Discussion Paper Series A No.739

Correspondence between Exploitation and Profits in General Neoclassical Production Economies

Naoki Yoshihara
(University of Massachusetts Amherst)

December 2022

Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

Correspondence between Exploitation and Profits in General Neoclassical Production Economies

Naoki Yoshihara*

Department of Economics, University of Massachusetts at Amherst; The Institute of Economic Research, Hitotsubashi University; and School of Management, Kochi University of Technology

December 4, 2022

Abstract

This paper axiomatically characterizes the appropriate definitions of ***exploitation as unequal exchange of labour*** (**UE exploitation**) that satisfy the axiom called **Profit-Exploitation Correspondence Principle** (**PECP**) in every equilibrium under a general model of capitalist economies with neoclassical production functions. **PECP** requires that, whatever the definition of exploitation is, it must follow that for any capitalist economy and any market equilibrium, total profits are positive if and only if any propertyless employee is exploited in terms of this definition, assuming the definition of exploitation is deemed admissible. The main result is that every admissible definition of **UE exploitation** can verify **PECP** under the neoclassical production economies whenever it satisfies an axiom called **Labor Value Theory of Exploitation** (**LVE**).

JEL classification: D63; D51.

Keywords: Domain Axiom of UE Exploitation; Profit-Exploitation Correspondence Principle; Neoclassical Production Economies.

1 Introduction

The issue of exploitative and dominance relations has been central in the capitalist society. It was Karl Marx (1867, 1894) who characterized the conflicting distributional relationship in the capitalist economy as exploitative. He argued

*Department of Economics, University of Massachusetts Amherst, Crotty Hall, 412 North Pleasant Street, Amherst, MA 01002, U.S.; The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-8603, Japan; and School of Management, Kochi University of Technology, 2-22, Eikokuji-Chou, Kochi, Kochi, 780-8515, Japan. (nyoshihara@econs.umass.edu)

that the relationship between capitalists and workers is mediated by a market contract that the worker is free to enter. Nonetheless, the worker cannot but spend part of his/her time working for a given capitalist by entering into a contract, that will be the source of profits the capitalists can receive as the return of his privately owned productive assets. Therefore, the *unequal exchange of labor* (UE) is a descriptive feature of exploitation in the capitalist economy, in that exploitative relations involve systematic differences between the amount of labor that individuals contribute to the economy in some relevant sense and the amount of labor they receive in some relevant sense through their income.

However, the application of the notion of UE exploitation to the capitalist economy involves a fundamental difficulty: unlike the case of feudal exploitation, the division of a worker's labor into working for him/herself and working for a capitalist is not a matter of observation. Therefore, the existence of UE due to the exploitative relationship should be measured through economic analysis in the capitalist economy. To promote such an analysis, one of the central issues in exploitation theory is to stipulate a suitable operational method to measure the difference between the labor expended and the labor received by an individual via his/her income.

As such a measure, Okishio (1963) provides a formal definition of exploitation, and then verifies the basic Marxian view by proving the so-called *Fundamental Marxian Theorem* (FMT). FMT shows that the capitalist economy is profitable if and only if the (average) rate of exploitation of the working class by the capitalist class is positive. In other words, FMT verifies the classical Marxian claim that the production of surplus value in the capitalist production process is the unique source of positive profits in market prices.

Applying Okishio's (1963) formal definition of exploitation, Roemer (1982) verifies the so-called *Class-Exploitation Correspondence Principle* (CECP) as a formal theorem. CECP establishes that for any capitalist economy and any competitive market equilibrium with a positive maximal profit rate, every member of the capitalist class is an exploiter while every member of the working class is exploited. Moreover, every member of the capitalist class is richer than every member of the working class. This states that in the equilibrium class membership and exploitation status emerge endogenously: the wealthy can rationally choose to belong to the capitalist class among other available options and become an exploiter, while the poor have no other option than being in the working class and are exploited.

However, the scope of both FMT and CECP seems to be limited within a simple economic model with a Leontief production technique, whenever the formal definition of exploitation is presumed according to Okishio (1963), which has led to several proposals for alternative definitions. For instance, some generalized versions of Okishio's (1963) definition, such as Morishima (1974) and Roemer (1982, chapter 5), have been proposed through a debate on the FMT in economies with more general production technology, while there is an alternative definition à la the New Interpretation (Duménil, 1980; Foley, 1982).

In contrast, Roemer (1982, 1994) even cricitizes the UE notion of exploitation and alternatively proposes the *property relation definition of exploitation* (PR exploitation). Though the main motive for Roemer (1982, 1994) to propose the PR theory of exploitation was to deny the relevance of exploitation as a primary normative concern, in that the injustice of the unequal distribution of productive assets should be the primary normative concern, rather than UE exploitation.

Since then, there have been developed some criticisms against the PR theory of exploitation, such as Cohen (1995), Wright (2000), and Vrousalis (2013), which are also to encourage the revival of the UE theory of exploitation. They claim that exploitation should be conceptualized as the systematic structure of economic transactions, in which some of the fruits of the labor of the exploited agents is appropriated by the exploiters under the institutional framework of asymmetric power relations resulting from private ownership.

Given these discussions, however, the issue of proper formal definitions of UE exploitation has yet remained unresolved. Axiomatic studies of UE exploitation, then, have been recently developed by Yoshihara (2010) and Veneziani and Yoshihara (2015, 2017a,b, 2018). These studies propose a domain axiom called *Labor Exploitation* (**LE**) that represents the minimal necessary condition for any definition of UE exploitation to be deemed admissible. Moreover, Veneziani and Yoshihara (2015, 2017a) propose a new axiom called *Profit-Exploitation Correspondence Principle* (**PECP**). This axiom requires that, whatever the appropriate definition of exploitation is, it must follow that for any capitalist economy and any market equilibrium, total profits are positive if and only if any propertyless employee is exploited in terms of this definition.

The underlying motive of axiom **PECP** is explained as follows. If a definition of UE exploitation is appropriate, it should point out the existence of a transfer mechanism by which UE is mediated: UE occurs by a mechanism that transfers (a part of) the productive fruits of the exploited to the exploiter. In perfectly competitive markets, neglecting the issue of rent, net outputs are distributed into wage income and profit income. Moreover, every party receives an equal wage per unit of (effective) labor. Therefore, the appropriation of more of the productive fruits by exploiters must be explained as a source of income other than wages, that is, profits. In other words, a proper formal definition of UE exploitation should be able to verify the correspondence between UE and profits.

Veneziani and Yoshihara (2015, 2017a) axiomatically characterize the measures of UE exploitation that satisfy **PECP** in every equilibrium under a general model of capitalist economies with a closed-convex cone production set. As a result, few admissible definitions of exploitation proposed in the literature preserve the axiom PECP, with only the definition à la the New Interpretation (Duménil, 1980; Foley, 1982) being an exception. However, as Yoshihara (2017) shows, if the production set is restricted to the one generated from a simple Leontief technique, every definition of UE exploitation preserves **PECP** whenever it meets the domain axiom **LE**.

Given these works, this paper examines how much of the admissible definitions of UE exploitation meeting the domain axiom **LE** can verify **PECP** when the production set is derived from the neoclassical production functions. That is, each sector is assumed to have a neoclassical production function which produces only one commodity as an output from each profile of material inputs and labor input, and is continuous, increasing, quasi-concave, and homogeneous degree of one. The production set derived from such functions represents an economy with the possibility of technical changes but without joint production.

Under such economies, this paper shows that any definition of exploitation can verify **PECP** whenever it meets **LE** and identifies the form of UE exploitation by means of a proper measure of labor values. Here, we need an additional axiom, called *Labor Value Theory of Exploitation* (**LVE**), to stipulate the class of *proper measures of labor values* in each capitalist economy. **LVE** requires that the amount of labor contents received by propertyless employees must be measured by a proper system of labor values, which should be computed from the information of an "*efficient*" production technique. This axiom stipulates that any technique is deemed "efficient" if its corresponding labor productivity is at least as high as that of the actually chosen technique in equilibrium.

The remainder of this paper is organized as follows. Section 2 introduces a basic model of capitalist economies. Section 3 provides a quick overview of axiomatic studies in Marxian exploitation theory. In particular, it povides a short survey on the axiomatic characterizations of proper definitions of UE exploitation by means of axioms **LE** and **PECP**, assuming the basic economic model with a convex come production set. Section 4 introduces a model of neoclassical production economies, in which the convex production set is representable by a profile of neoclassical production functions. Then, it develops an axiomatic analysis of UE exploitation by means of **LE**, **LVE**, and **PECP** under the neoclassical production economies. Finally, Appendix argues the characterization of UE exploitation by means of **PECP**, considering non-stationary equilibria.

2 Model

An economy comprises a set of agents, $\mathcal{N} = \{1, .., N\}$, with generic element $\nu \in \mathcal{N}$.[1] Denote the cardinal number of this set by N. There are n types of (purely private) commodities that are transferable in markets.

Production technology, commonly accessible by any agent, is represented by a production possibility set $P \subseteq \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+$ with generic element $\boldsymbol{\alpha} \equiv (-\alpha_l, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}})$, where $\alpha_l \in \mathbb{R}_+$ is the *effective* labor input; $\underline{\boldsymbol{\alpha}} \in \mathbb{R}^n_+$ are the inputs of the produced goods; and $\overline{\boldsymbol{\alpha}} \in \mathbb{R}^n_+$ are the outputs of the n goods. The net output vector arising from $\boldsymbol{\alpha}$ is denoted as $\widehat{\boldsymbol{\alpha}} \equiv \overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}}$. P is assumed to be closed and convex-cone such that (i) $\mathbf{0} \in P$; (ii) for any $\boldsymbol{\alpha} \in P$ with $\overline{\boldsymbol{\alpha}} \geq \mathbf{0}$, $\alpha_l > 0$ and $\underline{\boldsymbol{\alpha}} \geq \mathbf{0}$ hold; and (iii) for any $\boldsymbol{c} \in \mathbb{R}^n_+$, there exists $\boldsymbol{\alpha} \in P$ such that $\widehat{\boldsymbol{\alpha}} \geqq \boldsymbol{c}$. Property (ii) implies that labor and some of capital goods are indispensable for the production of a positive amount of a commodity, while property (iii) implies

[1] Let $\mathbb{R}$ be the set of real numbers and $\mathbb{R}_+$ (*resp.* $\mathbb{R}_-$) the set of non-negative (*resp.* non-positive) real numbers. For all x, $y \in \mathbb{R}^n$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, n)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; and $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, n)$. For any set, X and Y, $X \subseteq Y$ if and only if for any $x \in X$, $x \in Y$; $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$; and $X \subsetneq Y$ if and only if $X \subseteq Y$ and $X \neq Y$.

that any non-negative vector of commodities can be produced as a net output.

