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Takashi Kano

Hitotsubashi University

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Hitotsubashi Institute for Advanced Study, Hitotsubashi University
2-1, Naka, Kunitachi, Tokyo 186-8601, Japan
tel:+81 42 580 8668 http://hias.hit-u.ac.jp/

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Takashi Kano
Graduate School of Economics
Hitotsubashi University
Naka 2-1, Kunitachi, Tokyo
186-8601, JAPAN
Email: tkano@econ.hit-u.ac.jp

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Abstract

This study examines the exchange rate implications of trend inflation within a two-country New Keynesian (NK) model. An NK Phillips curve generalized by trend inflation makes the inflation differential smoother, more persistent, and less sensitive to the real exchange rate. A Bayesian analysis with post-Bretton Woods data for Canada and the U.S. shows that the model's equilibrium, which relies on Taylor rules with a persistent trend inflation shock and strong policy inertia, mimics empirical regularities in exchange rates that are difficult to reconcile within a standard NK model. Trend inflation helps explain the empirical puzzles of the exchange rate dynamics.

Key Words: Real and Nominal Exchange Rates; Trend Inflation; New Keynesian Model; Bayesian analysis
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1. Introduction

The empirical reality of exchange rates still confounds international finance researchers. Real exchange rates are persistent and volatile. Near random-walk nominal exchange rates have no significant dependence on current or past information on macroeconomic fundamentals. The real and nominal exchange rates closely move together. These stylized facts jointly refute the theoretical challenges of canonical open-economy models with price stickiness. The elucidation of the complicated life of exchange rates in the literature on New Keynesian (NK) open-economy models is still far from satisfactory.[1]

In this study, I examine the crucial role of trend inflation in exchange rate dynamics.[2] My investigation is based on an otherwise standard two-country NK model, as in Benigno (2004) and Engel (2014, 2019), equipped with the Calvo-type time-dependent pricing behaviors of monopolistic competitive final good firms conducting pricing-to-market strategies in terms of local currencies, Taylor rule-type monetary policies with high interest-rate smoothing, and incomplete international financial markets with debt-elastic risk premiums. I then extend the standard NK model by allowing for trend inflation that fluctuates stochastically around a positive long-run mean in the two countries. To the best of my knowledge, this study is the first attempt to embed trend inflation in a two-country NK model explicitly and extract its implications for exchange rate dynamics.[3] Such a natural modification of the canonical two-country NK model regarding the low-frequency property of inflation significantly impacts exchange rate dynamics.

Following Ireland (2007) and Cogley et al. (2010), I specify the source of the fluctuations in trend inflation in this model by using a time-varying inflation target set by the central banks of each country within the Taylor rule.[4] As in Cogley et al. (2010), the inflation target obeys a persistent stochastic process with a positive long-run mean. On the one hand, a positive long-run mean of trend inflation is undebatable

[1] Burstein and Gopinath (2014) and Engel (2014) provide excellent surveys of the recent empirical findings on international real and nominal relative prices.

[2] Ascari and Sbordone (2014) offer an excellent summary of recent research on trend inflation, focusing on the U.S. inflation dynamics.

[3] Zhao (2022) also examines the implications of trend inflation in an open economy, focusing mainly on the welfare implication of positive long-run inflation in a calibrated two-country NK model without stochastic variations in trend inflation. Zhao (2022), however, does not investigate the effects of a time-varying stochastic inflation target on exchange rate dynamics.

[4] Haque (2022) also investigates the role of a time-varying inflation target in the Taylor rule for the postwar U.S. inflation dynamics with the estimated closed-economy NK model with positive long-run trend inflation.

for post-war data in the world economy. On the other hand, there are three rationales behind the stochastic variation in the inflation target. First, as Cogley et al. (2010) discuss, a central bank only imperfectly knows the actual economic structure, and its learning process generates fluctuations in the inflation target. Second, private economic agents may suffer from imperfect information about inflation targets. Third, a persistent shock to the inflation target is observationally equivalent to a persistent monetary policy disturbance in the Taylor rule with high interest-rate smoothing, the importance of which the literature on monetary policy frequently emphasizes. Coibion and Gorodnichenko (2012) provide empirical evidence that the high persistence observed in the actual data of the short-run nominal interest rate in the U.S. stems largely from policy inertia, but not from the exogenous persistence of the monetary policy disturbance in the Taylor rule. Importantly, their empirical result suggests that the misspecification problem could lead to a high estimate of the serial correlation of monetary policy disturbances unless the Taylor rule correctly specifies the time-varying inflation target. Therefore, including a persistent shock to the inflation target in the model might resolve the Taylor rule's misspecification problem and better identify the critical role of monetary policy in exchange rate dynamics.

Allowing for persistent trend inflation with a positive long-run mean within a two-country NK model produces fundamentally richer equilibrium dynamics of real and nominal exchange rates. A positive long-run mean of trend inflation substantially alters the shape of the log-linearized New Keynesian Phillips curve (NKPC) in each country, as Cogley and Sbordone (2008) and Ascari and Sbordone (2014) demonstrate. In the NKPC generalized by a positive long-run trend inflation (hereafter, generalized NKPC, or GNKPC), the current inflation rate follows a higher-order expectational difference equation, which results in a persistent and smooth movement of each country's inflation rate with persistent trend inflation variations. High policy inertia in the Taylor rule yields near-non-stationary real interest-rate dynamics. With the real uncovered interest parity (RUIP) condition, Engel and West's (2005) proposition generates near-random-walk real exchange rate movements with high volatility. Hence, the proposed model reconciles with the persistence and volatility puzzle of the real exchange rate (Chari et al. 2002).

In this study, I draw Bayesian posterior inferences of the proposed log-linearized asymmetric two-country NK model augmented with trend inflation based on post-Bretton Woods (BW) Canada-U.S. data.

The data strictly require positive long-run trend inflation for the cross-equation restrictions of the trend inflation (TI) model: the overall data fit of the TI model outperforms that of a canonical model restricted by zero long-run trend inflation in terms of marginal likelihoods.

The estimated TI model generates a highly persistent and volatile real exchange rate, which I approximate as a near random-walk process. As recent studies on real exchange rate dynamics emphasize, including those by Cheung and Lai (2000), Steinsson (2008), Iversen and Söderström (2014), Burstein and Gopinath (2014), and Carvalho et al. (2018), the impulse response function (IRF) of the real exchange rate to a reduced-form disturbance is hump-shaped with a peak around a year, and the corresponding half-life of the real exchange rate is longer than four years. Decomposing the IRF and forecast error variance into structural shocks reveals that a shock to the Canadian trend inflation, even with a smaller standard deviation than those of other structural shocks, dominates the amplification and propagation mechanism of the real exchange rate by generating a persistent hump-shaped impulse response. Thus, the estimated TI model successfully reconciles with a significantly delayed overshooting of the exchange rate to a monetary easing shock identified as a surprise change in the long-run inflation target, as Eichenbaum and Evans (1995) and Bouakez and Normandin (2010) observe.

The hump-shaped real exchange rate response to a trend inflation shock stems from the theoretical outcome that the IRF of the real interest rate differential between the two countries flips its sign in several periods after the impact, as Steinsson (2008) demonstrates. The RUIP condition implies that the real exchange rate appreciates in the impact period, keeps appreciating for some periods, and then starts depreciating. This conditional covariance between the real exchange rate and the real interest rate differential explains two important empirical facts. First, the purchasing power parity (PPP) puzzle (Rogoff, 1996) can be reconciled with the strong hump-shaped IRF of the real exchange rate to an inflation target shock. Second, an inflation surprise generated by a positive shock to the inflation target corresponds to simultaneous currency appreciation (Clarida and Waldman, 2008).

The estimated TI model also replicates the empirical regularities of the joint dynamics of real and nominal exchange rates established by Mussa (1986) and investigated by Monacelli (2004). In particular, the real currency return (RCR) and nominal currency return (NCR) simulated by the estimated TI model are almost perfectly correlated, and the standard deviation ratio of the former to the latter is close to one under flexible exchange rate regimes.

The remainder of this paper is organized as follows. Section 2 introduces the proposed model. Section 3 reports the results of the Bayesian posterior inferences of the proposed TI model. Section 4 concludes.

2. The Model

To extract the theoretical implications of trend inflation for exchange rate dynamics as clearly as possible, I investigate a basic version of a two-country NK model. Because the NK model here is standard, except for trend inflation, I supply the full derivation and detailed explanation of the model in the online appendix. I focus mainly on deriving the model's primary propagation mechanism, that is, the open-economy GNKPC.

There are home (h) and foreign (f) countries in this model. Throughout this paper, any variable with an asterisk ($*$) corresponds to a foreign variable, while a variable without it denotes the home counterpart. To avoid redundancy in the model's description, I will describe only the home country's problem below. The same argument applies to the foreign country counterpart.

2.1. The household

The representative household maximizes the lifetime expected utility function subject to the budget constraint by supplying labor to the competitive domestic labor market elastically and forming external consumption habits. The representative household can access home and foreign state-non-contingent bond markets to smooth consumption surplus $H_t \equiv C_t - h\bar{C}_{t-1}$ across periods, where C_t is consumption and $h\bar{C}_{t-1}$ is the stock of external consumption habits with habit parameter $h \in (0, 1)$. The household's problem yields the Euler equation, utility-based uncovered interest parity condition, and intratemporal optimality conditions as first-order necessary conditions (FONCs).

Home aggregate consumption consists of home and foreign consumption bundles with constant substitution elasticity subject to home bias. The home and foreign consumption bundles comprise the continuum of domestically produced final goods. Dixit-Stiglitz-type aggregators provide home and foreign consumption bundles. The static cost minimization problem then yields the household's demand functions for individual home- and foreign-produced final goods.

2.2. The final goods firms

Facing the home and foreign representative households' demand functions, the home final good firms act as identical monopolistically competitive price setters. When setting its current optimal price, each firm follows a Calvo-type time-dependent pricing strategy, in which a firm cannot reset its optimal price with probability $\mu \in (0, 1]$. Moreover, to generate endogenous fluctuations in the real exchange rate in this model, I assume that all firms adopt a local currency pricing (LCP) strategy, as in Betts and Devereux (1996, 2000): each firm sets its optimal prices differently in the two countries in terms of local currencies.[5]

Under such a local-currency Calvo pricing strategy, the objective function of a home firm is

$$\max_{\mathbf{P}_{h,t},\mathbf{P}^*_{h,t}} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left\{ \left(\frac{\mathbf{P}_{h,t}}{P_{h,t+i}} \right) - mc_{t+i} \right\} \left(\frac{\mathbf{P}_{h,t}}{P_{h,t+i}} \right)^{-\zeta} C_{h,t+i}$$
$$+ E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left\{ \left(\frac{S_{t+i}\mathbf{P}^*_{h,t}}{P_{h,t+i}} \right) - mc_{t+i} \right\} \left(\frac{\mathbf{P}^*_{h,t}}{P^*_{h,t+i}} \right)^{-\zeta} C^*_{h,t+i},$$

where $\mathbf{P}_{h,t}$, $\mathbf{P}^*_{h,t}$, $P_{h,t}$, $P^*_{h,t}$, S_t, $C_{h,t}$, $C^*_{h,t}$, mc_t, and Γ_t are the optimal price of the home-produced goods in the home country, optimal price of the home-produced goods in the foreign country, home aggregate price over the home-produced final goods, foreign aggregate price over the home-produced final goods, nominal bilateral exchange rate, home consumption bundle comprising the home-produced final goods, foreign consumption bundle comprising home-produced final goods, real marginal cost of the home firm, and home real stochastic discount factor, respectively. In particular, the parameter ζ is the price elasticity of demand of the final home-produced good. The home country's real stochastic discount factor for future $t + i$ period is given by $\Gamma_{t+i} \equiv \beta^i (H_t/H_{t+i})$.

[5] As in the standard two-country NK models of Benigno (2004) and Engel (2014,2019), the sticky LCP is the fundamental mechanism for real exchange rate fluctuations in this model.

The FONCs for the optimal prices the home firm sets for the home and foreign markets are

$$\mathbf{P}_{h,t} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left(\frac{1}{P_{h,t+i}} \right)^{1-\zeta} C_{h,t+i} = \frac{\zeta}{\zeta - 1} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} mc_{t+i} \left(\frac{1}{P_{h,t+i}} \right)^{-\zeta} C_{h,t+i},$$

$$\mathbf{P}^*_{h,t} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left(\frac{S_{t+i} P^*_{h,t+i}}{P_{h,t+i}} \right) \left(\frac{1}{P^*_{h,t+i}} \right)^{1-\zeta} C^*_{h,t+i} = \frac{\zeta}{\zeta - 1} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} mc_{t+i} \left(\frac{1}{P^*_{h,t+i}} \right)^{-\zeta} C^*_{h,t+i},$$

respectively. Given the optimal prices, the price aggregators $P_{h,t}$ and $P^*_{h,t}$ follow the laws of motion:

$$P_{h,t}^{1-\zeta} = (1-\mu)\mathbf{P}_{h,t}^{1-\zeta} + \mu P_{h,t-1}^{1-\zeta}, \quad \text{and} \quad P_{h,t}^{*1-\zeta} = (1-\mu)\mathbf{P}_{h,t}^{*1-\zeta} + \mu P_{h,t-1}^{*1-\zeta}.$$

Each final good firm produces its product using only the labor input hired from the competitive domestic labor market. The production function of the home goods is $Y_t(z) = A_t L_t(z)$, where A_t is the labor productivity in the home country. In this case, the real marginal cost that the home-produced final good firms face is $mc_t = \frac{W_t}{A_t P_{h,t}}$, where W_t is the home competitive labor wage.

I assume that the logarithms of labor productivity, $\ln A_t$ and $\ln A^*_t$, are I(1). To guarantee a balanced growth path of this two-country model, I assume that the labor productivity differential $\ln a_t \equiv \ln A_t - \ln A^*_t$ is I(0). As Mandelman et al. (2011), Rabanal et al. (2011), and Ireland (2013) find, the I(1) labor productivity and stationary productivity differential jointly imply that the labor productivity of the home country must be cointegrated with that of the foreign country. Following Kano (2021), I assume that the home and foreign growth rates of labor productivity, $\ln \gamma_{A,t} \equiv \ln A_t - \ln A_{t-1}$ and $\ln \gamma^*_{A,t} \equiv \ln A^*_t - \ln A^*_{t-1}$, are generated by the error correction processes $\ln \gamma_{A,t} = \ln \gamma_A - \frac{\lambda}{2} \ln a_{t-1} + \epsilon_{A,t}$ and $\ln \gamma^*_{A,t} = \ln \gamma^*_A + \frac{\lambda}{2} \ln a_{t-1} + \epsilon^*_{A,t}$, where γ_A and γ^*_A are the drift terms and $\lambda \in [0, 1)$ is the speed of error correction. The error correction mechanism implies that the cross-country labor productivity differential is I(0) because $\ln a_t = \ln \gamma_A - \ln \gamma^*_A + (1-\lambda) \ln a_{t-1} + \epsilon_{A,t} - \epsilon^*_{A,t}$.[6] Importantly, if the adjustment speed λ is sufficiently close to zero, then the cross-country labor productivity differential can be realized near I(1).

2.3. Monetary policies with trend inflation

[6]As a result the two countries share the same unconditional mean of the labour productivity growth rate $\ln(\gamma_A \gamma^*_A)^{\frac{1}{2}}$.

The monetary policies of the two countries follow Taylor rules. The home country's central bank sets the short-term domestic nominal interest rate $(1+i_{h,t})$ depending on the past interest rate level $(1+i_{h,t-1})$, current inflation rate $\gamma_{\pi,t}$, detrended output level $y_t \equiv Y_t/A_t$, and nominal currency return S_t/S_{t-1}. As in the specifications of Ireland (2007) and Cogley et al. (2010), the central bank targets the time-varying trend inflation $\gamma_{\tau,t}$. The Taylor rule of the home country is

$$(1+i_{h,t}) = (1+i)^{1-\rho_i}(1+i_{h,t-1})^{\rho_i}\left[\left(\frac{\gamma_{\pi,t}}{\gamma_{\tau,t}}\right)^{a_\pi}(y_t)^{a_y}\left(\frac{S_t}{S_{t-1}}\right)^{\frac{a_s}{1-a_s}}\right]^{1-\rho_i}\exp(\epsilon_{i,t}),$$

where i is the deterministic steady-state value of the home nominal interest rate and $\rho_i \in (0,1)$ captures the degree of interest rate smoothing. The parameter $a_s \in [0,1)$ captures the degree of the managed exchange rate regime: the larger a_s is, the more stringent the managed regime is. If $a_s = 0$, then the model corresponds to a perfectly flexible exchange rate regime. If $a_s \to 1$, then the model is subject to a strict fixed nominal exchange rate regime with $S_t = 1$ for all t. If $a_s \in (0,1)$, then the model is characterized by a managed exchange rate regime, as specified by Monachelli (2004).

In this study, I allow the home trend inflation rate $\gamma_{\tau,t}$ to be stochastic with an exogenous AR(1) process in the logarithmic term:

$$\ln\gamma_{\tau,t} = (1-\rho_\tau)\ln\gamma_\tau + \rho_\tau\ln\gamma_{\tau,t-1} + \epsilon_{\tau,t},$$

where γ_τ is the long-run mean of the home trend inflation rate, $\rho_\tau \in [0,1)$ is the AR(1) root of the home trend inflation rate, and $\epsilon_{\tau,t}$ is the i.i.d. trend inflation shock.[7] As in the closed-economy NK model by Ascari and Sbordone (2014), the time-invariant long-run mean γ_τ makes the log-linearization exercise of this study tractable when characterizing the deterministic steady state.

I assume that the monetary policy disturbance $\epsilon_{i,t}$ is a serially uncorrelated i.i.d. shock. Coibion and Gorodnichenko (2012) provide empirical evidence that the high persistence observed in the actual short-run nominal interest rate data in the U.S. stems largely from the policy inertia caused by interest rate

[7]Ireland (2007) allows for an implicit time-varying inflation target with endogenous responses to supply shocks. He shows that in the US data the likelihood ratio test cannot reject the exogenous time-varying inflation target.

smoothing in the Taylor rule, but not from the exogenous persistence of the monetary policy disturbance, as Rudebusch (2002, 2006) claims. In particular, their empirical result suggests that the time-varying inflation target that the conventional Taylor rule misses could lead to a high estimate of the serial correlation of the monetary policy disturbance. Indeed, the persistent trend inflation $\gamma_{\tau,t}$ is indistinguishable from a persistent monetary policy disturbance if we set $\epsilon_{i,t}$ to $-(1-\rho_i)a_\pi \ln \gamma_{\tau,t}$. Following the above notion, I allow both interest rate smoothing and the time-varying inflation target in the Taylor rules.

2.4. Closing the model

To guarantee a stationary distribution of the two countries' net foreign asset positions within incomplete international financial markets, I allow for a debt-elastic risk premium in the interest rates faced only by the home country. The risk premium is given as an externality: the home household does not consider the effect of the debt position on the risk premium when maximizing its lifetime utility function. As in Bergin (2006) and Justiniano and Preston (2010), I set the risk premium subject to the exogenous risk premium shock $v_{r,t}$, which follows an AR(1) process, $v_{r,t} = \rho_r v_{r,t-1} + \epsilon_{r,t}$, where $\rho_r \in (0,1)$.

The home and foreign bond markets' equilibrium conditions require that the world net supply of nominal bonds be zero on a period-by-period basis along an equilibrium path. The home and foreign final-goods markets' equilibrium conditions depend on time-varying price dispersions in the final-good markets. The time-varying price dispersions represent resource costs. Given the output level, the higher the degree of price dispersion, the lower the amount allocated to consumption.

Because the model contains non-stationary components, I stochastically detrend the FONCs to characterize the unique deterministic steady state, as I show in detail in the accompanying appendix. To derive the corresponding linear rational expectations (LRE) models, I take a log-linear approximation of the stochastically detrended FONCs around the unique deterministic steady state. For the variable x_t, let $\hat{x}_t$ denote the percentage deviation from its deterministic steady-state value x; that is, $\hat{x}_t \equiv (x_t - x)/x$. $\tilde{x}_t$ is the deviation from the deterministic steady-state value; that is, $\tilde{x}_t \equiv x_t - x$.

2.5. GNKPC with trend inflation.

The online appendix provides the entire derivation of the GNKPC for the home good price inflation in the home country, $\Delta \ln P_{h,t}$, from the log-linearized FONC for the optimal price setting of the home final good firm. Let $\bar{\gamma} \equiv \gamma_\tau/(\gamma_A \gamma_A^*)^{\frac{1}{2}}$ denote the steady-state home gross inflation rate. With the definitions of the parameters, $\varphi_1 \equiv \beta\mu\bar{\gamma}^{\zeta-1} \in (0,1)$, $\varphi_2 \equiv \beta\mu\bar{\gamma}^{\zeta} \in (0,1)$, and $\varphi_0 \equiv \frac{1-\beta^{-1}\varphi_1}{\beta^{-1}\varphi_1}$, the corresponding GNKPC is

$$\Delta \ln P_{h,t} = \beta\bar{\gamma}E_t\Delta \ln P_{h,t+1} + \varphi_0(1-\varphi_2)\hat{mc}_t + \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}\sum_{i=1}^{\infty}\varphi_1^i E_t\Delta \ln P_{h,t+i}$$
$$- \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}\zeta\sum_{i=1}^{\infty}\varphi_1^i E_t\Delta \ln P_{t+i} - \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}\sum_{i=1}^{\infty}\varphi_1^i E_t\Delta(\hat{c}_{t+i} - \hat{h}_{t+i}) + \hat{v}_{p,t}^h, \quad (1)$$

where $\Delta \ln P_t$ is the home consumer price index (CPI) inflation rate, $\Delta\hat{c}_t$ is the change in the detrended consumption, and $\Delta\hat{h}_t$ is the change in the detrended consumption surplus. Therefore, as Cogley and Sbordone (2008) argue, GNKPC (1) depends on the higher-order leads of the home good price inflation as well as the home CPI inflation. The changes in consumption and consumption surplus affect the GNKPC through the final goods demand and stochastic discount factor.

Moreover, $\hat{v}_{p,t}^h$ is a reduced-form shock additive to the GNKPC, which follows an AR(1) process, $\hat{v}_{p,t}^h = \rho_p^h \hat{v}_{p,t-1}^h + \epsilon_{p,t}^h$, where $\rho_p^h \in (0,1)$. Steinsson (2008) interprets $\hat{v}_{p,t}^h$ as a mixture of different types of shocks, including labor supply shocks, government spending shocks, shocks to world demand, and cost-push shocks. His calibration study shows that $\hat{v}_{p,t}^h$ is the primary driver of the hump-shaped impulse response of the real exchange rate. The GNKPC shock $\hat{v}_{p,t}^h$, however, does not obey any cross-equation restrictions of the model and has no strict structural interpretation. As Lubik and Schorfheide (2006) argue, this nonstructural shock reflects the extent of model misspecification.

Note that the coefficients φ_0, φ_1, and φ_2 are given as nonlinear functions of the primitives β, μ, and $\bar{\gamma}$. In particular, higher steady-state inflation $\bar{\gamma}$ implies a smaller weight on the current real marginal cost $\hat{mc}_t$ and larger weights on the expected future home good price inflation $E_t\Delta \ln P_{h,t+i}$ and the expected future home CPI inflation $E_t\Delta \ln P_{t+i}$. This is because positive steady-state inflation affects the expected future profits of monopolistic competitive producers through its effect on the relative prices $P_{h,t+i}/P_{t+i}$ in the demand functions toward an infinite future. The higher the steady-state inflation, the larger the weight that producers place on expected future profits when they decide the current optimal price. Producers place a smaller weight on the current variable, such as the real marginal cost.

2.6. The open-economy GNKPC

The online appendix also provides the full derivation of the GNKPC for the home good price inflation in the foreign country, $\Delta \ln P^*_{h,t}$. To obtain an economic intuition for the role of the GNKPCs in the equilibrium of this two-country model, I tentatively assume that the two countries are symmetric in all structural parameters. Subtracting the GNKPC of $\Delta \ln P^*_{h,t}$ from that of $\Delta \ln P_{h,t}$ then provides the following GNKPC for the inflation differential between the two countries, $\Delta \ln P^d_t \equiv \Delta \ln P_t - \Delta \ln P^*_t$:

$$\begin{aligned}\Delta \ln P^d_t &= \beta\bar{\gamma} E_t \Delta \ln P^d_{t+1} + \varphi_0(1-\varphi_2)\hat{a}_t + \frac{\varphi_0\varphi_2(1-\varphi_1)}{\varphi_1}(\hat{q}_t - \hat{a}_t) \\ &\quad + \frac{\varphi_0(\varphi_1-\varphi_2)(1-\zeta)}{\varphi_1}\sum_{i=1}^{\infty}\varphi_1^i E_t \Delta \ln P^d_{t+i} - \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}\sum_{i=1}^{\infty}\varphi_1^i E_t \Delta \hat{c}^d_{t+i} \\ &\quad + \frac{\varphi_0(\varphi_1-\varphi_2)(1-\varphi_1)}{\varphi_1}\sum_{i=1}^{\infty}\varphi_1^i E_t(\hat{q}_{t+i} - \hat{a}_{t+i}) + \hat{v}^d_{p,t}, \quad (2)\end{aligned}$$

where $\hat{q}_t$ is the real exchange rate, $\Delta \hat{c}^d_{t+i}$ is the change in the cross-country differential of detrended consumption, and $\hat{v}^d_{p,t}$ is the cross-country differential of the GKNPC shocks.

An open-economy GNKPC (2) implies that the inflation differential becomes more persistent and less volatile in response to persistent trend inflation shocks, and less sensitive to the current real exchange rate than does the standard NKPC with $\bar{\gamma} = 1$. With positive steady-state inflation $\bar{\gamma} > 1$, the firms place more weight on future variables than current ones in its optimal price setting because of the implied higher discount factor φ_1. The inflation differential becomes less sensitive to the current variables, including the real exchange rate, but is more dependent on the present discounted values of the expected future variables. Therefore, only the persistent slow-moving components of the expected future variables matter for the inflation differential.[8]

[8]Under the condition $\bar{\gamma} = 1$, the GNKPC (2) is degenerated to the standard NKPC:

$$\Delta \ln P^d_t = \beta E_t \Delta \ln P^d_{t+1} + \kappa_\mu \hat{q}_t + \hat{v}^d_{p,t},$$

3. Bayesian posterior inferences with Canadian and U.S. data.

3.1. Data and prior distributions of the structural parameters

In this section, I empirically assess the proposed asymmetric two-country NK model with trend inflation by drawing formal Bayesian posterior inferences, as in Lubik and Schorfheide (2005) and Justiniano and Preston (2010).[9] Following the latter study, I analyze post-BW Canada-U.S. quarterly data, which contain nine time-series: Canadian and U.S. inflation rates, Canadian and U.S. real consumption growth rates, Canadian and U.S. hours worked, Canadian and U.S. short-term nominal interest rates, and the bilateral real exchange rate of the Canadian dollar per US dollar. The quarterly data cover Q1:1982 to Q1:2007. I use the first four quarters to initialize the Kalman filter. The appendix discusses the data sources and the construction.

Table 1 shows the prior distributions of the structural parameters of the model. The third, fourth, and fifth columns represent the corresponding density, mean, and standard deviation, respectively. These are fairly common across other studies that adopt Bayesian inference methods for dynamic stochastic general equilibrium (DSGE) models. I calibrate the subjective discount factor β to 0.990 for both countries and the debt elasticity of risk premium ψ to 0.00001 for Canada because these parameters are not well identified. Recognizing that the U.S. is a large foreign country, I also calibrate the U.S. home bias parameter τ^* to 0.999, which implies that U.S. households consume only the U.S. aggregate consumption basket.

Several parameters require further examination. I adopt normal prior densities for the drift parameters of the home and foreign labor productivities, γ_A and γ_A^*. I set the prior means of these parameters to the unconditional means of the Canadian and U.S. gross quarterly consumption growth rates, 1.0050 and 1.0061, along with an identical prior standard deviation of 0.0005. The prior means of the home and foreign steady-state gross inflation rates $\bar{\gamma}$ and $\bar{\gamma}^*$ stem from the unconditional means of the Canadian and U.S. gross inflation rates of 1.0072 and 1.0077, corresponding to 2.87% and 3.12%, respectively, at the annual

where $\kappa_\mu \equiv \frac{(1-\mu)(1-\beta\mu)}{\mu} \in (0,1)$. In this restrictive case, the current inflation differential depends on the one-period-ahead expected inflation differential and real exchange rate. This is the same representation of inflation differential dynamics as in Benigno (2004). The current inflation differential becomes sensitive to current changes in the real exchange rate.

[9] I thank Martin Uribe for his suggestion for the Bayesian estimation of the fully-fledged asymmetric two-country NK model with trend inflation.

rates. The model's crucial parameters are the unconditional means of the home and foreign unobservable trend inflation, γ_τ and γ_τ^*, respectively. For each country, I construct the prior mean of this parameter by taking the sum of the unconditional mean of the steady-state inflation rate and the unconditional mean of the consumption growth rate. The resulting prior density of γ_τ is normal with a mean of 1.0122 and a small standard deviation of 0.0005, whereas the prior density of γ_τ^* is normally distributed with a mean of 1.0138 and a standard deviation of 0.0005.

