MONEY MARKET STABILITY UNDER ALTERNATIVE
RESERVE REQUIREMENT RULES

AKIYOSHI HORIUCHI* AND SEIJI OGURI**

I. Introduction

In a number of major industrialized countries, such as the United States, West Germany, and Japan, commercial banks presently operate under a so-called lagged reserve accounting rule. This rule requires banks to hold reserves equal to certain fixed proportions of their deposit liabilities in the preceding period. Lagged reserve accounting, under which required reserves in any given period are predetermined, is alleged to weaken monetary control by the central bank, because banks have no way to avoid reserve deficiencies unless the central bank accommodatingly supplies at least enough reserves to meet the predetermined reserve requirements. With the purpose of modifying this defect some economists have made proposals for reforming the reserve accounting rule, but it still remains ambiguous how proposed reforms would change the adjustment mechanism in the money markets.

This paper, taking up three alternative rules of reserve accounting (i.e., lagged reserve accounting, contemporaneous reserve accounting, and marginal reserve accounting), investigates how alterations of reserve accounting rules affect stability of money markets. Specifically, responsiveness of money supply and interest rates to various disturbances are examined under the respective reserve accounting rules. Making use of a simplified money market model, it will be made clear that the marginal reserve accounting rule is not only most advantageous for money supply stability but also effective in mitigating fluctuations of money market rates caused by some disturbances. Thus, reforming the present lagged reserve accounting system would be helpful in promoting short run stability of money markets.

The organization of the paper is as follows. Our short run money market model is described in Section II. In Section III, we summarize stability implications of the money market model. We state two reservations concerning our analysis in Section IV.

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† Earlier drafts of this paper were reported at the TCER research meeting and seminar held by Monetary and Economic Studies Department of the Bank of Japan. We are indebted to many participants at these meetings for their very helpful comments. This paper represents the views of the authors and should not be interpreted as reflecting the views of the staff of the Bank of Japan.
1 See Holmes (1969). According to empirical studies by Pierce (1976) and Feige-McGee (1977), there was unidirectional causality from money supply to reserve supply under U.S. lagged reserve accounting.
2 Especially Poole (1976), Laurent (1979), and Fry (1979).
II. **A Short Run Money Market Model**

This section presents a simplified model of short run money markets on which our following discussions will be based. The model consists of three economic sectors (i.e., private banks, the non-bank public, and the central bank), and three markets (i.e., the market for high-powered money, the inter-bank money market in which the non-bank public cannot participate, and the general money market).\(^3\) We shall begin by explaining balance sheet conditions of each sector.

(A) The balance sheet of the private banks

Two assets are available to private banks; reserves \(R_B\) and loans to the non-bank public \(L_B\). Their liabilities are deposits supplied to the public \(D_B\), net borrowing in the inter-bank money market \(C_B\), net borrowing in the other money market \(F_B\), and borrowing from the central bank \(B_B\). The relation of these assets and liabilities is summarized in Fig. 1.

![Fig. 1. The Balance Sheet of the Private Banks](image)

The banks’ demand for reserves is assumed to be composed of both required reserves \(R_R\) and excess reserves \(u_E\) which is regarded as one of shift parameters in our discussions.\(^4\) Therefore,

\[
(1) \quad R_B = R_R + u_E.
\]

Specification of required reserves \(R_R\) is dependent upon which rule of reserve accounting is adopted. Since the amount of loans supplied to the non-bank public \(L_B\) is assumed to be unadjustable in the short run, \(L_B\) is exogenously given in this paper;

\[
(2) \quad L_B = L_B.
\]

It is assumed that bank deposits \(D_B\) are entirely determined by the public’s demand \(D_N\). Thus, supply of bank deposits is always equal to public’s demand;

\[
(3) \quad D_B = D_N.
\]

\(^3\) Specifically, the inter-bank money market and the general money market can be compared to the call loan market (including bill discount market) and the repurchase agreement market respectively in the Japanese financial system.

