}$$

The loci of (ρ, ρ^*) which satisfy (21) for various values of $\varepsilon(=5/4, 4/3, 3/2, ...)$ are drawn in Figure 2. These loci have two properties. (i) the transfer paradox occurs (i.e. $Q<0$) at combinations of (ρ, ρ^*) above each curve. For example, when $\varepsilon=4/3$ and (21) is represented by the second curve from the top of the figure, the transfer paradox occurs at a point like A, and does not occur at a point like B. (ii) the curve corresponding to the higher value of ε has the lower position in the figure. The high value of ε means that the degree of non-ERR (distortion) is large. Thus, the larger the degree of distortion, the lower position the curve has.

From Figure 2, it follows immediately that if the degree of the distortion is sufficiently large, the transfer paradox is possible for reasonable values of (ρ, ρ^*). For example, if $\varepsilon=3$, the paradox occurs for $\rho=\rho^*=2$. In addition, taking into account property (i) of the curves, we get the following relation. For any ε, the possibility that Q becomes negative will rise as the (ρ, ρ^*) become larger. This means that the larger are the values of (ρ, ρ^*), the more likely a transfer immiserizes the home country (the recipient). As to the case of $\varepsilon<1$, the same arguments hold. In this case, however, it is in the foreign country that the transfer paradox occurs. Then, we get the following proposition.

Proposition 2 *the more risk-averse consumers are in both countries, the more likely the transfer paradox is to occur.*

Furthermore, the property (ii) of the curve (21) means that as ε becomes large, the curve shifts downward, and therefore the possibility that (ρ, ρ^*) lie above the curve rises. It follows that the larger the distortion, the more likely it is that the transfer immiserizes the home country.

V. *The Cause of the Paradox*

In this section, we interpret the proposition 2. From (12), we already know that the cause of the transfer paradox in the home country is that the deterioration of the terms of trade outweighs the positive direct effect of the transfer. Therefore, the reason that the large values of (ρ, ρ^*) cause the paradox is that the large values of (ρ, ρ^*) make the changes in the terms of trade large. Then the next question is why the large values of (ρ, ρ^*) make the changes in the terms of trade large. This is because when (ρ, ρ^*) are high, the terms of trade must adjust by a large amount to get rid of excess demand for goods caused by the transfer. We can verify this by the equation (23) of appendix 3. A_{ss} of (23) represents how excess demand for the import good responds to a rise in its price. (23) shows that $|A_{ss}|$ is a decreasing function of (ρ, ρ^*). This means that, as (ρ, ρ^*) become large, the change in excess demand against the change in its price becomes small. This relation between risk aversion and excess demand change is based on the relation we saw in the section 2 that as consumers are more risk-averse, adjustment of portfolio choice to the change in good prices become small (see (8)).

Thinking the arguments above conversely, it follows, as mentioned above, that as consumers become more risk-averse, the terms of trade must change by a large amount to clear the goods markets, and therefore, the possibility of the paradox increases.

Since we use the specific model and consider a symmetric equilibrium, the above arguments do not seem to hold in general. However it may not be necessarily so. The reason is as follows. As shown by Figure 1, the degree of risk aversion has a close relationship with the curvature of the expected utility indifference curve. In other words, the degree of risk aversion corresponds to the degree of substitution between incomes in each state (especially, in the case of CRRA, the reciprocal of constant degree of risk aversion is equivalent to the elasticity of substitution) and the high degree of risk aversion implies the low elasticity of substitution. On the other hand, it is often observed that when elasticity is low, exogenous shocks to the economy cause the large change in price. The simplest example of this is a partial equilibrium model with demand and supply curves. It can be easily understood that in such a model, when demand and supply curves have low elasticities, shocks to both curves are likely to bring about large price adjustment.

Combining this relationship between elasticity and price change with the similarity between elasticity and risk aversion, we may conclude that the above arguments on the relationship between the degree of risk aversion and price change are likely to hold in general.