Recall that the NK literature conventionally calibrates the price elasticity of demand by fitting the NK model to a markup rate of approximately 10.00-12.00%. GNKPC (1) implies that the steady-state home markup rate is the inverse of the real marginal cost:

$$mc = \left(\frac{\zeta - 1}{\zeta}\right)\left(\frac{1 - \beta\mu\bar{\gamma}^{\zeta}}{1 - \beta\mu\bar{\gamma}^{\zeta-1}}\right)\left(\frac{1 - \mu\bar{\gamma}^{\zeta-1}}{1 - \mu}\right)^{\frac{1}{1-\zeta}}.$$

Given the calibrated value of β and the prior means of μ, γ_τ, and γ_A, a price elasticity of demand ζ equal to 22.00 provides a steady-state home markup rate of 8.66%. The same calculation with the foreign counterpart of ζ^* equal to 22.00 yields a steady-state foreign markup rate of 11.34%. Although the implied Canadian markup rate is slightly lower than the conventional value, I use an identical gamma density with a mean of 22.00 and a standard deviation of 1.000 as the prior distributions of ζ and ζ^*.

I construct the prior distributions of the parameters of the home and foreign Taylor rules as follows. I adopt identical gamma densities with a mean of 3.500 and a standard deviation of 0.500 for the inflation gap parameters a_π and a_π^*, identical gamma densities with a mean of 0.100 and a standard deviation of 0.010 for the output gap parameters a_y and a_y^*, and an identical beta density with a mean of 0.900 and a standard deviation of 0.200 for the interest rate smoothing parameters ρ_i and ρ_i^*. The prior means of a_π, a_π^*, ρ_i, and ρ_i^* are slightly larger than the conventional values under the specification for zero-trend inflation in the literature. Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011) show in a closed-economy NK model that high values of these Taylor rule parameters are necessary for equilibrium determinacy with positive trend inflation. I further allow for asymmetric policy attitudes toward nominal currency returns between the two central banks. On the one hand, I exploit the beta density with a mean of 0.250 with a standard deviation of 0.200 as the prior density of the Bank of Canada's policy sensitivity to the nominal currency return a_s. On the other hand, I use a beta density with a mean of 0.050 and a standard deviation of 0.010 as the prior density of the U.S. counterpart a_s^*, as the central bank of the large country, the Fed, puts a negligible weight on the nominal currency return when setting the short-term policy rate.

Lastly, I determine the prior distributions of the 11 exogenous impulses as follows. I assume that all structural shocks are normally distributed. The error correction speed of the cointegrated labor productivity process, λ, has a beta prior distribution with a mean of 0.010 and a standard distribution of 0.010 to reflect slow cross-country productivity diffusion. I set the prior distributions of the AR(1) coefficients of the home and foreign trend inflation processes ρ_τ and ρ_τ^*, respectively, to identical beta distributions with a mean of 0.990 and a small standard distribution of 0.001. The resulting tight prior of the high AR(1) coefficient acts as an identification restriction to extract the permanent components of the two countries' inflation rates. To construct the prior distribution of the AR(1) coefficient of the risk premium shock, ρ_r, I rely on Justiniano and Preston (2010) by adopting a beta density with a mean of 0.500 and a standard deviation of 0.100. As there is no prior information, I also apply the same beta prior density to the AR(1) coefficients of the four GNKPC shocks, ρ_p^h, ρ_p^{h*}, ρ_p^f, and ρ_p^{f*}. Following the conventional exercise in the literature on Bayesian inferences of DSGE models, I adopt inverse gamma densities for the prior distributions of the standard deviations of the 11 exogenous impulses, as summarized in Table 1. I refer to the above prior configuration the TI model.

To identify the role of trend inflation in the empirical performance of the TI model, I prepare another prior configuration by restricting the model with no long-run trend inflation $\bar{\gamma} = \bar{\gamma}^* = 1$. With these prior restrictions, the TI model degenerates to a standard two-country NK model. I also reset the prior mean of the price elasticity of demand ζ to 11.00, fitting the model to an empirical markup rate of 10.00%, as in the conventional literature. Throughout the rest of this paper, I refer to this alternative restricted prior configuration the non-TI model. The sixth and seventh columns of Table 1 summarize the prior densities of the structural parameters of the non-TI model.[10]

[10]The non-TI model sets smaller prior standard deviations of the habit, home-bias, and Calvo parameters than those in the TI model because larger prior standard deviations lead the posterior means of these structural parameters to the lower or upper boundaries of the admissible $[0\ 1)$ interval, respectively.

Given the prior distributions of the structural parameters, I draw the posterior inferences of the TI model and the non-TI model through a Markov chain Monte Carlo (MCMC) method implemented with the random-walk Metropolis–Hastings algorithm. During the MCMC sampling, I solve the rational expectations equilibrium of the log-linearized model, derive its state-space representation using the QZ algorithm proposed by Sims (2001), and construct the corresponding restricted unobserved component model that generates the likelihood function given the data. I keep only the MCMC samples that yield the equilibrium determinacy. In addition, to ensure that the posterior samplers are distributed around the conventional 12.00% markup rate, I prepare the prior distributions of the home and foreign markup rates with an identical gamma density with a mean of 1.120 and a standard deviation of 0.005.[11] The posterior distributions of the model's structural parameters are approximated with 500,000 MCMC samples, where the first 200,000 draws are burned in to guarantee posterior convergence.

3.2. Bayesian posterior inferences of the two models

The second, third, and fourth columns of Table 2 report the means, standard deviations, and 95% Bayesian highest probability density (HPD) intervals of the posterior distributions of the TI model's structural parameters, respectively. We should note at least five important results. First, the "Markup rate" rows show that the posterior sampler guarantees 10.92 % and 11.96 % gross markup rates for Canada and the U.S., respectively. Second, the posterior means of γ_τ, γ_A, γ_τ^*, and γ_A^* imply that the posterior Canadian and U.S. long-run annual inflation rates $\bar{\gamma}$ and $\bar{\gamma}^*$ are 2.92% and 3.61%, respectively. These positive long-run inflation rates support the empirical relevance of the GNKPCs in the TI model.

Third, the TI model reveals a smaller degree of Canadian home bias. The posterior mean of the Canadian home bias parameter τ is 0.7221, with a posterior standard deviation of 0.0121. Together with the calibration of perfect U.S. home bias ($\tau^* = 0.999$), this result reflects the small Canadian economy, relative to the large U.S. economy.

Fourth, the TI model does not depend much on the persistence of the risk premium and GNKPC

[11]The markup rate mc is a non-linear function of the other structural parameters θ; $mc = mc(\theta)$. Let $\mathbf{p}(mc)$ denote the prior distribution of the markup rate. Then with the prior distributions of the markup rate and the structural parameters, the posterior distribution of the structural parameters $\mathbf{p}(\theta|\mathbf{Y})$ is given by $\mathbf{p}(\theta|\mathbf{Y}) \propto \mathbf{p}(mc(\theta))\mathbf{p}(\theta)\mathbf{p}(\mathbf{Y}|\theta)$,where $\mathbf{Y}$ is the data and $\mathbf{p}(\mathbf{Y}|\theta)$ is the likelihood function.

shocks. The highest posterior mean among the AR(1) roots of these exogenous shocks is, at best, 0.9251 for ρ_p^{h*}. Past studies depend on these reduced-form shocks to explain the highly persistent real exchange rate data.[12] In contrast, to explain the real exchange rate data, the TI model in this study relies on the high persistence of neither the risk premium shock nor the GNKPC shocks.

Finally, as the fifth result, the posterior means of the standard deviations of the trend inflation shocks, σ_τ and σ_τ^*, remain at 0.0009 and 0.0006, which are smaller than those of the other shocks. With the high posterior means of the AR(1) coefficients of the trend inflation shocks, this result is consistent with the usual conjecture that changes in the inflation target are small and long-lasting. Figure 1(a) plots the posterior mean of the Kalman-smoothed inference of the Canadian trend inflation $\gamma_{\tau,t}$ as the solid blue line, with the corresponding 68% HPD interval as the dotted blue lines and the actual data of the Canadian inflation rate as the solid black line, while Figure 1(b) shows the U.S. counterparts. The Canadian smoothed trend inflation is less volatile with a small posterior uncertainty and can capture the actual inflation rate's long swing; indeed, it is highly correlated with the Hodrick-Prescott filtered trend component of the actual inflation rate with the correlation coefficient of 0.8236. In particular, the sharp change in the direction of the smoothed Canadian trend inflation around 1991 precisely detects the Bank of Canada's introduction of the inflation targeting policy. The U.S. smoothed trend inflation depicted in Figure 1(b) tracks the ups and downs of the slow-moving component of the actual U.S. inflation over the sample period. Thus, the TI model can plausibly identify the permanent components of the two countries' actual inflation rates.

The fifth, sixth, and seventh columns of Table 2 describe the posterior means, posterior standard deviations, and 95% HPD intervals of the structural parameters of the non-TI model. There are three crucial differences in the posterior distributions between the TI and non-TI models. First, the non-TI model infers a much higher degree of Canadian home bias than that of the TI model: the posterior mean of τ, estimated by the former model, is 0.9781, whereas that estimated by the TI model is 0.7221. This result casts doubt on the non-TI model's identification of the small openness of the Canadian economy.

[12] For instance, using quarterly Canada-U.S. data, Justiniano and Preston (2010) estimate the posterior mean of the AR(1) coefficient of the risk premium to be 0.980 and report that the risk premium shock accounts for roughly 90% of the forecast error variance of the real exchange rate. Lubik and Schorfheide (2005) also show the heavy dependence of their two-country open-economy DSGE model on reduced-form PPP shocks when accounting for real exchange rate variations in actual Euro and U.S. dollar data.

Second, the non-TI model estimates much smaller posterior mean of the Canadian habit parameter than do the TI model: the posterior mean of h for the former model is 0.1631, while that for the latter model is 0.6598. This inference about negligible Canadian consumption habit is inconsistent with the posterior inference by Justiniano and Preston (2010). Third, the non-TI model estimates a significantly larger posterior mean of the standard deviation of the NKPC shock than do the TI model: the posterior mean of σ_p^{h*} for the former model is 0.0374, while that for the latter model is 0.0007. Therefore, the posterior inferences above imply that the non-TI model fails to identify endogenous propagation mechanisms in Canada. Instead, the non-TI model relies heavily on a reduced-form NKPC shock to explain the real exchange rate dynamics, as Steinsson (2008) shows in his calibration exercise.

The TI model beats the non-TI model in terms of the overall data fit. The last row of Table 2 shows that the TI and Non-TI models' log marginal likelihoods are 2905.906 and 2801.289, respectively, estimated using Geweke's (1999) harmonic mean estimator. This significant log difference in marginal likelihood is "strongly" favorable for the TI model, according to Jeffreys' criterion (Kass and Raftery 1995). The positive long-run trend inflation estimated by the TI model significantly improves the overall empirical fit of the otherwise standard two-country NK model to the Canada-U.S. data.

Tables 3 (a) and (b) summarize the posterior means of the forecast error variance decompositions (FEVDs) of the real exchange rates estimated by the TI and non-TI models, respectively. On the one hand, Table 3(a) conveys the most outstanding finding from this study's FEVD exercise: the Canadian trend inflation shock explains the dominant portions of the stochastic variation in the real exchange rate for all horizons. In the impact period, 23.5% of the FEVD of the real exchange rate is due to the Canadian trend inflation shock. The same shock generates 25.2%, 28.2%, and 36.5% of the FEVDs of the real exchange rate for the 1-, 3-, and 10-year horizons, respectively. The TI model also admits other structural shocks, especially the Canadian productivity and monetary policy shocks and the U.S. trend inflation shock, which play relatively small but non-negligible roles in real exchange rate variations. The other reduced-form shocks, that is, the risk premium shock and the GNKPC shocks, jointly generate between 26.9% and 44.6% of stochastic variations in the real exchange rate over all the forecast horizons.

On the other hand, as Table 3(b) shows, the non-TI model estimates that the NKPC shocks predominantly explain the stochastic variations in the real exchange rate. The NKPC shocks jointly generate between 40.0% and 66.5% of the total variations in the real exchange rate across all the forecast horizons. In particular, the NKPC shock $\epsilon^h_{p,t}$ alone explains about 50% of the FEVD of the real exchange rate for longer horizons.

The results of the FEVD exercise strongly support this study's hypothesis that stochastic variation in trend inflation is fundamental for exchange rate dynamics and suggest that the Bank of Canada's inflation targeting policy, which crucially influenced Canada's long-run trend inflation, might have played a significant role in the Canada-U.S. real exchange rate after 1991. Figure 2 displays the demeaned Canada-U.S. RCR data (on the right axis) as a solid black line and the TI model's historical decompositions of the RCR into the six structural shocks (on the left axis) as the yellow bars for the Canadian trend inflation shocks, the purple bars for the U.S. trend inflation shocks, the blue bars for the Canadian productivity shocks, the red bars for the U.S. productivity shocks, the green bars for the Canadian monetary policy shocks, and the light blue bars for the U.S. monetary policy shocks, respectively. The vertical dashed black line indicates the period Q1:1991 for the Bank of Canada's introduction of the inflation-targeting policy.[13] Observe that the Canadian trend inflation shocks played quite significant roles in the historical movements of the RCR over the whole sample period; the absolute values of the RCR historical decompositions to the Canadian trend inflation shocks record around 20 % in many periods. Before Q1:1991, the historical decompositions for the Canadian trend inflation shocks went up and down toward positive and negative values. However, in most periods after Q1:1991, the historical decompositions for the Canadian trend inflation shocks remained positive until 2002. These results suggest that because the Bank of Canada's inflation targeting policy successfully anchored the Canadian long-run inflation toward the inflation target, the resulting sharp and steady declines of the Canadian inflation rate significantly contributed to the Canadian real depreciation realized by 2002.

Figure 3 plots the TI and non-TI models' IRFs of the percentage deviation of the real exchange rate from the long-run steady state to the eleven structural shocks: panel (a) for the Canadian productivity shock, panel (b) for the U.S. productivity shock, panel (c) for the Canadian trend inflation shock, panel

[13]Each historical decomposition for a structural shock is represented as a percentage ratio to the sum of the absolute values of the historical decompositions for all the eleven shocks.

(d) for the U.S. trend inflation shock, panel (e) for the Canadian monetary policy shock, panel (f) for the U.S. monetary policy shock, panel (g) the Canadian GNKPC shock to the Canadian price, panel (h) for the U.S. GNKPC shock to the Canadian price, panel (i) for the Canadian GNKPC shock to the U.S. price, panel (j) for the U.S. GNKPC shock to the U.S. price, and panel (k) for the risk premium shock. In all panels, the solid blue lines are the posterior mean of the IRFs estimated by the TI model, accompanied by the 95% HPD interval as dotted blue lines. Similarly, the solid red lines are the posterior means of the IRFs estimated by the non-TI model, accompanied by the 95% HPD interval as dotted red lines. To compare the magnitudes of the propagation mechanisms between the two models, I set the IRFs to one-unit shocks.

The most apparent finding of the TI model is persistent, nearly permanent IRFs. The Canadian productivity, trend inflation, and monetary policy shocks, shown in panels (a), (c) and (e), the U.S. trend inflation shock, shown in panel (d), and several GNKPC shocks, shown in panels (g), (h), and (i), generate strong hump-shaped IRFs. The two countries' trend inflation shocks in particular dominate the IRFs of the real exchange rate to the other structural shocks in terms of magnitude. The non-TI model, on the other hand, fails to generate persistent hump-shaped IRFs; the IRFs to most structural shocks quickly converge toward zero. Moreover, the magnitudes of the non-TI model's IRFs are much smaller than those of the TI model, except for the U.S. monetary policy shock in panel (f).

These IRF exercise results clearly show that the TI model possesses powerful amplification and propagation mechanisms, especially for the two countries' trend inflation shocks for the real exchange rate, which are absent in the standard two-country NK model.

3.3. Posterior predictive analysis of exchange rate moments

This subsection evaluates the TI and non-TI models' empirical performance in terms of empirical moments of the real and nominal exchange rates through posterior predictive checks. The first and second rows of Table 4 summarize the empirical moments I target in this study. The first empirical moment is the sum of the autoregressive coefficients of the AR(5) process of the real exchange rate, denoted by α. Applying the augmented Dickey-Fuller (ADF) regression with five lags for the U.S. trade-weighted real exchange rate, Steinsson (2008) estimates α to be 0.954 with a 90% confidence interval between 0.879 and

1.000.

Steinsson (2008) and Burstein and Gopinath (2014) report a half-life measure of the real exchange rate. The estimated ADF equation provides the IRFs of the real exchange rate to a unit reduced-form disturbance. I calculate the second empirical moment, denoted by HL, as the maximum period before the IRF reaches 0.500; that is, HL = T such that IRF(T-1) $>$ 0.500 and IRF(T) $\leq$ 0.500. Burstein and Gopinath (2014) report HLs for eight advanced countries: Canada, France, Germany, Italy, Japan, Switzerland, the U.K., and the U.S. The cross-country average HL is 4.425 years, with a minimum of 1.600 years for Switzerland and a maximum of 6.000 years for the U.S.

The third empirical moment is the correlation coefficient between the RCR and NCR. As Mussa (1986) observes, the real and nominal exchange rates move closely. Burstein and Gopinath (2014) calculate the correlation coefficients between the RCR and NCR for these eight advanced countries. The correlation coefficient, denoted as Corr, is 0.932 on average, with a minimum of 0.820 for Italy and a maximum of 0.990 for Japan. The tight comovement between the RCR and NCR implies that the exchange rate dynamics remain disconnected from the inflation differential, at least in the short run. Burstein and Gopinath (2014) also provide the standard deviation ratios of the RCR to the NCR, $\frac{\text{Std}(\Delta \ln q_t)}{\text{Std}(\Delta \ln S_t)}$ for these eight economies. The cross-country average is 0.956, with a minimum of 0.870 for France and a maximum of 1.040 for the U.K. A standard deviation ratio close to one implies that the exchange rate is much more volatile than the inflation differential. These moments confirm that the RCR and NCR move almost one-to-one and are much more volatile than the inflation differential. The third row of Table 4 reports the corresponding sample moments for the Canada-U.S. data I use in this study.

The fourth row of Table 4 presents the posterior means of the targeted moments inferred by the TI model, accompanied by the corresponding 95% HPD intervals in the fifth row. Given a particular posterior draw of the structural parameters, I simulate the state variables of 100 periods from the state-space representation of the TI model and calculate the model counterparts of the targeted moments. I generate ten repetitions of the targeted moments and keep the ensemble average to mitigate possible Monte Carlo errors. Repeating this process over 300,000 MCMC samplers constructs the posterior distributions of the targeted moments. The TI model infers a large AR root of the real exchange rate; the posterior mean of α is 0.939. The HPD interval between 0.912 and 0.963 covers the empirical counterparts of 0.954 for the cross-country average and 0.961 for the Canada-U.S. data. The TI model also estimates the posterior mean of the HL measure to be 4.615 years, closely mimicking the empirical counterpart of 4.425 years for the cross-country average and 5.250 for the Canada-U.S. data. The corresponding HPD interval between 3.365 and 6.000 covers the two empirical HL measures. Hence, the TI model successfully generates the highly persistent real exchange rate observed in the actual data. The TI model is also consistent with Mussa's (1986) observations of one-to-one comovement between the real and nominal exchange rates. The TI model estimates a high positive correlation coefficient between the RCR and NCR, and the posterior mean of the correlation coefficient of 0.980 is close to its empirical and Canada-U.S. counterparts. The TI model also replicates the relative volatility of the RCR to the NCR, as in the actual data. The TI model estimates the posterior mean of the standard deviation ratio of the RCR to NCR to be 1.029, with a 95% HPD interval between 1.022 and 1.046. This posterior inference reasonably matches the Canadian and U.S. data counterpart of 1.037.

The sixth and seventh rows of Table 4 report the targeted moments inferred by the non-TI model. First, the sum of the AR(5) coefficients, α, decreases sharply to 0.747 in the non-TI model from 0.939 in the TI model. Furthermore, the posterior mean of the HL measure of 1.245 years indicates low persistence in the real exchange rate, as this model implies. The non-TI model also fails to reconcile Mussa's (1986) observations. The posterior mean of the correlation coefficient between the RCR and NCR is 0.883, which is far below the corresponding Canada-U.S. point estimates of 0.957. The reason behind the non-TI model's apparent failure in Mussa's (1986) observations is the implied excess volatility of the two countries' inflation differential; the standard NKPC cannot generate smoothed inflation dynamics observed in actual data.

Figure 4 plots, as the solid blue line, the posterior mean of the IRF of the real exchange rate to a reduced-form disturbance from the ADF regression inferred by the TI model. The two dotted blue lines represent the corresponding 95% HPD interval. The inferred IRF is hump-shaped with a peak around a year and monotonically, but slowly, declining toward zero over time. The solid red line, on the other hand, displays the non-TI model's posterior mean of the IRF of the real exchange rate from the ADF regression with the corresponding 95% HPD interval as the dotted red lines. Note that the IRF monotonically and quickly declines toward zero with no hump. The HL measure's significant difference between the two models reported in Table 4 reflects this difference in the IRF of the real exchange rate.

The same figure plots the point estimate of the IRF of the real exchange rate for the Canada-U.S. data as a solid black line. The grey-shaded area represents the corresponding 95% bootstrapping confidence interval. The hump-shaped IRF estimated by the TI model is included in the shaded area. However, its absolute size is much smaller than that of the Canadian and U.S. counterparts. This result is an apparent failure of the TI model; the propagation mechanism embedded in the TI model is not sufficiently strong to mimic the large hump-shaped real exchange rate response observed in the Canada-U.S. data.

3.4. Why can the TI model replicate the exchange rate dynamics?

To intuitively capture the economic mechanism underlying the successful outcomes of the TI model in terms of real exchange rate dynamics, it is useful to solve the RUIP forward:

$$\hat{q}_t = -\sum_{j=0}^{\infty} E_t\{(1+\hat{r}_{t+j}) - (1+\hat{r}^*_{t+j})\} + \lim_{j\to\infty} E_t\hat{q}_{t+j}, \tag{3}$$

where $(1+\hat{r}_t)$ and $(1+\hat{r}^*_t)$ are the home and foreign real interest rates, respectively. The last limiting term on the right-hand side (RHS) converges toward zero. Hence, the real exchange rate response depends negatively on the expected present values of the future real interest rate differentials with the unit discount factor.[14]

To understand the high persistence of the real exchange rate implied by the TI model, suppose that the real interest rate differential $(1+\hat{r}_t)^d \equiv (1+\hat{r}_t) - (1+\hat{r}^*_t)$ follows an AR(1) process exogenous to the

[14] Eq.(3) is a restricted case of the fundamental asset pricing equation in Engel and West (2005).

$$z_t = (1-b)\sum_{j=0}^{\infty} b^j E_t(\mathbf{a}_1'\mathbf{x}_{1,t+i}) + b\sum_{j=0}^{\infty} b^j E_t(\mathbf{a}_2'\mathbf{x}_{2,t+j}),$$

where z_t is the asset price, b is the discount factor, $\mathbf{x}_{1,t}$ is a vector of the observable economic fundamentals, $\mathbf{x}_{2,t}$ is a vector of the unobservable economic fundamentals, and $\mathbf{a}_1$ and $\mathbf{a}_2$ are the corresponding coefficient vectors. I obtain Eq.(3) by imposing $\mathbf{a}_1 = \mathbf{0}$ and $b \to 1$ in the fundamental asset pricing equation. As Engel and West (2005) prove, if an element of the economic fundamentals, which is the real interest rate differential in this case, is I(1), then the real exchange rate converges to a random walk with an infinite variance of the RCR.

real exchange rate: $(1+\hat{r}_t)^d = \rho(1+\hat{r}_{t-1})^d + u_t$, where $\rho \in (0,1)$ and u_t is an i.i.d. shock. The RUIP condition (3) implies that if the real interest rate differential $(1+\hat{r}_t)^d$ approaches a unit root process, then the real exchange rate approximately follows a random walk:

$$\lim_{\rho\to 1} \Delta q_t = -\lim_{\rho\to 1}\left[\frac{1}{1-\rho}u_t + \rho(1+\hat{r}_{t-1})^d\right] = -\lim_{\rho\to 1}\frac{1}{1-\rho}u_t,$$

where the second equality stems from the fact that $\lim_{\rho\to 1}\frac{1}{1-\rho}u_t$ dominates $(1+\hat{r}_{t-1})^d$ in terms of volatility at the limit $\rho \to 1$. Therefore, the TI model implies that a persistent real interest rate differential is sufficient to yield the near random-walk behavior of the real exchange rate accompanied by high volatility. Large and permanent IRFs of the real exchange rates to the two-countries' trend inflation shocks, displayed in Figure 3, indeed confirm this near random-walk property that the TI model reflects the real exchange rate dynamics.

Why can the TI model generate the highly persistent real interest rate differential, independent of real exchange rate movements? Figures 5(a) and (b) display the posterior means of the IRFs of selected variables to a unit standard deviation shock to the Canadian trend inflation in the TI model. In particular, Figure 5(a) depicts the posterior means of the IRFs of the real interest rate differential (solid red line), expected inflation rate differential (circle-bar green line), inflation gap differential (cross-bar blue line), and nominal interest rate differential (solid black line). Figure 5(b) displays the IRF of the real exchange rate (solid blue line).

Observe the persistent response of the nominal interest rate differential. In response to a positive Canadian trend inflation shock, the nominal interest rate differential rises in the impact period and continues to increase for 28 consecutive quarters thereafter. The nominal interest rate differential then begins to decline toward the long-run value at a slow rate, and even after ten years, the corresponding IRF stays at 93.7% of the peak value.

This highly persistent response of the nominal interest rate differential stems from the slow-moving dynamics of the two countries' inflation gaps, $\hat{\gamma}_{\pi,t} - \hat{\gamma}_{\tau,t}$ and $\hat{\gamma}^*_{\pi,t} - \hat{\gamma}^*_{\tau,t}$, which are an inherent outcome of the GNKPCs. Moreover, the inflation gaps are insensitive to any developments in the current real exchange rate because of the primary property of GNKPCs, as discussed in Section 2. The Taylor rules then propagate the inflation gaps into the nominal interest rate differential; the high degrees of policy inertia in the Taylor rules then cumulatively and persistently feed these positive responses of the inflation gaps into the nominal interest rate differential. In summary, the GNKPCs and high policy inertia jointly imply the high persistence of the nominal interest rate differential, exogenous to real exchange rate developments. The expected inflation rate differential response is also highly persistent. Thus, the persistence of the nominal interest rate differential results in the real interest rate differential. Indeed, the IRF of the latter is almost a mirror image of that of the former.

The IRF of the real interest rate differential in Figure 5(a) further reveals an outstanding TI model property. In their complete market sticky-price models, Steinsson (2008) and Iversen and Söderström (2014) state that to generate a hump-shaped response of the real exchange rate, the response of the real interest rate differential needs to flip its sign within several quarters of the impact period. Figure 5(a) confirms that this is the case for the TI model. The IRF of the nominal interest rate differential has a persistent hump shape. While the IRF of the expected inflation rate differential is greater than that of the nominal interest rate differential up to 5.5 years, the latter gradually overcomes the former over time. In particular, as Steinsson (2008) and Iversen and Söderström (2014) claim, the IRF of the real interest rate differential is negative for up to 5.5 years but turns positive subsequently for long periods. Therefore, as in Figure 5(b), the real exchange rate appreciates in the impact period because the expected present value of the future real interest rate differential is positive. The real exchange rate keeps appreciating when the response of the real interest rate differential remains negative; when the response of the real interest rate differential becomes zero, the real exchange rate stops appreciating, and when the response of the real interest rate differential turns positive, the real exchange rate starts depreciating. Consequently, the model generates a hump-shaped IRF of the real exchange rate to a positive shock to the Canadian trend inflation.

The TI model implies that, other things being equal, monetary easing with a positive trend inflation shock in Canada results in a sudden rise in the inflation rate differential and real exchange rate appreciation in the impact period. This conditional covariance between the inflation rate differential and the real exchange rate is consistent with the well-known empirical finding of Clarida and Waldman (2008): bad news about inflation is good news for the real exchange rate. In the Canada-U.S. data, the correlation coefficient between the RCR and inflation differential is -0.417.

Figures 5(c) and (d) show the non-TI model counterparts of the selected IRFs to the Canadian trend inflation shock. Figure 5(d) shows the non-hump-shaped IRF of the real exchange rate in the non-TI model. Figure 5(c) reveals that this failure of the non-TI model regarding the IRF of the real exchange rate stems from the monotonically increasing IRF of the real interest rate differential to zero without sign flipping. The lack of sign flipping of the IRF of the real interest rate differential results from a sharp increase in the expected inflation rate differential and a quick convergence of the nominal interest rate to the expected inflation rate differential due to the weak persistence of the inflation gap differential. The non-TI model fails to reconcile Clarida and Waldman's (2008) findings.