A specific type of production technology P is of a *Leontief type* if there exists a pair (A, L), where A is an $n \times n$ non-negative square matrix of material input coefficients and L is a $1 \times n$ positive vector of labor input coefficients, such that P is represented by the following form:

$$P_{(A,L)} \equiv \left\{ \boldsymbol{\alpha} = (-\alpha_l, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists \boldsymbol{x} \in \mathbb{R}^n_+ : \boldsymbol{\alpha} \leqq (-L\boldsymbol{x}, -A\boldsymbol{x}, \boldsymbol{x}) \right\}.$$

Here, A is assumed to be productive and indecomposable. Another specific type of production technology P is of a *von Neumann type* if there exists a profile (A, B, L) such that P is represented by the following form:

$$P_{(A,B,L)} \equiv \left\{ \boldsymbol{\alpha} = (-\alpha_l, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}) \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists \boldsymbol{x} \in \mathbb{R}^n_+ : \boldsymbol{\alpha} \leqq (-L\boldsymbol{x}, -A\boldsymbol{x}, B\boldsymbol{x}) \right\}.$$

where A is an $n \times m$ matrix, the generic component of which, $a_{ij} \geqq 0$, represents the amount of commodity i used as an input to operate one unit of the j-th production process; B is an $n \times m$ matrix, the generic component of which, $b_{ij} \geqq 0$, represents the amount of commodity i produced as an output by operating one unit of the j-th production process; and L is a $1 \times m$ positive row vector of direct labor input coefficients. In the following discussion, we sometimes use the notation A_i (resp. B_i) to refer to the i-th row vector of A (resp. B).

Given $\mathcal{N}$ and P, agents commonly have one unit of labour endowment, but can be heterogeneous in terms of their capital endowments $(\omega^\nu_t)_{\nu \in \mathcal{N}}$ in each period t. In the following discussion, let $\mathcal{W}_t \equiv \{\nu \in \mathcal{N} \mid \boldsymbol{\omega}^\nu_t = \mathbf{0}\}$ be the set of propertyless agents at period t. Typically, $\mathcal{W}_t$ would represent the set of workers who own no material means of production. Let $C \subseteq \mathbb{R}^n_+ \times [0, 1]$ be the consumption space common to all agents, and for each $\nu \in \mathcal{N}$, let $u^\nu : C \to \mathbb{R}_+$ be his/her welfare function. All available welfare functions are assumed to be strongly increasing in consumption bundles and decreasing in the supply of labor hours. Thus, one capitalist economy is defined by the list $\mathcal{E} \equiv \left\langle \mathcal{N}; P; (u^\nu, \boldsymbol{\omega}^\nu_0)_{\nu \in \mathcal{N}} \right\rangle$.

As in Roemer (1982), the time structure of production is explicitly considered and production activities are financed with current wealth. Agent ν's wealth, at the beginning of period t, is given by $\boldsymbol{p}_{t-1}\boldsymbol{\omega}^\nu_t$: this is fixed at the end of period $t-1$ given market prices $\boldsymbol{p}_{t-1}$. Thus, given a price system $\left\langle \left\{ \boldsymbol{p}_{t-1}, \boldsymbol{p}_t \right\}, w_t \right\rangle$ in period t, each agent $\nu \in \mathcal{N}$ engages in an optimal choice of production plan $\boldsymbol{\alpha}^\nu_t \in P$. Here, each agent (i) purchases a bundle of capital goods $\underline{\boldsymbol{\alpha}}^\nu_t$ at price $\boldsymbol{p}_{t-1}$ under his/her wealth constraint, $\boldsymbol{p}_{t-1}\boldsymbol{\omega}^\nu_t$, and employs labor power, α^ν_{lt}, at the beginning of this period; (ii) purchases an optimal amount of commodity bundle $\boldsymbol{\delta}^\nu_t$ at price $\boldsymbol{p}_{t-1}$ under budget constraint $\boldsymbol{p}_{t-1}(\boldsymbol{\omega}^\nu_t - \underline{\boldsymbol{\alpha}}^\nu_t)$ for speculative purposes, to be sold at the end of the period with an expected price $\boldsymbol{p}_t$; and (iii) chooses an optimal labor supply and consumption plan, $(\boldsymbol{c}^\nu_t, \Lambda^\nu_t) \in C$, where $\boldsymbol{c}^\nu_t$ will be purchased at the end of this period with an expected price $(\boldsymbol{p}_t, w_t)$ under the budget constraint of his/her revenue from both production and speculation, and Λ^ν_t is the labor supplied in period t. This choice behavior is determined as a solution to the optimization problem (MP^ν_t), as follows:

$$MP^\nu_t : \max_{(\boldsymbol{c}^\nu_t, \Lambda^\nu_t) \in C;\ \boldsymbol{\delta}^\nu_t \in \mathbb{R}^n_+;\ \boldsymbol{\alpha}^\nu_t \in P} u^\nu(\boldsymbol{c}^\nu_t, \Lambda^\nu_t)$$

$$\text{s.t.} \quad [\boldsymbol{p}_t\overline{\boldsymbol{\alpha}}_t^\nu - w_t\alpha_{lt}^\nu] + w_t^\nu\Lambda_t^\nu + \boldsymbol{p}_t\boldsymbol{\delta}_t^\nu \geqq \boldsymbol{p}_t\boldsymbol{c}_t^\nu + \boldsymbol{p}_t\boldsymbol{\omega}_{t+1}^\nu;$$
$$\boldsymbol{p}_{t-1}\boldsymbol{\delta}_t^\nu + \boldsymbol{p}_{t-1}\underline{\boldsymbol{\alpha}}_t^\nu \leqq \boldsymbol{p}_{t-1}\boldsymbol{\omega}_t^\nu;$$
$$\boldsymbol{p}_t\boldsymbol{\omega}_{t+1}^\nu \geqq \boldsymbol{p}_{t-1}\boldsymbol{\omega}_t^\nu.$$

Then, denote the set of solutions to the problem (MP_t^ν) by $\boldsymbol{O}_t^\nu\left(\left\{\boldsymbol{p}_{t-1}, \boldsymbol{p}_t\right\}, w_t\right)$.

We focus on the stationary equilibrium price vector, $\boldsymbol{p}^* = \boldsymbol{p}_{t-1} = \boldsymbol{p}_t$ $(\forall t)$. Moreover, we focus on the non-trivial equilibrium satisfying $\pi \equiv \max_{\boldsymbol{\alpha}' \in P} \frac{\boldsymbol{p}^*\overline{\boldsymbol{\alpha}}' - \boldsymbol{p}^*\underline{\boldsymbol{\alpha}}' - w_t\alpha_l'}{\boldsymbol{p}^*\underline{\boldsymbol{\alpha}}'} \geqq 0$. In this case, according to the monotone increasing characteristic of u^ν at $\boldsymbol{c}_t^\nu$, there always exists an optimal solution having $\boldsymbol{\delta}_t^\nu = \mathbf{0}$. By focusing on this optimal solution, we can remove the description of $\boldsymbol{\delta}_t^\nu$ without loss of generality. Therefore, we consider the following equilibrium notion:

Definition 1: For a capitalist economy, $\mathcal{E}$, a *reproducible solution (RS)* is a profile $\left((\boldsymbol{p}^*, w_t^*); ((\boldsymbol{c}_t^{*\nu}, \Lambda_t^{*\nu}); \boldsymbol{\alpha}_t^{*\nu})_{\nu \in \mathcal{N}}\right)$ of a price system and economic activities in each period, t, satisfying the following conditions:
(i) $((\boldsymbol{c}_t^{*\nu}, \Lambda_t^{*\nu}); \boldsymbol{\alpha}_t^{*\nu}) \in \boldsymbol{O}_t^\nu(\boldsymbol{p}^*, w_t^*)$ $(\forall t)$ (each agent's optimization);
(ii) $\sum_{\nu \in \mathcal{N}} \widehat{\boldsymbol{\alpha}}_t^{*\nu} \geqq \sum_{\nu \in \mathcal{N}} \boldsymbol{c}_t^{*\nu}$ $(\forall t)$ (demand-supply matching at the end of each period);
(iii) $\sum_{\nu \in \mathcal{N}} \alpha_{lt}^{*\nu} = \sum_{\nu \in \mathcal{N}}^{*\nu} \Lambda_t^{*\nu}$ $(\forall t)$ (labor market equilibrium);
(iv) $\sum_{\nu \in \mathcal{N}} \underline{\boldsymbol{\alpha}}_t^{*\nu} \leqq \sum_{\nu \in \mathcal{N}} \boldsymbol{\omega}_t^\nu$ $(\forall t)$ (social feasibility of production at the beginning of each period).

Henceforth, we assume the stationary state on economic activities of agents and delete the time description, t.

3 Axiomatic Analysis of UE Exploitation

Here, we discuss the axiom proposed by Yoshihara (2010) and Veneziani and Yoshihara (2015, 2017a,b, 2018), which represents the minimal necessary condition for admissible definitions of UE exploitation. Then, we introduce alternative definitions of exploitation proposed in the literature on mathematical Marxian economics.