\(^4\) This assumption means that the banks’ demand for excess reserves is independent of money market interest rates. Though clearly inconsistent with the traditional model, which assumes banks’ excess reserves to be a decreasing function of money market interest rates, this assumption seems to fit Japanese banks very well because, as is well known, their holdings of excess reserves are almost negligible.
Net borrowings by the banks in the inter-bank money market vary negatively with the interest rate in this market \( i \), and positively with the general money market rate \( r \). Thus,

\[
C_B = -a_1i + a_2r + u_C; \quad (a_1, a_2 \geq 0),
\]

where \( u_C \) represents a disturbance term in this demand function.\(^5\)

With regard to banks' borrowing from the central bank \( B_B \), we assume passive supply of the central bank loans. That is, the central bank accommodates banks' demand for its loans, which is represented by (5);

\[
B_B = b_1i + b_2r + u_B; \quad (b_1, b_2 \geq 0).
\]

This means that rises in the interest rate of either the inter-bank money market or the other money market induce the banks to decrease their borrowing in those money markets, and borrow more from the central bank. Coefficients \( b_1 \) and \( b_2 \) indicate the extent to which banks are inclined to borrow from the central bank; i.e., the larger \( b_1 \) and \( b_2 \) are, the more aggressively the banks borrow from the central bank. An exogenous shift parameter in this demand function is represented by \( u_B \).

From the balance sheet condition summarized in Fig. 1, we can derive the following relation with respect to private banks' net borrowing in the other money market \( F_B \);

\[
F_B = R_B + L_B - D_B - C_B - B_B.
\]

Utilizing the assumptions (1)-(5), \( F_B \) can be represented by (6).

\[
F_B = R_B + R_B - D_N + (a_1 - b_1)i - (a_2 + b_2)r + u_E - u_C - u_B; \quad (a_1 - b_1 > 0).
\]

While \( F_B \) is unambiguously a decreasing function of \( r \), the assumptions we have made so far cannot determine how \( F_B \) responds to variations in the inter-bank money market rate \( i \), because the sign of \( (a_1 - b_1) \), the coefficient on \( i \), is indeterminate. In the following discussion, we shall assume \( (a_1 - b_1) \) to be positive; i.e. the banks' demand for borrowing in the other money market \( F_B \) is an increasing function of the inter-bank money market rate \( i \).

(B) The balance sheet of the non-bank public

The balance sheet of the non-bank public is given in Fig. 2. The non-bank public holds cash \( R_N \), bank deposits \( D_N \), and the general money market instruments \( F_N \) as assets, and borrows \( L_N \) from the banks.

![FIG. 2. THE BALANCE SHEET OF THE NON-BANK PUBLIC](image)

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash ((R_N))</td>
<td>Borrowing from banks ((L_N))</td>
</tr>
<tr>
<td>Deposits ((D_N))</td>
<td></td>
</tr>
<tr>
<td>Money market instrument ((F_N))</td>
<td></td>
</tr>
</tbody>
</table>

The public's demand for cash and deposits are represented by (7) and (8) respectively;

\[
(7) \quad R_N = -c_2r + u_R; \quad (c_2 \geq 0),
\]

\[
(8) \quad D_N = -d_2r + u_D; \quad (d_2 \geq 0).
\]

In our money market model, interest rates on bank deposits are institutionally fixed, and

\(^5\) For analytical simplicity, all behavioral equations of the respective sectors will be presented as linear functions without constant terms. Our conclusions are unaffected by this formulation.
the public is not allowed to participate in the inter-bank money market. Therefore, it is quite natural to assume that the public's demands for both cash and deposits are decreasing functions of the general money market rate $r$. Disturbances in these demand functions are represented by $u_R$ and $u_D$ respectively.

As we have already explained, the loans the banks supply to the public cannot be adjusted in the short-run, so the public's borrowing $L_N$ must equal $L_B$ which is exogenously given in our model; i.e.,

$$L_N = L_B.$$  \hspace{1cm} (9)

From the balance sheet of Fig. 2 and the assumptions of portfolio behavior (7), (8), and (9), the public's demand for the general money market instrument $F_N$ can be derived as follows;

$$F_N = L_N - R_N - D_N = (c_2 + d_2)r + L_B - u_R - u_D.$$  \hspace{1cm} (10)

That is, the public's demand for the general money market instrument is an increasing function of the interest rate on that instrument $r$.

(C) The balance sheet of the central bank

The balance sheet of the central bank is presented by Fig. 3. The central bank can supply high-powered money $R_C$ through two routes; i.e., by increasing its holding of the inter-bank money market instrument $C_C$ and by increasing the amount of the loans supplied to private banks $B_C$. Total supply of high-powered money can, therefore, be represented by (11);

$$R_C = C_C + B_C.$$  \hspace{1cm} (11)

As already explained we shall assume that the central bank always accommodates private banks' demand for its loans $B_B$. Thus $B_C$ must always be equal to $B_B$ represented by (5);\(^6\)

$$B_C = B_B = b_1 + b_2r + u_B; \quad (b_1, b_2 \geq 0).$$  \hspace{1cm} (12)