4. Conclusion

This paper argues that allowing trend inflation in an otherwise standard two-country NK model essentially changes the exchange rate dynamics. Positive long-run trend inflation implies a more persistent, but less volatile, inflation differential with the GNKPCs of the two countries. Simultaneously, the GNKPCs weaken the link between the inflation differential and the real exchange rate. The RUIP condition, jointly with the Taylor rule with high policy inertia, generates a nearly permanent real exchange rate with high volatility. Combined with the less volatile inflation differential, the RCR and NCR move almost one-to-one in the equilibrium of the model.

The results of this study highlight the crucial role of monetary policy frameworks in exchange rate dynamics. In particular, the model identifies small stochastic variation in trend inflation around the long-run inflation target as the primary driver of volatile and persistent exchange rate fluctuations: under positive trend inflation, revisions in market participants' perceptions of the long-term policy goal of the central bank could be the primary fundamental for the real and nominal exchange rates. This result stands in stark contrast to that of the influential calibration study by Steinsson (2008), which advocates real shocks to the Phillips curve to resolve the PPP puzzle. Future research is thus needed to delve into the structural interpretations of time-varying trend inflation and inflation targets.

The model I present in this paper still lacks many of the theoretical features used to approach exchange rate anomalies emphasized in the recent literature on international relative prices. For example, even when equipped with strong home bias, the TI model estimates a positive correspondence between the real exchange rate and the consumption differential in incomplete international financial markets. Thus, it is essential to address Backus and Smith's (1993) anomaly. Moreover, the risk-neutral UIP condition nearly holds along the equilibrium of the TI model. This property implies that the TI model cannot describe an auto-covariance structure between the expected excess currency return and the nominal interest rate differential frequently and robustly observed in actual data of advanced currencies. To reconcile the empirical puzzles of Fama (1984) and Engel (2016), future work should augment the proposed TI model to generate an endogenous currency risk premium consistent with the empirical reality of exchange rates.

Appendix. Data description and construction.

Most data for Canada and the U.S. are distributed by the Federal Reserve Economic Data (FRED), operated by the Federal Reserve Bank of St. Louis (http://http://research.stlouisfed.org/fred2/). I construct the U.S. consumption data as the sum of real personal consumption expenditure on non-durables and services. However, FRED distributes only the nominal values of the two categories of personal consumption expenditure as *Personal Consumption Expenditure on NonDurables* and *Personal Consumption Expenditure on Services*. To construct real total personal consumption expenditure, I first calculate the share of the two nominal consumption categories in nominal total personal consumption expenditure (*Personal Consumption Expenditure*) and then multiply real total personal consumption expenditure (*Real Personal Consumption Expenditure at Chained 2005 Dollars*) by the calculated share. The U.S. aggregate price data are provided by the *Consumer Price Index for All Urban Consumers: All Items in U.S. City Average*. The U.S. nominal interest rate is provided by the *Effective Federal Funds Rate*. The U.S. labor supply is provided by *Weekly Hours Worked for Manufacturing*. All variables except for the nominal interest rate are seasonally adjusted at annual rates and converted into the corresponding per-capita terms by the *Total Population*. The Canadian dollar/U.S. dollar nominal exchange rate is distributed as *Canada/U.S. Foreign Exchange Rate*.

FRED also distributes the Canadian CPI (*Consumer Price Index: Total, All Items for Canada*) and policy rate (*Immediate Rates: Less than 24 Hours: Central Bank Rates for Canada*). The consumption data for Canada are distributed by Statistics Canada (http://www5.statcan.gc.ca/cansim/). I construct the real consumption data as the sum of *Personal Expenditure on NonDurables at Chained 2002 Dollars*, *Personal Expenditure on Semi-Durables at Chained 2002 Dollars*, and *Personal Expenditure on Services at Chained 2002 Dollars*. The Canadian labor supply is provided by *Business Sector Average Hours Worked*. All variables except for the nominal interest rate are seasonally adjusted at annual rates and converted into the corresponding per-capita terms by the *Estimate of Total Population*.

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Table 1: Prior Distributions: Structural Parameters

	Parameters	Density	TI model Mean	TI model Std	Non-TI model Mean	Non-TI model Std
	Canadian Parameters					
γ_τ	Mean Trend Inflation Rate	Normal	1.0122	0.0005	—	—
γ_A	Mean Productivity Growth	Normal	1.0050	0.0005	1.0050	0.0005
μ	Calvo Probability of No Price Resetting	Beta	0.8000	0.1000	0.8000	0.0100
h	Consumption Habit	Beta	0.5000	0.2000	0.5000	0.0500
τ	Home Bias	Beta	0.7000	0.0500	0.7000	0.0300
η	Labor Supply Elasticity	Gamma	2.0000	0.5000	2.0000	0.5000
ζ	Price Elasticity of Final Goods Demand	Gamma	22.000	1.0000	11.000	1.0000
a_π	Taylor Rule Parameter for the Inflation Gap	Gamma	3.5000	0.5000	3.5000	0.5000
a_y	Taylor Rule Parameter on the Output Gap	Gamma	0.1000	0.0100	0.1000	0.0100
a_s	Taylor Rule Parameter for NCR	Beta	0.2500	0.2000	0.2500	0.2000
ρ_i	Interest Rate Smoothing Parameter	Beta	0.9000	0.2000	0.9000	0.2000
λ	Error Correction Speed of Productivity Shock	Beta	0.0100	0.0100	0.0100	0.0100
ρ_τ	Trend Inflation AR(1) Coefficient (Coef)	Beta	0.9900	0.0010	0.9900	0.0010
ρ_r	Risk Premium Shock AR(1) Coef	Beta	0.5000	0.1000	0.5000	0.1000
ρ_p^h	Home Market GNKPC Shock AR(1) Coef	Beta	0.5000	0.1000	0.5000	0.1000
ρ_p^f	Foreign Market GNKPC Shock AR(1) Coef	Beta	0.5000	0.1000	0.5000	0.1000
σ_A	Productivity Shock Standard Deviation (Std)	InvGamma	0.0050	0.0050	0.0050	0.0050
σ_τ	Trend Inflation Shock Std	InvGamma	0.0007	0.00005	0.0007	0.00005
σ_i	Monetary policy shock Std	InvGamma	0.0010	0.0005	0.0010	0.0005
σ_r	Risk Premium Shock Std	InvGamma	0.0010	0.0005	0.0010	0.0005
σ_p^h	Home Market GNKPC Shock Std	InvGamma	0.0010	0.0005	0.0010	0.0005
σ_p^f	Foreign Market GNKPC Shock Std	InvGamma	0.0010	0.0005	0.0010	0.0005
	U.S. Parameters					
γ_τ^*	Mean Trend Inflation Rate	Normal	1.0138	0.0005	—	—
γ_A^*	Mean Productivity Growth	Normal	1.0061	0.0005	1.0061	0.0005
μ^*	Calvo Probability of No Price Resetting	Beta	0.8000	0.1000	0.8000	0.0100
h^*	Consumption Habit	Beta	0.5000	0.2000	0.5000	0.2000
η^*	Labor Supply Elasticity	Gamma	2.0000	0.5000	2.0000	0.5000
ζ^*	Price Elasticity of Final Goods Demand	Gamma	22.000	1.0000	11.000	1.0000
a_π^*	Taylor Rule Parameter for the Inflation Gap	Gamma	3.5000	0.5000	3.5000	0.5000
a_y^*	Taylor Rule Parameter for the Output Gap	Gamma	0.1000	0.0100	0.1000	0.0100
a_s^*	Taylor Rule Parameter on NCR	Beta	0.0500	0.0100	0.0500	0.0100
ρ_i^*	Interest Rate Smoothing Parameter	Beta	0.9000	0.2000	0.9000	0.2000
ρ_τ^*	Trend Inflation AR(1) Coef	Beta	0.9900	0.0010	0.9900	0.0010
ρ_p^{h*}	Home Market GNKPC Shock AR(1) Coef	Beta	0.5000	0.1000	0.5000	0.1000
ρ_p^{f*}	Foreign Market GNKPC Shock AR(1) Coef	Beta	0.5000	0.1000	0.5000	0.1000
σ_A^*	Productivity Shock Std	InvGamma	0.0050	0.0050	0.0050	0.0050
σ_τ^*	Trend Inflation Shock Std	InvGamma	0.0007	0.00005	0.0007	0.00005
σ_i^*	Monetary Policy Shock Std	InvGamma	0.0010	0.0005	0.0010	0.0005
σ_p^{h*}	Home Market GNKPC Shock Std	InvGamma	0.0010	0.0005	0.0010	0.0005
σ_p^{f*}	Foreign Market GNKPC Shock Std	InvGamma	0.0010	0.0005	0.0010	0.0005

Note: The subjective discount factor β is calibrated to 0.99 for both countries. The sensitivity of the debt-elastic risk premium ψ is calibrated to 0.00001. The U.S. home bias τ^* is calibrated to 0.999 by assuming that the U.S. is a large, nearly closed country.

Table 2: Posterior Distributions: Structural Parameters

Parameter	TI model Mean	TI model Std	TI model 95% HPD	Non-TI model Mean	Non-TI model Std	Non-TI model 95% HPD
			Canadian parameters			
γ_τ	1.0120	0.0002	[1.0116 1.0125]	—	—	—
γ_A	1.0035	0.0003	[1.0028 1.0041]	1.0049	0.0002	[1.0045 1.0053]
μ	0.8112	0.0044	[0.8023 0.8186]	0.8297	0.0107	[0.8084 0.8503]
h	0.6598	0.0177	[0.6318 0.6882]	0.1631	0.0205	[0.1172 0.2016]
τ	0.7221	0.0121	[0.6985 0.7466]	0.9781	0.0033	[0.9716 0.9842]
η	2.8782	0.0010	[2.8765 2.8797]	2.8896	0.0091	[2.8704 2.9010]
ζ	22.539	0.0105	[22.521 22.557]	9.2488	0.2437	[8.7312 9.6414]
a_π	3.4588	0.0080	[3.4421 3.4732]	3.2675	0.0670	[3.1576 3.3637]
a_y	0.0821	0.0084	[0.0665 0.0994]	0.1064	0.0106	[0.0858 0.1259]
a_s	0.4205	0.0280	[0.3747 0.4681]	0.5344	0.0220	[0.4862 0.5719]
ρ_i	0.9737	0.0026	[0.9682 0.9783]	0.9414	0.0063	[0.9282 0.9525]
λ	0.0012	0.0002	[0.0007 0.0017]	0.0000	0.0000	[0.0000 0.0000]
ρ_τ	0.9914	0.0008	[0.9897 0.9930]	0.9908	0.0007	[0.9896 0.9923]
ρ_r	0.5163	0.0028	[0.5114 0.5213]	0.5070	0.0083	[0.4961 0.5229]
ρ_p^h	0.5742	0.0258	[0.5307 0.6179]	0.6992	0.0071	[0.6853 0.7082]
ρ_p^f	0.7159	0.0116	[0.6946 0.7377]	0.6814	0.0047	[0.6737 0.6888]
σ_A	0.0217	0.0017	[0.0184 0.0250]	0.0076	0.0006	[0.0065 0.0088]
σ_τ	0.0009	0.0001	[0.0008 0.0010]	0.0013	0.0001	[0.0012 0.0014]
σ_i	0.0031	0.0003	[0.0025 0.0036]	0.0033	0.0003	[0.0026 0.0040]
σ_r	0.0010	0.0005	[0.0003 0.0020]	0.0009	0.0004	[0.0004 0.0016]
σ_p^h	0.0032	0.0003	[0.0026 0.0038]	0.0073	0.0009	[0.0056 0.0090]
σ_p^f	0.0128	0.0011	[0.0107 0.0152]	0.0051	0.0008	[0.0035 0.0066]
Markup rate	1.1092	0.0033	[1.1029 1.1159]	1.1214	0.0037	[1.1155 1.1291]
			U.S. Parameters			
γ_τ^*	1.0137	0.0002	[1.0132 1.0142]	—	—	—
γ_A^*	1.0060	0.0004	[1.0053 1.0067]	1.0049	0.0002	[1.0045 1.0053]
μ^*	0.7774	0.0046	[0.7679 0.7857]	0.8171	0.0092	[0.8000 0.8356]
h^*	0.5593	0.0210	[0.5285 0.5957]	0.6829	0.0173	[0.6506 0.7140]
η^*	1.6182	0.0008	[1.6167 1.6197]	1.6538	0.0074	[1.6424 1.6660]
ζ^*	22.392	0.0165	[22.361 22.420]	8.9252	0.2804	[8.3021 9.4182]
a_π^*	3.5175	0.0230	[3.4776 3.5571]	3.2305	0.0468	[3.1491 3.3272]
a_y^*	0.0841	0.0097	[0.0654 0.1029]	0.0906	0.0093	[0.0724 0.1088]
a_s^*	0.0530	0.0105	[0.0332 0.0736]	0.0547	0.0109	[0.0339 0.0760]
ρ_i^*	0.9170	0.0072	[0.9030 0.9288]	0.9627	0.0041	[0.9546 0.9707]
ρ_τ^*	0.9872	0.0010	[0.9852 0.9891]	0.9920	0.0005	[0.9909 0.9929]
ρ_p^{h*}	0.9251	0.0084	[0.9086 0.9403]	0.8555	0.0044	[0.8473 0.8640]
ρ_p^{f*}	0.4762	0.0163	[0.4384 0.4982]	0.4794	0.0048	[0.4711 0.4879]
σ_A^*	0.0017	0.0004	[0.0009 0.0025]	0.0050	0.0005	[0.0040 0.0060]
σ_τ^*	0.0006	0.0000	[0.0006 0.0007]	0.0009	0.0000	[0.0008 0.0010]
σ_i^*	0.0013	0.0001	[0.0011 0.0015]	0.0020	0.0002	[0.0016 0.0024]
σ_p^{h*}	0.0007	0.0001	[0.0005 0.0010]	0.0374	0.0054	[0.0281 0.0492]
σ_p^{f*}	0.0023	0.0002	[0.0019 0.0026]	0.0030	0.0003	[0.0025 0.0036]
Markup rate	1.1196	0.0035	[1.1122 1.1259]	1.1260	0.0042	[1.1191 1.1364]
Log Marginal Likelihood		2905.906			2801.289	

Note 1: "HPD" represents the Bayesian highest probability density intervals.
Note 2: The marginal log likelihoods are estimated based on Geweke's (1999) harmonic mean estimator.

Table 3: Posterior Mean FEVDs (%): Real Exchange Rate

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
(a) TI model											
Impact	5.38	0.02	23.5	4.67	19.3	2.18	10.4	16.3	7.55	10.43	0.00
1 yr	13.0	0.02	25.2	7.65	5.75	0.85	12.3	17.6	10.0	7.48	0.00
3 yr	21.3	0.06	28.2	11.3	4.17	0.28	6.06	13.0	12.6	2.92	0.00
10 yr	22.9	0.08	36.5	14.5	3.45	0.09	1.99	4.92	14.2	0.95	0.00
(b) Non-TI model											
Impact	0.13	1.10	3.19	5.38	26.1	23.8	33.1	1.99	0.45	4.67	0.00
1 yr	0.21	0.52	1.97	3.68	15.3	15.3	50.4	5.05	1.44	6.04	0.00
3 yr	0.43	0.54	1.83	3.31	13.7	13.6	49.5	7.26	4.14	5.64	0.00
10 yr	1.22	0.86	1.86	3.27	13.3	13.2	48.3	7.24	5.05	5.50	0.00

Note: Part (a) reports the posterior means of the FEVDs of the real exchange rate in the structural shocks at the impact period, 1-, 3-, and 10-year forecast horizons inferred by the TI model; and Part (b) reports those for the non-TI model.

Table 4: Posterior Predictive Analysis for Exchange Rate Moments

	α	HL	Corr	$\frac{\text{Std}(\Delta \ln q_t)}{\text{Std}(\Delta \ln S_t)}$
Empirical	0.954	4.425	0.932	0.956
90% CI or $\{min, max\}$	[0.879 1.000]	$\{$1.600 6.000$\}$	$\{$0.820 0.990$\}$	$\{$0.870 1.040$\}$
Canada-U.S. data	0.961	5.250	0.957	1.037
TI model	0.939	4.615	0.980	1.029
95% HPD	[0.912 0.963]	[3.365 6.000]	[0.973 0.986]	[1.022 1.046]
Non-TI model	0.747	1.245	0.883	1.043
95% HPD	[0.684 0.808]	[1.050 1.450]	[0.858 0.907]	[0.989 1.101]

Note 1. α is the sum of the AR coefficients in the AR(5) process of $\ln q_t$ estimated using the ADF regression.
Note 2. Half-life (HL) is the maximum period at which the IRF of $\ln q_t$ is greater than 0.5, measured in years.
Note 3. Corr represents the correlation coefficient between $\Delta \ln q_t$ and $\Delta \ln S_t$.
Note 4. $\frac{\text{Std}(\Delta \ln q_t)}{\text{Std}(\Delta \ln S_t)}$ represents the ratio of the standard deviations of $\Delta \ln q_t$ to $\Delta \ln S_t$.
Note 5. In the "Empirical" row, α comes from Steinsson's (2008) estimate using the U.S. trade-weighted real exchange rate. The other statistics are from Burstein and Gopinath (2014). Both report the average point estimates for eight advanced countries.

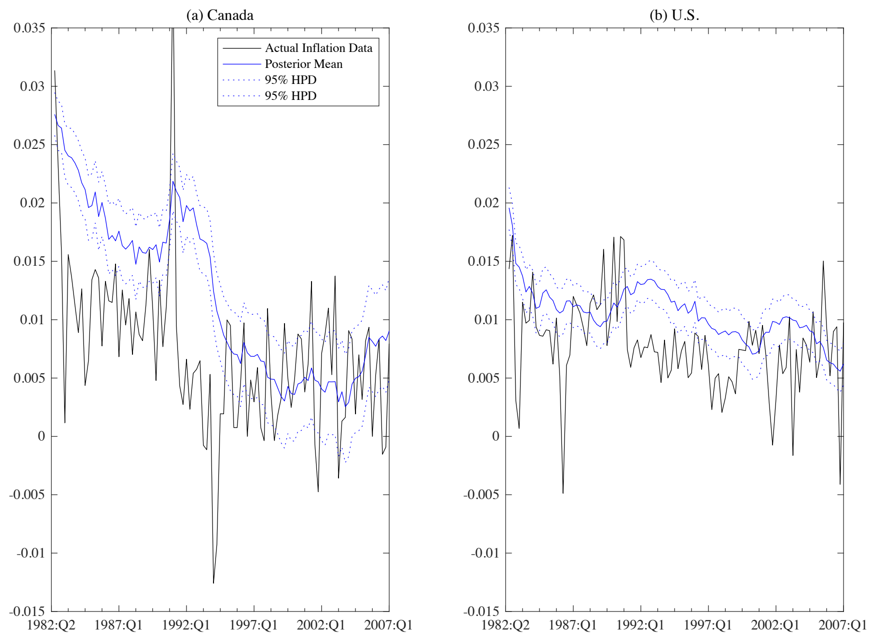

Figure 1: Posterior Inferences of Kalman-smoothed Trend Inflation

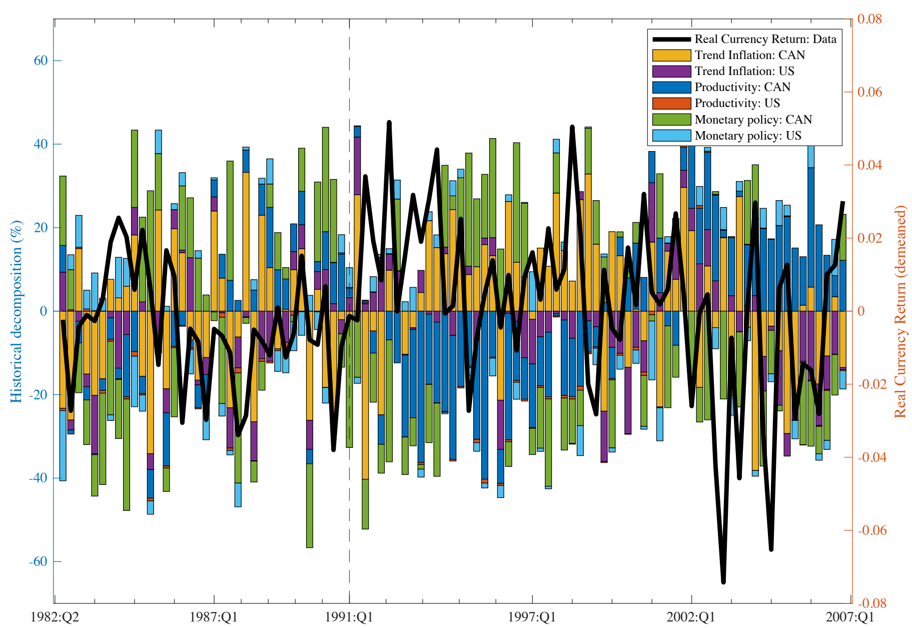

Figure 2: Historical Decomposition of Real Currency Return into Structural Shocks

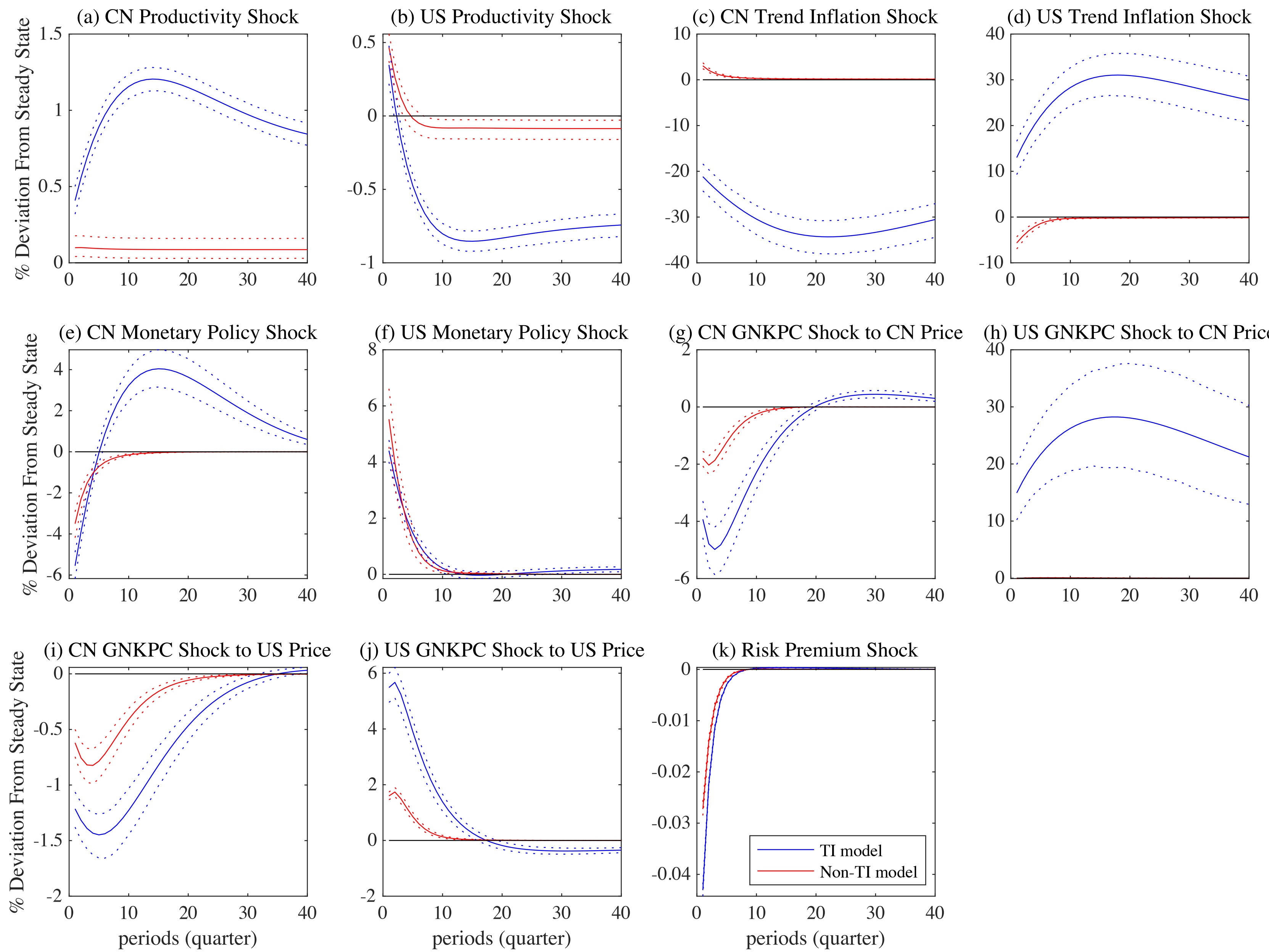

Figure 3: IRFs of the Real Exchange Rate to Structural Shocks

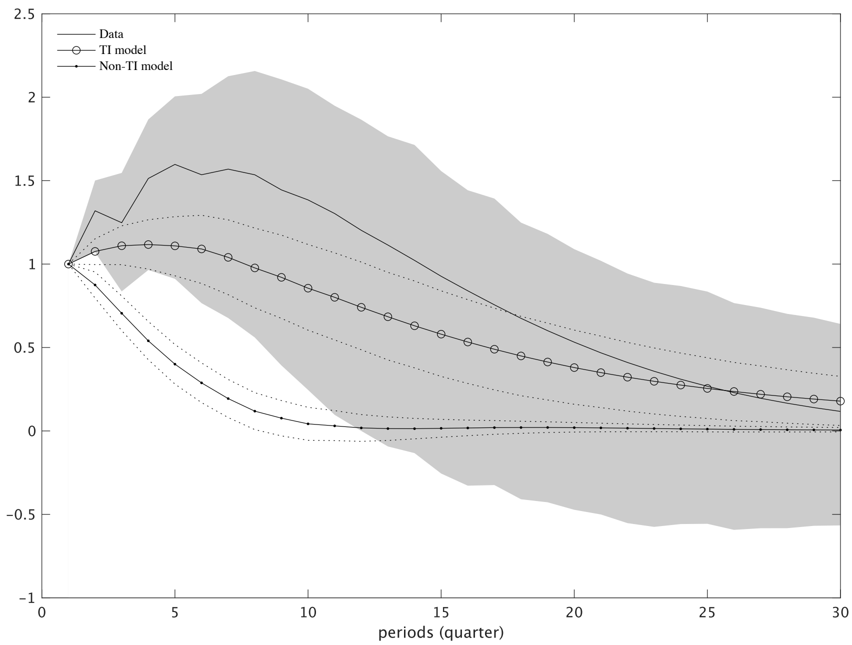

Figure 4: Reduced-Form IRFs of the Real Exchange Rate

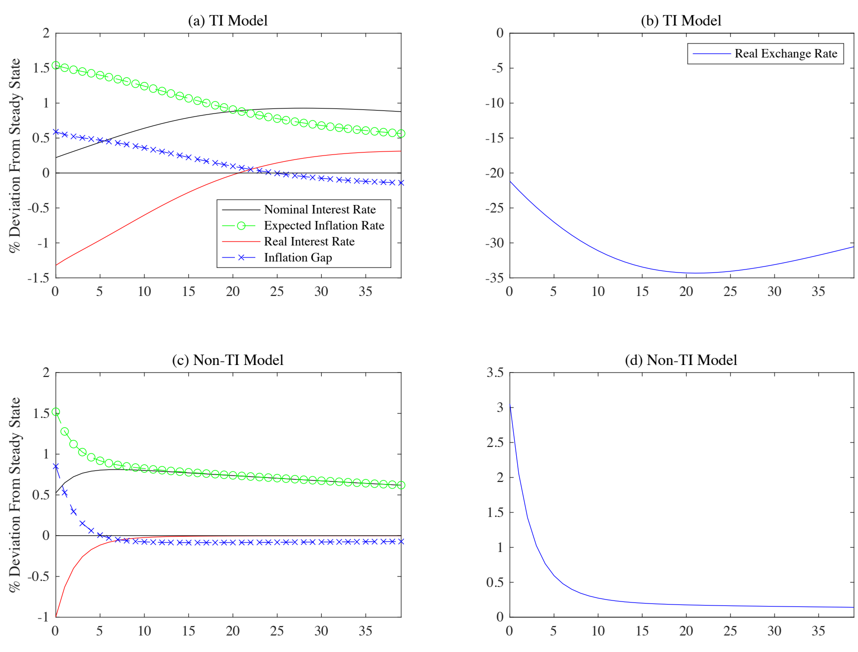

Figure 5: IRFs of Two-country Differentials in the Nominal Interest Rate, Expected Inflation Rate, Inflation Gap, and Real Interest Rate to a Canadian Trend Inflation Shock

Online Appendix

Trend Inflation and Exchange Rate Dynamics: A New Keynesian Approach

Takashi Kano
Graduate School of Economics
Hitotsubashi University
Naka 2-1, Kunitachi, Tokyo
186-8608, JAPAN
Tel/fax: +81-42-580-8283
Email : tkano@econ.hit-u.ac.jp

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Appendix A: The two-country new Keynesian model

A.1. Household sectors

There are home and foreign countries in this model. Throughout this paper, any variable with an asterisk corresponds to a foreign variable, while a variable without an asterisk denotes the home counterpart. The two countries are endowed with representative households whose objectives are the following lifetime utility functions:

$$\sum_{j=0}^{\infty}\beta^j E_t\left\{\ln(C_{t+j}-h\bar{C}_{t+j-1})-\frac{(N_{t+j})^{1+\eta}}{1+\eta}\right\} \quad \text{and} \quad \sum_{j=0}^{\infty}\beta^j E_t\left\{\ln(C^*_{t+j}-h^*\bar{C}^*_{t+j-1})-\frac{(N^*_{t+j})^{1+\eta^*}}{1+\eta^*}\right\},$$

for the home and foreign countries, respectively, where C_t and N_t represent the home country's consumption basket and hours worked, and C^*_t and N^*_t represent their foreign country's counterparts. The two countries share the same subjective discount factor $\beta \in (0,1)$ and have different Frisch elasticities of labor supply η and $\eta^* > 1$.[1]

The representative households in the two countries consume both home (h) and foreign (f) final goods. Home consumption basket C_t consists of home and foreign product aggregators $C_{h,t}$ and $C_{f,t}$, while foreign consumption basket C^*_t consists of home and foreign product aggregators $C^*_{h,t}$ and $C^*_{f,t}$:

$$C_t=\left[\tau^{\frac{1}{\zeta}}(C_{h,t})^{\frac{\zeta-1}{\zeta}}+(1-\tau)^{\frac{1}{\zeta}}(C_{f,t})^{\frac{\zeta-1}{\zeta}}\right]^{\frac{\zeta}{\zeta-1}} \quad \text{and} \quad C^*_t=\left[(1-\tau^*)^{\frac{1}{\zeta^*}}(C^*_{h,t})^{\frac{\zeta^*-1}{\zeta^*}}+\tau^{*\frac{1}{\zeta^*}}(C^*_{f,t})^{\frac{\zeta^*-1}{\zeta^*}}\right]^{\frac{\zeta^*}{\zeta^*-1}},$$

where ζ and $\zeta^* > 0$ are the elasticities of substitution between the home and foreign product aggregators, and τ and τ^* are the degrees of home bias. The static cost minimization problems of the home and foreign households derive the demand functions for $C_{h,t}$, $C_{f,t}$, $C^*_{h,t}$, and $C^*_{f,t}$:

$$C_{h,t}=\tau\left(\frac{P_{h,t}}{P_t}\right)^{-\zeta}C_t, \qquad C_{f,t}=(1-\tau)\left(\frac{P_{f,t}}{P_t}\right)^{-\zeta}C_t,$$

$$C^*_{h,t}=(1-\tau^*)\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\zeta^*}C^*_t, \quad \text{and} \quad C^*_{f,t}=\tau^*\left(\frac{P^*_{f,t}}{P^*_t}\right)^{-\zeta^*}C^*_t,$$

where P_t and P^*_t are the aggregate consumer price indices (CPIs) of the home and foreign countries:

$$P_t=\left[\tau(P_{h,t})^{1-\zeta}+(1-\tau)(P_{f,t})^{1-\zeta}\right]^{\frac{1}{1-\zeta}} \quad \text{and} \quad P^*_t=\left[(1-\tau^*)(P^*_{h,t})^{1-\zeta^*}+\tau^*(P^*_{f,t})^{1-\zeta^*}\right]^{\frac{1}{1-\zeta^*}}.$$

Here, $P_{h,t}$ is the aggregate price over the home goods in the home country, $P_{f,t}$ the aggregate price over

[1] Because of the stochastic trends due to the permanent labor productivity shocks in the two countries, the model assumes the log-utility functions over consumption to guarantee the existence of a balanced growth path in the two-country equilibrium.

the foreign goods in the home country, $P^*_{h,t}$ the aggregate price over the home goods in the foreign country, and $P^*_{f,t}$ the aggregate price over the foreign goods in the foreign country.