VI. *Concluding Remarks*

In this paper, we investigate the effects of international income transfers under uncertainty with no international trade in assets and show the following results. First, non-equalization of the rates of return from real-equities (assets) is the necessary condition for the transfer paradox to arise (proposition 1). Second, if ERR does not hold, the transfer paradox is possible. In addition, we attained to the following relation (proposition 2): the more risk-averse consumers are in both countries, the more likely the transfer paradox is to occur.

Finally, two comments are needed. First, in this model, a transfer changes the goods supply under the constant terms of trade. Fries (1983) thought that when (ρ, ρ^*) are high, this change in goods supply under the constant terms of trade becomes large, and hence, the change in the terms of trade also becomes large, and this causes the transfer paradox. However, it is easy to show that this idea is wrong because, as we have already seen by the figure 1, under the constant terms of trade, changes in the goods supply do not depend on (ρ, ρ^*) at least in the case of CRRA. Furthermore, there is nothing to indicate that the change in the goods supply under the constant terms of trade has any systematic relations with the degree of risk aversion. The true reason for the transfer paradox is, as shown in the previous section, that as consumers are more risk-averse, the change in excess demand of goods against *the changes in good prices* becomes larger.

Second, since the existence of non-ERR is based on the assumption of no international trade in assets, we can infer that opening the trade in assets will sweep away the possibility of the paradox. But we cannot analyze the case with trade in assets explicitly.

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APPENDIX 1

In this appendix, we only derive $\partial y_1/\partial t(1)$ and $\partial y_1/\partial p(1)$ as examples. Following the same procedures, the others results can be derived. Let Φ denote the left hand side of (6). $\partial y_1/\partial t(1)$ and $\partial y_1/\partial p(1)$ are given by

$$\partial y_1/\partial t(1) = -\Phi_{t(1)}/\Phi_{y(1)}, \qquad \partial y_1/\partial p(1) = -\Phi_{p(1)}/\Phi_{y(1)}.$$

$\Phi_{t(1)}$, $\Phi_{p(1)}$, and $\Phi_{y(1)}$ are respectively given by

$$\Phi_{t(1)} = -\pi(2)v_{yy}(2)q^2, \qquad \Phi_{p(1)} = \pi(1)v_{yp}(1), \qquad \Phi_{y(1)} = \pi(1)v_{yy}(1) + \pi(2)v_{yy}(2)q^2.$$

Since $c(s) = -v_p(s)/v_y(s)$ and $v_{yp}(s) = v_{py}(s)$, $\Phi_{p(1)}$ is rewritten as

$$\Phi_{p(1)} = \pi(1)\frac{v_y(s)c(s)}{y(s)}[\rho - \sigma(s)].$$

Moreover, using $\rho = -v_{yy}(s)y(s)/v_y(s)$ and $\pi(1)v_y(1)/\pi(2)v_y(2) = q$, we can rewrite $\Phi_{y(1)}$ as

$$\begin{aligned}\Phi_{y(1)} &= \pi(1)v_{yy}(1) + \pi(2)v_{yy}(2)q^2 = \pi(2)v_{yy}(2)q^2\left(\frac{\pi(1)v_{yy}(1)}{\pi(2)v_{yy}(2)q^2} + 1\right) \\ &= \pi(2)v_{yy}(2)q^2\left(\frac{\pi(1)v_y(1)y(2)}{\pi(2)v_y(2)y(1)q^2} + 1\right) = \pi(2)v_{yy}(2)q^2\left(\frac{y(2)}{qy(1)} + 1\right) \\ &= \frac{\pi(2)v_{yy}(2)q}{y(1)}(qy(1) + y(2)).\end{aligned}$$

Thus

$$\frac{\partial y_1}{\partial t(1)} = \frac{qy(1)}{qy(1) + y(2)} = \frac{q_1 y(1)}{q_1 y(1) + q_2 y(2)} = \omega(1).$$

Similarly,

$$\begin{aligned}\frac{\partial y_1}{\partial p(1)} &= -\frac{c(1)\pi(1)v_y(1)}{\pi(2)v_{yy}(2)q}\frac{(\rho - \sigma(1))}{(qy(1) + y(2))} = \frac{c(1)(\rho - \sigma(1))y(2)}{\rho(qy(1) + y(2))} \\ &= m(1)\left(1 - \frac{\sigma(1)}{\rho}\right)\omega(2).\end{aligned}$$