(D) Short run equilibrium of the money markets

Equilibrium of our money market model can be described by three conditions; i.e., equilibria of demand and supply of high-powered money, the inter-bank money market, and the general money market. The equilibrium condition for high-powered money is represented by the following equation;

---

\(^6\) By setting both $b_1$ and $b_2$ to zero, we can obtain a non-accommodating case; i.e., the case in which the central bank rigidly fixes the amount of its loans to private banks $B_C$ at a certain level $u_B$ regardless of interest rates in the money markets.
\[ R_B + R_N = R_C, \]

where the left-hand side is total demand for high-powered money, and the right-hand side is its supply. Using (1), (7), (11), and (12), this equation can be rewritten as

\[ (13) \quad R_R + u_E - c_2r + u_R = C_C + b_1i + b_2r + u_B. \]

The equilibrium condition of the inter-bank money market will be fulfilled when net borrowing by the private banks \( C_B \) equals the amount of the inter-bank money market instrument held by the central bank \( C_C \); thus

\[ C_B = C_C. \]

Substituting (4) into this equation, we obtain (14).

\[ (14) \quad a_i - a_2r = -C_C + u_C. \]

When the amount of net borrowing by the banks in the general money market \( F_B \) equals the public's demand for the instrument of this market \( F_N \), equilibrium will be achieved in this market. Therefore, the equilibrium condition of the general money market is

\[ F_B = F_N. \]

Utilizing balance sheet conditions of both private banks and the public, and relations (3), (9), (11), and (12), this equilibrium condition of the general money market can be rewritten as follows;

\[ (R_B + R_N - R_C) + (C_C - C_B) = 0. \]

This equation means that if both high-powered money and the inter-bank money markets clear, then equilibrium of the other money market will be automatic. This is of course "Walras's Law," and so the equilibrium condition of the general money market is redundant in the following our discussion.

Since money supply is one of the most important money market variables, we shall be particularly concerned with its short run stability. Money supply \( M \) is defined to consist of cash and deposits held by the non-bank public; i.e.,

\[ M = R_N + D_N. \]

This definition can be rewritten as follows;

\[ (15) \quad (c_2 + d_2)r + M = u_R + u_D. \]

Now, in order for our money market model to be completed, we have to add two more conditions; the first is specification of required reserves \( R_R \), the second is specification of the operating target pursued by the central bank.

(E) Alternative rules of required reserve accounting

In this paper, we take up three rules of required reserve accounting, i.e., lagged reserve accounting (LRA), contemporaneous reserve accounting (CRA), and marginal reserve accounting (MRA).

(i) Lagged reserve accounting (LRA)

Under this rule, the required reserve level \( R_R \) depends upon the banks' deposit liabilities in the previous period. Thus,

\[ (16) \quad R_R = k_L D_B; \quad (0 < k_L < 1), \]
where $k_L$ is a required reserve ratio, and $\bar{D}_B$ is the deposits in the previous period. Clearly, under this LRA rule, required reserves $R_R$ are predetermined, and hence perfectly independent of any adjustments made in the money market during the current period.

(ii) Contemporaneous reserve accounting (CRA)

The CRA rule bases reserve requirements on the current level of bank deposits. Specifically, denoting the required reserve ratio as $k_C$, $R_R$ is given by (17).

\begin{equation}
R_R = k_C \bar{D}_B
= -k_C d_2 r + k_C u_D; \quad (0 < k_C < 1).
\end{equation}

Under this CRA rule, adjustments in the money markets made during the current period will influence required reserves $R_R$ through variations in the interest rate $r$.

(iii) Marginal reserve accounting (MRA)

Under the MRA rule, which has been proposed by Poole (1976), the banks are required to hold reserves according to the LRA, plus 100 per cent of the change in deposits from the previous period. That is,

\begin{equation}
R_R = k_M \bar{D}_B + (D_N - \bar{D}_B)
= -d_2 r + u_D - (1 - k_M) \bar{D}_B; \quad (0 < k_M < 1),
\end{equation}

where $k_M$ is the required reserve ratio, and $\bar{D}_B$ is the previous period’s deposits.

(F) Operating targets of the central bank

In our model, there are three candidates for the central bank’s operating target; i.e., the amount of the inter-bank money market instrument held by the central bank $C_C$, total supply of high-powered money $R_C$, and the inter-bank money market rate $i$. The variable chosen as the operating target is fixed by the central bank at a certain level, so that it must be regarded as an exogenous variable. Since our model is substantially changed by which variables are considered exogenous, specification of the central bank’s operating target is critical in the following discussions.