The home and foreign aggregators of the home and foreign final goods, $C_{h,t}$, $C_{f,t}$, $C^*_{h,t}$, and $C^*_{f,t}$, consist of a continuum of the home and foreign final goods, each of which is produced by a monopolistically competitive firm residing either in the home or in the foreign countries. The home and foreign final good aggregators in the home country are of the following Dixit–Stiglitz type: for $j = \{h, f\}$,

$$C_{j,t} = \left(\int_0^1 C_{j,t}(z)^{\frac{\zeta-1}{\zeta}} dz \right)^{\frac{\zeta}{\zeta-1}} \text{ and } C^*_{j,t} = \left(\int_0^1 C^*_{j,t}(z)^{\frac{\zeta^*-1}{\zeta^*}} dz \right)^{\frac{\zeta^*}{\zeta^*-1}},$$

where $C_{j,t}(z)$ and $C^*_{j,t}(z)$ are home and foreign demand for the particular home and foreign final good indexed by $z \in [0,1]$, and ζ and $\zeta^* > 0$ represent the home and foreign price elasticities of demand for each final good.

The static cost minimization problems of the representative households in the two countries derive the two countries' demand functions for the home and foreign final goods. Given the home and foreign prices of the home and foreign goods indexed by z, $P_{h,t}(z)$, $P_{f,t}(z)$, $P^*_{h,t}(z)$, and $P^*_{f,t}(z)$, they are

$$C_{j,t}(z) = \left(\frac{P_{j,t}(z)}{P_{j,t}} \right)^{-\zeta} C_{j,t} \quad \text{and} \quad C^*_{j,t}(z) = \left(\frac{P^*_{j,t}(z)}{P^*_{j,t}} \right)^{-\zeta^*} C^*_{j,t},$$

for $j = \{h, f\}$. Price indices $P_{h,t}$, $P_{f,t}$, $P^*_{h,t}$, and $P^*_{f,t}$ are then given as

$$P_{j,t} = \left(\int_0^1 P_{j,t}(z)^{1-\zeta} dz \right)^{\frac{1}{1-\zeta}} \quad \text{and} \quad P^*_{j,t} = \left(\int_0^1 P^*_{j,t}(z)^{1-\zeta^*} dz \right)^{\frac{1}{1-\zeta^*}},$$

for $j = \{h, f\}$.

The home representative household maximizes the lifetime utility subject to the following budget constraint:

$$B_{h,t} + S_t B_{f,t} + P_t C_t \leq (1 + i_{h,t-1}) B_{h,t-1} + S_t (1 + i_{f,t-1}) B_{f,t-1} + W_t N_t + \Lambda_t,$$

where $B_{h,t}$, $B_{f,t}$, $i_{h,t}$, $i_{f,t}$, W_t, Λ_t, and S_t denote the home country's holdings of the home country's nominal bonds, the home country's holdings of the foreign country's nominal bonds, the home country's nominal interest rate for the home country's bonds, the home country's nominal interest rate for the foreign country's bonds, the home country's nominal wage, the static profits from the home country's monopolistically competitive firms, and the bilateral nominal exchange rate of the home currency per the foreign country, respectively. Similarly, the representative household in the foreign country maximizes the lifetime utility subject to the following budget constraint:

$$\frac{B^*_{h,t}}{S_t} + B^*_{f,t} + P^*_t C^*_t \leq (1 + i^*_{h,t-1}) \frac{B^*_{h,t-1}}{S_t} + (1 + i^*_{f,t-1}) B^*_{f,t-1} + W^*_t N^*_t + \Lambda^*_t,$$

where $B^*_{h,t}$, $B^*_{f,t}$, $i^*_{h,t}$, $i^*_{f,t}$, W^*_t, and Λ^*_t denote the foreign country's holdings of the home country's nominal bonds, the foreign country's holdings of the foreign country's nominal bonds, the foreign country's nominal interest rate for the home country's bonds, the foreign country's nominal interest rate for the foreign country's bonds, the foreign country's nominal wage, and the static profits from the foreign country's monopolistically competitive final good firms.

Define habit-adjusted consumption $H_t \equiv C_t - h\bar{C}_{t-1}$ and $H^*_t \equiv C^*_t - h^*\bar{C}^*_{t-1}$. The first-order necessary conditions (FONCs) for the home and foreign households' lifetime utility maximization problems consist of the home and foreign Euler equations:

$$\frac{1}{P_t H_t} = \beta(1 + i_{h,t}) E_t \frac{1}{P_{t+1} H_{t+1}} \quad \text{and} \quad \frac{1}{P^*_t H^*_t} = \beta(1 + i^*_{f,t}) E_t \frac{1}{P^*_{t+1} H^*_{t+1}};$$

the home and foreign utility-based uncovered interest parity conditions (UIPs):

$$E_t \frac{1}{P_{t+1} H_{t+1}} \left\{ (1 + i_{h,t}) - (1 + i_{f,t}) \frac{S_{t+1}}{S_t} \right\} = 0 \quad \text{and} \quad E_t \frac{1}{P^*_{t+1} H^*_{t+1}} \left\{ (1 + i^*_{f,t}) - (1 + i^*_{h,t}) \frac{S_t}{S_{t+1}} \right\} = 0;$$

and the home and foreign intratemporal optimality conditions for hours worked

$$N_t^{\eta} = \frac{W_t}{P_t H_t} \quad \text{and} \quad N_t^{*\eta^*} = \frac{W^*_t}{P^*_t H^*_t}.$$

Suitable transversality conditions for international bond holdings should be satisfied.

A.2. Final good sectors

Facing the corresponding demand functions in the home and foreign countries, a final good firm with index $z \in [0, 1]$ acts as a monopolistically competitive price setter. When setting its current optimal price, each firm follows a Calvo-type time-dependent pricing strategy: in each period, a firm cannot reset its optimal price with a probability $\mu \in (0, 1]$. Moreover, to generate endogenous fluctuations of the real exchange rate in this model, I assume that each firm adopts the local currency pricing strategy as in Betts and Devereux (1996, 2000): each firm sets its optimal price differently between the two countries in terms of the local currencies.

Under such a local currency pricing–Calvo pricing strategy, the objective function for the home firm's profit maximization problem is

$$\max_{\mathbf{P}_{h,t},\mathbf{P}^*_{h,t}} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left\{ \left(\frac{\mathbf{P}_{h,t}}{P_{h,t+i}} \right) - mc_{t+i} \right\} \left(\frac{\mathbf{P}_{h,t}}{P_{h,t+i}} \right)^{-\zeta} C_{h,t+i}$$
$$+ E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left\{ \left(\frac{S_{t+i}\mathbf{P}^*_{h,t}}{P_{h,t+i}} \right) - mc_{t+i} \right\} \left(\frac{\mathbf{P}^*_{h,t}}{P^*_{h,t+i}} \right)^{-\zeta^*} C^*_{h,t+i},$$

where $\mathbf{P}_{h,t}$, $\mathbf{P}^*_{h,t}$, mc_t, and Γ_t are the optimal price of the home good in the home country, the optimal price of the home good in the foreign country, the real marginal cost of the home firm, and the home country's real stochastic discount factor $\Gamma_{t+i} \equiv \beta^i (H_t/H_{t+i})$, respectively. Similarly, the objective function for the foreign firm's profit maximization problem is

$$\max_{\mathbf{P}_{f,t},\mathbf{P}^*_{f,t}} E_t \sum_{i=0}^{\infty} \mu^{*i} \Gamma^*_{t+i} \left\{ \left(\frac{\mathbf{P}_{f,t}}{S_{t+i}P^*_{f,t+i}} \right) - mc^*_{t+i} \right\} \left(\frac{\mathbf{P}_{f,t}}{P_{f,t+i}} \right)^{-\zeta} C_{f,t+i}$$
$$+ E_t \sum_{i=0}^{\infty} \mu^{*i} \Gamma^*_{t+i} \left\{ \left(\frac{\mathbf{P}^*_{f,t}}{P^*_{f,t+i}} \right) - mc^*_{t+i} \right\} \left(\frac{\mathbf{P}^*_{f,t}}{P^*_{f,t+i}} \right)^{-\zeta^*} C^*_{f,t+i},$$

where $\mathbf{P}_{f,t}$, $\mathbf{P}^*_{f,t}$, mc^*_t, and Γ^*_t are the optimal price of the foreign good in the home country, the optimal price of the foreign good in the foreign country, the real marginal cost of the foreign firm, and the foreign country's real stochastic discount factor $\Gamma^*_{t+i} = \beta^i (H^*_t/H^*_{t+i})$, respectively.

The FONCs for the optimal prices that the home firm sets for the home and foreign countries are

$$\mathbf{P}_{h,t} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left(\frac{1}{P_{h,t+i}} \right)^{1-\zeta} C_{h,t+i} = \frac{\zeta}{\zeta - 1} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} mc_{t+i} \left(\frac{1}{P_{h,t+i}} \right)^{-\zeta} C_{h,t+i},$$
$$\mathbf{P}^*_{h,t} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} \left(\frac{S_{t+i}P^*_{h,t+i}}{P_{h,t+i}} \right) \left(\frac{1}{P^*_{h,t+i}} \right)^{1-\zeta^*} C^*_{h,t+i} = \frac{\zeta^*}{\zeta^* - 1} E_t \sum_{i=0}^{\infty} \mu^i \Gamma_{t+i} mc_{t+i} \left(\frac{1}{P^*_{h,t+i}} \right)^{-\zeta^*} C^*_{h,t+i},$$

respectively. Similarly, the FONCs for the optimal prices that the foreign firm sets for the home and foreign countries are

$$\mathbf{P}_{f,t} E_t \sum_{i=0}^{\infty} \mu^{*i} \Gamma^*_{t+i} \left(\frac{P_{f,t+i}}{S_{t+i}P^*_{f,t+i}} \right) \left(\frac{1}{P_{f,t+i}} \right)^{1-\zeta} C_{f,t+i} = \frac{\zeta}{\zeta - 1} E_t \sum_{i=0}^{\infty} \mu^{*i} \Gamma^*_{t+i} mc^*_{t+i} \left(\frac{1}{P_{f,t+i}} \right)^{-\zeta} C_{f,t+i},$$
$$\mathbf{P}^*_{f,t} E_t \sum_{i=0}^{\infty} \mu^{*i} \Gamma^*_{t+i} \left(\frac{1}{P^*_{f,t+i}} \right)^{1-\zeta^*} C^*_{f,t+i} = \frac{\zeta^*}{\zeta^* - 1} E_t \sum_{i=0}^{\infty} \mu^{*i} \Gamma^*_{t+i} mc^*_{t+i} \left(\frac{1}{P^*_{f,t+i}} \right)^{-\zeta^*} C^*_{f,t+i}.$$

Given the optimal prices, price indices $P_{h,t}$, $P_{f,t}$, $P^*_{h,t}$, and $P^*_{f,t}$ follow the laws of motion:

$$P_{j,t}^{1-\zeta} = (1-\mu)\mathbf{P}_{j,t}^{1-\zeta} + \mu P_{j,t-1}^{1-\zeta}, \quad \text{and} \quad P_{j,t}^{*1-\zeta^*} = (1-\mu^*)\mathbf{P}_{j,t}^{*1-\zeta^*} + \mu^* P_{j,t-1}^{*1-\zeta^*},$$

where $j = \{h, f\}$.

Each firm produces its final good by using labor input hired from a domestic competitive labor market. The production functions of the home and foreign goods are $Y_t(z) = A_t L_t(z)$ and $Y_t^*(z) = A_t^* L_t^*(z)$, where A_t and A_t^* are the labor productivities in the home and foreign countries. In this case, the real marginal costs that the home and foreign firms face are

$$mc_t = \frac{W_t}{A_t P_{h,t}} \quad \text{and} \quad mc_t^* = \frac{W_t^*}{A_t^* P_{f,t}^*}.$$

The home and foreign static profits are

$$\begin{aligned}
\Lambda_t &\equiv \int_0^1 \{P_{h,t}(z)C_{h,t}(z) + S_t P^*_{h,t}(z) C^*_{h,t}(z) - W_t N_t(z)\} dz \\
&= P_{h,t}\tau \left(\frac{P_{h,t}}{P_t}\right)^{-\zeta} C_t + S_t P^*_{h,t}(1-\tau^*) \left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\zeta^*} C^*_t - W_t N_t, \\
S_t \Lambda^*_t &\equiv \int_0^1 \{P_{f,t}(z)C_{f,t}(z) + S_t P^*_{f,t}(z) C^*_{f,t}(z) - S_t W^*_t N^*_t(z)\} dz \\
&= P_{f,t}(1-\tau) \left(\frac{P_{f,t}}{P_t}\right)^{-\zeta} C_t + S_t P^*_{f,t}\tau^* \left(\frac{P^*_{f,t}}{P^*_t}\right)^{-\zeta^*} C^*_t - S_t W^*_t N^*_t,
\end{aligned}$$

respectively.

I assume that the logarithms of labor productivities, $\ln A_t$ and $\ln A_t^*$, are of I(1). To guarantee a balanced growth path of the two-country model, the labor productivity differential $\ln a_t \equiv \ln A_t - \ln A_t^*$ should be of I(0). As investigated by Mandelman et al. (2011), Rabanal et al. (2011), and Ireland (2013), the I(1) labor productivities and stationary productivity differential jointly imply that the labor productivity of the home country must be cointegrated with that of the foreign country. Following Kano (2020), I assume that the home and foreign growth rates of labor productivity, $\gamma_{A,t} \equiv \ln A_t - \ln A_{t-1}$ and $\gamma^*_{A,t} \equiv \ln A^*_t - \ln A^*_{t-1}$, are generated by error correction processes:

$$\ln \gamma_{A,t} = \ln \gamma_A - \frac{\lambda}{2} \ln a_{t-1} + \epsilon_{A,t}, \quad \text{and} \quad \ln \gamma^*_{A,t} = \ln \gamma^*_A + \frac{\lambda}{2} \ln a_{t-1} + \epsilon^*_{A,t},$$

where $\gamma_A, \gamma_A^* > 1$ are the drift terms and $\lambda \in [0, 1)$ are the speed of error correction. The error correction mechanism implies that the cross-country labor productivity differential is of I(0) because

$$\ln a_t = \ln \gamma_A / \gamma_A^* + (1-\lambda) \ln a_{t-1} + \epsilon_{A,t} - \epsilon^*_{A,t}.$$

Importantly, if the adjustment speed λ is sufficiently close to zero, the cross-country labor productivity differential follows a near I(1) process.

A.3. Monetary policies with trend inflation

The monetary policies in the two countries are characterized by Taylor rules. The central banks of the two countries set their short-term domestic nominal interest rate as $(1 + i_{h,t})$ and $(1 + i^*_{f,t})$ depending on the past interest rate levels $(1 + i_{h,t-1})$ and $(1 + i^*_{f,t-1})$, current inflation rates $\gamma_{\pi,t} \equiv P_t/P_{t-1}$ and $\gamma^*_{\pi,t} \equiv P^*_t/P^*_{t-1}$, and detrended output levels $y_t \equiv Y_t/A_t$ and $y^*_t \equiv Y^*_t/A^*_t$. In this paper, I allow for stochastic trend inflation $\gamma_{\tau,t} \equiv P_{\tau,t}/P_{\tau,t-1}$ and $\gamma^*_{\tau,t} \equiv P^*_{\tau,t}/P^*_{\tau,t-1}$ in the two countries, where $P_{\tau,t}$ and $P^*_{\tau,t}$ are the exogenous permanent components of the aggregate price levels of the two countries generated by trend inflation. As specified by Ireland (2007) and Cogley et al. (2010), the central banks of the two countries target the time-varying trend inflation levels in the Taylor rules:

$$(1 + i_{h,t}) = (1 + i)^{1-\rho_i}(1 + i_{h,t-1})^{\rho_i} \left[\left(\frac{\gamma_{\pi,t}}{\gamma_{\tau,t}}\right)^{a_\pi} (y_t)^{a_y} \left(\frac{q_t \gamma_{\pi,t}}{q_{t-1}\gamma^*_{\pi,t}}\right)^{\frac{a_s}{1-a_s}} \right]^{1-\rho_i} \exp(\epsilon_{i,t}),$$

$$(1 + i^*_{f,t}) = (1 + i^*)^{1-\rho^*_i}(1 + i^*_{f,t-1})^{\rho^*_i} \left[\left(\frac{\gamma^*_{\pi,t}}{\gamma^*_{\tau,t}}\right)^{a^*_\pi} (y^*_t)^{a^*_y} \left(\frac{q_t \gamma_{\pi,t}}{q_{t-1}\gamma^*_{\pi,t}}\right)^{-\frac{a^*_s}{1-a^*_s}} \right]^{1-\rho^*_i} \exp(\epsilon^*_{i,t}),$$

where i and i^* are the deterministic steady-state values of the home and foreign nominal interest rates and ρ_i and $\rho^*_i \in (0, 1)$ capture the home and foreign degrees of interest rate smoothing. In this paper, the trend inflation rates $\gamma_{\tau,t}$ and $\gamma^*_{\tau,t}$ follow exogenous AR(1) processes in the logarithmic term:

$$\ln \gamma_{\tau,t} = (1 - \rho_\tau) \ln \gamma_\tau + \rho_\tau \ln \gamma_{\tau,t-1} + \epsilon_{\tau,t} \quad \text{and} \quad \ln \gamma^*_{\tau,t} = (1 - \rho^*_\tau) \ln \gamma^*_\tau + \rho^*_\tau \ln \gamma^*_{\tau,t-1} + \epsilon^*_{\tau,t},$$

where γ_τ and γ^*_τ are the long-run means of the home and foreign trend inflation rates, ρ_τ and $\rho^*_\tau \in [0, 1)$ are the AR(1) roots of the home and foreign trend inflation rates, and $\epsilon_{\tau,t}$ and $\epsilon^*_{\tau,t}$ are the i.i.d. trend inflation shocks.[2] As in a closed-economy NK model by Ascari and Sbordone (2014), the time-invariant unconditional means γ_τ and γ^*_τ make the log-linearization exercise of this study tractable when characterizing the deterministic steady state.[3]

I assume that the monetary policy disturbances in the home and foreign countries, $\epsilon_{i,t}$ and $\epsilon^*_{i,t}$, are i.i.d. shocks. Coibion and Gorodnichenko (2012) provide a series of empirical evidence that the large persistence observed in the actual data on the short-run nominal interest rate in the United States stems largely from

[2]Cogley et al. (2010) discuss as a primary reason for the stochastic variations in the central banks' long-run inflation targets that the central banks only imperfectly know the true economic structure and their learning process generates endogenous updating of the inflation targets.

[3]Ascari and Sbordone (2014) do not allow for stochastic variations in trend inflation.

the policy inertia caused by interest rate smoothing following the Taylor rule, but not from the exogenous persistence of the monetary policy disturbance, as claimed by Rudebusch (2002, 2006). In particular, their empirical result suggests that the time-varying inflation target that the conventional Taylor rule misses could lead to a high estimate of the serial correlation of the monetary policy disturbance. Indeed, the persistent trend inflation shock $\gamma_{\tau,t}$ is indistinguishable from a persistent monetary policy disturbance if $\epsilon_{i,t}$ is set to $-(1-\rho_i)a_\pi \ln \gamma_{\tau,t}$. Following the above notion, I allow for both interest rate smoothing and the time-varying inflation target in the Taylor rules in this study.

A.4. Market clearing and productivity shocks

To guarantee a stationary distribution of the net foreign asset positions of the two countries within incomplete international financial markets, I allow for a debt-elastic risk premium in the interest rates faced only by the home country: for $j = \{h, f\}$,

$$i_{j,t} = i^*_{j,t}[1 + \psi\{\exp(-b_{j,t} + \bar{d}) - 1\}] \exp(v_{r,t}), \quad \bar{d} \leq 0, \quad \psi > 0,$$

where $b_{j,t}$ is the transitory component of the home country's holdings of country j's bonds, which is precisely defined below. The home country needs to pay a risk premium over the interest rate level the foreign household faces when the transitory component of the home country's net borrowing positions $b_{j,t} < 0$ is beyond its threshold level $\bar{d}$. The risk premium is given as an externality: the household does not take into account the effect of the debt position on the risk premium when maximizing its lifetime utility function. As in Bergin (2006) and Justiniano and Preston (2010), the risk premium is also subject to exogenous risk premium shock $v_{r,t}$, which follows an AR(1) process:

$$\ln v_{r,t} = \rho_r \ln v_{r,t-1} + \epsilon_{r,t}.$$

On the contrary, I do not attach a risk premium to the foreign country's interest rates.[4]

The market-clearing conditions of the two countries' bond markets are

$$B_{h,t} + B^*_{h,t} = 0 \quad \text{and} \quad B_{f,t} + B^*_{f,t} = 0.$$

In other words, along an equilibrium path, the world net supply of nominal bonds is zero on a period-by-

[4]The elasticity of the risk premium toward the debt position, ψ, is set to a small number 0.00001.

period basis. The market-clearing conditions of the home and foreign final goods are

$$A_t N_t = \int_0^1 \{C_{h,t}(z) + C^*_{h,t}(z)\}dz = \Omega_{h,t} C_{h,t} + \Omega^*_{h,t} C^*_{h,t},$$
$$A^*_t N^*_t = \int_0^1 \{C_{f,t}(z) + C^*_{f,t}(z)\}dz = \Omega_{f,t} C_{f,t} + \Omega^*_{f,t} C^*_{f,t},$$

where the variables $\Omega_{h,t}$, $\Omega^*_{h,t}$, $\Omega_{f,t}$, and $\Omega^*_{f,t}$, which are defined by

$$\Omega_{j,t} \equiv \int_0^1 \left(\frac{P_{j,t}(z)}{P_{j,t}}\right)^{-\zeta} dz, \quad \text{and} \quad \Omega^*_{j,t} \equiv \int_0^1 \left(\frac{P^*_{j,t}(z)}{P^*_{j,t}}\right)^{-\zeta^*} dz, \quad \text{for } j = h, f$$

capture the degrees of price dispersion in the four final good markets. As discussed by Ascari and Sbordone (2014), the price dispersion variables are greater than one under price stickiness, but are one if all the prices are identical within each final good market in each country under flexible price adjustments. The market-clearing conditions then imply that variables $\Omega_{j,t}$ and $\Omega^*_{j,t}$ represent the resource costs of price dispersion: given the output level, the higher the price dispersion variable, the lower the amount allocated to consumption. It is shown that the price dispersion variables follow the transitions

$$\Omega_{j,t} = (1-\mu)\left(\frac{\mathbf{P}_{j,t}}{P_{j,t}}\right)^{-\zeta} + \mu\left(\frac{P_{j,t}}{P_{j,t-1}}\right)^{\zeta} \Omega_{j,t-1},$$
$$\Omega^*_{j,t} = (1-\mu^*)\left(\frac{\mathbf{P}^*_{j,t}}{P^*_{j,t}}\right)^{-\zeta^*} + \mu^*\left(\frac{P^*_{j,t}}{P^*_{j,t-1}}\right)^{\zeta^*} \Omega^*_{j,t-1},$$

for $j = h, f$.[5]

Appendix B: log-linear approximation of FONCs

Because the model contains non-stationary components, A_t, A^*_t, $P_{\tau,t}$, and $P^*_{\tau,t}$, I stochastically detrend the FONCs by these stochastic trend components to characterize the unique deterministic steady state. In doing so, I define the stochastically detrended versions of home consumption by $c_t \equiv C_t/A_t$; foreign consumption $c^*_t \equiv C^*_t/A^*_t$; home consumption habit $h_t \equiv H_t/A_t$; foreign consumption habit $h^*_t \equiv H^*_t/A^*_t$; the home price of home goods $p_{h,t} \equiv P_{h,t}A_t/P_{\tau,t}$; the home price of foreign goods $p_{f,t} \equiv P_{f,t}A_t/P_{\tau,t}$; the foreign price of home goods $p^*_{h,t} \equiv P^*_{h,t}A^*_t/P^*_{\tau,t}$; the foreign price of foreign goods $p^*_{f,t} \equiv P^*_{f,t}A^*_t/P^*_{\tau,t}$; the optimal home price of home goods $\mathbf{p}_{h,t} \equiv \mathbf{P}_{h,t}A_t/P_{\tau,t}$; the optimal home price of foreign goods $\mathbf{p}_{f,t} \equiv \mathbf{P}_{f,t}A_t/P_{\tau,t}$; the optimal foreign price of home goods $\mathbf{p}^*_{h,t} \equiv \mathbf{P}^*_{h,t}A^*_t/P^*_{\tau,t}$; the optimal foreign price of foreign goods $\mathbf{p}^*_{f,t} \equiv \mathbf{P}^*_{f,t}A^*_t/P^*_{\tau,t}$; the home CPI $p_t \equiv P_tA_t/P_{\tau,t}$; the foreign

[5] The full derivation of the transition equations of the price dispersion variables is found in Ascari and Sbordone (2014).