As a preliminary step, given any P, let us define the set of production activities feasible with k units of labor inputs by $P(\alpha_l = k) \equiv \{(-\alpha_l', -\underline{\boldsymbol{\alpha}}', \overline{\boldsymbol{\alpha}}') \in P \mid \alpha_l' = k\}$. Given $\boldsymbol{c} \in \mathbb{R}_+^n$, define the set of efficient production activities to produce $\boldsymbol{c}$ as a net output by $\phi(\boldsymbol{c}) \equiv \{\boldsymbol{\alpha} \in P \mid \widehat{\boldsymbol{\alpha}} \geqq \boldsymbol{c}\}$. Moreover, the set of efficient production activities to produce $\boldsymbol{c}$ as a net output is denoted by $\partial\underline{\phi}(\boldsymbol{c}) \equiv \{\boldsymbol{\alpha} \in \phi(\boldsymbol{c}) \mid \forall \boldsymbol{\alpha}' \in \phi(\boldsymbol{c}),\ (-\alpha_l' > -\alpha_l \Rightarrow \exists i : -\underline{\alpha}_i' \leqq -\underline{\alpha}_i < 0)\}$.[2] Finally, let $\phi(\boldsymbol{c}; \boldsymbol{p}, w) \equiv \left\{\boldsymbol{\alpha} \in \arg\max_{\boldsymbol{\alpha}' \in P} \frac{\boldsymbol{p}\overline{\boldsymbol{\alpha}}' - w\alpha_l'}{\boldsymbol{p}\underline{\boldsymbol{\alpha}}'} \mid \widehat{\boldsymbol{\alpha}} \geqq \boldsymbol{c}\right\}$.

Any definition of exploitation should be able to identify, associated with each equilibrium allocation, the set of exploiting agents, $\mathcal{N}^{ter} \subseteq \mathcal{N}$, and the set of exploited agents, $\mathcal{N}^{ted} \subseteq \mathcal{N}$, such that $\mathcal{N}^{ter} \cap \mathcal{N}^{ted} = \varnothing$ holds. Moreover,

[2] By this definition, for the frontier of the production possibility set P, $\partial P \equiv \{\alpha \in P \mid \nexists \alpha' \in P : \alpha' > \alpha\}$, we have $\partial\underline{\phi}(c) \subseteq \partial P \cap \{\alpha \in \phi(c) \mid \widehat{\alpha} \ngtr c\}$.

it should capture the feature of UE as the difference between the amount of labor supplied by each agent and the amount of labor "received" through each agent's income. In particular, the supplied labor amount should be greater than the received labor amount for each exploited agent. Such properties should be preserved as a core feature of exploitation regardless of the way in which UE exploitation is measured.

Note that for the capitalist economies considered herein, each agent's supply of labor is identified by Λ^ν. By contrast, how to formulate the labor amount that each agent can "receive" through his/her earned income remains open to debate. Based on the forms of "received" labor, a number of possible definitions of exploitation exist.

Summarizing the above arguments, due to Yoshihara (2010) and Veneziani and Yoshihara (2015, 2017a,b, 2018), an axiom can be proposed to represent the minimal necessary condition for any definition of exploitation to be deemed admissible:

Labor Exploitation (LE): Given any definition of exploitation, for any capitalist economy $\mathcal{E}$ and any RS $\left((\boldsymbol{p}, w) ; ((\boldsymbol{c}^\nu, \Lambda^\nu) ; \boldsymbol{\alpha}^\nu)_{\nu \in \mathcal{N}}\right)$, the set of exploited agents, $\mathcal{N}^{ted} \subseteq \mathcal{N}$, should have the following property: there exists a profile of commodity bundles, $(\boldsymbol{c}_e^\nu)_{\nu \in \mathcal{W}} \in \mathbb{R}_+^{nW}$, such that, for any $\nu \in \mathcal{W}$, $\boldsymbol{p}\boldsymbol{c}_e^\nu = w\Lambda^\nu$ holds, and for some productive activity $\boldsymbol{\alpha}^{\boldsymbol{c}_e^\nu} \in \partial \underline{\phi}(\boldsymbol{c}_e^\nu)$:

$$\nu \in \mathcal{N}^{ted} \Leftrightarrow \alpha_l^{\boldsymbol{c}_e^\nu} < \Lambda^\nu.$$

That is, axiom **LE** requires that any admissible definition of UE exploitation must identify whether each propertyless agent is exploited for each RS under any economy. More specifically, the axiom stipulates that the set of propertyless exploited agents be identified as follows: according to each specific admissible definition, there should be a profile, $(\boldsymbol{c}_e^\nu)_{\nu \in \mathcal{W}}$, for each propertyless agent's commodity bundle affordable by that agent's revenue, and its corresponding profile $\left(\boldsymbol{\alpha}^{\boldsymbol{c}_e^\nu}\right)_{\nu \in \mathcal{W}}$ of production activities, where each $\boldsymbol{\alpha}^{\boldsymbol{c}_e^\nu}$ can produce the corresponding commodity bundle $\boldsymbol{c}_e^\nu$ as a net output in a technologically efficient way. Then, the exploitation status of each propertyless agent can be identified by comparing his/her amount of labor supply Λ^ν with the amount of labor input $\alpha_l^{\boldsymbol{c}_e^\nu}$ that he/she is able to "receive" through his/her income $w\Lambda^\nu$.

Axiom **LE** is a rather weak condition in that it only refers to the exploitation status of propertyless agents in each RS. This should be reasonable as a minimal necessary condition for the admissible domain. In other words, a definition of exploitation is not necessarily deemed to be proper, even if it satisfies **LE**. In fact, there may be infinitely many definitions of exploitation that satisfy **LE**, and all the main definitions proposed in the mathematical Marxian economics literature satisfy this axiom.[3]

[3] Of course, this does not imply that the axiom **LE** is trivial. For instance, the definition proposed by Matsuo (2008) does not satisfy **LE.**

To see the last point, let us consider three main definitions. First, the following two definitions are respectively natural extensions of Okishio's definition of UE exploitation into economies with a general convex cone production set:

Definition 2 (Morishima, 1974)**:** For any capitalist economy, $\mathcal{E}$, and any $\nu \in \mathcal{W}$, who supplies Λ^ν and purchases $\boldsymbol{c}^\nu \in \mathbb{R}^n_+$, $\nu \in \mathcal{N}^{ted}$ if and only if $\Lambda^\nu > \min_{\boldsymbol{\alpha} \in \phi(\boldsymbol{c}^\nu)} \alpha_l$.

Definition 3 (Roemer, 1982, chapter 5)**:** For any capitalist economy, $\mathcal{E}$, any RS, $\left((\boldsymbol{p}, w) ; ((\boldsymbol{c}^\nu, \Lambda^\nu) ; \boldsymbol{\alpha}^\nu)_{\nu \in \mathcal{N}}\right)$, and any $\nu \in \mathcal{W}$, who supplies Λ^ν and purchases $\boldsymbol{c}^\nu \in \mathbb{R}^n_+$, $\nu \in \mathcal{N}^{ted}$ if and only if $\Lambda^\nu > \min_{\boldsymbol{\alpha} \in \phi(\boldsymbol{c}^\nu; \boldsymbol{p}, w)} \alpha_l$.

Finally, for any capitalist economy, $\mathcal{E}$, and any RS, $\left((\boldsymbol{p}, w) ; ((\boldsymbol{c}^\nu, \Lambda^\nu) ; \boldsymbol{\alpha}^\nu)_{\nu \in \mathcal{N}}\right)$, let $\boldsymbol{\alpha}^{\boldsymbol{p},w} \equiv \sum_{\nu \in \mathcal{N}} \boldsymbol{\alpha}^\nu$. Moreover, for any $\boldsymbol{c} \in \mathbb{R}^n_+$, we define a non-negative number, $\tau^{\boldsymbol{c}} \in \mathbb{R}_+$, as satisfying $\tau^{\boldsymbol{c}} \boldsymbol{p} \widehat{\boldsymbol{\alpha}}^{\boldsymbol{p},w} = \boldsymbol{pc}$. Then:

Definition 4 (Veneziani and Yoshihara, 2015)**:** For any capitalist economy, $\mathcal{E}$, any RS, $\left((\boldsymbol{p}, w) ; ((\boldsymbol{c}^\nu, \Lambda^\nu) ; \boldsymbol{\alpha}^\nu)_{\nu \in \mathcal{N}}\right)$, and any $\nu \in \mathcal{W}$, who supplies Λ^ν and can purchase $\boldsymbol{c}^\nu \in \mathbb{R}^n_+$, $\nu \in \mathcal{N}^{ted}$ if and only if $\Lambda^\nu > \tau^{\boldsymbol{c}^\nu} \alpha_l^{\boldsymbol{p},w}$.

In Definition 4, for each $\nu \in \mathcal{W}$, $\tau^{\boldsymbol{c}^\nu}$ represents ν's share of national income, and thus $\tau^{\boldsymbol{c}^\nu} \alpha_l^{\boldsymbol{p},w}$ is the share of social labor that this agent receives through the wage income sufficient to purchase $\boldsymbol{c}^\nu$. It is conceptually related to the New Interpretation (NI) definition of exploitation à la Duménil (1980) and Foley (1982), which was originally defined in Leontief economies.[4]

3.1 Profit-Exploitation Correspondence Principle

Following Veneziani and Yoshihara (2015, 2017a), the axiom of Profit-Exploitation Correspondence Principle is given as follows:

Profit-Exploitation Correspondence Principle (PECP): For any capitalist economy, $\mathcal{E}$, and any RS, $\left((\boldsymbol{p}, w) ; ((\boldsymbol{c}^\nu, \Lambda^\nu) ; \boldsymbol{\alpha}^\nu)_{\nu \in \mathcal{N}}\right)$:

$$\left[\boldsymbol{p}\widehat{\boldsymbol{\alpha}}^{\boldsymbol{p},w} - w\alpha_l^{\boldsymbol{p},w} > 0 \Leftrightarrow \mathcal{N}^{ted} \supseteq \mathcal{W}_+\right],$$

where $\mathcal{W}_+ \equiv \{\nu \in \mathcal{W} \mid \Lambda^\nu > 0\} \neq \varnothing$.

[4] In the NI, *the value of money* is defined by the labor amount per unit of national income, and the wage multiplied by the value of money is the value of labor power, as Foley (1986, p. 43) states: "the amount of average social labor workers receive a claim to in the wage for each hour they actually work—that is, as the average wage multiplied by the value of money." In Definition 4, for each $\nu \in \mathcal{W}$, $\tau^{c^\nu} \alpha_l^{p,w} = w\Lambda^\nu \frac{\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}}$ holds by $w\Lambda^\nu = pc^\nu$. Since $w\Lambda^\nu$ is ν's wage income and $\frac{\alpha_l^{p,w}}{p\widehat{\alpha}^{p,w}}$ corresponds to the value of money in the NI, $\Lambda^\nu > \tau^{c^\nu} \alpha_l^{p,w}$ means that ν is exploited as "a worker expends more labor hours than he or she receives an equivalent for in wages" (Foley 1986, p.122).