When the central bank chooses $C_C$ as its operating target, the following equation (19) holds:

\begin{equation}
C_C = \bar{C}_C.
\end{equation}

In this case, total supply of high-powered money $R_C$ is endogenously determined in the money markets, because the central bank is assumed to supply its loans $B_C$ accommodatingly.

When total supply of high-powered money $R_C$ is the operating target for the central bank, $C_C$ becomes an endogenous variable, because in order to attain target level of $R_C$ the central bank has to adjust $C_C$ to offset changes in its loans passively supplied to the private-banks $B_C$. Therefore, in this case,

\[ R_C = C_C + B_C = C_C + b_1 i + b_2 r + u_B, \]

or rearranging this equation,

\begin{equation}
C_C = R_C - b_1 i - b_2 r + u_B.
\end{equation}

When the interbank money market rate $i$ is chosen as the operating target, both $C_C$ and
$R_C$ are endogenous. The central bank adjusts $C_C$ so as to always attain target level of the interest $i$; therefore the following equation holds.

$$i = i.$$ 

As adjustments in $C_C$ cannot be devoted to pursuing a fixed level of $R_C$, total supply of high-powered money $R_C$ is also endogenously determined in the money markets.

### III. Stability Implications of Alternative Reserve Accounting Rules

Our money market model determines equilibrium values of money supply $M$, and interest rates of the money markets when both a specific reserve accounting rule and the central bank’s operating target are given. Strictly speaking, our model is completed by adding an equation concerning the reserve accounting rule (any one of (16), (17), or (18)) and a condition on the operating target (any one of (19), (20), or (21)) to the money market equilibrium conditions (13), (14), and (15).

An equilibrium value of each money market variable can be represented by a linear function of exogenous disturbance terms $u_E, \ldots, u_D$, and other exogenous variables. In this linear function, the coefficient on each disturbance term indicates responsiveness of the money market variable to that disturbance. That is, the smaller is the absolute value of the coefficient, the less sensitive the variable is to the disturbance. We may conclude that the money markets are stable as the money market variables such as $M$ are less responsive to exogenous disturbances. In this section, we investigate implications of different reserve accounting rules with respect to stability of the money markets, and compare influences exerted on that stability by the choice of the operating target.

**(A) The case of $C_C$ as the operating target**

When the central bank chooses the amount of $C_C$ as its operating target, $C_C$ is fixed exogenously by the central bank, and thus, total supply of high-powered money must be regarded as endogenous because of accommodating supply of central bank loans. In this case, endogenous variables in our money market model are interest rates of both the inter-bank and the general money markets $i, r$, money supply $M$, and total supply of high-powered money $R_C$. These are determined under the respective reserve accounting rules by the following matrix relations.

**(i) Under the LRA rule;**

\[
\begin{pmatrix}
 b_1 & b_2 + c_2 & 0 & 0 \\
 a_1 & -a_2 & 0 & 0 \\
 0 & c_2 + d_2 & 1 & 0 \\
 b_1 & b_2 & 0 & -1
\end{pmatrix}
\begin{pmatrix}
 i \\
 r \\
 M \\
 R_C
\end{pmatrix}
= 
\begin{pmatrix}
 k_L \delta_B - \tilde{C}_C + u_E - u_B + u_R \\
 -\tilde{C}_C + u_C \\
 u_R + u_D \\
 -\tilde{C}_C - u_B
\end{pmatrix}
\]

\[^7\text{We shall not consider exogenous variables other than those disturbance terms } u_E, \ldots, u_D \text{ because they do not play any important role in our analysis.}\]
(ii) Under the CRA rule:

\[
\begin{pmatrix} b_1 & b_2 + c_2 + kcd_2 & 0 & 0 \\ a_1 & -a_2 & 0 & 0 \\ 0 & c_2 + d_2 & 1 & 0 \\ b_1 & b_2 & 0 & -1 \end{pmatrix} \begin{pmatrix} i \\ r \\ M \\ R_C \end{pmatrix} = \begin{pmatrix} -\bar{C}_C + u_E - u_B + u_R + kCU_D \\ -\bar{C}_C + u_C \\ u_R + u_D \\ -\bar{C}_C - u_B \end{pmatrix}
\]

(iii) Under the MRA rule:

\[
\begin{pmatrix} b_1 & b_2 + c_2 + d_2 & 0 & 0 \\ a_1 & -a_2 & 0 & 0 \\ 0 & c_2 + d_2 & 1 & 0 \\ b_1 & b_2 & 0 & -1 \end{pmatrix} \begin{pmatrix} i \\ r \\ M \\ R_C \end{pmatrix} = \begin{pmatrix} -\bar{C}_C + u_E - u_B + u_R + u_D \\ -\bar{C}_C + u_C \\ u_R + u_D \\ -\bar{C}_C - u_B \end{pmatrix}
\]

From the above relations we can derive responsiveness of each money market variable to disturbances under the respective reserve accounting rules, and these results are summarized in Table 1. For example, the first row of this table shows response of the inter-bank money market rate \( i \) to each disturbance term under the LRA rule.