CPI $p_t^* \equiv P_t^* A_t^* / P_{\tau,t}^*$; the home holdings of the home bond $b_{h,t} = B_{h,t}/P_{\tau,t}$; the home holdings of the foreign bond $b_{f,t} = B_{f,t}/P_{\tau,t}^*$; the foreign holdings of the home bond $b_{h,t}^* = B_{h,t}^*/P_{\tau,t}$; the foreign holdings of the foreign bond $b_{f,t}^* \equiv B_{f,t}^*/P_{\tau,t}^*$; the home nominal wage $w_t \equiv W_t/P_{\tau,t}$; the foreign nominal wage $w_t^* \equiv W_t^*/P_{\tau,t}^*$; and the nominal exchange rate $s_t \equiv S_t P_{\tau,t}^*/P_{\tau,t}$. The real exchange rate is given by $q_t \equiv S_t P_t^*/P_t = s_t a_t p_t^*/p_t$.

B.1. Stochastically detrended FONCs

The stochastically detrended versions of the FONCs for the home country consist of the budget constraint

$$p_t c_t + b_{h,t} + s_t b_{f,t} = \frac{(1+i_{h,t-1}) b_{h,t-1}}{\gamma_{\tau,t}} + \frac{(1+i_{f,t-1}) s_t b_{f,t-1}}{\gamma_{\tau,t}^*} + p_{h,t} \tau \left(\frac{p_{h,t}}{p_t}\right)^{-\zeta} c_t + s_t p_{h,t}^* (1-\tau^*) \left(\frac{p_{h,t}^*}{p_t^*}\right)^{-\zeta^*} c_t^*; \tag{B.1}$$

the home intratemporal optimality condition

$$N_t^\eta = \frac{w_t}{p_t h_t}; \tag{B.2}$$

the home Euler equation

$$\frac{1}{p_t h_t} = \beta(1+i_{h,t}) E_t \left(\frac{1}{\gamma_{\tau,t+1} p_{t+1} h_{t+1}}\right); \tag{B.3}$$

the home UIP condition

$$s_t (1+i_{h,t}) E_t \left(\frac{1}{p_{t+1} h_{t+1} \gamma_{\tau,t+1}}\right) = (1+i_{f,t}) E_t \left(\frac{s_{t+1}}{p_{t+1} h_{t+1} \gamma_{\tau,t+1}^*}\right); \tag{B.4}$$

the debt elastic risk premium that the home country faces

$$i_{h,t} = i_{h,t}^* [1 + \psi\{\exp(-b_{h,t} + \bar{d}) - 1\}] \exp(v_{r,t}), \quad \text{and} \quad i_{f,t} = i_{f,t}^* [1 + \psi\{\exp(-b_{f,t} + \bar{d}) - 1\}] \exp(v_{r,t}); \tag{B.5}$$

the market clearing condition for the home country

$$N_t = \Omega_{h,t} c_{h,t} + \Omega_{h,t}^* c_{h,t}^* a_t^{-1}; \tag{B.6}$$

the transition equations of price dispersions $\Omega_{h,t}$ and $\Omega^*_{h,t}$

$$\Omega_{h,t} = (1-\mu)\left(\frac{\mathbf{p}_{h,t}}{p_{h,t}}\right)^{-\zeta} + \mu\left(\frac{p_{h,t}}{p_{h,t-1}}\right)^{\zeta}\left(\frac{\gamma_{\tau,t}}{\gamma_{A,t}}\right)^{\zeta}\Omega_{h,t-1},$$

$$\text{and}\quad \Omega^*_{h,t} = (1-\mu^*)\left(\frac{\mathbf{p}^*_{h,t}}{p^*_{h,t}}\right)^{-\zeta^*} + \mu^*\left(\frac{p^*_{h,t}}{p^*_{h,t-1}}\right)^{\zeta^*}\left(\frac{\gamma^*_{\tau,t}}{\gamma^*_{A,t}}\right)^{\zeta^*}\Omega^*_{h,t-1}; \quad \text{(B.7)}$$

the optimal price setting rules of the home firm for the home and foreign markets

$$\frac{\mathbf{p}_{h,t}}{p_{h,t}}E_t\sum_{i=0}^{\infty}(\beta\mu)^i\phi_{h,t+i}^{\zeta-1}\left(\frac{c_{h,t+i}}{h_{t+i}}\right) = \frac{\zeta}{\zeta-1}E_t\sum_{i=0}^{\infty}(\beta\mu)^i\phi_{h,t+i}^{\zeta}mc_{t+i}\left(\frac{c_{h,t+i}}{h_{t+i}}\right), \quad \text{and} \quad \text{(B.8)}$$

$$\frac{\mathbf{p}^*_{h,t}}{p^*_{h,t}}E_t\sum_{i=0}^{\infty}(\beta\mu)^i\frac{s_{t+i}p^*_{h,t+i}}{p_{h,t+i}}\phi_{h,t+i}^{*\zeta^*-1}\left(\frac{c^*_{h,t+i}}{h_{t+i}}\right) = \frac{\zeta^*}{\zeta^*-1}E_t\sum_{i=0}^{\infty}(\beta\mu)^i\phi_{h,t+i}^{*\zeta^*}\frac{mc_{t+i}}{a_{t+i}}\left(\frac{c^*_{h,t+i}}{h_{t+i}}\right), \quad \text{(B.9)}$$

where cumulative inflations ϕ_t, $\phi_{h,t}$, and $\phi^*_{h,t}$ are defined as

$$\phi_{t+i} \equiv \Pi_{s=1}^{i}\left(\frac{P_{t+s}}{P_t}\right) = \left(\frac{p_{t+i}}{p_t}\right)\Pi_{s=1}^{i}\left(\frac{\gamma_{\tau,t+s}}{\gamma_{A,t+s}}\right), \quad \phi_{h,t+i} \equiv \Pi_{s=1}^{i}\left(\frac{P_{h,t+s}}{P_{h,t}}\right) = \left(\frac{p_{h,t+i}}{p_{h,t}}\right)\Pi_{s=1}^{i}\left(\frac{\gamma_{\tau,t+s}}{\gamma_{A,t+s}}\right),$$

$$\text{and}\quad \phi^*_{h,t+i} \equiv \Pi_{s=1}^{i}\left(\frac{P^*_{h,t+s}}{P^*_{h,t}}\right) = \left(\frac{p^*_{h,t+i}}{p^*_{h,t}}\right)\Pi_{s=1}^{i}\left(\frac{\gamma^*_{\tau,t+s}}{\gamma^*_{A,t+s}}\right); \quad \text{(B.10)}$$

the home and foreign demand functions for the home good

$$c_{h,t} = \tau\left(\frac{p_{h,t}}{p_t}\right)^{-\zeta}c_t, \quad \text{and} \quad c^*_{h,t} = (1-\tau^*)\left(\frac{p^*_{h,t}}{p^*_t}\right)^{-\zeta^*}c^*_t; \quad \text{(B.11)}$$

the home CPI

$$p_t^{1-\zeta} = \tau p_{h,t}^{1-\zeta} + (1-\tau)p_{f,t}^{1-\zeta}; \quad \text{(B.12)}$$

the laws of motion of the home final good prices at the home and foreign countries

$$p_{h,t}^{1-\zeta} = (1-\mu)\mathbf{p}_{h,t}^{1-\zeta} + \mu p_{h,t-1}^{1-\zeta}\left(\frac{\gamma_{A,t}}{\gamma_{\tau,t}}\right)^{1-\zeta}, \quad \text{and} \quad p_{h,t}^{*1-\zeta^*} = (1-\mu)\mathbf{p}_{h,t}^{*1-\zeta^*} + \mu p_{h,t-1}^{*1-\zeta^*}\left(\frac{\gamma^*_{A,t}}{\gamma^*_{\tau,t}}\right)^{1-\zeta^*}; \quad \text{(B.13)}$$

the real marginal cost of the home firm

$$mc_t = \frac{w_t}{p_{h,t}}; \quad \text{(B.14)}$$

the Taylor rule the home central bank follows

$$(1+i_{h,t}) = (1+i)^{1-\rho_i}(1+i_{h,t-1})^{\rho_i} \left[\left(\frac{p_t}{p_{t-1}} \gamma_{A,t}^{-1} \right)^{a_\pi} N_t^{a_y} \left(\frac{q_t}{q_{t-1}} \right)^{\frac{a_s}{1-a_s}} \left(\frac{\gamma_{\pi,t}}{\gamma_{\pi,t}^*} \right)^{\frac{a_s}{1-a_s}} \right]^{1-\rho_i} \exp(\epsilon_{i,t}); \quad \text{(B.15)}$$

and home consumption habit

$$h_t = c_t - h\gamma_{A,t}^{-1} c_{t-1}. \quad \text{(B.16)}$$

Similarly, the stochastically detrended versions of the FONCs for the foreign country consist of the budget constraint

$$\frac{q_t p_t c_t^*}{a_t} - b_{h,t} - s_t b_{f,t} = -\frac{(1+i_{h,t-1}^*) b_{h,t-1}}{\gamma_{\tau,t}} - \frac{(1+i_{f,t-1}^*) s_t b_{f,t-1}}{\gamma_{\tau,t}^*} + p_{f,t}(1-\tau) \left(\frac{p_{f,t}}{p_t} \right)^{-\zeta} c_t + s_t p_{f,t}^* \tau^* \left(\frac{p_{f,t}^*}{p_t^*} \right)^{-\zeta^*} c_t^*; \quad \text{(B.17)}$$

the foreign intratemporal optimality condition

$$N_t^{*\eta^*} = \frac{w_t^* s_t a_t}{q_t p_t h_t^*}; \quad \text{(B.18)}$$

the foreign Euler equation

$$\frac{a_t s_t}{q_t p_t h_t^*} = \beta(1+i_{f,t}^*) E_t \frac{a_{t+1} s_{t+1}}{\gamma_{\tau,t+1}^* q_{t+1} p_{t+1} h_{t+1}^*}; \quad \text{(B.19)}$$

the foreign UIP condition

$$s_t(1+i_{h,t}^*) E_t \left(\frac{a_{t+1}}{q_{t+1} p_{t+1} h_{t+1}^* \gamma_{\tau,t+1}} \right) = (1+i_{f,t}^*) E_t \left(\frac{a_{t+1} s_{t+1}}{q_{t+1} p_{t+1} h_{t+1}^* \gamma_{\tau,t+1}^*}. \right), \quad \text{(B.20)}$$

the market clearing condition for the foreign country

$$N_t^* = \Omega_{f,t} a_t c_{f,t} + \Omega_{f,t}^* c_{f,t}^*; \quad \text{(B.21)}$$

the transition equations of price dispersions $\Omega_{f,t}$ and $\Omega_{f,t}^*$

$$\Omega_{f,t} = (1-\mu^*) \left(\frac{\mathbf{p}_{f,t}}{p_{f,t}} \right)^{-\zeta} + \mu^* \left(\frac{p_{f,t}}{p_{f,t-1}} \right)^{\zeta} \left(\frac{\gamma_{\tau,t}}{\gamma_{A,t}} \right)^{\zeta} \Omega_{f,t-1},$$

$$\text{and} \quad \Omega_{f,t}^* = (1-\mu^*) \left(\frac{\mathbf{p}_{f,t}^*}{p_{f,t}^*} \right)^{-\zeta^*} + \mu^* \left(\frac{p_{f,t}^*}{p_{f,t-1}^*} \right)^{\zeta^*} \left(\frac{\gamma_{\tau,t}^*}{\gamma_{A,t}^*} \right)^{\zeta^*} \Omega_{f,t-1}^*; \quad \text{(B.22)}$$

the optimal price setting rules of the foreign firm

$$\frac{\mathbf{p}_{f,t}}{p_{f,t}} E_t \sum_{i=0}^{\infty} (\beta\mu^*)^i \frac{p_{f,t+i}}{s_{t+i} p^*_{f,t+i}} \phi_{f,t+i}^{\zeta-1} \left(\frac{c_{f,t+i}}{h^*_{t+i}} \right) = \frac{\zeta}{\zeta-1} E_t \sum_{i=0}^{\infty} (\beta\mu^*)^i mc^*_{t+i} a_{t+i} \phi_{f,t+i}^{\zeta} \left(\frac{c_{f,t+i}}{h^*_{t+i}} \right), \quad \text{and} \tag{B.23}$$

$$\frac{\mathbf{p}^*_{f,t}}{p^*_{f,t}} E_t \sum_{i=0}^{\infty} (\beta\mu^*)^i \phi_{f,t+i}^{*\zeta^*-1} \left(\frac{c^*_{f,t+i}}{h^*_{t+i}} \right) = \frac{\zeta^*}{\zeta^*-1} E_t \sum_{i=0}^{\infty} (\beta\mu^*)^i mc^*_{t+i} \phi_{f,t+i}^{*\zeta^*} \left(\frac{c^*_{f,t+i}}{h^*_{t+i}} \right), \tag{B.24}$$

where cumulative inflations ϕ^*_t, $\phi_{f,t}$, and $\phi^*_{f,t}$ are defined as

$$\phi^*_{t+i} \equiv \Pi_{s=1}^i \left(\frac{P^*_{t+s}}{P^*_t} \right) = \left(\frac{p^*_{t+i}}{p^*_t} \right) \Pi_{s=1}^i \left(\frac{\gamma^*_{\tau,t+s}}{\gamma^*_{A,t+s}} \right), \quad \phi_{f,t+i} \equiv \Pi_{s=1}^i \left(\frac{P_{f,t+s}}{P_{f,t}} \right) = \left(\frac{p_{f,t+i}}{p_{f,t}} \right) \Pi_{s=1}^i \left(\frac{\gamma_{\tau,t+s}}{\gamma_{A,t+s}} \right),$$

$$\text{and} \quad \phi^*_{f,t+i} \equiv \Pi_{s=1}^i \left(\frac{P^*_{f,t+s}}{P^*_{f,t}} \right) = \left(\frac{p^*_{f,t+i}}{p^*_{f,t}} \right) \Pi_{s=1}^i \left(\frac{\gamma^*_{\tau,t+s}}{\gamma^*_{A,t+s}} \right); \tag{B.25}$$

the demand functions for the foreign goods

$$c_{f,t} = (1-\tau) \left(\frac{p_{f,t}}{p_t} \right)^{-\zeta} c_t, \quad \text{and} \quad c^*_{f,t} = \tau^* \left(\frac{p^*_{f,t}}{p^*_t} \right)^{-\zeta^*} c^*_t; \tag{B.26}$$

the foreign CPI

$$p_t^{*1-\zeta^*} = (1-\tau^*) p_{h,t}^{*1-\zeta^*} + \tau^* p_{f,t}^{*1-\zeta^*}; \tag{B.27}$$

the laws of motion of the foreign final good prices at the home and foreign countries

$$p_{f,t}^{1-\zeta} = (1-\mu^*) \mathbf{p}_{f,t}^{1-\zeta} + \mu^* p_{f,t-1}^{1-\zeta} \left(\frac{\gamma_{A,t}}{\gamma_{\tau,t}} \right)^{1-\zeta}, \quad \text{and} \quad p_{f,t}^{*1-\zeta^*} = (1-\mu^*) \mathbf{p}_{f,t}^{*1-\zeta^*} + \mu^* p_{f,t-1}^{*1-\zeta^*} \left(\frac{\gamma^*_{A,t}}{\gamma^*_{\tau,t}} \right)^{1-\zeta^*}; \tag{B.28}$$

the real marginal cost the foreign firm faces

$$mc^*_t = \frac{w^*_t}{p^*_{f,t}}; \tag{B.29}$$

the Taylor rule the foreign central bank follows

$$(1+i^*_{f,t}) = (1+i^*)^{1-\rho^*_i} (1+i^*_{f,t-1})^{\rho^*_i} \left[\left(\frac{p^*_t}{p^*_{t-1}} \gamma^{*-1}_{A,t} \right)^{a^*_\pi} N_t^{*a^*_y} \left(\frac{q_t}{q_{t-1}} \right)^{-\frac{a^*_s}{1-a^*_s}} \left(\frac{\gamma_{\pi,t}}{\gamma^*_{\pi,t}} \right)^{-\frac{a^*_s}{1-a^*_s}} \right]^{1-\rho^*_i} \exp(\epsilon^*_{i,t}); \tag{B.30}$$

and foreign consumption habit

$$h^*_t = c^*_t - h^* \gamma^{*-1}_{A,t} c^*_{t-1}. \tag{B.31}$$

B.2. Deterministic steady state of the stochastically detrended system

Let ω_h, ω_f, ω_h^*, and ω_f^* denote the steady-state home and foreign relative prices, p_h/p, p_f/p, p_h^*/p^*, and p_f^*/p^*. Let Φ and Φ^* denote the home and foreign countries' terms of trade p_h/p_f and p_f^*/p_h^*. Also let Ψ and Ψ^* denote the deviations of the home and foreign goods from the law of one price, sap_h^*/p_h and $p_f/(sap_f^*)$. In this paper, the deterministic steady state of the stochastically detrended system (B.1) - (B.31) is characterized under a restriction that the law of one price holds for the imported goods of the home and foreign countries at the deterministic steady state; $sap_h^* = p_f$. Under this condition, the law of one price deviation of the home good is equal to the inverse of the home terms of trade; $\Psi = \Phi^{-1}$. Similarly, the law of one price deviation of the foreign good is equal to the foreign terms of trade; $\Psi^* = \Phi^{*-1}$.

Define $\bar{\gamma} \equiv \gamma_\tau/(\gamma_A\gamma_A^*)^{\frac{1}{2}}$, $\bar{\gamma}^* \equiv \gamma_\tau^*/(\gamma_A\gamma_A^*)^{\frac{1}{2}}$, $\bar{h} \equiv h/(\gamma_A\gamma_A^*)^{\frac{1}{2}}$, $\bar{h}^* \equiv h^*/(\gamma_A\gamma_A^*)^{\frac{1}{2}}$, and $a \equiv (\gamma_A/\gamma_A^*)^{\frac{1}{\lambda}}$. The model then determines the steady state values of the 19 detrended variables, q, c, c^*, N, N^*, ω_h, ω_h^*, ω_f, ω_f^*, mc, mc^*, Ω_h, Ω_h^*, Ω_f, Ω_f^*, Ψ, Ψ^*, Φ, and Φ^*, by the following 19 equations:

$$\Phi = \frac{1}{\Psi},$$

$$\Phi^* = \frac{1}{\Psi^*},$$

$$q = \Psi\frac{w_h^*}{w_h}$$

$$\omega_h^{\zeta-1} = \tau + (1-\tau)\Phi^{\zeta-1},$$

$$\omega_h^{*\zeta^*-1} = 1-\tau^* + \tau^*(\Psi\Psi^*\Phi)^{\zeta^*-1},$$

$$\omega_f^{\zeta-1} = 1-\tau+\tau\Phi^{1-\zeta},$$

$$\omega_f^{*\zeta^*-1} = \tau^* + (1-\tau^*)(\Psi\Psi^*\Phi)^{1-\zeta^*},$$

$$(1-\bar{h})cN^\eta = \omega_h mc,$$

$$N = [\Omega_h\tau\omega_h^{-\zeta} + \Omega_h^*(1-\tau)\omega_f^{-\zeta}]c$$

$$(1-\bar{h}^*)c^*N^{*\eta^*} = \omega_f^* mc^*,$$

$$N^* = [\Omega_f(1-\tau^*)\omega_h^{*-\zeta^*} + \Omega_f^*\tau^*\omega_f^{*-\zeta^*}]c^*,$$

$$mc = \left(\frac{\zeta-1}{\zeta}\right)\left(\frac{1-\beta\mu\bar{\gamma}^{\zeta}}{1-\beta\mu\bar{\gamma}^{\zeta-1}}\right)\left(\frac{1-\mu\bar{\gamma}^{\zeta-1}}{1-\mu}\right)^{\frac{1}{1-\zeta}},$$

$$mc^* = \left(\frac{\zeta^*-1}{\zeta^*}\right)\left(\frac{1-\beta\mu^*\bar{\gamma}^{*\zeta^*}}{1-\beta\mu^*\bar{\gamma}^{*\zeta^*-1}}\right)\left(\frac{1-\mu^*\bar{\gamma}^{*\zeta^*-1}}{1-\mu^*}\right)^{\frac{1}{1-\zeta^*}},$$

$$\Omega_h = \left(\frac{1-\mu}{1-\mu\bar{\gamma}^{\zeta}}\right)\left(\frac{1-\mu\bar{\gamma}^{\zeta-1}}{1-\mu}\right)^{-\frac{\zeta}{1-\zeta}},$$

$$\Omega_h^* = \left(\frac{1-\mu}{1-\mu\bar{\gamma}^{*\zeta^*}}\right)\left(\frac{1-\mu\bar{\gamma}^{*\zeta^*-1}}{1-\mu}\right)^{-\frac{\zeta^*}{1-\zeta^*}},$$

$$\Omega_f = \left(\frac{1-\mu^*}{1-\mu^*\bar{\gamma}^{\zeta}}\right)\left(\frac{1-\mu^*\bar{\gamma}^{\zeta-1}}{1-\mu^*}\right)^{-\frac{\zeta}{1-\zeta}},$$

$$\Omega_f^* = \left(\frac{1-\mu^*}{1-\mu^*\bar{\gamma}^{*\zeta^*}}\right)\left(\frac{1-\mu^*\bar{\gamma}^{*\zeta^*-1}}{1-\mu^*}\right)^{-\frac{\zeta^*}{1-\zeta}{}^{*}},$$

$$\Psi = \left(\frac{\zeta-1}{\zeta}\right)\left(\frac{1-\beta\mu\bar{\gamma}^{\zeta}}{1-\beta\mu\bar{\gamma}^{\zeta-1}}\right)\left(\frac{1-\mu\bar{\gamma}^{\zeta-1}}{1-\mu}\right)^{\frac{1}{1-\zeta}}\left(\frac{\zeta^*}{\zeta^*-1}\right)\left(\frac{1-\beta\mu\bar{\gamma}^{*\zeta^*-1}}{1-\beta\mu\bar{\gamma}^{*\zeta^*}}\right)\left(\frac{1-\mu\bar{\gamma}^{*\zeta^*-1}}{1-\mu}\right)^{-\frac{1}{1-\zeta^*}},$$

$$\Psi^* = \left(\frac{\zeta^*-1}{\zeta^*}\right)\left(\frac{1-\beta\mu^*\bar{\gamma}^{*\zeta^*}}{1-\beta\mu^*\bar{\gamma}^{*\zeta^*-1}}\right)\left(\frac{1-\mu^*\bar{\gamma}^{*\zeta^*-1}}{1-\mu^*}\right)^{\frac{1}{1-\zeta^*}}\left(\frac{\zeta}{\zeta-1}\right)\left(\frac{1-\beta\mu^*\bar{\gamma}^{\zeta-1}}{1-\beta\mu^*\bar{\gamma}^{\zeta}}\right)\left(\frac{1-\mu^*\bar{\gamma}^{\zeta-1}}{1-\mu^*}\right)^{-\frac{1}{1-\zeta}}.$$

B.3. Log-linear approximation of the stochastically detrended system

To derive the corresponding linear rational expectations (LRE) models, I take a log-linear approximation of the stochastically detrended FONCs around the unique deterministic steady state. For a variable x_t, let $\hat{x}_t$ denote the percentage deviation from its deterministic steady-state value x, i.e., $\hat{x}_t \equiv (x_t - x)/x$, and $\tilde{x}_t$ the deviation from the deterministic steady-state value, i.e., $\tilde{x}_t \equiv x_t - x$.

Given the above stochastically detrended FONCs (B.1)-(B.31), the corresponding log-linearized FONCs around the deterministic steady state are derived as follows. The log-linear approximation of the home budget constraint is

$$\tilde{b}_{h,t} + \tilde{b}_{f,t} + s\bar{d}\hat{s}_t = \beta^{-1}\bar{d}\left[(1+\hat{i}_{h,t}) - \hat{\gamma}_{\tau,t}\right] + \beta^{-1}\tilde{b}_{h,t-1} + \beta^{-1}\bar{d}\left[(1+\hat{i}_{f,t}) - \hat{\gamma}^*_{\tau,t} + \hat{s}_t\right] + \beta^{-1}s\tilde{b}_{f,t-1} + \\ \tau p^{\zeta} p_h^{1-\zeta} c\left[(1-\zeta)\hat{p}_{h,t} + \zeta\hat{p}_t + \hat{c}_t\right] + (1-\tau^*)sp^{*\zeta^*} p_h^{*1-\zeta^*} c^*\left[\hat{s}_t + (1-\zeta^*)\hat{p}^*_{h,t} + \zeta^*\hat{p}^*_t + \hat{c}^*_t\right] - pc(\hat{p}_t + \hat{c}_t); \tag{B.32}$$

that of the home intratemporal optimality condition is

$$\eta\hat{N}_t = \hat{w}_t - \hat{p}_t - \hat{h}_t; \tag{B.33}$$

that of the home Euler equation is

$$\hat{p}_t + \hat{h}_t + (1+\hat{i}_{h,t}) = E_t(\hat{p}_{t+1} + \hat{h}_{t+1} + \hat{\gamma}_{\tau,t+1}); \tag{B.34}$$

that of the home UIP condition is

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1+\hat{i}_{h,t}) - (1+\hat{i}_{f,t}) - E_t(\hat{\gamma}_{\tau,t+1} - \hat{\gamma}^*_{\tau,t+1}); \tag{B.35}$$

those of the home risk premiums are

$$(1+\hat{i}_{h,t}) = (1+\hat{i}^*_{h,t}) - \psi(1-\kappa)\tilde{b}_{h,t} + (1-\kappa)v_{r,t}, \quad \text{and} \quad (1+\hat{i}_{f,t}) = (1+\hat{i}^*_{f,t}) - \psi(1-\kappa)\tilde{b}_{f,t} + (1-\kappa)v_{r,t}, \tag{B.36}$$

where κ is defined by $\kappa \equiv \beta/\gamma_\tau$; that of the optimal price setting rule of the home firm for the home market is

$$\hat{\mathbf{p}}_{h,t} - \hat{p}_{h,t} + (1-\mu\varphi_1)\sum_{i=0}^{\infty}(\mu\varphi_1)^i E_t(\hat{c}_{h,t+i} - \hat{h}_{t+i}) + (1-\mu\varphi_1)(\zeta-1)\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\hat{\phi}_{h,t+i} =$$
$$(1-\mu\varphi_2)\sum_{i=0}^{\infty}(\mu\varphi_2)^i E_t(\hat{c}_{h,t+i} - \hat{h}_{t+i}) + (1-\mu\varphi_2)\sum_{i=0}^{\infty}(\mu\varphi_2)^i E_t\hat{mc}_{t+i} + (1-\mu\varphi_2)\zeta\sum_{i=1}^{\infty}(\mu\varphi_2)^i E_t\hat{\phi}_{h,t+i}, \tag{B.37}$$

and that for the foreign market is

$$\hat{\mathbf{p}}^*_{h,t} - \hat{p}^*_{h,t} + (1-\mu\varphi^*_1)\sum_{i=0}^{\infty}(\mu\varphi^*_1)^i E_t(\hat{c}^*_{h,t+i} - \hat{h}_{t+i}) + (1-\mu\varphi^*_1)\sum_{i=0}^{\infty}(\mu\varphi^*_1)^i E_t(\hat{s}_{t+i} + \hat{p}^*_{h,t+i} - \hat{p}_{h,t+i}) +$$
$$(1-\mu\varphi^*_1)(\zeta^*-1)\sum_{i=1}^{\infty}(\mu\varphi^*_1)^i E_t\hat{\phi}^*_{h,t+i} = (1-\mu\varphi^*_2)\sum_{i=0}^{\infty}(\mu\varphi_2)^{*i} E_t(\hat{c}^*_{h,t+i} - \hat{h}_{t+i})$$
$$+ (1-\mu\varphi^*_2)\sum_{i=0}^{\infty}(\mu\varphi^*_2)^i E_t(\hat{mc}_{t+i} - \hat{a}_{t+i}) + (1-\mu\varphi^*_2)\zeta^*\sum_{i=1}^{\infty}(\mu\varphi^*_2)^i E_t\hat{\phi}^*_{h,t+i}, \tag{B.38}$$

where the parameters φ_1, φ_2, φ^*_1, and φ^*_2 are defined by $\varphi_1 \equiv \beta\bar{\gamma}^{\zeta-1}$, $\varphi_2 \equiv \beta\bar{\gamma}^{\zeta}$, $\varphi^*_1 \equiv \beta\bar{\gamma}^{*\zeta^*-1}$, and $\varphi^*_2 \equiv \beta\bar{\gamma}^{*\zeta^*}$, respectively; those of the home cumulative inflation rates are