That is, whatever the definition of exploitation is, it must follow that for any capitalist economy and any RS, total profits are positive if and only if any propertyless employee is exploited in terms of this definition, assuming the definition of exploitation is deemed appropriate. This is required by **PECP**.

For the available class of capitalist economies considered here, there is no requirement of a restriction that excludes the existence of fixed capital goods, the possibility of joint production, or of technical changes. Therefore, the correspondence between profits and exploitation is required for a large class of economies, as assumed by the standard general equilibrium theory.

However, **PECP** *per se* is not so strong. Indeed, it even allows for a situation in which some propertyless employees are exploited in an equilibrium with zero total profit.[5] This finding implies that, at least within the class of economies with homogeneous agents, **PECP** is logically weaker than the statement of the FMT, as within such economies, the latter implies that no propertyless employee is exploited in any equilibrium with zero profit. By contrast, while the FMT implies that the rate of exploitation for the whole working class is positive in any equilibrium with positive total profits, **PECP** requests that every propertyless worker is exploited, which is a stronger claim than that of the FMT.

Veneziani and Yoshihara (2015, 2017a) study the necessary and sufficient condition for **PECP**, as stated in the following proposition:[6]

Proposition 1 (Veneziani and Yoshihara, 2015, 2017a): For any definition of exploitation satisfying **LE**, the following two statements are equivalent for any capitalist economy, $\mathcal{E}$, and any RS, $\left((\boldsymbol{p}, w); ((\boldsymbol{c}^{\nu}, \Lambda^{\nu}); \boldsymbol{\alpha}^{\nu})_{\nu \in \mathcal{N}}\right)$:
(1) **PECP** holds under this definition of exploitation;
(2) If $\boldsymbol{p}\widehat{\boldsymbol{\alpha}}^{\boldsymbol{p},w} - w\alpha_l^{\boldsymbol{p},w} > 0$, then for any $\nu \in \mathcal{W}_+$, there exists a production activity $\boldsymbol{\alpha}_{\pi}^{\nu} \in P(\alpha_l = \Lambda^{\nu}) \cap \partial P$ such that $\widehat{\boldsymbol{\alpha}}_{\pi}^{\nu} \in \mathbb{R}_+^n$, $\boldsymbol{p}\widehat{\boldsymbol{\alpha}}_{\pi}^{\nu} > w\Lambda^{\nu}$, and $(\alpha_{\pi l}^{\nu}, \underline{\boldsymbol{\alpha}}_{\pi}^{\nu}, \overline{\boldsymbol{\alpha}}_{\pi}^{\nu}) \geqq \eta^{\nu}\left(\alpha_l^{\boldsymbol{c}_e^{\nu}}, \underline{\boldsymbol{\alpha}}^{\boldsymbol{c}_e^{\nu}}, \overline{\boldsymbol{\alpha}}^{\boldsymbol{c}_e^{\nu}}\right)$ hold for some $\eta^{\nu} > 1$.

That is, condition (2) of Proposition 1 is the necessary and sufficient condition for any definition of exploitation satisfying **LE** to preserve **PECP**. Condition (2) states that if total profits are positive in the present equilibrium, then for each propertyless employee, $\nu \in \mathcal{W}_+$, there exists a suitable efficient production point, $\boldsymbol{\alpha}_{\pi}^{\nu}$, activated by the present amount of labor supply, Λ^{ν}, which in conjunction with production activity, $\boldsymbol{\alpha}^{\boldsymbol{c}_e^{\nu}}$, can verify that this agent is being exploited. Recall that, according to axiom **LE**, production activity $\boldsymbol{\alpha}^{\boldsymbol{c}_e^{\nu}}$ is identified by the presumed definition of exploitation, and the corresponding labor input $\alpha_l^{\boldsymbol{c}_e^{\nu}}$ represents agent ν's "received" labor. Production activity $\boldsymbol{\alpha}_{\pi}^{\nu} \in P(\alpha_l = \Lambda^{\nu}) \cap \partial P$ is defined as the proportional expansion of production point $\boldsymbol{\alpha}^{\boldsymbol{c}_e^{\nu}}$ up to the point of his/her present labor supply, Λ^{ν}, and that produces a non-negative net

[5] However, any definition of exploitation satisfying **LE** does not allow the existence of exploited propertyless employees in conjunction with zero profit.

[6] Note that, though all of the following analyses herein presume economies with homogeneous labor, the completely parallel results can be obtained even if we consider economies with heterogeneous labor, as shown in Veneziani and Yoshihara (2015, 2017a).

output, $\widehat{\boldsymbol{\alpha}}_\pi^\nu \in \mathbb{R}_+^n$, that is non-affordable by ν at the present equilibrium because $\boldsymbol{p}\widehat{\boldsymbol{\alpha}}_\pi^\nu > w\Lambda^\nu$. Therefore, since $\Lambda^\nu = \alpha_{\pi l}^\nu > \alpha_l^{\boldsymbol{c}_e^\nu}$ holds for such a selection of $\boldsymbol{\alpha}_\pi^\nu$, we can confirm that agent $\nu \in \mathcal{W}_+$ is exploited at this RS, according to the given definition satisfying **LE**.

Proposition 1 does not provide a normative characterization of the presumed definition of exploitation, but rather a demarcation line (condition (2)) by which one can test which of infinitely many potential definitions preserves the essential relation of exploitation and profits in capitalist economies. Thus, if a definition of exploitation satisfying **LE** does not generally meet condition (2), then it will not satisfy **PECP**, which implies that it is not a proper definition of UE exploitation.

Some may criticize the methodological positions of **PECP** and Proposition 1, claiming that **PECP** should be proved as a theorem rather than treated as an axiom. In fact, as Okishio and Morishima did, the methodological standpoint of the FMT was, assuming a specific definition of exploitation, to verify that a capitalist economy can be conceived of as exploitative.

By contrast, Proposition 1 presumes a correspondence between positive profits and exploitation for every propertyless employee as an axiom and then tests the validity of each alternative definition of UE exploitation by checking whether it satisfies this axiom. Such a methodology has been implicitly adopted within debates on the FMT. Typically, whenever a counterexample has been raised against the FMT with a major definition of exploitation by generalizing the model of economies, this criticism has been resolved by proposing an alternative definition and proving that the FMT is held with this alternative form under the generalized economic model. This implicitly suggests that in the overall debate on the FMT, the validity of each form of exploitation has been tested by the robustness of the equivalence between exploitation and positive profits. However, even if such an interpretation is acceptable, the structure of the debate on the FMT could not function as such, because it may involve an infinite repetition of counterexample and alternate proposal. By contrast, by providing an axiomatic characterization such as Proposition 1, the validity of every form of UE exploitation is testable simply by checking condition (2).

There are economies in which, for all $\nu \in \mathcal{W}_+$, condition (2) is never satisfied if $\boldsymbol{\alpha}^{\boldsymbol{c}_e^\nu}$ is given by Definition 2 or 3, and thus **PECP** does not hold. By contrast, Definition 4 satisfies condition (2), and thus **PECP** holds *for all $\mathcal{E}$ and all* RS:

Corollary 1 (Veneziani and Yoshihara, 2015, 2017a): There exists a capitalist economy, $\mathcal{E}$, and an RS for this economy such that neither Definition 2 nor Definition 3 satisfies **PECP**.

Corollary 2 (Veneziani and Yoshihara, 2015, 2017a): For any capitalist economy, $\mathcal{E}$, and any RS, Definition 4 satisfies **PECP**.

These corollaries suggest that, at least among the main competing proposals of exploitation forms, Definition 4 is the sole appropriate form.

However, the above implications can be revised whenever we restrict our attention into the class of economies with a simple Leontief technique. Indeed, in such a class of economies, the equivalence of positive profits and exploitation of each propertyless employee and the equivalence of zero profit and no exploitation are preserved for any definition of exploitation, as long as it satisfies **LE**.

Proposition 2 (Veneziani and Yoshihara, 2015): For any capitalist economy, $\left\langle \mathcal{N}; P_{(A,L)}; (u^\nu, \boldsymbol{\omega}_0^\nu)_{\nu\in\mathcal{N}} \right\rangle$, and any RS, $\left((\boldsymbol{p}, w)\,; ((\boldsymbol{c}^\nu, \Lambda^\nu)\,; \boldsymbol{\alpha}^\nu)_{\nu\in\mathcal{N}}\right)$, **PECP** holds for any definition of exploitation satisfying **LE**.

4 Axiomatic Analysis of UE Exploitation in Neoclassical Production Economies

The above characterization results of Propositions 1 and 2 suggest an issue of whether the validity of the basic Marxian perception of capitalist economies as exploitative crucially depends on the degree of the complexity of the production technology or not. To examine this issue, we will consider a model of economies with neoclassical production functions, where each commodity has its own production sector and the latter is endowed with a production possibility set represented by a neoclassical production function. In such an economy, there is no joint production, but each sector has a plenty of alternative techniques to produce its corresponding commodity as an output. Therefore, economies with neoclassical production functions are less general than the generalized von Neumann production economies, but much more general than the simple Leontief production economies.

If we obtained the same general result as Proposition 2 under the economies with neoclassical production functions, we may say that the main source for the difficulty in preserving the validity of the basic Marxian perception would be the difficulty of how to measure the (socially necessary) labor content received by agents via their income when the underlying economy involves joint production.

4.1 A Model for Neoclassical Production Economies

Assume that there are n types of physical commodities. For each commodity $j = 1, \ldots, n$, there is a production function $f_j : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$ such that f_j is (i) continuous; (ii) increasing; (iii) homogeneous of degree one; and (iv) quasi-concave. Then, the set of alternative techniques to produce one unit of commodity $j = 1, \ldots, n$, is given by:

$$\mathcal{T}_j \equiv \left\{ (a_{1j}, \ldots, a_{nj}, L_j) \in \mathbb{R}_+^{n+1} \mid f_j\,(a_{1j}, \ldots, a_{nj}, L_j) = 1 \right\}.$$

Let $\mathcal{T} \equiv \mathcal{T}_1 \times \ldots \times \mathcal{T}_n$.