(B) The case of \( R_C \) as the operating target

In this case, equilibrium conditions of the money markets can be described by equations (13), (14), (15), (20), and any one of (16)–(18) with endogenous variables of \( i \), \( r \), \( M \), and \( C_C \).

**Table 1. Responsiveness of Money Market Variables to Exogenous Disturbances under Alternative Reserve Accounting Rules: The Case of \( C_C \) as the Target**

<table>
<thead>
<tr>
<th></th>
<th>( u_E )</th>
<th>( u_B )</th>
<th>( u_C )</th>
<th>( u_R )</th>
<th>( u_D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( LRA )</td>
<td>( \frac{a_2}{A_1} )</td>
<td>( \frac{a_2}{A_1} )</td>
<td>( \frac{a_2}{A_1} )</td>
<td>( \frac{a_2}{A_1} )</td>
<td>0</td>
</tr>
<tr>
<td>( i )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{kca_d}{A_1+kca_d} )</td>
</tr>
<tr>
<td>( MRA )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
</tr>
<tr>
<td>( r )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{a_2}{A_1+kca_d} )</td>
<td>( \frac{kca_d}{A_1+kca_d} )</td>
</tr>
<tr>
<td>( MRA )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
<td>( \frac{a_2}{A_1+a_d} )</td>
</tr>
<tr>
<td>( LRA )</td>
<td>( \frac{a_1}{A_1} )</td>
<td>( \frac{a_1}{A_1} )</td>
<td>( \frac{-b_1}{A_1} )</td>
<td>( \frac{a_1}{A_1} )</td>
<td>0</td>
</tr>
<tr>
<td>( M )</td>
<td>( \frac{a_1}{A_1+kca_d} )</td>
<td>( \frac{a_1}{A_1+kca_d} )</td>
<td>( \frac{a_1}{A_1+kca_d} )</td>
<td>( \frac{a_1}{A_1+kca_d} )</td>
<td>( \frac{a_1}{A_1+kca_d} )</td>
</tr>
<tr>
<td>( MRA )</td>
<td>( \frac{a_1}{A_1+a_d} )</td>
<td>( \frac{a_1}{A_1+a_d} )</td>
<td>( \frac{a_1}{A_1+a_d} )</td>
<td>( \frac{a_1}{A_1+a_d} )</td>
<td>( \frac{a_1}{A_1+a_d} )</td>
</tr>
<tr>
<td>( M )</td>
<td>( \frac{(c_2+d_2)a_1}{A_1} )</td>
<td>( \frac{(c_2+d_2)a_1}{A_1} )</td>
<td>( \frac{(c_2+d_2)a_1}{A_1} )</td>
<td>( \frac{(c_2+d_2)a_1}{A_1} )</td>
<td>( \frac{(c_2+d_2)a_1}{A_1} )</td>
</tr>
</tbody>
</table>

Note: \( a_1 = a_2b_2 + a_3b_1 + a_1c_2 \).
Thus, money market equilibria under the respective reserve accounting rules will be represented by the following relations.

(i) Under the LRA rule;

\[
\begin{pmatrix}
  b_1 & b_2 + c_2 & 0 & 1 \\
  a_1 & -a_2 & 0 & 1 \\
  0 & c_2 + d_2 & 1 & 0 \\
  b_1 & b_2 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
i \\
r \\
M \\
C
\end{pmatrix}
= \begin{pmatrix}
k_L \delta_e + u_E - u_B + u_R \\
uc \\
 u_R + u_D \\
\bar{R}_C - u_B
\end{pmatrix}
\]

(ii) Under the CRA rule;

\[
\begin{pmatrix}
  b_1 & b_2 + c_2 + k_cd_2 & 0 & 1 \\
  a_1 & -a_2 & 0 & 1 \\
  0 & c_3 + d_2 & 1 & 0 \\
  b_1 & b_2 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
i \\
r \\
M \\
C
\end{pmatrix}
= \begin{pmatrix}
u_e - u_B + u_R + k_cu_D \\
uc \\
 u_R + u_D \\
\bar{R}_C - u_B
\end{pmatrix}
\]