$$\hat{\phi}_{t+i} = \hat{p}_{t+i} - \hat{p}_t + \sum_{s=1}^{i}(\hat{\gamma}_{\tau,t+s} - \hat{\gamma}_{A,t+s}) = \sum_{s=1}^{i}(\hat{\pi}_{t+s} + \hat{\gamma}_{\tau,t+s} - \hat{\gamma}_{A,t+s}) = \sum_{s=1}^{i}\hat{\chi}_{t+s},$$
$$\hat{\phi}_{h,t+i} = \hat{p}_{h,t+i} - \hat{p}_{h,t} + \sum_{s=1}^{i}(\hat{\gamma}_{\tau,t+s} - \hat{\gamma}_{A,t+s}) = \sum_{s=1}^{i}(\hat{\pi}_{h,t+s} + \hat{\gamma}_{\tau,t+s} - \hat{\gamma}_{A,t+s}) = \sum_{s=1}^{i}\hat{\chi}_{h,t+s},$$
$$\hat{\phi}^*_{h,t+i} = \hat{p}^*_{h,t+i} - \hat{p}^*_{h,t} + \sum_{s=1}^{i}(\hat{\gamma}^*_{\tau,t+s} - \hat{\gamma}^*_{A,t+s}) = \sum_{s=1}^{i}(\hat{\pi}^*_{h,t+s} + \hat{\gamma}^*_{\tau,t+s} - \hat{\gamma}^*_{A,t+s}) = \sum_{s=1}^{i}\hat{\chi}^*_{h,t+s}, \tag{B.39}$$

where $\hat{\pi}_t \equiv \hat{p}_t - \hat{p}_{t-1}$, $\hat{\pi}_{h,t} \equiv \hat{p}_{h,t} - \hat{p}_{h,t-1}$, $\hat{\pi}^*_{h,t} \equiv \hat{p}^*_{h,t} - \hat{p}^*_{h,t-1}$, $\hat{\chi}_t \equiv \hat{\pi}_t + \hat{\gamma}_{\tau,t} - \hat{\gamma}_{A,t}$, $\hat{\chi}_{h,t} \equiv \hat{\pi}_{h,t} + \hat{\gamma}_{\tau,t} - \hat{\gamma}_{A,t}$, and $\hat{\chi}^*_{h,t} \equiv \hat{\pi}^*_{h,t} + \hat{\gamma}^*_{\tau,t} - \hat{\gamma}^*_{A,t}$; that of the market clearing condition for the home country is

$$N\hat{N}_t = \Omega_h c_h(\hat{\Omega}_{h,t} + \hat{c}_{h,t}) + \Omega^*_h c^*_h a^{-1}(\hat{\Omega}^*_{h,t} + \hat{c}^*_{h,t} - \hat{a}_t); \tag{B.40}$$

those of the transition equations of price dispersions $\hat{\Omega}_{h,t}$ and $\hat{\Omega}^*_{h,t}$ are

$$\begin{aligned} \hat{\Omega}_{h,t} &= \zeta\mu\bar{\gamma}^{\zeta-1}\Theta\left(\hat{\pi}_{h,t} + \hat{\gamma}_{\tau,t} - \hat{\gamma}_{A,t}\right) + \mu\bar{\gamma}^{\zeta}\hat{\Omega}_{h,t-1}, \quad \text{and} \\ \hat{\Omega}^*_{h,t} &= \zeta^*\mu\bar{\gamma}^{*\zeta^*-1}\Theta^*\left(\hat{\pi}^*_{h,t} + \hat{\gamma}^*_{\tau,t} - \hat{\gamma}^*_{A,t}\right) + \mu\bar{\gamma}^{*\zeta^*}\hat{\Omega}^*_{h,t-1}, \end{aligned} \tag{B.41}$$

where the parameters Θ and Θ^* are defined by $\Theta \equiv \bar{\gamma} - \frac{1-\mu\bar{\gamma}^{\zeta}}{1-\mu\bar{\gamma}^{\zeta-1}}$ and $\Theta^* \equiv \bar{\gamma}^* - \frac{1-\mu\bar{\gamma}^{*\zeta^*}}{1-\mu\bar{\gamma}^{*\zeta^*-1}}$; those of the demand functions for the home goods are

$$\hat{c}_{h,t} = \hat{c}_t - \zeta\left(\hat{p}_{h,t} - \hat{p}_t\right), \quad \text{and} \quad \hat{c}^*_{h,t} = \hat{c}^*_t - \zeta^*\left(\hat{p}^*_{h,t} - \hat{p}^*_t\right); \tag{B.42}$$

that of the home CPI is

$$\hat{p}_t = \tau\omega_h^{1-\zeta}\hat{p}_{h,t} + (1 - \tau\omega_h^{1-\zeta})\hat{p}_{f,t}; \tag{B.43}$$

those of the laws of motion of the home good prices in the home and foreign countries are

$$\begin{aligned} \hat{p}_{h,t} &= (1-\mu)\left(\frac{\mathbf{p}_h}{p_h}\right)^{1-\zeta}\hat{\mathbf{p}}_{h,t} + \mu\bar{\gamma}^{\zeta-1}\left[\hat{p}_{h,t-1} + \hat{\gamma}_{A,t} - \hat{\gamma}_{\tau,t}\right], \quad \text{and} \\ \hat{p}^*_{h,t} &= (1-\mu)\left(\frac{\mathbf{p}^*_h}{p^*_h}\right)^{1-\zeta^*}\hat{\mathbf{p}}^*_{h,t} + \mu\bar{\gamma}^{*\zeta^*-1}\left[\hat{p}^*_{h,t-1} + \hat{\gamma}^*_{A,t} - \hat{\gamma}^*_{\tau,t}\right]; \end{aligned} \tag{B.44}$$

that of the real marginal cost of the home firm is

$$\hat{mc}_t = \hat{w}_t - \hat{p}_{h,t}; \tag{B.45}$$

that of the Taylor rule of the home central bank is

$$(1+\hat{i}_{h,t}) = \rho_i(1+\hat{i}_{h,t-1}) + (1-\rho_i)\left[a_\pi\left(\hat{\chi}_t - \hat{\gamma}_{\tau,t}\right) + a_y\hat{y}_t + \frac{a_s}{1-a_s}(\hat{q}_t - \hat{q}_{t-1} + \hat{\chi}_t - \hat{\chi}^*_t)\right] + \epsilon_{i,t}; \tag{B.46}$$

and that of home consumption habit is

$$(1-\bar{h})\hat{h}_t = \hat{c}_t - \bar{h}\hat{c}_{t-1} + \bar{h}\hat{\gamma}_{A,t}, \tag{B.47}$$

where $\bar{h} \equiv h\gamma_A^{-1}$.

The foreign country's counterparts are given as follows. The log-linear approximation of the foreign country's budget constraint is

$$-\tilde{b}_{h,t}-\tilde{b}_{f,t}-s\bar{d}\hat{s}_t = -\beta^{-1}\bar{d}\left[(1+\hat{i}^*_{h,t})-\hat{\gamma}_{\tau,t}\right]-\beta^{-1}\tilde{b}_{h,t-1}-\beta^{-1}\bar{d}\left[(1+\hat{i}^*_{f,t})-\hat{\gamma}^*_{\tau,t}+\hat{s}_t\right]-\beta^{-1}s\tilde{b}_{f,t-1}+ (1-\tau)p^{\zeta}p_f^{1-\zeta}c\left[(1-\zeta)\hat{p}_{f,t}+\zeta\hat{p}_t+\hat{c}_t\right]+\tau^* s p^{*\zeta^*}p_f^{*1-\zeta^*}c^*\left[\hat{s}_t+(1-\zeta^*)\hat{p}^*_{f,t}+\zeta^*\hat{p}^*_t+\hat{c}^*_t\right]-sp^*c^*(\hat{s}_t+\hat{p}^*_t+\hat{c}^*_t); \tag{B.48}$$

that of the foreign intratemporal optimality condition is

$$\eta^*\hat{N}^*_t = \hat{w}^*_t - \hat{p}^*_t - \hat{h}^*_t; \tag{B.49}$$

that of the foreign Euler equation is

$$\hat{p}_t + \hat{h}^*_t + \hat{q}_t - \hat{a}_t - \hat{s}_t + (1+\hat{i}^*_{f,t}) = E_t(\hat{p}_{t+1} + \hat{h}^*_{t+1} + \hat{q}_{t+1} - \hat{a}_{t+1} - \hat{s}_{t+1} + \hat{\gamma}^*_{\tau,t+1}); \tag{B.50}$$

that of the foreign UIP condition is

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1+\hat{i}^*_{h,t}) - (1+\hat{i}^*_{f,t}) - E_t(\hat{\gamma}_{\tau,t+1} - \hat{\gamma}^*_{\tau,t+1}); \tag{B.51}$$

those of the optimal price setting rules of the foreign firm are

$$\hat{\mathbf{p}}_{f,t} - \hat{p}_{f,t} + (1-\mu^*\varphi_1)\sum_{i=0}^{\infty}(\mu^*\varphi_1)^i E_t(\hat{c}_{f,t+i} - \hat{h}^*_{t+i}) + (1-\mu^*\varphi_1)\sum_{i=0}^{\infty}(\mu^*\varphi_1)^i E_t(\hat{p}_{f,t+i} - \hat{s}_{t+i} - \hat{p}^*_{f,t+i}) + (1-\mu^*\varphi_1)(\zeta-1)\sum_{i=1}^{\infty}(\mu^*\varphi_1)^i E_t\hat{\phi}_{f,t+i} = (1-\mu^*\varphi_2)\sum_{i=0}^{\infty}(\mu^*\varphi_2)^i E_t(\hat{c}_{f,t+i} - \hat{h}^*_{t+i}) + (1-\mu^*\varphi_2)\sum_{i=0}^{\infty}(\mu^*\varphi_2)^i E_t(\hat{mc}^*_{t+i} + \hat{a}_{t+i}) + (1-\mu^*\varphi_2)\zeta\sum_{i=1}^{\infty}(\mu^*\varphi_2)^i E_t\hat{\phi}_{f,t+i}, \tag{B.52}$$

and

$$\hat{\mathbf{p}}^*_{f,t} - \hat{p}^*_{f,t} + (1-\mu^*\varphi^*_1)\sum_{i=0}^{\infty}(\mu^*\varphi^*_1)^i E_t(\hat{c}^*_{f,t+i} - \hat{h}^*_{t+i}) + (1-\mu^*\varphi^*_1)(\zeta^*-1)\sum_{i=1}^{\infty}(\mu^*\varphi^*_1)^i E_t\hat{\phi}^*_{f,t+i} = (1-\mu^*\varphi^*_2)\sum_{i=0}^{\infty}(\mu^*\varphi^*_2)^i E_t(\hat{c}^*_{f,t+i} - \hat{h}^*_{t+i}) + (1-\mu^*\varphi^*_2)\sum_{i=0}^{\infty}(\mu^*\varphi^*_2)^i E_t\hat{mc}^*_{t+i} + (1-\mu^*\varphi^*_2)\zeta^*\sum_{i=1}^{\infty}(\mu^*\varphi^*_2)^i E_t\hat{\phi}^*_{f,t+i}; \tag{B.53}$$

those of the foreign cumulative inflation rates are

$$\hat{\phi}^*_{t+i} = \hat{p}^*_{t+i} - \hat{p}^*_t + \sum_{s=1}^{i}(\hat{\gamma}^*_{\tau,t+s} - \hat{\gamma}^*_{A,t+s}) = \sum_{s=1}^{i}(\hat{\pi}^*_{t+s} + \hat{\gamma}^*_{\tau,t+s} - \hat{\gamma}^*_{A,t+s}) = \sum_{s=1}^{i}\hat{\chi}^*_{t+s},$$

$$\hat{\phi}_{f,t+i} = \hat{p}_{f,t+i} - \hat{p}_{f,t} + \sum_{s=1}^{i}(\hat{\gamma}_{\tau,t+s} - \hat{\gamma}_{A,t+s}) = \sum_{s=1}^{i}(\hat{\pi}_{f,t+s} + \hat{\gamma}_{\tau,t+s} - \hat{\gamma}_{A,t+s}) = \sum_{s=1}^{i}\hat{\chi}_{f,t+s},$$

$$\hat{\phi}^*_{f,t+i} = \hat{p}^*_{f,t+i} - \hat{p}^*_{f,t} + \sum_{s=1}^{i}(\hat{\gamma}^*_{\tau,t+s} - \hat{\gamma}^*_{A,t+s}) = \sum_{s=1}^{i}(\hat{\pi}^*_{f,t+s} + \hat{\gamma}^*_{\tau,t+s} - \hat{\gamma}^*_{A,t+s}) = \sum_{s=1}^{i}\hat{\chi}^*_{f,t+s}, \tag{B.54}$$

where $\hat{\pi}^*_t \equiv \hat{p}^*_t - \hat{p}^*_{t-1}$, $\hat{\pi}_{f,t} \equiv \hat{p}_{f,t} - \hat{p}_{f,t-1}$, $\hat{\pi}^*_{f,t} \equiv \hat{p}^*_{f,t} - \hat{p}^*_{f,t-1}$, $\hat{\chi}^*_t \equiv \hat{\pi}^*_t + \hat{\gamma}^*_{\tau,t} - \hat{\gamma}^*_{A,t}$, $\hat{\chi}_{f,t} \equiv \hat{\pi}_{f,t} + \hat{\gamma}_{\tau,t} - \hat{\gamma}_{A,t}$, and $\hat{\chi}^*_{f,t} \equiv \hat{\pi}^*_{f,t} + \hat{\gamma}^*_{\tau,t} - \hat{\gamma}^*_{A,t}$; that of the market clearing condition for the foreign country is

$$N^*\hat{N}^*_t = \Omega_f c_f a(\hat{\Omega}_{f,t} + \hat{c}_{f,t} + \hat{a}_t) + \Omega^*_f c^*_f(\hat{\Omega}^*_{f,t} + \hat{c}^*_{f,t}); \tag{B.55}$$

those of the transition equations of price dispersions $\hat{\Omega}_{f,t}$ and $\hat{\Omega}^*_{f,t}$ are

$$\hat{\Omega}_{f,t} = \zeta\mu^*\bar{\gamma}^{\zeta-1}\Xi\left(\hat{\pi}_{f,t} + \hat{\gamma}_{\tau,t} - \hat{\gamma}_{A,t}\right) + \mu^*\bar{\gamma}^{\zeta}\hat{\Omega}_{f,t-1}, \quad \text{and}$$

$$\hat{\Omega}^*_{f,t} = \zeta^*\mu^*\bar{\gamma}^{*\zeta^*-1}\Xi^*\left(\hat{\pi}^*_{f,t} + \hat{\gamma}^*_{\tau,t} - \hat{\gamma}^*_{A,t}\right) + \mu^*\bar{\gamma}^{*\zeta^*}\hat{\Omega}^*_{f,t-1}, \tag{B.56}$$

where the parameters Ξ and Ξ^* are defined by $\Xi \equiv \bar{\gamma} - \frac{1-\mu^*\bar{\gamma}^{\zeta}}{1-\mu^*\bar{\gamma}^{\zeta-1}}$ and $\Xi^* \equiv \bar{\gamma}^* - \frac{1-\mu^*\bar{\gamma}^{*\zeta^*}}{1-\mu^*\bar{\gamma}^{*\zeta^*-1}}$; those of the demand functions for the foreign goods are

$$\hat{c}_{f,t} = \hat{c}_t - \xi\left(\hat{p}_{f,t} - \hat{p}_t\right), \quad \text{and} \quad \hat{c}^*_{f,t} = \hat{c}^*_t - \zeta^*\left(\hat{p}^*_{f,t} - \hat{p}^*_t\right); \tag{B.57}$$

that of the foreign CPI is

$$\hat{p}^*_t = (1 - \tau^*\omega_f^{*1-\zeta^*})\hat{p}^*_{h,t} + \tau^*\omega_f^{*1-\zeta^*}\hat{p}^*_{f,t}; \tag{B.58}$$

those of the laws of motion of the home good prices in the home and foreign countries are

$$\hat{p}_{f,t} = (1-\mu^*)\left(\frac{\mathbf{p}_f}{p_f}\right)^{1-\zeta}\hat{\mathbf{p}}_{f,t} + \mu^*\bar{\gamma}^{\zeta-1}\left[\hat{p}_{f,t-1} + \hat{\gamma}_{A,t} - \hat{\gamma}_{\tau,t}\right], \quad \text{and}$$

$$\hat{p}^*_{f,t} = (1-\mu^*)\left(\frac{\mathbf{p}^*_f}{p^*_f}\right)^{1-\zeta^*}\hat{\mathbf{p}}^*_{f,t} + \mu^*\bar{\gamma}^{*\zeta^*-1}\left[\hat{p}^*_{f,t-1} + \hat{\gamma}^*_{A,t} - \hat{\gamma}^*_{\tau,t}\right]; \tag{B.56}$$

that of the marginal cost of the foreign firm is

$$\hat{mc}^*_t = \hat{w}^*_t - \hat{p}^*_{f,t}; \tag{B.59}$$

that of the Taylor rule of the foreign central bank is

$$(1+\hat{i}^*_{f,t}) = \rho^*_i(1+\hat{i}^*_{f,t-1})+(1-\rho^*_i)\left[a^*_\pi\left(\hat{\chi}^*_t - \hat{\gamma}^*_{\tau,t}\right) + a^*_y\hat{y}^*_t - \frac{a^*_s}{1-a^*_s}(\hat{q}_t - \hat{q}_{t-1} + \hat{\chi}_t - \hat{\chi}^*_t)\right]+\epsilon^*_{i,t}; \quad \text{(B.58)}$$

and that of foreign consumption habit is

$$(1-\bar{h}^*)\hat{h}^*_t = \hat{c}^*_t - \bar{h}^*\hat{c}^*_{t-1} + \bar{h}^*\hat{\gamma}^*_{A,t}. \quad \text{(B.59)}$$

where $\bar{h}^* \equiv h^*\gamma_A^{*-1}$; finally, that of the real exchange rate is

$$\hat{q}_t = \hat{s}_t + \hat{a}_t + \hat{p}^*_t - \hat{p}_t. \quad \text{(B.60)}$$

Comparing the home and foreign UIPs (B.35) and (B.51) reveals that the two countries must face the same interest rate differential: $(1+\hat{i}_{h,t}) - (1+\hat{i}_{f,t}) = (1+\hat{i}^*_{h,t}) - (1+\hat{i}^*_{f,t})$. The home risk premium equations (B.36) then imply that the home holdings of the home issued bond must be equal to the home holdings of the foreign issued bond: $\hat{b}_t = \hat{b}_{h,t} = \hat{b}_{f,t}$ in equilibrium.

Appendix C: GNKPCs with trend inflation

Now I derive the generalized new Keynesian Phillips curves (GNKPCs). In doing so, I assume that the parameters φ_1, φ_2, φ^*_1, and φ^*_2 take values between zero and one. At the deterministic steady state, the law of motion of the home good price at the home market (B.13) implies that

$$\left(\frac{\mathbf{p}_h}{p_h}\right)^{1-\zeta} = \frac{1-\mu\bar{\gamma}^{\zeta-1}}{1-\mu}. \quad \text{(C.1)}$$

Substituting the above equation (C.1) into the log-linear approximation of the law of motion of the home good price at the home market, eq.(B.44), gives

$$\hat{\chi}_{h,t} = \varphi_0(\hat{\mathbf{p}}_{h,t} - \hat{p}_{h,t}), \quad \text{(C.2)}$$

where the parameter φ_0 is defined by $\varphi_0 \equiv (1-\mu\bar{\gamma}^{\zeta-1})/\mu\bar{\gamma}^{\zeta-1}$. Cumulative inflation rate processes (B.39) rewrite the optimal price setting of the home firm for the home market (B.37) as

$$\hat{\mathbf{p}}_{h,t} - \hat{p}_{h,t} = (1-\mu\varphi_2)D_{h,t} - (1-\mu\varphi_1)Z_{h,t}, \quad \text{(C.3)}$$

where random variables $D_{h,t}$ and $Z_{h,t}$ follow the stochastic difference equations

$$D_{h,t} = \hat{c}_{h,t} - \hat{h}_t + \hat{mc}_t + \frac{\zeta\mu\varphi_2}{1-\mu\varphi_2} E_t\hat{\chi}_{h,t+1} + \mu\varphi_2 E_t D_{h,t+1},$$
$$Z_{h,t} = \hat{c}_{h,t} - \hat{h}_t + \frac{(\zeta-1)\mu\varphi_1}{1-\mu\varphi_1} E_t\hat{\chi}_{h,t+1} + \mu\varphi_1 E_t Z_{h,t+1}.$$

After several calculations, it is straightforward to show that eqs. (C.1) – (C.3) provide

$$\hat{\chi}_{h,t} = \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}\left[\hat{c}_{h,t} - \hat{h}_t - (1-\mu\varphi_1)\sum_{i=0}^{\infty}(\mu\varphi_1)^i E_t(\hat{c}_{h,t+i} - \hat{h}_{t+i})\right] + \varphi_0(1-\mu\varphi_2)\hat{mc}_t$$
$$+ \mu\varphi_2(1+\varphi_0)E_t\hat{\chi}_{h,t+1} - \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}(\zeta-1)\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\hat{\chi}_{h,t+i}. \quad \text{(C.4)}$$

Further, the demand function for the home good (B.42) implies

$$\begin{aligned}
&\hat{c}_{h,t} - \hat{h}_t - (1-\mu\varphi_1)\sum_{i=0}^{\infty}(\mu\varphi_1)^i E_t(\hat{c}_{h,t+i} - \hat{h}_{t+i}) \\
&= \hat{c}_{h,t} - \hat{c}_t + \hat{c}_t - \hat{h}_t - (1-\mu\varphi_1)\sum_{i=0}^{\infty}(\mu\varphi_1)^i E_t(\hat{c}_{h,t+i} - \hat{c}_{t+i} + \hat{c}_{t+i} - \hat{h}_{t+i}) \\
&= -\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\Delta(\hat{c}_{h,t+i} - \hat{c}_{t+i}) - \sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\Delta(\hat{c}_{t+i} - \hat{h}_{t+i}) \\
&= -\zeta\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t(\hat{\pi}_{t+i} - \hat{\pi}_{h,t+i}) - \sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\Delta(\hat{c}_{t+i} - \hat{h}_{t+i}).
\end{aligned}$$

Substituting this relation into eq.(C.4) yields the GNKPC for the home good price at the home market

$$\hat{\chi}_{h,t} = \mu\varphi_2(1+\varphi_0)E_t\hat{\chi}_{h,t+1} + \varphi_0(1-\mu\varphi_2)\hat{mc}_t - \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}\zeta\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t(\hat{\chi}_{t+i} - \hat{\chi}_{h,t+i})$$
$$- \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}(\zeta-1)\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\hat{\chi}_{h,t+i} - \frac{\varphi_0(\varphi_1-\varphi_2)}{\varphi_1}\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\Delta(\hat{c}_{t+i} - \hat{h}_{t+i}) + \hat{v}^h_{p,t}. \quad \text{(C.5)}$$

Following Steinsson (2008), I add a reduced-form cost push shock $\hat{v}_{cp,h,t}$ to the RHS of the GNKPC (C.5). Steinsson (2008) discusses that this additive shock to the NKPC plays a crucial role in replicating the hump-shaped impulse response of the real exchange rate. The cost push shock follows an AR(1) process

$$\hat{v}^h_{p,t} = \rho^h_p \hat{v}^h_{p,t-1} + \epsilon^h_{p,t}.$$

The same steps of calculation above is applicable to derive the GNKPCs for the home good price inflation at the foreign market, $\chi^*_{h,t}$, the foreign good price inflation at the home market, $\chi_{f,t}$, and the foreign good price inflation at the foreign market, $\chi^*_{f,t}$ as follows.

$$\hat{\chi}^*_{h,t} = \mu\varphi^*_2(1+\vartheta^*_0)E_t\hat{\chi}^*_{h,t+1} + \vartheta^*_0(1-\mu\varphi^*_2)(\hat{mc}_t - \hat{a}_t) - \frac{\vartheta^*_0\varphi^*_2(1-\mu\varphi^*_1)}{\varphi^*_1}[\hat{q}_t - \hat{a}_t - \tau^*\omega^{*1-\zeta^*}_f\hat{\Psi}^*_t - (1-\tau\omega^{1-\zeta}_h)\hat{\Psi}_t]$$
$$+ \frac{\vartheta^*_0(\varphi^*_1-\varphi^*_2)}{\varphi^*_1}\zeta^*\sum_{i=1}^{\infty}(\mu\varphi^*_1)^i E_t(\hat{\chi}^*_{h,t+i} - \hat{\chi}^*_{t+i}) - \frac{\vartheta^*_0(\varphi_1-\varphi_2)}{\varphi_1}\sum_{i=1}^{\infty}(\mu\varphi_1)^i E_t\Delta(\hat{c}^*_{t+i} - \hat{h}_{t+i})$$
$$- \frac{\vartheta^*_0(\varphi^*_1-\varphi^*_2)(1-\mu\varphi^*_1)}{\varphi^*_1}\sum_{i=0}^{\infty}(\mu\varphi^*_1)^i E_t[\hat{q}_{t+i} - \hat{a}_{t+i} - \tau^*\omega^{*1-\zeta^*}_f\hat{\Psi}^*_{t+i} - (1-\tau\omega^{1-\zeta}_h)\hat{\Psi}_{t+i}]$$
$$- \frac{\vartheta^*_0(\varphi^*_1-\varphi^*_2)}{\varphi^*_1}(\zeta^*-1)\sum_{i=1}^{\infty}(\mu\varphi^*_1)^i E_t\hat{\chi}^*_{h,t+i} + \hat{v}^{h*}_{p,t}, \quad \text{(C.6)}$$

$$\hat{\chi}_{f,t} = \mu^*\varphi_2(1+\vartheta_0)E_t\hat{\chi}_{f,t+1} + \vartheta_0(1-\mu^*\varphi_2)(\hat{mc}^*_t + \hat{a}_t) + \frac{\vartheta_0\varphi_2(1-\mu^*\varphi_1)}{\varphi_1}[\hat{q}_t - \hat{a}_t + (1-\tau^*\omega^{*1-\zeta^*}_f)\hat{\Psi}^*_t + \tau\omega^{1-\zeta}_h\hat{\Psi}_t]$$
$$+ \frac{\vartheta_0(\varphi_1-\varphi_2)}{\varphi_1}\zeta\sum_{i=1}^{\infty}(\mu^*\varphi_1)^i E_t(\hat{\chi}_{f,t+i} - \hat{\chi}_{t+i}) - \frac{\vartheta_0(\varphi_1-\varphi_2)}{\varphi_1}\sum_{i=1}^{\infty}(\mu^*\varphi_1)^i E_t\Delta(\hat{c}_{t+i} - \hat{h}^*_{t+i})$$
$$+ \frac{\vartheta_0(\varphi_1-\varphi_2)(1-\mu^*\varphi_1)}{\varphi_1}\sum_{i=0}^{\infty}(\mu^*\varphi_1)^i E_t[\hat{q}_{t+i} - \hat{a}_{t+i} + (1-\tau^*\omega^{*1-\zeta^*}_f)\hat{\Psi}^*_{t+i} + \tau\omega^{1-\zeta}_h\hat{\Psi}_{t+i}]$$
$$- \frac{\vartheta_0(\varphi_1-\varphi_2)}{\varphi_1}(\zeta-1)\sum_{i=1}^{\infty}(\mu^*\varphi_1)^i E_t\hat{\chi}_{f,t+i} + \hat{v}^{f}_{p,t}, \quad \text{(C.7)}$$

and

$$\hat{\chi}^*_{f,t} = \mu^*\varphi^*_2(1+\varphi^*_0)E_t\hat{\chi}^*_{f,t+1} + \varphi^*_0(1-\mu^*\varphi^*_2)\hat{mc}^*_t - \frac{\varphi^*_0(\varphi^*_1-\varphi^*_2)}{\varphi^*_1}\zeta^*\sum_{i=1}^{\infty}(\mu^*\varphi^*_1)^i E_t(\hat{\chi}^*_{t+i} - \hat{\chi}^*_{f,t+i})$$
$$- \frac{\varphi^*_0(\varphi^*_1-\varphi^*_2)}{\varphi^*_1}(\zeta^*-1)\sum_{i=1}^{\infty}(\mu^*\varphi^*_1)^i E_t\hat{\chi}^*_{f,t+i} - \frac{\varphi^*_0(\varphi^*_1-\varphi^*_2)}{\varphi^*_1}\sum_{i=1}^{\infty}(\mu^*\varphi^*_1)^i E_t\Delta(\hat{c}^*_{t+i} - \hat{h}^*_{t+i}) + \hat{v}^{f*}_{p,t}, \quad \text{(C.8)}$$

where the parameters ϑ^*_0, ϑ_0, and φ^*_0 are defined by $\vartheta^*_0 \equiv (1-\mu\bar{\gamma}^{*\zeta^*-1})/\mu\bar{\gamma}^{*\zeta^*-1}$, $\vartheta_0 \equiv (1-\mu^*\bar{\gamma}^{\zeta-1})/\mu^*\bar{\gamma}^{\zeta-1}$, and $\varphi^*_0 \equiv (1-\mu^*\bar{\gamma}^{*\zeta^*-1})/\mu^*\bar{\gamma}^{*\zeta*-1}$. Similar to the GNKPC for the home good price at the home market, eq.(C.5), the GNKPCs (C.6) - (C.8) are subject to reduced-form cost push shocks $\hat{v}^*_{cp,h,t}$, $\hat{v}_{cp,f,t}$, and $\hat{v}^*_{cp,f,t}$

which follows independent AR(1) processes

$$\hat{v}^{h*}_{p,t} = \rho^{h*}_p \hat{v}^{h*}_{p,t-1} + \epsilon^{h*}_{p,t},$$
$$\hat{v}^{f}_{p,t} = \rho^{f}_p \hat{v}^{f}_{p,t-1} + \epsilon^{f}_{p,t},$$
$$\hat{v}^{f*}_{p,t} = \rho^{f*}_p \hat{v}^{f*}_{p,t-1} + \epsilon^{f*}_{p,t}.$$