For each pair $\left((a_{1j},\ldots,a_{nj},L_j)_{j=1,\ldots,n}\right)\in\mathcal{T}$, a unique Leontief production technique (A,L) can be specified as

$$A=\begin{bmatrix} a_{11} & \ldots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \ldots & a_{nn} \end{bmatrix}, \text{ and } L=(L_1,\ldots,L_n).$$

With abuse of notation, let us denote $(A,L)\in\mathcal{T}$ whenever $(a_{1j},\ldots,a_{nj},L_j)\in\mathcal{T}_j$ for every $j=1,\ldots,n$. Assume that for every $(A,L)\in\mathcal{T}$, A is productive and indecomposable, and L is positive. Then, as in the standard argument, each Leontief production technique $(A,L)\in\mathcal{T}$ can specify the maximal profit rate $\Pi_{(A,L)}>0$, which satisfies the property that there is positive vector $\boldsymbol{x}>\mathbf{0}$ unique up to scale such that $\boldsymbol{x}=\left(1+\Pi_{(A,L)}\right)A\boldsymbol{x}$. Then, by Kurose and Yoshihara (2019), there exist $(A^*,L^*)\in\mathcal{T}$ and a positive scalar $\Pi^*>0$ associated with (A^*,L^*) such that for some $\boldsymbol{x}^*>\mathbf{0}$, $\boldsymbol{x}^*=(1+\Pi^*)A^*\boldsymbol{x}^*$ holds, and for any $(A,L)\in\mathcal{T}$, $\Pi^*\geqq\Pi_{(A,L)}$ holds.

Let $\mathcal{N}$ be the set of population in the capitalist economy. Following the convention of the classical and Marxian approach, assume that $\mathcal{N}$ can be partitioned into two-tier classes, the class of capitalists $\mathcal{K}$ and the class of workers $\mathcal{W}$. The difference between these classes is that every capitalist has a positive endowment vector of capital goods and seeks to maximize the accumulation of the value magnitude of her own capital goods in every period of production, while every worker has no endowment of capital goods and seeks to maximize the welfare of her consumption available from her wage revenue in each period. Each worker has one unit of labor endowment. There is no skill difference among workers, and there is no heterogeneity in the type of labor. Thus, the aggregate labor endowment of this society is $W\equiv\#\mathcal{W}$.

Every worker has a common consumption space, which is defined as follows. First, let $\boldsymbol{b}\in\mathbb{R}^n_{++}$ be the subsistence consumption bundle that every worker must consume for her own survival per unit of working time. Let $\zeta:[0,1]\twoheadrightarrow\mathbb{R}^n_+$ be a continuous correspondence such that for each working time $\Lambda\in[0,1]$, $\zeta(\Lambda)\equiv\left\{\boldsymbol{c}\in\mathbb{R}^n_+\mid\boldsymbol{c}\geqq\Lambda\boldsymbol{b}\right\}$. Then, the consumption space C is specified by the graph of this correspondence, that is, $C\equiv graph(\zeta)\equiv\left\{(\boldsymbol{c},\Lambda)\in\mathbb{R}^n_+\times[0,1]\mid\boldsymbol{c}\in\zeta(\Lambda)\right\}$.

Each worker $\mu\in\mathcal{W}$ has her own preference ordering over C which is represented by a utility function $u^\mu:C\to\mathbb{R}$ which has the following specific form: for any $(\boldsymbol{c},\Lambda)\in C$, $u^\mu(\boldsymbol{c},\Lambda)=\phi^\mu(\boldsymbol{c})-\Lambda$, where ϕ^μ is continuous, strongly monotonic, and quasi-concave on $\zeta(\Lambda)$ for every $\Lambda\in[0,1]$, such that for each $(\Lambda\boldsymbol{b},\Lambda)\in C$, $\phi^\mu(\Lambda\boldsymbol{b})=\Lambda$. By this construction, $u^\mu(\Lambda\boldsymbol{b},\Lambda)=0$ for every $\Lambda\in[0,1]$, which specifies her minimal level of utility necessary for surviving.

Let $\left(\boldsymbol{p}_t,\boldsymbol{p}_{t-1},w_t\right)$ be a market price system prevailed at the beginning of the present period t, where $\boldsymbol{p}_{t-1}\in\mathbb{R}^n_+$ is the price of n commodities expected to be prevailed at the end of period $t-1$; $\boldsymbol{p}_t\in\mathbb{R}^n_+$ is the price of n commodities prevailed at the end of period t; $w_t\in\mathbb{R}_+$ is the wage rate expected to be prevailed at the end of the present period t. Here, the subsistence consumption bundle is chosen as the numeraire, so that $\boldsymbol{p}_t\boldsymbol{b}=1$ and $\boldsymbol{p}_{t-1}\boldsymbol{b}=1$.

Then, each worker $\mu \in \mathcal{W}$ should solve the following optimization program $MP_t^{\mu}\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$:

$$\max_{(\boldsymbol{c},\Lambda)\in C} u^{\mu}\left(\boldsymbol{c}, \Lambda\right) \text{ subject to } \boldsymbol{p}_t \boldsymbol{c} \leqq w_t \Lambda.$$

By the above construction, if the wage rate is the subsistence level, $w_t = 1$, then μ is indifferent to the supply of labor time and her optimal consumption bundle is simply $\boldsymbol{b}\Lambda$ for every $\Lambda \in [0,1]$. In contrast, if $w_t > 1$, then $\Lambda = 1$ is her optimal labor supply and her optimal consumption bundle meets $\boldsymbol{c}_t^{\mu} > \boldsymbol{b}$ whenever $\boldsymbol{p}_t > \mathbf{0}$.

In contrast, each capitalist $\nu \in \mathcal{K}$ should solve the following optimization program $MP_t^{\nu}\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$:

$$\max_{(\boldsymbol{y}_t^{\nu},\boldsymbol{\delta}_t^{\nu},(A_t^{\nu},L_t^{\nu}))\in\mathbb{R}_+^n\times\mathbb{R}_+^n\times\mathcal{T}} \boldsymbol{p}_t\boldsymbol{\omega}_{t+1}^{\nu}$$

subject to

$$\begin{aligned} \boldsymbol{p}_t\boldsymbol{\omega}_{t+1}^{\nu} &= \boldsymbol{p}_t\boldsymbol{y}_t^{\nu} + \boldsymbol{p}_t\boldsymbol{\delta}_t^{\nu} + \left(\boldsymbol{p}_{t-1}\left[\boldsymbol{\omega}_t^{\nu} - A_t^{\nu}\boldsymbol{y}_t^{\nu} - \boldsymbol{\delta}_t^{\nu}\right]\right) - w_t L_t^{\nu}\boldsymbol{y}_t^{\nu}; \\ \boldsymbol{p}_{t-1}\boldsymbol{\omega}_t^{\nu} &\geqq \boldsymbol{p}_{t-1}A_t^{\nu}\boldsymbol{y}_t^{\nu} + \boldsymbol{p}_{t-1}\boldsymbol{\delta}_t^{\nu}. \end{aligned}$$

Here, $(A_t^{\nu}, L_t^{\nu}) \in \mathcal{T}$ is the optimal Leontief technique chosen by the capitalist ν from the universal set of production techniques $\mathcal{T}$ at prices $\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$, and $\boldsymbol{\delta}_t^{\nu} \in \mathbb{R}_+^2$ is the commodity bundle which is purchased with prices $\boldsymbol{p}_{t-1}$ at the beginning of the period t and sold with prices $\boldsymbol{p}_t$ at the end of the same period for the sake of speculation. Note that for any capitalist $\nu \in \mathcal{K}$ and for each commodity $i = 1, \ldots, n$, $\delta_{it}^{\nu} > 0$ if and only if $\frac{p_{it}}{p_{it-1}} > \max_{j=1,\ldots,n} \frac{p_{jt} - w_t L_{jt}^{\nu}}{p_{t-1}A_{jt}^{\nu}}$.

Let $\boldsymbol{\omega}_t \equiv \sum_{\nu\in\mathcal{K}} \boldsymbol{\omega}_t^{\nu}$, and Λ_t be the aggregate labor endowment of the society at period t. Then, an equilibrium solution concept for this economy can be specified as follows:

Definition 5. A profile $\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$ associated with $\left(\left(\boldsymbol{y}_t^{\nu}, \boldsymbol{\delta}_t^{\nu}, (A_t^{\nu}, L_t^{\nu})\right)_{\nu\in\mathcal{K}}, \left(\boldsymbol{c}_t^{\mu}, \Lambda_t^{\mu}\right)_{\mu\in\mathcal{W}}\right)$ is a *reproducible solution (RS) for the economy* $\left\langle(\mathcal{K},\mathcal{W}); \mathcal{T}; (u^{\mu})_{\mu\in\mathcal{W}}; \boldsymbol{\omega}_t; W\right\rangle$ *at period* t if and only if it satisfies the following conditions:
(1) for each $\mu \in \mathcal{W}$, $(\boldsymbol{c}_t^{\mu}, \Lambda_t^{\mu})$ is a solution to the program $MP_t^{\mu}\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$;
(2) for each $\nu \in \mathcal{K}$, $(\boldsymbol{y}_t^{\nu}, \boldsymbol{\delta}_t^{\nu}, (A_t^{\nu}, L_t^{\nu}))$ is a solution to the program $MP_t^{\nu}\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$;
(3) $\sum_{\nu\in\mathcal{K}} \boldsymbol{y}_t^{\nu} + \sum_{\nu\in\mathcal{K}} \boldsymbol{\delta}_t^{\nu} \geqq \sum_{\nu\in\mathcal{K}} \boldsymbol{\omega}_{t+1}^{\nu} + \sum_{\mu\in\mathcal{W}} \boldsymbol{c}_t^{\mu}$ (excess demand condition for commodity markets);
(4) $\sum_{\nu\in\mathcal{K}} L_t^{\nu}\boldsymbol{y}_t^{\nu} = \sum_{\mu\in\mathcal{W}} \Lambda_t^{\mu}$ (equilibrium for labor market);
(5) $\sum_{\nu\in\mathcal{K}} A_t^{\nu}\boldsymbol{y}_t^{\nu} + \sum_{\nu\in\mathcal{K}} \boldsymbol{\delta}_t^{\nu} \leqq \sum_{\nu\in\mathcal{K}} \boldsymbol{\omega}_t^{\nu}$ (social feasibility of production); and
(6) $\sum_{\nu\in\mathcal{K}} \boldsymbol{\omega}_{t+1}^{\nu} \geqq \sum_{\nu\in\mathcal{K}} \boldsymbol{\omega}_t^{\nu}$ (reproducibility).