APPENDIX 2

Here, we show that under (B-1) and (B-2), (17) holds at an initial equilibrium. First, by (B-2), $y(2)=F(y(1);0)$ and $y^*(2)=F^*(y^*(1);0)$ cross on the 45° line. Suppose that $p(s)$ is equal to one for $\forall s$, then the marginal rate of substitution between $(y(1), y(2))$ is equal to the ratio of subjective probabilities on the 45° line, that is, $\text{MRS}_{12}=\pi(1)/\pi(2)$ and $\text{MRS}^*_{12}=\pi^*(1)/\pi^*(2)$. By (B-1), $\text{MRS}_{12}=q$ and $\text{MRS}^*_{12}=q^*$ on the 45° line. Since these are precisely the first-order conditions for the expected utility maximization, (B-1) and (B-2) means that, if $p(s)=1$, both countries choose the same level of income in both states. Let y denote this same income level.

Consider the demand and supply of good s in state s under $p(s)=1$ and y. Since each country demands $y/2$ units of good s, total demand is y. On the other hand, good s is supplied only by the home country: $x_s(s)=Z_s=y$. Since demand is equal to supply, $p(s)=1$ and y are exactly the equilibrium price and income. Thus, under (B-1) and (B-2), (17) holds.

APPENDIX 3

In this appendix, we derive (18) and (19).

Step 1.

Totally differentiating the equilibrium conditions (9)−(11) and considering $e_p(s)=m(s)$, $e_{pp}(s)=m_p(s)$, $e_{pu}(s)=c_y(s)e_u(s)$, and $\partial p^*(s)/\partial p(s)=-1/p(s)^2$, we get

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} dp(1) \\ dp(2) \end{pmatrix}=\begin{pmatrix} T_1 \\ T_2 \end{pmatrix}dt(1), \tag{22}$$

where for $\forall s, s'=1, 2\ s\neq s'$

$$\begin{aligned}
A_{ss}=&-m(s)\Bigg(\phi(s)-\psi(s)e_u(s)\frac{1}{m(s)}\frac{\partial u_s}{\partial p(s)} \\
&\qquad+\phi^*(s)-\psi^*(s)e^*_u(s)\frac{1}{m^*(s)}\frac{\partial u^*_s}{\partial p^*(s)}-1\Bigg), \\
A_{ss'}=&\,\psi(s)e_u(s)\frac{\partial u_s}{\partial p(s')}+\psi^*(s)e^*_u(s)\frac{\partial u^*_s}{\partial p^*(s')}\frac{p(s)}{p(s')^2}, \\
T_1=&-\psi(1)e_u(1)\frac{\partial u_1}{\partial t(1)}-\psi^*(1)e^*_u(1)\frac{\partial u^*_1}{\partial t^*(1)}+1, \\
T_2=&-\psi(2)e_u(2)\frac{\partial u_2}{\partial t(1)}-\psi^*(2)e^*_u(2)\frac{\partial u^*_2}{\partial t^*(1)}\frac{p(2)}{p(1)},
\end{aligned}$$

$$\phi(s)\equiv -p(s)m_p(s)/m(s), \qquad \psi(s)\equiv p(s)c_y(s).$$

Step 2.

Next, we derive $\partial u_s/\partial t(s')$ and $\partial u_s/\partial p(s')$ in (22). For this, we use (7), (8) and the relation $u_s(p,t)=v(p(s), y_s(p,t))$. As an example, we derive $\partial u_1/\partial t(1)$ and $\partial u_s/\partial p(s)$.

$$\partial u_1/\partial t(1)=v_y(1)\,\partial y_1/\partial t(1)=\omega(1)/e_u(1),$$

and

$$\frac{\partial u_s}{\partial p(s)}=v_p(s)+v_y(s)\frac{\partial y_s}{\partial p(s)}=v_p(s)+v_y(s)m(s)(1-\sigma(s)\rho^{-1})\omega(s')$$
$$=-v_y(s)m(s)[1-\omega(s')+\sigma(s')\rho^{-1}\omega(s')]=-\frac{m(s)}{e_u(s)}\left(\omega(s)+\frac{\sigma(s)}{\rho}\omega(s')\right).$$