Appendix D: The linear rational expectations model

The log-linear approximation of the stochastically detrended FONCs, eqs.(B.32)-(B.60), which is augmented by the second-order expectational difference equations of the GNKPCs (C.5)-(C.8), provide the following tractable linear rational expectations (LRE) model. The LRE model contains the UIP condition

$$E_t\hat{s}_{t+1} - \hat{s}_t = (1 + \hat{i}_{h,t}) - (1 + \hat{i}^*_{f,t}) + \psi(1-\kappa)\tilde{b}_t + (1-\kappa)v_{r,t} - E_t(\hat{\gamma}_{\tau,t+1} - \hat{\gamma}^*_{\tau,t+1}); \tag{D.1}$$

the transition equation of bond holdings

$$\tilde{b}_t = \beta^{-1}\tilde{b}_{t-1} + \bar{p}c(1 - \tau\omega_h^{1-\zeta})[\tau\omega_h^{1-\zeta}(1-\zeta)\hat{\Psi}_t - \tau^*\omega_f^{*1-\zeta^*}(1-\zeta^*)\hat{\Psi}^*_t + \hat{q}_t - \hat{a}_t - \hat{c}_t + \hat{c}^*_t], \tag{D.2}$$

where $\bar{p} \equiv p/2$; the home Euler equation

$$\hat{h}_t + (1 + \hat{i}_{h,t}) = E_t(\hat{h}_{t+1} + \hat{\chi}_{t+1} + \hat{\gamma}_{A,t+1}); \tag{D.3}$$

the foreign Euler equation

$$\hat{h}^*_t + (1 + \hat{i}^*_{f,t}) = E_t(\hat{h}^*_{t+1} + \hat{\chi}^*_{t+1} + \hat{\gamma}^*_{A,t+1}); \tag{D.4}$$

the home intratemporal optimality condition

$$\eta\hat{N}_t = \hat{mc}_t + (1 - \tau\omega_h^{1-\zeta})\hat{\Psi}_t - \hat{h}_t; \tag{D.5}$$

the foreign intratemporal optimality condition

$$\eta^*\hat{N}^*_t = \hat{mc}^*_t + (1 - \tau^*\omega_f^{*1-\zeta^*})\hat{\Psi}^*_t - \hat{h}^*_t; \tag{D.6}$$

the home market clearing condition

$$N\hat{N}_t = \Omega_h c_h(\hat{\Omega}_{h,t} + \hat{c}_{h,t}) + \Omega^*_h c^*_h a^{-1}(\hat{\Omega}^*_{h,t} + \hat{c}^*_{h,t} - \hat{a}_t); \tag{D.7}$$

the foreign market clearing condition

$$N^* \hat{N}_t^* = \Omega_f c_f a(\hat{\Omega}_{f,t} + \hat{c}_{f,t} + \hat{a}_t) + \Omega_f^* c_f^* (\hat{\Omega}_{f,t}^* + \hat{c}_{f,t}^*); \tag{D.8}$$

the transition equation of price dispersion $\hat{\Omega}_{h,t}$

$$\hat{\Omega}_{h,t} = \mu \bar{\gamma}^{\zeta} \hat{\Omega}_{h,t-1} + \zeta \mu \bar{\gamma}^{\zeta-1} \Theta \chi_{h,t}; \tag{D.9}$$

the transition equation of price dispersion $\hat{\Omega}_{h,t}^*$

$$\hat{\Omega}_{h,t}^* = \mu \bar{\gamma}^{*\zeta^*} \hat{\Omega}_{h,t-1}^* + \zeta^* \mu \bar{\gamma}^{*\zeta^*-1} \Theta^* \chi_{h,t}^*; \tag{D.10}$$

the transition equation of price dispersion $\hat{\Omega}_{f,t}$

$$\hat{\Omega}_{f,t} = \mu^* \bar{\gamma}^{\zeta} \hat{\Omega}_{f,t-1} + \zeta \mu^* \bar{\gamma}^{\zeta-1} \Xi \chi_{f,t}; \tag{D.11}$$

the transition equation of price dispersion $\hat{\Omega}_{f,t}^*$

$$\hat{\Omega}_{f,t}^* = \mu^* \bar{\gamma}^{*\zeta^*} \hat{\Omega}_{f,t-1}^* + \zeta^* \mu^* \bar{\gamma}^{*\zeta^*-1} \Xi^* \chi_{f,t}^*; \tag{D.12}$$

the home general inflation rate

$$\hat{\chi}_t = \tau \omega_h^{1-\zeta} \hat{\chi}_{h,t} + (1 - \tau \omega_h^{1-\zeta}) \hat{\chi}_{f,t}; \tag{D.13}$$

the foreign general inflation rate

$$\hat{\chi}_t^* = (1 - \tau^* \omega_f^{*1-\zeta^*}) \hat{\chi}_{h,t}^* + \tau^* \omega_f^{*1-\zeta^*} \hat{\chi}_{f,t}^*; \tag{D.14}$$

the home terms of trade

$$\hat{\Psi}_t = \hat{\Psi}_{t-1} + \hat{\chi}_{h,t} - \hat{\chi}_{f,t}; \tag{D.15}$$

the foreign terms of trade

$$\hat{\Psi}_t^* = \hat{\Psi}_{t-1}^* + \hat{\chi}_{f,t}^* - \hat{\chi}_{h,t}^*; \tag{D.16}$$

the GNKPC for $\hat{\chi}_{h,t}$

$$\begin{aligned} \beta \mu \varphi_2 E_t \hat{\chi}_{h,t+2} - [\mu \varphi_1 + \beta \bar{\gamma} + \varphi_0 \mu (\varphi_1 - \varphi_2)] E_t \hat{\chi}_{h,t+1} + \hat{\chi}_{h,t} \\ = \varphi_0 (1 - \mu \varphi_2) \hat{mc}_t - \varphi_0 (1 - \mu \varphi_2) \mu \varphi_1 E_t \hat{mc}_{t+1} - \varphi_0 \mu (\varphi_1 - \varphi_2) \zeta E_t \hat{\chi}_{t+1} \\ - \varphi_0 \mu (\varphi_1 - \varphi_2) E_t \Delta (\hat{c}_{t+1} - \hat{h}_{t+1}) + (1 - \mu \varphi_1 \rho_{cp,h}) \hat{v}_{cp,h,t}; \end{aligned} \tag{D.17}$$

the GNKPC for $\hat{\chi}^*_{h,t}$

$$\begin{aligned}&\beta\mu\varphi_2^* E_t\hat{\chi}^*_{h,t+2} - [\mu\varphi_1^* + \beta\bar{\gamma}^* + \vartheta_0^*\mu(\varphi_1^* - \varphi_2^*)]E_t\hat{\chi}^*_{h,t+1} + \hat{\chi}^*_{h,t}\\
&= \vartheta_0^*(1-\mu\varphi_2^*)(\hat{mc}_t - \hat{a}_t) - \vartheta_0^*(1-\mu\varphi_2^*)\mu\varphi_1^* E_t(\hat{mc}_{t+1} - \hat{a}_{t+1}) - \vartheta_0^*(1-\mu\varphi_1^*)[\hat{q}_t - \hat{a}_t - \tau^*\omega_f^{*1-\zeta^*}\hat{\Psi}^*_t - (1-\tau\omega_h^{1-\zeta})\hat{\Psi}_t]\\
&\quad + \vartheta_0^*\mu\varphi_2^*(1-\mu\varphi_1^*)E_t[\hat{q}_{t+1} - \hat{a}_{t+1} - \tau^*\omega_f^{*1-\zeta^*}\hat{\Psi}^*_{t+1} - (1-\tau\omega_h^{1-\zeta})\hat{\Psi}_{t+1}] - \vartheta_0^*\mu(\varphi_1^* - \varphi_2^*)\zeta^* E_t\hat{\chi}^*_{t+1}\\
&\qquad - \vartheta_0^*\mu(\varphi_1^* - \varphi_2^*)E_t\Delta(\hat{c}^*_{t+1} - \hat{h}_{t+1}) + (1-\mu\varphi_1^*\rho^*_{cp,h})\hat{v}^*_{cp,h,t}; \quad \text{(D.18)}\end{aligned}$$

the GNKPC for $\hat{\chi}_{f,t}$

$$\begin{aligned}&\beta\mu^*\varphi_2 E_t\hat{\chi}_{f,t+2} - [\mu^*\varphi_1 + \beta\bar{\gamma} + \vartheta_0\mu^*(\varphi_1 - \varphi_2)]E_t\hat{\chi}_{f,t+1} + \hat{\chi}_{f,t}\\
&= \vartheta_0(1-\mu^*\varphi_2)(\hat{mc}^*_t + \hat{a}_t) - \vartheta_0(1-\mu^*\varphi_2)\mu^*\varphi_1 E_t(\hat{mc}^*_{t+1} + \hat{a}_{t+1}) + \vartheta_0(1-\mu^*\varphi_1)[\hat{q}_t - \hat{a}_t + (1-\tau^*\omega_f^{*1-\zeta^*})\hat{\Psi}^*_t + \tau\omega_h^{1-\zeta}\hat{\Psi}_t]\\
&\quad - \vartheta_0\mu^*\varphi_2(1-\mu^*\varphi_1)E_t[\hat{q}_{t+1} - \hat{a}_{t+1} + (1-\tau^*\omega_f^{*1-\zeta^*})\hat{\Psi}^*_{t+1} + \tau\omega_h^{1-\zeta}\hat{\Psi}_{t+1}] - \vartheta_0\mu^*(\varphi_1 - \varphi_2)\zeta E_t\hat{\chi}_{t+1}\\
&\qquad - \vartheta_0\mu^*(\varphi_1 - \varphi_2)E_t\Delta(\hat{c}_{t+1} - \hat{h}^*_{t+1}) + (1-\mu^*\varphi_1\rho_{cp,f})\hat{v}_{cp,f,t}; \quad \text{(D.19)}\end{aligned}$$

the GNKPC for $\hat{\chi}^*_{f,t}$

$$\begin{aligned}&\beta\mu^*\varphi_2^* E_t\hat{\chi}^*_{f,t+2} - [\mu^*\varphi_1^* + \beta\bar{\gamma}^* + \varphi_0^*\mu^*(\varphi_1^* - \varphi_2^*)]E_t\hat{\chi}^*_{f,t+1} + \hat{\chi}^*_{f,t}\\
&= \varphi_0^*(1-\mu^*\varphi_2^*)\hat{mc}^*_t - \varphi_0^*(1-\mu^*\varphi_2^*)\mu^*\varphi_1^* E_t\hat{mc}^*_{t+1} - \varphi_0^*\mu^*(\varphi_1^* - \varphi_2^*)\zeta^* E_t\hat{\chi}^*_{t+1}\\
&\qquad - \varphi_0^*\mu^*(\varphi_1^* - \varphi_2^*)E_t\Delta(\hat{c}^*_{t+1} - \hat{h}^*_{t+1}) + (1-\mu^*\varphi_1^*\rho^*_{cp,f})\hat{v}^*_{cp,f,t}; \quad \text{(D.20)}\end{aligned}$$

the demand function of $\hat{c}_{h,t}$

$$\hat{c}_{h,t} = \hat{c}_t - \zeta(1-\tau\omega_h^{1-\zeta})\hat{\Psi}_t; \quad \text{(D.21)}$$

the demand function of $\hat{c}^*_{h,t}$

$$\hat{c}^*_{h,t} = \hat{c}^*_t + \zeta^*\tau^*\omega_f^{*1-\zeta^*}\hat{\Psi}^*_t; \quad \text{(D.22)}$$

the demand function of $\hat{c}_{f,t}$

$$\hat{c}_{f,t} = \hat{c}_t + \zeta\tau\omega_h^{1-\zeta}\hat{\Psi}_t; \quad \text{(D.23)}$$

the demand function of $\hat{c}^*_{f,t}$

$$\hat{c}^*_{f,t} = \hat{c}^*_t - \zeta^*(1-\tau^*\omega_f^{*1-\zeta^*})\hat{\Psi}^*_t; \quad \text{(D.24)}$$

the home habit formation

$$(1-\bar{h})\hat{h}_t = \hat{c}_t - \bar{h}\hat{c}_{t-1} + \bar{h}\hat{\gamma}_{A,t}; \quad \text{(D.25)}$$

the foreign habit formation

$$(1-\bar{h}^*)\hat{h}^*_t = \hat{c}^*_t - \bar{h}^*\hat{c}^*_{t-1} + \bar{h}^*\hat{\gamma}^*_{A,t}; \quad \text{(D.26)}$$

the home Taylor rule

$$(1+\hat{i}_{h,t}) = \rho_i(1+\hat{i}_{h,t-1})+(1-\rho_i)\left[a_\pi(\hat{\chi}_t - \hat{\gamma}_{\tau,t}) + a_y\hat{N}_t + \frac{a_s}{1-a_s}(\hat{q}_t - \hat{q}_{t-1} + \hat{\chi}_t - \hat{\chi}^*_t)\right]+v_{i,t}; \quad \text{(D.27)}$$

the foreign Taylor rule

$$(1+\hat{i}^*_{f,t}) = \rho^*_i(1+\hat{i}^*_{f,t-1})+(1-\rho^*_i)\left[a^*_\pi(\hat{\chi}^*_t - \hat{\gamma}^*_{\tau,t}) + a^*_y\hat{N}^*_t - \frac{a^*_s}{1-a^*_s}(\hat{q}_t - \hat{q}_{t-1} + \hat{\chi}_t - \hat{\chi}^*_t)\right]+v^*_{i,t}; \quad \text{(D.28)}$$

the real exchange rate

$$\Delta\hat{q}_t = \Delta\hat{s}_t - (\hat{\chi}_t - \hat{\gamma}_{\tau,t}) + (\hat{\chi}^*_t - \hat{\gamma}^*_{\tau,t}); \quad \text{(D.29)}$$

Appendix E. The restricted UC model and posterior simulation strategy

Let $\mathbf{X}_t$ denote an 66×1 unobserved state column vector defined as

$$\begin{aligned}\mathbf{X}_t \equiv [&\hat{c}_t, \hat{c}^*_t, \hat{c}_{h,t}, \hat{c}^*_{h,t}, \hat{c}_{f,t}, \hat{c}^*_{f,t}, \hat{h}_t, \hat{h}^*_t, \hat{N}_t, \hat{N}^*_t, \hat{mc}_t, \hat{mc}^*_t, \Delta\hat{p}_t, \Delta\hat{p}^*_t, \hat{\chi}_t, \chi^*_t, \hat{\chi}_{h,t}, \hat{\chi}^*_{h,t}, \hat{\chi}_{f,t}, \hat{\chi}^*_{f,t}, \hat{\Psi}_t,\ \hat{\Psi}^*_t,\\
&\hat{\omega}_{h,t}, \hat{\omega}^*_{h,t}, \hat{\omega}_{f,t}, \hat{\omega}^*_{f,t}, (1+\hat{i}_{h,t}), (1+\hat{i}^*_{f,t}), \hat{b}_t, \Delta\hat{s}_t, \hat{q}_t, \hat{a}_t, \hat{\gamma}_{A,t}, \hat{\gamma}^*_{A,t}, \hat{\gamma}_{\tau,t}, \hat{\gamma}^*_{\tau,t}, \epsilon_{i,t}, \epsilon^*_{i,t}, v^h_{p,t}, v^f_{p,t}, v^{h*}_{p,t}, v^{f*}_{p,t}, v_{r,t},\\
&E_t\hat{c}_{t+1}, E_t\hat{c}^*_{t+1}, E_t\hat{h}_{t+1}, E_t\hat{h}^*_{t+1}, E_t\hat{mc}_{t+1}, E_t\hat{mc}^*_{t+1}, E_t\hat{\chi}_{t+1}, E_t\hat{\chi}^*_{t+1}, E_t\hat{\chi}_{h,t+1}, E_t\hat{\chi}_{h,t+2}, E_t\hat{\chi}^*_{h,t+1}, E_t\hat{\chi}^*_{h,t+2},\\
&E_t\hat{\chi}_{f,t+1}, E_t\hat{\chi}_{f,t+2}, E_t\hat{\chi}^*_{f,t+1}, E_t\hat{\chi}^*_{f,t+2}, E_t\hat{\Psi}_{t+1}, E_t\hat{\Psi}^*_{t+1}, E_t\Delta\hat{s}_{t+1}, E_t\hat{q}_{t+1}, \hat{q}_{t-1}, \hat{c}_{t-1}, \hat{c}^*_{t-1}]'.\end{aligned}$$

Furthermore, let ϵ_t and η_t denote 11×1 and 20×1 random vectors consisting of structural shocks and rational expectations errors:

$$\epsilon_t \equiv [\epsilon_{A,t}, \epsilon^*_{A,t}, \epsilon_{\tau,t}, \epsilon^*_{\tau,t}, \epsilon_{i,t}, \epsilon^*_{i,t}, \epsilon^h_{p,t}, \epsilon^f_{p,t}, \epsilon^{h*}_{p,t}, \epsilon^{f*}_{p,t}, \epsilon_{r,t}]'$$

and

$$\begin{aligned}\eta_t \equiv [&\hat{c}_t - E_{t-1}\hat{c}_t,\ \hat{c}^*_t - E_{t-1}\hat{c}^*_t,\ \hat{h}_t - E_{t-1}\hat{h}_t,\ \hat{h}^*_t - E_{t-1}\hat{h}^*_t,\ \hat{mc}_t - E_{t-1}\hat{mc}_t,\ \hat{mc}^*_t - E_{t-1}\hat{mc}^*_t,\\
&\chi_t - E_{t-1}\chi_t,\ \chi^*_t - E_{t-1}\chi^*_t,\ \chi_{h,t} - E_{t-1}\chi_{h,t},\ E_t\chi_{h,t+1} - E_{t-1}\chi_{h,t+1},\ \chi^*_{h,t} - E_{t-1}\chi^*_{h,t},\ E_t\chi^*_{h,t+1} - E_{t-1}\chi^*_{h,t+1},\\
&\chi_{f,t} - E_{t-1}\chi_{f,t},\ E_t\chi_{f,t+1} - E_{t-1}\chi_{f,t+1},\ \chi^*_{f,t} - E_{t-1}\chi^*_{f,t},\ E_t\chi^*_{f,t+1} - E_{t-1}\chi^*_{f,t+1},\ \hat{\Psi}_t, -E_{t-1}\hat{\Psi}_t,\ \hat{\Psi}^*_t - E_{t-1}\hat{\Psi}^*_t,\\
&\Delta\hat{s}_t - E_{t-1}\Delta\hat{s}_t,\ \hat{q}_t - E_{t-1}\hat{q}_t]',\end{aligned}$$

respectively. In particular, for empirical investigation purposes, I presume that the structural shock vector ϵ_t is normally distributed, with a mean of zero and a diagonal variance-covariance matrix Σ: $\epsilon_t \sim i.i.d.N(\mathbf{0}, \Sigma)$ with $diag(\Sigma) = [\sigma_A, \sigma^*_A, \sigma_\tau, \sigma^*_\tau, \sigma_i, \sigma^*_i, \sigma^h_p, \sigma^f_p, \sigma^{h*}_p, \sigma^{f*}_p, \sigma_r]'$.

Accompanied by the stochastic processes of the exogenous forcing variables, the LRE model then implies that

$$\mathbf{G}_0(\theta)\mathbf{X}_t = \mathbf{G}_1(\theta)\mathbf{X}_{t-1} + \mathbf{Q}(\theta)\eta_t + \mathbf{R}(\theta)\epsilon_t,$$

where $\mathbf{G}_0$, $\mathbf{G}_1$, $\mathbf{Q}$, and $\mathbf{R}$ are the corresponding coefficient matrices and θ is the vector of the structural parameters. Applying Sims's (2001) QZ algorithm to the linear rational expectations model above yields a unique solution as the following stationary transition equation of the unobservable state vector:

$$\mathbf{X}_t = \mathbf{F}(\theta)\mathbf{X}_{t-1} + \Phi(\theta)\epsilon_t, \tag{E.1}$$

where $\mathbf{F}$ and Φ are confirmable coefficient matrices.

Let $\mathbf{Y}_t$ denote the information set that consists of the nine time series:

$$\mathbf{Y}_t \equiv [\Delta \ln P_t,\ \Delta \ln P_t^*,\ \Delta \ln C_t,\ \Delta \ln C_t^*,\ (1 + i_{h,t}),\ (1 + i_{f,t}^*), \ln q_t,\ \ln N_t,\ \ln N_t^*]'.$$

It is then straightforward to show that the information set $\mathbf{Y}_t$ is linearly related to the unobservable state vector $\mathbf{X}_t$ as

$$\mathbf{Y}_t = \mathbf{d}(\theta) + \mathbf{H}(\theta)\mathbf{X}_t, \tag{E.2}$$

where $\mathbf{d}$ and $\mathbf{H}$ are confirmable coefficient vector and matrix. The transition equation of the unobserved state (E.1) and the observation equation (E.2) jointly consist of a non-stationary state-space representation of the two-country model.

Given the data set $\mathbf{Y}^T \equiv \{\mathbf{Y}_t\}_{t=0}^T$, applying the Kalman filter to the UC model provides model likelihood $\mathbf{L}(\mathbf{Y}^T|\theta)$, where θ is the structural parameter vector of the two-country model. Multiplying the likelihood by a prior probability of the structural parameters, $\mathbf{p}(\theta)$, is proportional to the corresponding posterior distribution $\mathbf{p}(\theta|\mathbf{Y}^\mathbf{T}) \propto \mathbf{p}(\theta)\mathbf{L}(\mathbf{Y}^T|\theta)$ through Bayes' law. The posterior distribution $\mathbf{p}(\theta|\mathbf{Y}^T)$ is simulated by the random-walk Metropolis-Hastings algorithm, as implemented by Schorfheide (2000), Bouakez and Kano (2006), Nason and Rogers (2008), and Kano (2009; 2020).

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Table A.1: Posterior Mean FEVDs (%): Home Inflation Rate

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
(a) TI model											
Impact	8.01	0.00	3.40	0.29	16.8	0.00	70.5	0.77	0.09	0.05	0.00
1yr	13.2	0.00	7.77	0.46	30.2	0.01	46.9	0.99	0.25	0.08	0.00
3yr	13.3	0.00	13.7	0.41	31.9	0.01	39.2	0.83	0.38	0.08	0.00
10yr	11.9	0.00	21.7	0.98	28.5	0.01	34.9	0.88	0.83	0.08	0.00
(b) Non-TI model											
Impact	0.02	0.13	6.81	2.47	16.1	1.61	69.7	0.20	2.91	0.00	0.00
1yr	0.01	0.15	13.3	3.75	19.5	1.88	57.7	0.23	3.34	0.01	0.00
3yr	0.01	0.13	19.0	4.15	17.6	1.69	54.0	0.23	3.00	0.01	0.00
10yr	0.01	0.11	29.9	4.99	14.9	1.42	45.7	0.20	2.54	0.01	0.00

Table A.2: Posterior Mean FEVDs (%): Foreign Inflation Rate

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
(a) TI model											
Impact	0.00	0.15	0.00	6.67	0.03	0.91	0.05	0.66	1.26	90.2	0.00
1yr	0.01	0.26	0.01	19.7	0.03	1.75	0.13	1.78	2.92	73.3	0.00
3yr	0.06	0.22	0.01	33.7	0.06	1.54	0.12	1.57	3.61	59.1	0.00
10yr	0.16	0.17	0.13	48.9	0.10	1.13	0.09	1.26	4.71	43.2	0.00
(b) Non-TI model											
Impact	0.00	5.10	0.04	27.8	0.06	36.6	0.34	0.45	1.15	28.3	0.00
1yr	0.00	4.49	0.04	38.9	0.05	39.0	0.29	0.45	1.17	15.5	0.00
3yr	0.00	3.90	0.04	45.6	0.05	34.6	0.30	0.47	1.06	13.8	0.00
10yr	0.00	3.12	0.04	56.3	0.04	27.7	0.24	0.39	1.02	11.0	0.00

Table A.3: Posterior Mean FEVDs (%): Home Consumption Growth

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
					(a) TI model						
Impact	16.1	0.00	36.7	3.61	14.6	0.02	21.6	0.65	5.71	0.88	0.00
1yr	24.2	0.01	34.1	2.28	10.4	0.04	21.3	0.42	6.40	0.69	0.00
3yr	24.5	0.01	30.1	1.86	16.5	0.04	19.3	0.46	6.54	0.56	0.00
10yr	24.0	0.01	28.8	1.84	17.5	0.04	19.9	0.54	6.62	0.55	0.00
					(b) Non-TI model						
Impact	12.5	0.16	3.58	2.36	27.2	2.61	44.8	0.18	6.45	0.01	0.00
1yr	11.3	0.18	3.59	2.48	27.7	2.81	45.5	0.23	6.15	0.01	0.00
3yr	10.8	0.17	3.46	2.39	26.7	2.70	47.5	0.25	5.89	0.01	0.00
10yr	10.8	0.17	3.46	2.38	26.7	2.70	47.5	0.26	5.90	0.01	0.00

Table A.4: Posterior Mean FEVDs (%): Foreign Consumption Growth

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
					(a) TI model						
Impact	0.75	0.01	0.02	22.7	1.18	3.40	1.69	36.7	0.31	33.1	0.00
1yr	0.78	0.16	0.01	25.8	1.24	2.74	1.62	36.0	0.25	31.3	0.00
3yr	0.76	0.23	0.03	25.5	1.20	3.04	1.59	35.1	0.27	32.2	0.00
10yr	0.78	0.22	0.07	24.7	1.31	2.93	1.67	36.4	0.33	31.4	0.00
					(b) Non-TI model						
Impact	0.00	0.65	0.00	22.3	0.02	64.3	0.51	1.38	3.57	7.11	0.00
1yr	0.01	5.58	0.01	20.1	0.07	57.9	0.88	2.36	3.77	9.24	0.00
3yr	0.01	5.95	0.01	19.7	0.07	57.1	0.99	2.49	3.56	10.0	0.00
10yr	0.01	5.92	0.01	19.6	0.08	56.9	1.04	2.68	3.70	10.0	0.00

Table A.5: Posterior Mean FEVDs (%): Home Interest Rate

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
					(a) TI model						
Impact	24.1	0.03	0.33	1.61	60.4	0.20	3.30	2.73	6.90	0.31	0.00
1yr	19.6	0.02	1.01	1.47	51.8	0.13	14.5	2.34	8.62	0.38	0.00
3yr	15.6	0.02	4.99	1.30	42.2	0.10	23.0	1.93	10.2	0.44	0.00
10yr	12.6	0.01	30.2	1.19	32.2	0.08	15.3	1.23	6.74	0.35	0.00
					(b) Non-TI model						
Impact	0.00	0.65	9.91	0.67	42.9	9.59	29.5	0.19	6.33	0.19	0.00
1yr	0.00	0.23	21.6	1.04	16.9	3.62	53.9	0.08	2.31	0.10	0.00
3yr	0.00	0.14	41.6	3.56	10.5	2.23	40.2	0.13	1.43	0.06	0.00
10yr	0.00	0.08	63.2	6.35	5.83	1.23	22.3	0.08	0.79	0.03	0.00

Table A.6: Posterior Mean FEVDs (%): Foreign Interest Rate

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
					(a) TI model						
Impact	0.29	6.91	0.04	0.02	0.80	47.7	0.13	0.06	0.18	43.7	0.00
1yr	0.23	3.20	0.07	0.47	0.66	22.6	0.66	6.76	1.07	64.1	0.00
3yr	0.16	1.78	0.19	3.39	0.44	12.6	1.72	27.3	3.96	48.3	0.00
10yr	0.78	1.10	0.24	23.6	1.68	7.83	1.61	27.3	5.00	30.7	0.00
					(b) Non-TI model						
Impact	0.00	6.27	0.00	4.37	0.01	82.1	0.16	0.12	0.13	6.77	0.00
1yr	0.00	3.23	0.03	34.9	0.03	48.7	0.77	1.13	1.42	9.66	0.00
3yr	0.00	1.47	0.06	67.4	0.03	22.4	0.58	1.09	2.18	4.66	0.00
10yr	0.00	0.60	0.05	85.7	0.01	9.19	0.24	0.46	1.79	1.91	0.00