Note that the condition (5) should be satisfied only with equality at RS with $\boldsymbol{p}_{t-1} > \mathbf{0}$, since every capitalist would like to sell all of her own capital $\boldsymbol{\omega}_t^{\nu}$ to meet $\boldsymbol{p}_{t-1}A_t^{\nu}\boldsymbol{y}_t^{\nu} + \boldsymbol{p}_{t-1}\boldsymbol{\delta}_t^{\nu} = \boldsymbol{p}_{t-1}\boldsymbol{\omega}_t^{\nu}$ in her program $MP_t^{\nu}\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$.

Assume that this RS is *non-trivial*, that is, $\sum_{\nu \in \mathcal{K}} \boldsymbol{y}_t^\nu \geq \mathbf{0}$ holds. This implies that, $\max_{\nu \in \mathcal{K}} \max_{j=1,\ldots,n} \frac{p_{jt} - w_t L_{jt}^\nu}{p_{t-1} A_{jt}^\nu} \geqq \max_{j=1,\ldots,n} \frac{p_{jt}}{p_{jt-1}}$ holds. In this case, without loss of generality, we can assume that every agent's competitive choice of technique is identical: $(A_t^\nu, L_t^\nu) = \left(A_t^{\nu'}, L_t^{\nu'}\right)$ for any $\nu, \nu' \in \mathcal{K}$. Let $\left(A^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}, L^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}\right) \equiv (A_t^\nu, L_t^\nu)$ for any $\nu \in \mathcal{K}$. Then,

$$\max_{j=1,\ldots,n} \frac{p_{jt} - w_t L_j^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}}{p_{t-1} A_j^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}} \geqq \max_{j=1,\ldots,n} \frac{p_{jt}}{p_{jt-1}}$$

holds, which further implies that $\boldsymbol{\delta}_t^\nu = \mathbf{0}$ constitutes an optimal solution for every $\nu \in \mathcal{K}$. Therefore, without loss of generality, consider $\boldsymbol{\delta}_t^\nu = \mathbf{0}$ for every $\nu \in \mathcal{K}$ in the non-trivial RS.

Then, as $\sum_{\mu \in \mathcal{W}} \boldsymbol{c}_t^\mu \geqq \sum_{\nu \in \mathcal{K}} \boldsymbol{b} L_t^\nu \boldsymbol{y}_t^\nu > \mathbf{0}$ follows from the definition of workers' consumption space C, Definition 1-(3) with $\sum_{\nu \in \mathcal{K}} \boldsymbol{\delta}_t^\nu = \mathbf{0}$ and 1-(6) together imply that $\sum_{\nu \in \mathcal{K}} \boldsymbol{y}_t^\nu > \mathbf{0}$. This implies that all sector must be activated in the non-trivial RS, which implies that all sectors are equally profitable: there exists $\pi_t \in \mathbb{R}$ such that

$$1 + \pi_t = \frac{p_{jt} - w_t L_j^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}}{p_{t-1} A_j^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}} \text{ for any } j = 1, \ldots, n.$$

In other words, in the non-trivial RS with $\sum_{\nu \in \mathcal{K}} \boldsymbol{\delta}_t^\nu = \mathbf{0}$, the equilibrium price system $\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$ satisfies:

$$\boldsymbol{p}_t = (1 + \pi_t)\, \boldsymbol{p}_{t-1} A^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} + w_t L^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}.$$

Then, as $\sum_{\nu \in \mathcal{K}} w_t L_t^\nu \boldsymbol{y}_t^\nu = \sum_{\mu \in \mathcal{W}} \boldsymbol{p}_t \boldsymbol{c}_t^\mu$ holds in equilibrium, it follows that

$$\sum_{\nu \in \mathcal{K}} \boldsymbol{p}_t \boldsymbol{\omega}_{t+1}^\nu = (1 + \pi_t) \sum_{\nu \in \mathcal{K}} \boldsymbol{p}_{t-1} \boldsymbol{\omega}_t^\nu.$$

Finally, if $\boldsymbol{p}_{t-1} = \boldsymbol{p}_t$, then by $\sum_{\nu \in \mathcal{K}} A_t^\nu \boldsymbol{y}_t^\nu = \sum_{\nu \in \mathcal{K}} \boldsymbol{\omega}_t^\nu$, the condition (6) is equivalent to $\sum_{\nu \in \mathcal{K}} (I - A_t^\nu)\, \boldsymbol{y}_t^\nu \geqq \sum_{\mu \in \mathcal{W}} \boldsymbol{c}_t^\mu$.

Let a RS $\left(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t\right)$ be called *stationary* if $\boldsymbol{p}_{t-1} = \boldsymbol{p}_t$. In the following discussion, we will focus on a stationary RS. Note that at a stationary RS, any subscript to present time can be removed. Therefore, a stationary RS is represented by $(\boldsymbol{p}, w)$ associated with $\left(\left(\boldsymbol{y}^\nu, A^{(\boldsymbol{p}, w)}, L^{(\boldsymbol{p}, w)}\right)_{\nu \in \mathcal{K}}, (\boldsymbol{c}^\mu, \Lambda^\mu)_{\mu \in \mathcal{W}}\right)$. Denote the maximal profit rate associated with the stationary RS $(\boldsymbol{p}, w)$ by π.

Let us introduce the following assumption.

Assumption 1: For any $(A, L) \in \mathcal{T}$, $L (I - A)^{-1} \boldsymbol{b} < 1$ holds.

This condition is to ensure the possibility of surplus products for this society. This is because $L (I - A)^{-1} \boldsymbol{b}$ represents the minimal amount of labor time necessary to produce $\boldsymbol{b}$ as the net output by using the technique (A, L). If $L\,(I-A)^{-1}\,\boldsymbol{b} \geqq 1$, this implies that by using the technique (A, L), the society can at most make the workers with the subsistence level, and no room for capital accumulation. The capitalist economic system is not interesting in such a low productivity level of technique.

Let $\boldsymbol{y}_t \equiv \sum_{\nu \in \mathcal{K}} \boldsymbol{y}_t^\nu$. Then:

Theorem 1. Let $(\boldsymbol{p}, w)$ associated with $\left(\left(\boldsymbol{y}_t^\nu, A^{(\boldsymbol{p},w)}, L^{(\boldsymbol{p},w)}\right)_{\nu \in \mathcal{K}}, \left(\boldsymbol{c}^\mu, \Lambda^\mu\right)_{\mu \in \mathcal{W}}\right)$ be a stationary RS for the economy $\left\langle (\mathcal{K}, \mathcal{W})\,; \mathcal{T}; (u^\mu)_{\mu \in \mathcal{W}}\,; \boldsymbol{\omega}_t; W \right\rangle$ at period t. Then, the following condition holds:
(i) If $(w, \pi) > (1, 0)$, then $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$ and $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = W$;
(ii) If $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$ and $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = W$, then $\pi \geqq 0$ and $w \geqq 1$. In particular, if $w = 1$, then $\pi > 0$;
(iii) If $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$ and $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t < W$, then $\pi > 0$ and $w = 1$;
(iv) If $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t < \boldsymbol{\omega}_t$ and $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = W$, then $\pi = 0$ and $w > 1$.

Proof. For (i). As $w > 1$, for every $\mu \in \mathcal{W}$, $\Lambda^\mu = 1$ is the unique optimal labor supply at prices $(\boldsymbol{p}, w)$. Therefore, in the stationary RS, condition (3) of Definition 5 implies that $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = W$. As $\boldsymbol{p}\boldsymbol{\omega}_{t+1}^\nu = (1+\pi)\,\boldsymbol{p}A^{(\boldsymbol{p},w)}\boldsymbol{y}_t^\nu + \left[\boldsymbol{p}\boldsymbol{\omega}_t^\nu - \boldsymbol{p}A^{(\boldsymbol{p},w)}\boldsymbol{y}_t^\nu\right]$ is the optimal capital accumulated by each and every capitalist $\nu \in \mathcal{K}$. Note that $\boldsymbol{p} > \boldsymbol{0}$ holds from

$$\boldsymbol{p} = wL^{(\boldsymbol{p},w)}\left[I - (1+\pi)A^{(\boldsymbol{p},w)}\right]^{-1} > \boldsymbol{0}$$

in the stationary RS. This follows from $L^{(\boldsymbol{p},w)} > \boldsymbol{0}$ and $\left[I - (1+\pi)A^{(\boldsymbol{p},w)}\right]^{-1} > \boldsymbol{0}$, where the latter two strict inequalities follow from the presumption that $\mathcal{T}_1 \times \mathcal{T}_2$ consists of Leontief techniques whose labor coefficient vectors are positive and whose material input coefficient matrices are productive and indecomposable. Note that $w > 1$ implies that π is less than the maximal profit rate obtained from the Frobenius eigenvalue of $A^{(\boldsymbol{p},w)}$. Thus, as $\boldsymbol{p} > \boldsymbol{0}$ and $\pi > 0$, every capitalist's optimal solution $\boldsymbol{y}_t^\nu$ should meet that $\boldsymbol{p}A^{(\boldsymbol{p},w)}\boldsymbol{y}_t^\nu = \boldsymbol{p}\boldsymbol{\omega}_t^\nu$. Therefore, in this RS, we have $\boldsymbol{p}A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{p}\boldsymbol{\omega}_t$. As $\boldsymbol{p} > \boldsymbol{0}$, we have $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$.