(iii) Under the MRA rule;

\[
\begin{pmatrix}
  b_1 & b_2 + c_2 + d_2 & 0 & 1 \\
  a_1 & -a_2 & 0 & 1 \\
  0 & c_2 + d_2 & 1 & 0 \\
  b_1 & b_2 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
i \\
r \\
M \\
C
\end{pmatrix}
= \begin{pmatrix}
-(1 - k_M) \delta_e + u_E - u_B + u_R + u_D \\
uc \\
 u_R + u_D \\
\bar{R}_C - u_B
\end{pmatrix}
\]

Table 2. Responsiveness of Money Market Variables to Exogenous Disturbances under Alternative Reserve Accounting Rules: The Case of $R_C$ as the Target

<table>
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<tr>
<th></th>
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<th>$u_D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRA</td>
<td>$\frac{a_2+b_2}{A_2}$</td>
<td>$\frac{c_2}{A_2}$</td>
<td>$\frac{c_2}{A_2}$</td>
<td>$\frac{a_2+b_2}{A_2}$</td>
<td>0</td>
</tr>
<tr>
<td>$i$</td>
<td>$\frac{a_2+b_2}{A_2}$</td>
<td>$\frac{c_2+k_c d_2}{A_2}$</td>
<td>$\frac{c_2+k_c d_2}{A_2}$</td>
<td>$\frac{a_2+b_2}{A_2}$</td>
<td>$\frac{(a_2+b_2)k_c}{A_2}$</td>
</tr>
<tr>
<td>CRA</td>
<td>$\frac{a_2+(a_1-b_1)k_c d_2}{A_2}$</td>
<td>$\frac{A_2+(a_1-b_1)k_c d_2}{A_2}$</td>
<td>$\frac{A_2+(a_1-b_1)k_c d_2}{A_2}$</td>
<td>$\frac{A_2+(a_1-b_1)k_c d_2}{A_2}$</td>
<td>$\frac{A_2+(a_1-b_1)k_c d_2}{A_2}$</td>
</tr>
<tr>
<td>MRA</td>
<td>$\frac{1}{c_2}$</td>
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<td>0</td>
<td>$\frac{1}{c_2}$</td>
<td>0</td>
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</tbody>
</table>

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<th>$u_C$</th>
<th>$u_R$</th>
<th>$u_D$</th>
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<td>0</td>
<td>$\frac{1}{c_2}$</td>
<td>0</td>
</tr>
<tr>
<td>$r$</td>
<td>$\frac{c_2+k_c d_2}{c_2}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{c_2+k_c d_2}$</td>
<td>$\frac{k_c}{c_2+k_c d_2}$</td>
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<tr>
<td>CRA</td>
<td>$\frac{1}{c_2+k_c d_2}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{c_2+k_c d_2}$</td>
<td>$\frac{1}{c_2+k_c d_2}$</td>
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<td>$\frac{1}{c_2+k_c d_2}$</td>
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<th>$u_C$</th>
<th>$u_R$</th>
<th>$u_D$</th>
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</thead>
<tbody>
<tr>
<td>LRA</td>
<td>$\frac{c_2+d_2}{c_2}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{(1-k_c)d_2}{c_2+k_c d_2}$</td>
<td>$\frac{(1-k_c)c_2}{c_2+k_c d_2}$</td>
</tr>
<tr>
<td>$M$</td>
<td>$\frac{c_2+d_2}{c_2+k_c d_2}$</td>
<td>0</td>
<td>0</td>
<td>$\frac{(1-k_c)c_2}{c_2+k_c d_2}$</td>
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<td>$\frac{1}{c_2+k_c d_2}$</td>
<td>$\frac{1}{c_2+k_c d_2}$</td>
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</tbody>
</table>

Note: $A_e=(a_1-b_1)c_2$. 


Table 2 shows response of each of the money market variables $i$, $r$, and $M$ to disturbances under the respective reserve accounting rules.

(C) The case of $i$ as the operating target

When the central bank chooses the inter-bank money market rate $i$ as its operating target, both $C_c$ and $R_c$ must be adjusted endogenously so as to secure the target level of the interest rate $i$. Thus, equilibrium conditions for the money markets under each rule of reserve accounting are described by the following relations respectively.