The other results are obtained similarly

$$\frac{\partial u_2}{\partial t(1)}=\frac{1}{e_u(2)}\frac{q_1}{q_2}\omega(2),\qquad \frac{\partial u_s}{\partial p(s')}=-\frac{m(s')}{e_u(s)}\left(1-\frac{\sigma(s')}{\rho}\right)\frac{q_{s'}}{q_s}\omega(s).$$

Attaching asterisks, exchanging q_s for $q_{s'}^*$ and $\omega(s)$ for $\omega^*(s')$, we get the results corresponding to the foreign country (see also footnote 9).[21]

Step 3.

Lastly, we substitute the results in Step 2 into (22), and evaluate them at a symmetric equilibrium. By (16), we have $\phi(s)=\psi(s)=\phi^*(s)=\psi^*(s)=1/2$, and $\sigma(s)=\sigma^*(s)=1$ (see footnote 16). Moreover, at the symmetric equilibrium, $p(s)=p^*(s)=1$ (see (17)). Therefore, (22) can be written as

$$A_{ss}=-\frac{m(s)}{2}\left[2-\left(1-\frac{1}{\rho}\right)\omega(s')-\left(1-\frac{1}{\rho^*}\right)\omega^*(s)\right], \tag{23}$$

$$A_{ss'}=-\frac{m(s')}{2}\left[\left(1-\frac{1}{\rho}\right)\frac{q_{s'}}{q_s}\omega(s')+\left(1-\frac{1}{\rho^*}\right)\frac{q_s^*}{q_{s'}^*}\omega^*(s)\right], \tag{24}$$

$$T_1=(\omega(2)+\omega^*(1))/2,\qquad T_2=-(q\omega(2)+q^*\omega^*(1))/2. \tag{25}$$

Since $y(1)=y(2)$ and $y^*(1)=y^*(2)$ at a symmetric equilibrium, we have $\omega(1)/q_1=\omega(2)/q_2$ and $\omega^*(1)/q_1^*=\omega^*(2)/q_2^*$. Moreover, considering $\omega(1)+\omega(2)=1$ and $\omega^*(1)+\omega^*(2)=1$, we get $\omega(1)=q/(1+q)$, $\omega(2)=1/(1+q)$, $\omega^*(1)=1/(1+q^*)$, and $\omega^*(2)=q^*/(1+q^*)$. From this, we can rewrite all omegas in terms of $\omega(2)(=1/(1+q))$ as follows, $\omega(1)=q\omega(2)$, $\omega^*(1)=\varepsilon\omega(2)/\tau$, $\omega^*(2)=\varepsilon q\omega(2)$.

Using the above results and the next relation, we can get (18) and (19).

$$\partial p(s)/\partial t(1)=\Delta^{-1}(A_{s's'}T_s-A_{ss'}T_{s'})\quad s, s'=1, 2\ s\neq s',$$

where $\Delta\equiv A_{11}A_{22}-A_{12}A_{21}$.

[21] The results are

$$\frac{\partial u_1^*}{\partial t^*(1)}=\frac{1}{e_u^*(1)}\omega^*(2),\qquad \frac{\partial u_2^*}{\partial t^*(1)}=\frac{1}{e_u^*(2)}\frac{q_2^*}{q_1^*}\omega^*(1),$$
$$\frac{\partial u_s^*}{\partial p^*(s)}=-\frac{m^*(s)}{e_u^*(s)}\left(\omega^*(s')+\frac{\sigma^*(s)}{\rho^*}\omega^*(s)\right),\qquad \frac{\partial u_s^*}{\partial p^*(s')}=-\frac{m^*(s')}{e_u^*(s)}\left(1-\frac{\sigma^*(s')}{\rho^*}\right)\frac{q_s^*}{q_{s'}^*}\omega^*(s').$$