For (ii). Let $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$ and $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = W$ in this RS. Suppose $\pi < 0$. Then, $\boldsymbol{p}\boldsymbol{\omega}_{t+1}^\nu = (1+\pi)\,\boldsymbol{p}A^{(\boldsymbol{p},w)}\boldsymbol{y}_t^\nu + \left[\boldsymbol{p}\boldsymbol{\omega}_t^\nu - \boldsymbol{p}A^{(\boldsymbol{p},w)}\boldsymbol{y}_t^\nu\right] \leqq \boldsymbol{p}\boldsymbol{\omega}_t^\nu$ holds for any $\nu \in \mathcal{K}$. Therefore, her optimal solution is only $\boldsymbol{y}_t^\nu = \boldsymbol{0}$ for any $\nu \in \mathcal{K}$. Thus, $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t < \boldsymbol{\omega}_t$, which is a contradiction. Suppose $w < 1$. Then, $l^\mu = 0$ is the unique optimal labor supply at prices $(\boldsymbol{p}, w)$. Thus, to preserve the condition (3) of Definition 5, $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = 0$ must hold. However, it contradicts $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = W$. Thus, $\pi \geqq 0$ and $w \geqq 1$. Let, in particular, $w = 1$. Suppose $r = 0$. In this case, $\boldsymbol{p} = L^{(\boldsymbol{p},w)}\left[I - A^{(\boldsymbol{p},w)}\right]^{-1}$ holds. Then, $1 = \boldsymbol{p}\boldsymbol{b} = L^{(\boldsymbol{p},w)}\left[I - A^{(\boldsymbol{p},w)}\right]^{-1}\boldsymbol{b}$, which contradicts Assumption 1. Therefore, $\pi > 0$ should hold.

For (iii). By (ii), $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$ implies that $\pi \geqq 0$. By $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t < W$, $w \leqq 1$ holds from (i). Indeed, if $w > 1$, we have to have $L^{(\boldsymbol{p},w)}\boldsymbol{y}_t = W$ as shown in the proof of (i). Suppose $\pi = 0$ or $w < 1$. Let $w < 1$. Then, for every worker

$\mu \in \mathcal{W}$, $\Lambda^{\mu} = 0$ is the unique optimal labor supply at prices $(\boldsymbol{p}, w)$. Therefore, the condition (3) of Definition 5 implies that $\boldsymbol{y}_t = \mathbf{0}$, which contradicts from $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$. Therefore, $w = 1$ should hold. Thus, suppose $\pi = 0$. In this case, $\boldsymbol{p} = L^{(\boldsymbol{p},w)}\left[I - A^{(\boldsymbol{p},w)}\right]^{-1}$, and so by the same argument as the last in the proof for part (ii), we derive a contradiction from Assumption 1. Therefore, $\pi > 0$ should hold.

For (iv). By (i), if $\pi > 0$, then $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t = \boldsymbol{\omega}_t$, which contradicts from $A^{(\boldsymbol{p},w)}\boldsymbol{y}_t < \boldsymbol{\omega}_t$. Therefore, $\pi = 0$ should hold. By (ii), $w \geqq 1$. If $w = 1$, then again we have $\boldsymbol{p} = L^{(\boldsymbol{p},w)}\left[I - A^{(\boldsymbol{p},w)}\right]^{-1}$, and so $1 = L^{(\boldsymbol{p},w)}\left[I - A^{(\boldsymbol{p},w)}\right]^{-1}\boldsymbol{b}$, a contradiction. Therefore, $w > 1$ holds. ■

4.2 Characterizations of UE Exploitation in Neoclassical Production Economies

Let us characterize the class of the definitions for UE exploitation in neoclassical production economies. As a preliminary step, let $\boldsymbol{v}^{(\boldsymbol{p},w)} \equiv L^{(\boldsymbol{p},w)}\left(I - A^{(\boldsymbol{p},w)}\right)^{-1}$.

Then, consider the following new axiom:

Labor Value Theory of Exploitation (LVE): Consider a definition of exploitation satisfying **LE**. For any capitalist economy $\left\langle (\mathcal{K}, \mathcal{W}); \mathcal{T}; (u^{\mu})_{\mu\in\mathcal{W}}; \boldsymbol{\omega}_t; W \right\rangle$ and any RS $(\boldsymbol{p}, w)$, $\alpha_l^{\boldsymbol{c}_e^{\nu}} \leqq \boldsymbol{v}^{(\boldsymbol{p},w)}\boldsymbol{c}_e^{\nu}$ holds for any $\nu \in \mathcal{W}$.

This axiom requires that any admissible definition of exploitation should measure the labor content received by propertyless agents according to the labor theory of values. That is, the socially necessary labor time for the reproduction of labor power should be evaluated by means of a properly defined vector of labor values.

Given a RS $(\boldsymbol{p}, w)$, the vector $\boldsymbol{v}^{(\boldsymbol{p},w)}$ represents a profile of labor contents inputted directly as well as indirectly to (re)produce one unit of each sector's net output, which is determined by means of the competitively chosen technique $\left(A^{(\boldsymbol{p},w)}, L^{(\boldsymbol{p},w)}\right)$. If an economy with a simple Leontief technique is presumed, the vector $\boldsymbol{v}^{(\boldsymbol{p},w)}$ should be definitely conceived as the proper definition of labor values. However, in an economy with many alternative techniques, like the neoclassical production economies with $\#\mathcal{T} > 1$, a serious debate has been occurred regarding how to formulate the labor values under such an economy, and so the vector $\boldsymbol{v}^{(\boldsymbol{p},w)}$ would not necessarily constitute the appropriate formulation of labor values.[7] Having such a situation, the axiom **LVE** stipulates the upper bound of admissible forms of labor values by means of $\boldsymbol{v}^{(\boldsymbol{p},w)}$.

This upper bound is weak enough for allowing a rather broad class of admissible labor theories of values, in that there are infinitely many potential forms

[7] For instance, following Marx (1847), Morishima (1974) defines the labor values as the "*optimum values,*" which are defined as the minimizer of the labor expenditure: $\boldsymbol{v}^{\min} \equiv \arg\min_{(A,L)\in\mathcal{T}} L(I-A)^{-1}\boldsymbol{b}$ like Definition 2. In contrast, Morishima (1974) calls the vector $\boldsymbol{v}^{(\boldsymbol{p},w)}$ the "actual values."

of labor values satisfying this constraint. Indeed, all of Definitions 2, 3, and 4 satisfy **LVE**.

With this additional axiom, we are now ready to characterize the admissible definitions of exploitation under which **PECP** holds:

Theorem 2: For any capitalist economy, $\left\langle (\mathcal{K}, \mathcal{W}) ; \mathcal{T}; (u^{\mu})_{\mu \in \mathcal{W}} ; \boldsymbol{\omega}_t; W \right\rangle$, and any stationary RS $(\boldsymbol{p}, w)$, **PECP** holds for any definition of exploitation satisfying **LE** and **LVE**.

Proof. For a stationary RS $(\boldsymbol{p}, w)$, let $\pi > 0$. Take any definition of exploitation satisfying **LE** and **LVE**. Then, there exists a profile of commodity bundles, $(\boldsymbol{c}_e^{\nu})_{\nu \in \mathcal{W}} \in \mathbb{R}_+^{nW}$, such that, for any $\nu \in \mathcal{W}$, $\boldsymbol{p}\boldsymbol{c}_e^{\nu} = w\Lambda^{\nu}$ holds, and for some production point, $\boldsymbol{\alpha}^{\boldsymbol{c}_e^{\nu}} \in \partial \underline{\phi} (\boldsymbol{c}_e^{\nu})$, it follows that $\nu \in \mathcal{N}^{ted} \Leftrightarrow \alpha_l^{\boldsymbol{c}_e^{\nu}} < \Lambda^{\nu}$.

As the production possibility set in this economy is given by

$$P_{\mathcal{T}} \equiv \left\{ \boldsymbol{\alpha} = (-\alpha_l, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}) \in \mathbb{R}_- \times \mathbb{R}_-^n \times \mathbb{R}_+^n \mid \exists (A, L) \in \mathcal{T} \ \& \ \exists \boldsymbol{x} \in \mathbb{R}_+^n : \boldsymbol{\alpha} \leqq (-L\boldsymbol{x}, -A\boldsymbol{x}, \boldsymbol{x}) \right\},$$

there exist $(A, L) \in \mathcal{T}$ and $\boldsymbol{y} \geq \mathbf{0}$ such that $\boldsymbol{\alpha}^{\boldsymbol{c}_e^{\nu}} = (-L\boldsymbol{y}, -A\boldsymbol{y}, \boldsymbol{y})$. Note that by the assumption of $\mathcal{T}$, for any $\boldsymbol{c}_e^{\nu} \geq \mathbf{0}$ and any $(A, L) \in \mathcal{T}$, there exists $\boldsymbol{y} \geq \mathbf{0}$ such that $(-L\boldsymbol{y}, -A\boldsymbol{y}, \boldsymbol{y}) \in \partial \underline{\phi} (\boldsymbol{c}_e^{\nu})$.

Without loss of generality, take $\mu \in \mathcal{W}_+$. Then,

$$\begin{aligned}
\boldsymbol{p} &= (1+\pi) \boldsymbol{p} A^{(\boldsymbol{p},w)} + wL^{(\boldsymbol{p},w)} \\
&= \pi \boldsymbol{p} A^{(\boldsymbol{p},w)} \left(I - A^{(\boldsymbol{p},w)}\right)^{-1} + w\boldsymbol{v}^{(\boldsymbol{p},w)} \\
&\Leftrightarrow \\
\boldsymbol{p}\boldsymbol{c}_e^{\mu} &= \pi \boldsymbol{p} A^{(\boldsymbol{p},w)} \left(I - A^{(\boldsymbol{p},w)}\right)^{-1} \boldsymbol{c}_e^{\mu} + w\boldsymbol{v}^{(\boldsymbol{p},w)} \boldsymbol{c}_e^{\mu} \\
&\Leftrightarrow \\
w\Lambda^{\mu} &= \pi \boldsymbol{p} A^{(\boldsymbol{p},w)} \left(I - A^{(\boldsymbol{p},w)}\right)^{-1} \boldsymbol{c}_e^{\mu} + w\boldsymbol{v}^{(\boldsymbol{p},w)} \boldsymbol{c}_e^{\mu}.
\end{aligned}$$

Thus, $\Lambda^{\mu} > \boldsymbol{v}^{(\boldsymbol{p},w)} \boldsymbol{c}_e^{\mu}$ holds by $\pi > 0$. By **LVE**, it follows that $\alpha_l^{\boldsymbol{c}_e^{\mu}} < \Lambda^{\mu}$, which implies $\mu \in \mathcal{N}^{ted}$.