(i) Under the LRA rule;
\[
\begin{pmatrix}
 b_2 + c_2 & 0 & 1 & 0 \\
-a_2 & 0 & 1 & 0 \\
 c_2 + d_2 & 1 & 0 & 0 \\
 b_2 & 0 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
 r \\
 M \\
 C_c \\
 R_c
\end{pmatrix}
= 
\begin{pmatrix}
 k_L \bar{B} - b_1 i + u_E - u_B + u_R \\
 -a_1 i + u_C \\
 u_R + u_D \\
 -b_1 i - u_B
\end{pmatrix}
\]

(ii) Under the CRA rule;
\[
\begin{pmatrix}
 b_2 + c_2 + k_c d_2 & 0 & 1 & 0 \\
-a_2 & 0 & 1 & 0 \\
 c_2 + d_2 & 1 & 0 & 0 \\
 b_2 & 0 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
 r \\
 M \\
 C_c \\
 R_c
\end{pmatrix}
= 
\begin{pmatrix}
 -b_1 i + u_E - u_B + u_R + k_c u_D \\
 -a_1 i + u_C \\
 u_R + u_D \\
 -b_1 i - u_B
\end{pmatrix}
\]

(iii) Under the MRA rule;
\[
\begin{pmatrix}
 b_2 + c_2 + d_2 & 0 & 1 & 0 \\
-a_2 & 0 & 1 & 0 \\
 c_2 + d_2 & 1 & 0 & 0 \\
 b_2 & 0 & 1 & -1
\end{pmatrix}
\begin{pmatrix}
 r \\
 M \\
 C_c \\
 R_c
\end{pmatrix}
= 
\begin{pmatrix}
 -b_1 i + u_E - u_B + u_R + u_D \\
 -a_1 i + u_C \\
 u_R + u_D \\
 -b_1 i - u_B
\end{pmatrix}
\]

We can determine responses of the money market variables $r$ and $M$ to disturbances under the respective reserve accounting rules from these relations. Table 3 summarizes the results.

Some propositions concerning short run stability of the money markets can be derived from Tables 1, 2, and 3:

1. Regardless of choice of the central bank's operating target, money supply $M$ is least responsive to disturbances arising in the money markets under the MRA rule. In particular, under the MRA rule money supply $M$ is independent from all disturbances except for $u_E$ (disturbance in banks' demand for reserves), when total supply of high-powered money $R_c$ is chosen as the operating target. We may conclude from this result that, if short run stability of the money supply $M$ is to be given highest priority in the central bank's monetary management, the MRA rule should be adopted.

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8 Exceptions exist. In the case in which the central bank's operating target is either $C_c$ or $i$, responsiveness of money supply to $u_B$ will becomes lower under the LRA rule.
2. Regardless of the central bank’s operating target, the LRA rule can prevent disturbances in the public’s demand for deposits \( u_D \) from affecting money market rate \( i \). However, when disturbances arise either in the banks’ behavior (i.e., \( u_E, u_C, u_B \)) or in the public’s demand for cash (i.e., \( u_R \)), money market interest rates \( i \) and \( r \) are most unstable under the LRA rule, and most stable under the MRA rule. Thus, which accounting rule makes the money market interest rates least responsive to a disturbance is essentially dependent upon where the disturbance takes place. Therefore, we cannot generally determine the most preferable accounting rule when highest priority is given to short run stability of money market rates.

3. In our model, the reserve control rule is more important as the operating target shifts from \( i \) to \( C_C \), and \( C_C \) to \( R_C \). It has been generally feared that more rigid reserve control will make money market interest rates more vulnerable to disturbances, and make them fluctuate more. This apprehension is most likely justified when the central bank’s monetary management shifts to more rigid control of high-powered money without alteration of the reserve accounting rule. For example, suppose that the central bank changes its operating target from \( i \) to \( C_C \), but that the reserve accounting rule remains LRA. Comparing responsiveness of the general money market rate \( r \) to disturbances under the \( i \) target case (the 1-st row of Table 3) with those under the \( C_C \) target case (the 4-th row of Table 3), it is clear that \( r \) is more responsive to all disturbances except for

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Note: \( A_3 = a_2 + b_2 + c_2 \).
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Similarly, when the operating target is changed from \( C_C \) to \( R_C \) under the LRA rule, the inter-bank money market rate \( i \) is likely to become more responsive to disturbances.\(^{13}\) However, a shift to more rigid reserve control need not make interest rates less stable if it is accompanied with conversion of reserve accounting rule from the LRA to the MRA. For example, it is indeterminate whether \( r \) is less stable in the \( C_C \) target cum MRA rule case (the 6-th row of Table 1) than in the \( i \) target cum LRA rule case (the 1-st row of Table 3); \( r \) is independent of disturbance \( u_D \) in the latter case, but not in the former.\(^{14}\) Furthermore, by comparing the 6-th row of Table 2 with the 1-st row of Table 3, we can prove that responsiveness of \( r \) to all disturbances except for \( u_D \) is smaller in the \( R_C \) target cum MRA rule case than in the \( i \) target cum LRA rule case. In short, even if more rigid reserve control is to be pursued by the central bank, the resultant cost of wider fluctuation of money market interest rates can be mitigated by conversion of the reserve accounting rule from the LRA to the MRA.