Next, let $\pi = 0$. Then, we have $\Lambda^{\mu} = \boldsymbol{v}^{(\boldsymbol{p},w)} \boldsymbol{c}_e^{\mu}$. Moreover, for any $(A, L) \in \mathcal{T}$, it follows that

$$\begin{aligned}
\boldsymbol{p} &\leqq (1+\pi) \boldsymbol{p} A + wL \\
&\Leftrightarrow \\
\boldsymbol{p} &\leqq \pi \boldsymbol{p} A (I - A)^{-1} + w\boldsymbol{v} = w\boldsymbol{v} \\
&\Leftrightarrow \\
\boldsymbol{p}\boldsymbol{c}_e^{\mu} &\leqq w\boldsymbol{v}\boldsymbol{c}_e^{\mu}.
\end{aligned}$$

Thus, $w\Lambda^{\mu} \leqq w\alpha_l^{\boldsymbol{c}_e^{\mu}}$ follows from **LE** and **LVE**, which implies $\mu \notin \mathcal{N}^{ted}$.

In summary, **PECP** holds. ■

The above theorem shows that under the class of neoclassical production economies, the equivalence of positive profits and exploitation of each propertyless employee and the equivalence of zero profit and no exploitation are preserved for any definition of exploitation, as long as it satisfies **LE** and **LVE**. Combined with Propositions 1 and 2, this theorem may suggest that the main source for the difficulty in preserving the validity of the basic Marxian perception would be the presence of joint production in the underlying model of capitalist economies.

Remember that, as discussed by Morishima (1974), the presence of joint production makes it difficult to measure labor value of each individual commodity. Given this difficulty, Morishima (1973, 1974) and Roemer (1982, chapter 5) propose alternative definitions of exploitation. They are defined over the class of more general economic models such as the von Neumann production or the general convex production set, without relying on the determination of individual labor values. However, as shown in Corollary 1, these solutions do not successfully work to verify the basic Marxian perception represented by axiom **PECP** whenever the presumed economic models are such general types.

Note that the model of neoclassical production economies considered here is a special case of the von Neumann model as well as the general convex economic model. However, because of the lack of joint production, neoclassical production economies allow us to define individual labor values as the vector of the standard employment multipliers. In such a case, Theorem 2 implies that the basic Marxian perception can be verified under many admissible definitions of exploitation including the ones proposed by Morishima (1974) and Roemer (1982, chapter 5).

5 References

Cohen, G. A. (1995): *Self-ownership, Freedom and Equality*, Cambridge University Press, Cambridge.

Duménil, G. (1980): *De la Valeur aux Prix de Production*, Paris: Economica.

Foley, D. K. (1982): "The Value of Money, the Value of Labor Power, and the Marxian Transformation Problem," *Review of Radical Political Economics* 14, pp. 37-47.

Foley, D.K. (1986): *Understanding Capital*. Harvard University Press, Cambridge, MA.

Marx, K. (1847, 1976): *Poverty of Philosophy*, in Marx-Engels Collected Works: Volume 6: Marx and Engels, 1845–1848. New York: International Publishers.

Marx, K. (1867, 1976): *Capital. A Critique of Political Economy, Vol.I.*, trans. by Fowkes, B., Penguin Books, Harmondsworth, UK.

Marx, K. (1894, 1981): *Capital. A Critique of Political Economy, Vol.3*, trans. by Fernbach, D., Penguin Books, Harmondsworth, UK.

Matsuo, T. (2008): "Profit, Surplus Product, Exploitation and Less than Maximized Utility," *Metroeconomica* 59, pp. 249-265.

Morishima, M. (1973): *Marx's Economics*, Cambridge University Press, Cambridge.

Morishima, M. (1974): "Marx in the Light of Modern Economic Theory," *Econometrica* 42, pp. 611-632.

Okishio, N. (1963): "A Mathematical Note on Marxian Theorems," *Weltwirtschaftliches Archiv* 91, pp. 287-299.

Roemer, J. E. (1982): *A General Theory of Exploitation and Class*, Harvard University Press.

Roemer, J. E. (1994): *Egalitarian Perspectives: Essays in Philosophical Economics*, Cambridge University Press, Cambridge.

Veneziani, R., and N. Yoshihara (2015): "Exploitation in Economies with Heterogeneous Preferences, Skills and Assets: An Axiomatic Approach," *Journal of Theoretical Politics*, 27, pp. 8–33.

Veneziani, R., and N. Yoshihara (2017a): "One million miles to go: taking the axiomatic road to defining exploitation," *Cambridge Journal of Economics*, 41, pp. 1607–1626.

Veneziani, R., and N. Yoshihara (2017b): "Globalisation and Inequality in a Dynamic Economy: An Axiomatic Analysis of Unequal Exchange," *Social Choice and Welfare*, 49, pp. 445–468.

Vrousalis, N. (2013): "Exploitation, Vulnerability, and Social Domination," *Philosophy and Public Affairs* 41, pp. 131-157.

Wright, E. O. (2000): "Class, Exploitation, and Economic Rents: Reflections on Sorensen's 'Sounder Basis'," *The American Journal of Sociology* 105, pp. 1559-1571.

Yoshihara, N. (2010): "Class and Exploitation in General Convex Cone Economies," *Journal of Economic Behavior & Organization*, 75, pp. 281–296.

Yoshihara, N. (2017): "A Progress Report on Marxian Economic Theory: On the Controversies in Exploitation Theory since Okishio (1963)," *Journal of Economic Surveys*, 31, pp. 632–659.

Yoshihara, N., and R. Veneziani (2018): "The theory of exploitation as the unequal exchange of labour," *Economics and Philosophy*, 34, pp. 381-409.

6 Appendix: PECP for non-stationary RS in Neoclassical Production Economies

Theorem 2 in section 4.2 characterizes the class of appropriate definitions of UE exploitation in terms of **PECP** under neoclassical production economies by presuming the stationary RS. In this appendix, we will examine **PECP** under neoclassical production economies by taking a non-stationary RS which was given in Definition 5.

Consider a neoclassical production economy $\left\langle (\mathcal{K}, \mathcal{W}); \mathcal{T}; (u^\mu)_{\mu \in \mathcal{W}}; \boldsymbol{\omega}_t; W \right\rangle$, and let $(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)$ associated with $\left((\boldsymbol{y}_t^\nu, \boldsymbol{\delta}_t^\nu, (A_t^\nu, L_t^\nu))_{\nu \in \mathcal{K}}, (\boldsymbol{c}_t^\mu, \Lambda_t^\mu)_{\mu \in \mathcal{W}} \right)$ be a non-trivial RS in period t, which is not necessarily stationary. By the non-triviality, it follows that $\boldsymbol{p}_t \leqq (1+\pi_t)\, \boldsymbol{p}_{t-1}$. We may say that for the non-trivial RS, the *equilibrium real profit rates* are *positive* if $\boldsymbol{p}_t \leq (1+\pi_t)\, \boldsymbol{p}_{t-1}$ holds. This is because the equilibrium profit rate π_t is nominal in nature and so the nominal return rate minus inflation rates would be identical to the real return rates.

We will show the correspondence between UE exploitation and real profits for a non-trivial RS as follows, which verifies **PECP** for non-stationary RSs:

Theorem A1: Take any definition of exploitation satisfying **LE** and **LVE**. For any capitalist economy, $\left\langle (\mathcal{K}, \mathcal{W}); \mathcal{T}; (u^\mu)_{\mu \in \mathcal{W}}; \boldsymbol{\omega}_t; W \right\rangle$, and any non-trivial *RS* $(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)$, the following statement holds true: the *equilibrium real profit rates are positive*: $(1+\pi_t)\, \boldsymbol{p}_{t-1} - \boldsymbol{p}_t \geq \mathbf{0}$ *if and only if for any* $\mu \in \mathcal{W}_+$, μ *is exploited in terms of this definition.*

Proof. For a non-trivial *RS* $(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)$, the equilibrium price system meets the following equations:

$$\boldsymbol{p}_t = (1+\pi_t)\, \boldsymbol{p}_{t-1} A^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} + w_t L^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}.$$

Let $(1+\pi_t)\, \boldsymbol{p}_{t-1} - \boldsymbol{p}_t \geq \mathbf{0}$. Then, from the above system of equations, we have:

$$\boldsymbol{p}_t \geq \boldsymbol{p}_t A^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} + w_t L^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)}.$$

Thus, we have:

$$\boldsymbol{p}_t \left(I - A^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} \right) \geq w_t L^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)},$$

which further implies by $\left(I - A^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} \right)^{-1} > \mathbf{0}$ that

$$\boldsymbol{p}_t > w_t \boldsymbol{v}^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} > \mathbf{0}.$$

Hence, by multiplying $\boldsymbol{c}_e^\mu \geq \mathbf{0}$ from the right in both sides, we have:

$$\boldsymbol{p}_t \boldsymbol{c}_e^\mu > w_t \boldsymbol{v}^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} \boldsymbol{c}_e^\mu.$$

As $w_t \Lambda^\mu = \boldsymbol{p}_t \boldsymbol{c}_e^\mu$ by axiom **LE**, we have $w_t \Lambda^\mu > w_t \boldsymbol{v}^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} \boldsymbol{c}_e^\mu \geqq w_t \alpha_l^{\boldsymbol{c}_e^\mu}$ by **LVE**, which implies for any $\mu \in \mathcal{W}_+$, $\mu \in \mathcal{N}^{ted}$ holds.

Next, let $(1+\pi_t)\, \boldsymbol{p}_{t-1} - \boldsymbol{p}_t = \mathbf{0}$. Then, following the similar analysis to that in the proof of Theorem 2, we obtain $\boldsymbol{p}_t = w_t \boldsymbol{v}^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} \leqq w_t \boldsymbol{v}$ for any $(A, L) \in \mathcal{T}$ and so $w_t \Lambda^\mu = w_t \boldsymbol{v}^{(\boldsymbol{p}_t, \boldsymbol{p}_{t-1}, w_t)} \boldsymbol{c}_e^\mu \leqq w_t \alpha_l^{\boldsymbol{c}_e^\mu}$ for any definition satisfying **LE** and **LVE**, which implies for any $\mu \in \mathcal{W}_+$, $\mu \notin \mathcal{N}^{ted}$ holds. ■