4. Generally speaking, the more sensitive are the public’s demands for cash and deposits to changes in the interest rate \( r \) (i.e., the larger \( c_2 \) and \( d_2 \) are), the less responsive the money market rates \( i \) and \( r \) become to exogenous disturbances.\(^{15}\) On the other hand, the above tables show that it is ambiguous whether increases in interest rate sensitivity of the public’s demand for cash and deposits will make the money supply more vulnerable to exogenous disturbances. Because development of money markets promotes substitutability between cash, deposits, and general money market instruments and thus makes the values of \( c_2 \) and \( d_2 \) rise, such development can be expected to make money markets more resistant to exogenous disturbances.\(^{16}\)

IV. Concluding Comments

Having already summed up implications derived from our model at the end of the last section, we comment here only on two reservations concerning our analysis.

1. Although we have not referred to it so far, the reverse lag accounting (RLA) rule proposed by Laurent (1979) has attracted some attention among economists.\(^{17}\) Under the RLA rule, banks’ required reserves are determined by an identical formula to the CRA, but the reserves the banks can use to meet these requirements are those they actually held in the

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\(^{12}\) Using \( a_i > b_i \), we can easily prove that \( a_i/A_i > 1/A_i > b_i/A_i \).

\(^{13}\) Only with respect to the disturbance \( u_B \), the result is ambiguous. If \( (a_i - b_i)a_i > A_i \), then responsiveness of \( i \) to \( u_B \) is smaller in the \( R_C \) target case than in the \( C_C \) target case.

\(^{14}\) If \( d_2 \) is large enough to ensure \( d_2 > (a_i - b_i)a_i/A_i \), then \( a_i/(A_i + a_i d_2) < 1/A_2 \). In this case, as the relation \( b_i/(A_i + a_i d_2) < 1/A_2 \) always holds, the interest rate \( r \) is less responsive to all disturbances except for \( u_D \) in the former case.

\(^{15}\) This is not true of the case in which \( C_C \) is the operating target and, at the same time, the disturbance arises in \( u_C \). It is clear from Table 1 that in the \( C_C \) target case coefficients of the money market rates on \( u_C \) will become larger as either \( c_2 \) or \( d_2 \) rises.

\(^{16}\) As is well known, rapid increase in substitutability among monetary assets will hinder efficient monetary control because it will become very difficult for the central bank to predict its policy impact accurately. We cannot investigate this important topic here.

\(^{17}\) For example, feasibility of this RLA rule is at some length discussed by the Chancellor of the Exchequer (1980).
preceding period. Thus, reserves supplied in any given period are predetermined, and current deposits must change so that required reserves are brought into line with the fixed reserves supplied; i.e.,

\[ R_R = k_R D_N \]
\[ = -k_R d_R + k_R u_D; \quad (0 < k_R < 1), \]

where \( R_R \) is the amount of reserves actually held by the banks in the preceding period.

Under the RLA rule, our money market model is given one additional constraint represented by the above (22), so if our model is to be consistent, one more endogenous variable will have to be added. Perhaps the simplest way is to regard \( R_B \), reserves actually held by banks in the current period, as a residual term endogenously determined so as to balance demand and supply of high-powered money in the current period. However, as this \( R_B \) fundamentally limits the amount of the banks’ deposit liabilities in the next period, it is quite natural to consider explicitly the banks’ demand for current reserves \( R_B \). Thus, banks’ demand for current reserves should be assumed to depend on planning with respect to deposit liabilities in the next period, which is a factor we have not considered.

At any rate, a money market model using the RLA rule would be so different from those under the other rules examined in this paper that straightforward comparison between equilibrium under the RLA and those under the other rules is impossible. Implications of the RLA rule would have to be investigated in a somewhat different context.

2. We have throughout assumed that portfolio behavior of the respective private sectors does not change with alteration of the reserve accounting rule. Though this assumption has been useful for comparing alternative reserve accounting rules, it seems rather difficult to justify. Particularly, private banks behavior represented by equations (4), (5), etc., may be so sensitive to alterations of the reserve accounting rule that it is not permissible to neglect ‘rational’ responses to the rule changes. In this respect, we must frankly acknowledge an insufficiency. Our discussions will have to be supplemented by careful analysis about possible changes in behavior patterns of private sectors, in particular private banks, induced by alterations of reserve accounting rules.
REFERENCES