TABLE A.7: POSTERIOR MEAN FEVDS (%): HOME HOURS WORKED

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
					(a) TI model						
Impact	57.4	0.00	1.01	1.28	18.6	0.02	17.0	3.60	0.14	0.81	0.00
1yr	34.5	0.01	0.99	2.63	41.7	0.05	11.9	5.99	1.26	0.81	0.00
3yr	28.5	0.01	0.58	2.84	52.1	0.07	4.96	4.58	5.63	0.66	0.00
10yr	22.9	0.01	4.12	2.17	45.9	0.08	3.71	3.04	17.3	0.59	0.00
					(b) Non-TI model						
Impact	0.47	0.21	3.45	2.49	26.3	2.88	58.4	0.22	5.46	0.00	0.00
1yr	0.10	0.10	1.58	1.12	11.5	1.28	81.6	0.32	2.28	0.00	0.00
3yr	0.09	0.09	1.40	0.99	10.0	1.12	83.8	0.33	2.06	0.00	0.00
10yr	0.09	0.09	1.41	0.98	9.97	1.11	82.9	0.33	3.05	0.00	0.00

TABLE A.8: POSTERIOR MEAN FEVDS (%): FOREIGN HOURS WORKED

Horizon	ϵ_A	ϵ_A^*	ϵ_τ	ϵ_τ^*	ϵ_i	ϵ_i^*	ϵ_p^h	ϵ_p^f	ϵ_p^{h*}	ϵ_p^{f*}	ϵ_r
					(a) TI model						
Impact	0.42	4.22	0.09	9.35	0.55	9.14	3.57	54.4	0.18	18.0	0.00
1yr	0.21	1.56	0.24	6.60	0.22	7.06	3.50	75.9	0.28	4.39	0.00
3yr	0.16	1.15	0.54	4.53	0.40	6.06	3.38	77.6	3.64	2.48	0.00
10yr	0.36	0.88	0.58	7.84	1.33	4.71	2.51	56.3	23.2	2.16	0.00
					(b) Non-TI model						
Impact	0.03	33.8	0.21	12.2	1.15	32.8	7.36	9.39	0.02	2.93	0.00
1yr	0.02	15.0	0.19	15.0	0.90	39.6	7.32	12.7	0.50	8.51	0.00
3yr	0.02	12.7	0.18	14.3	0.79	36.8	6.40	12.3	7.33	9.07	0.00
10yr	0.03	8.89	0.13	10.1	0.55	25.7	4.45	8.51	35.1	6.34	0.00

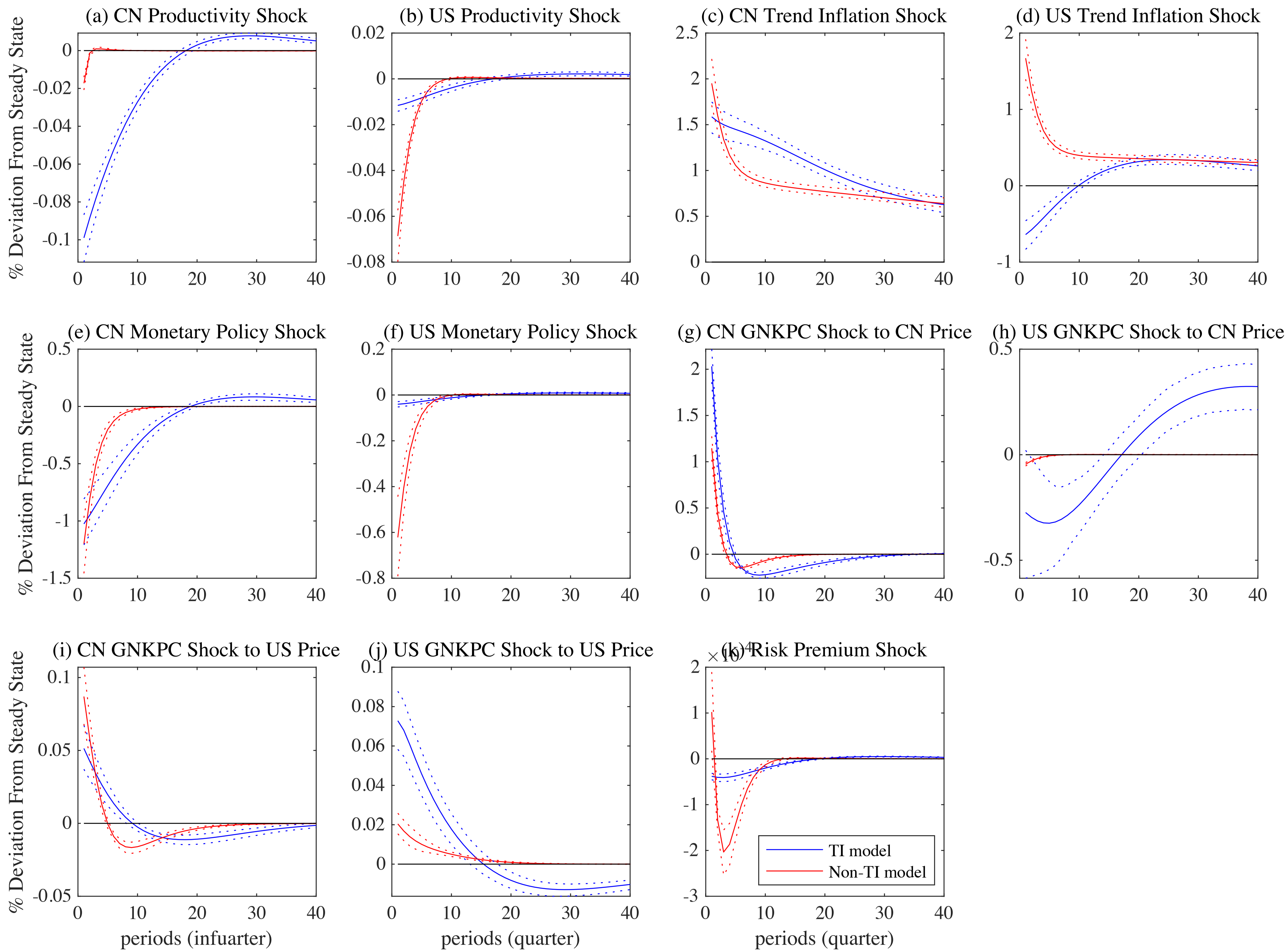

Figure 1: IRFs of the Home Inflation Rate to Structural Shocks

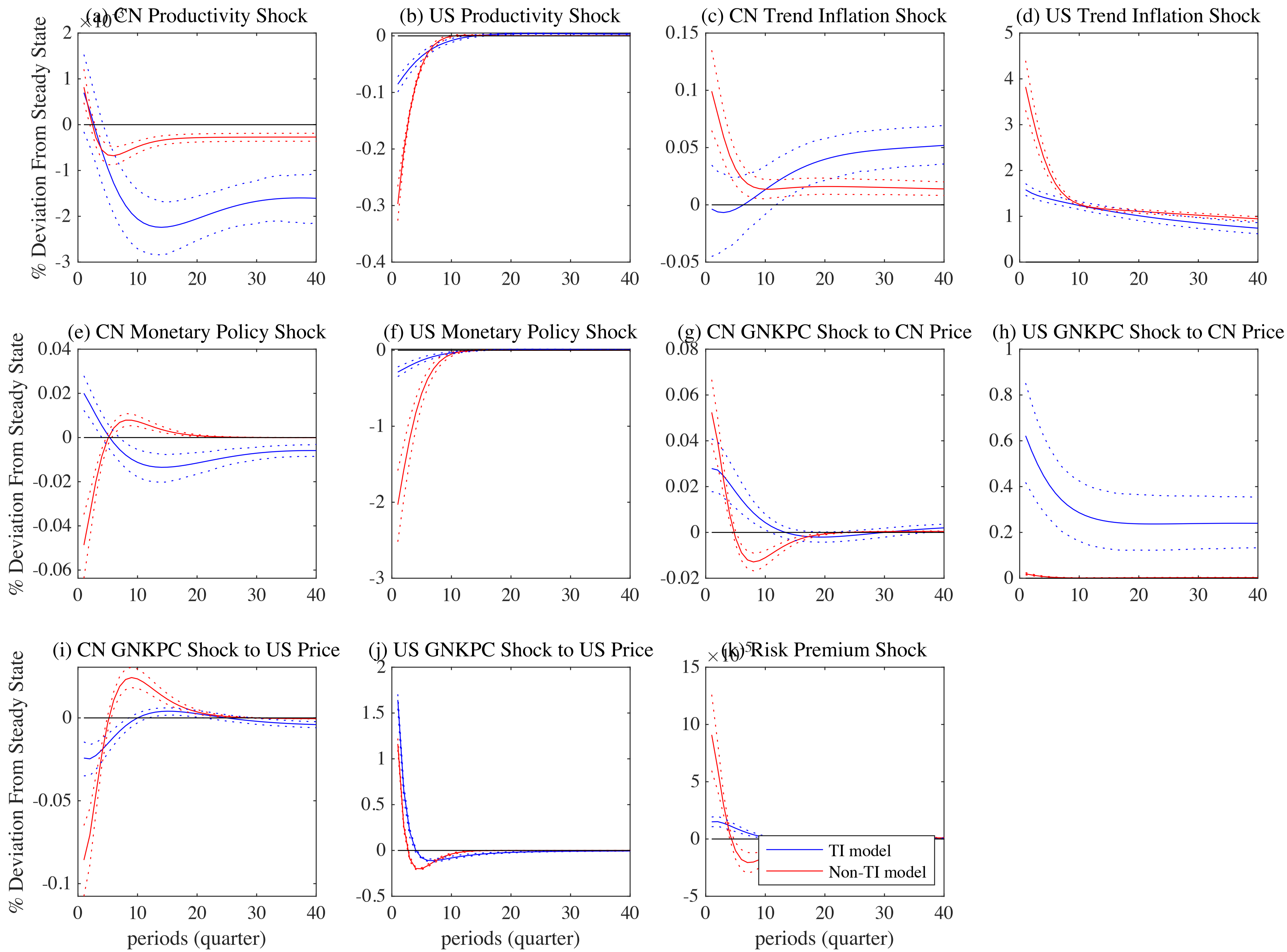

Figure 2: IRFs of the Foreign Inflation Rate to Structural Shocks

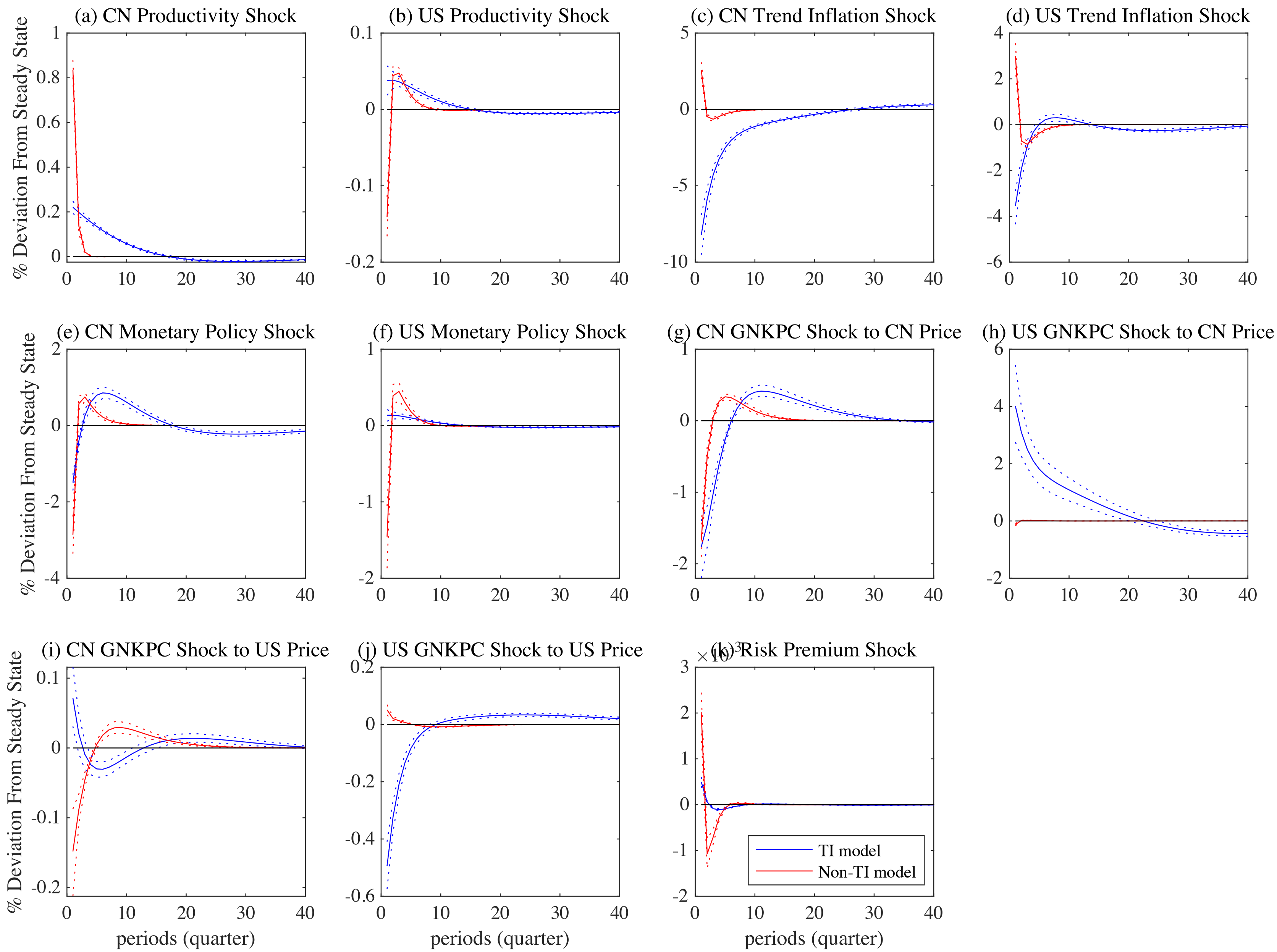

Figure 3: IRFs of the Home Consumption Growth Rate to Structural Shocks

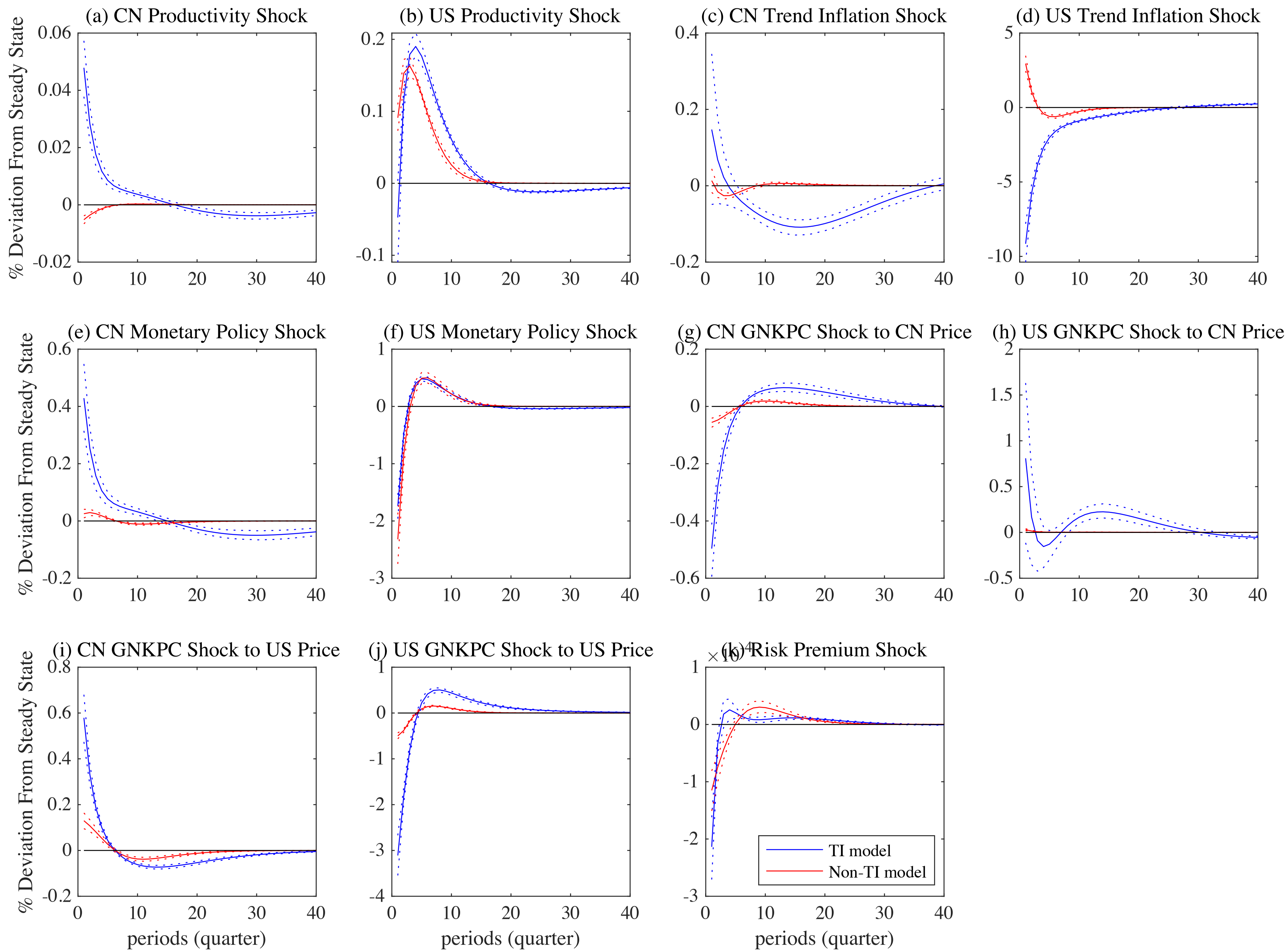

Figure 4: IRFs of the Foreign Consumption Growth Rate to Structural Shocks

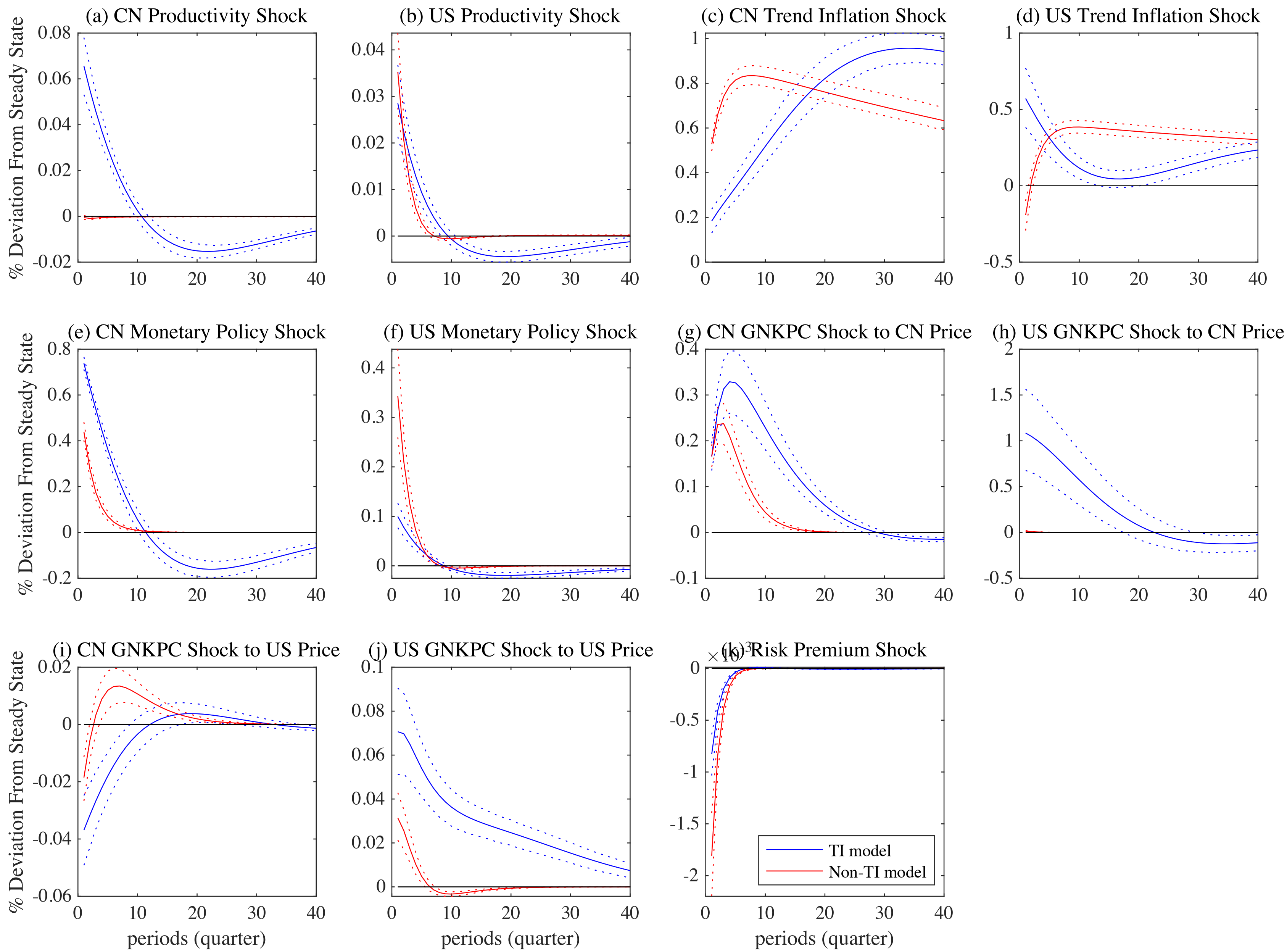

Figure 5: IRFs of the Home Nominal Interest Rate to Structural Shocks

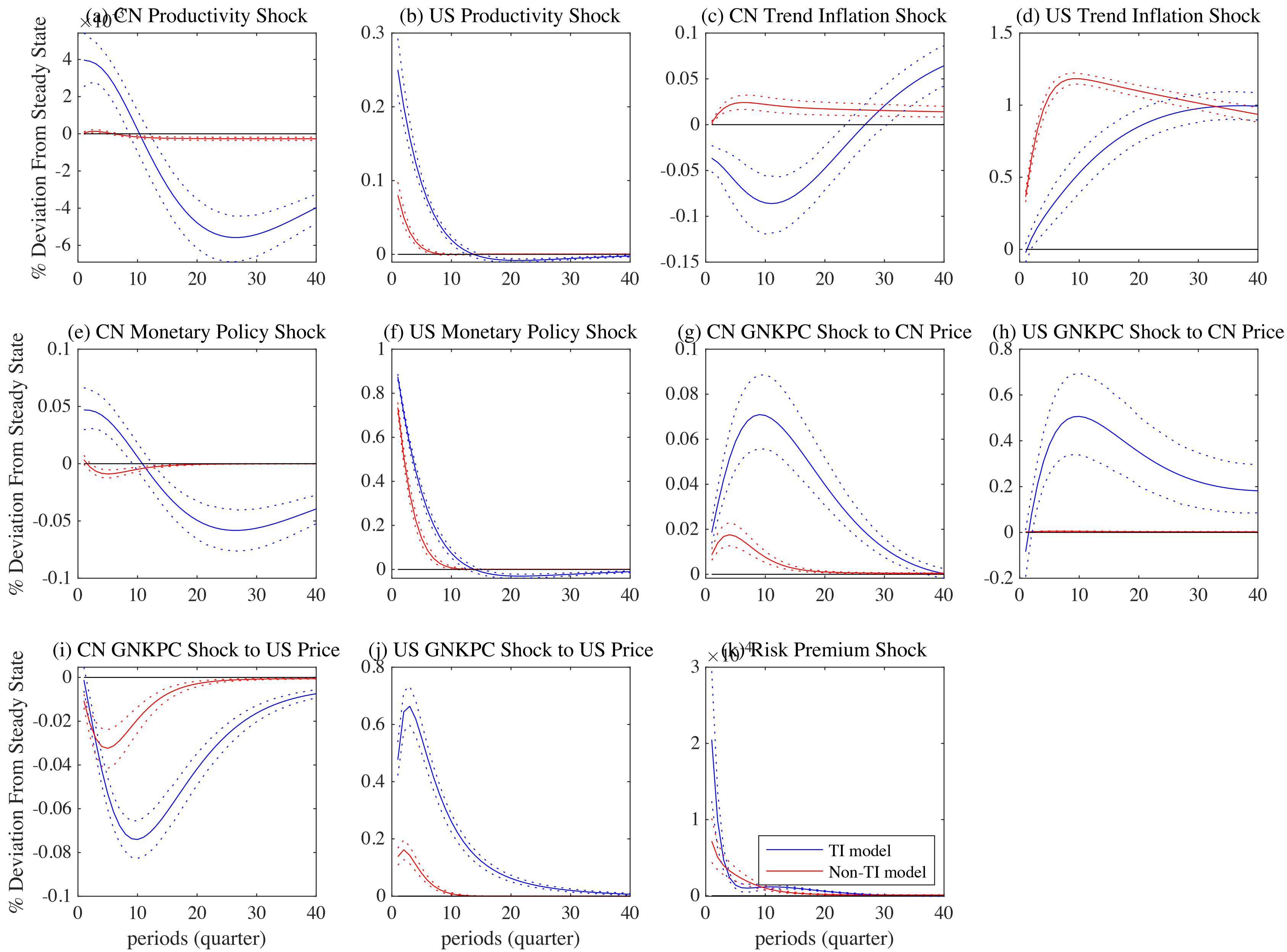

Figure 6: IRFs of the Foreign Nominal Interest Rate to Structural Shocks

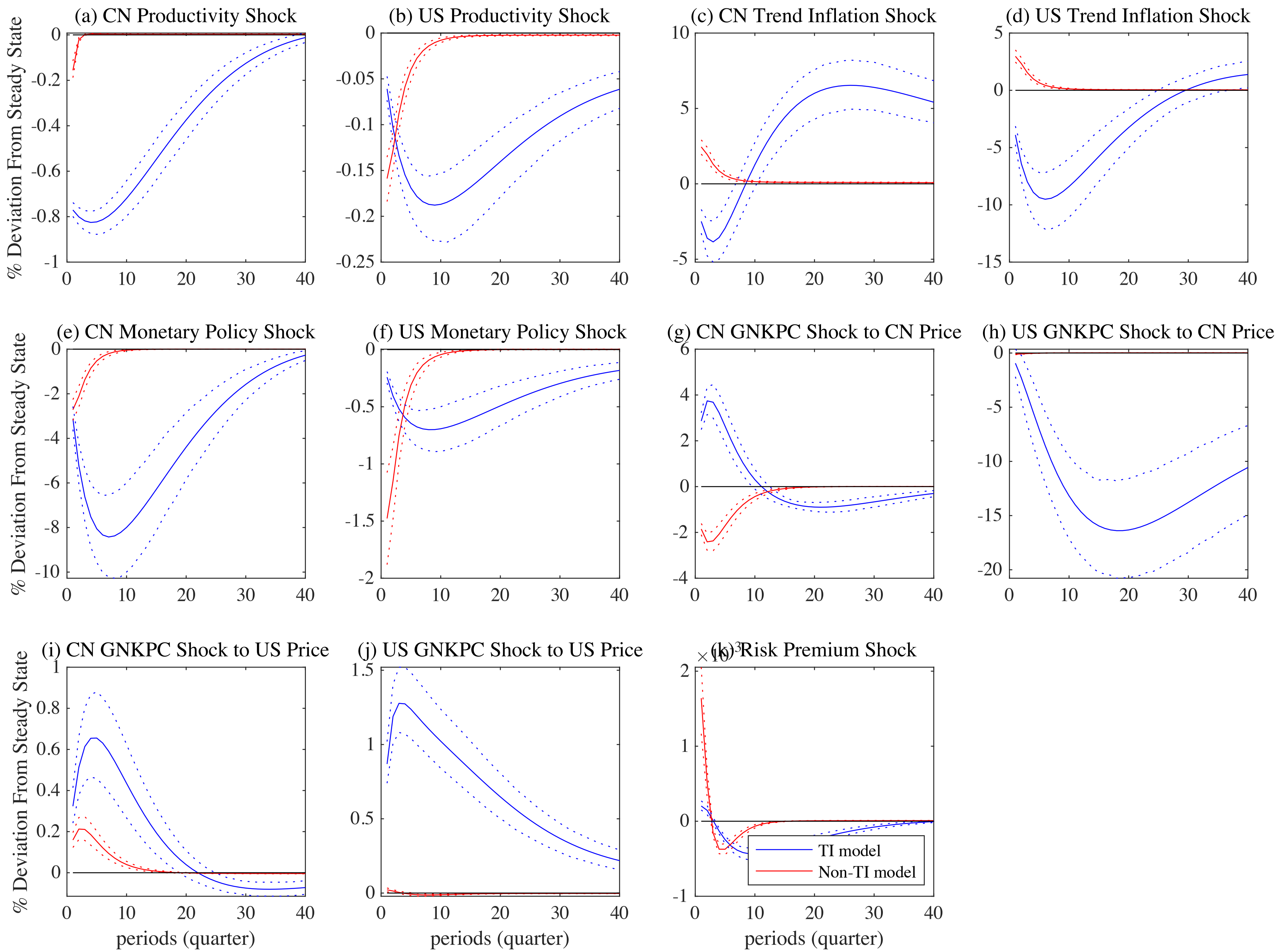

Figure 7: IRFs of the Home Hours Worked to Structural Shocks

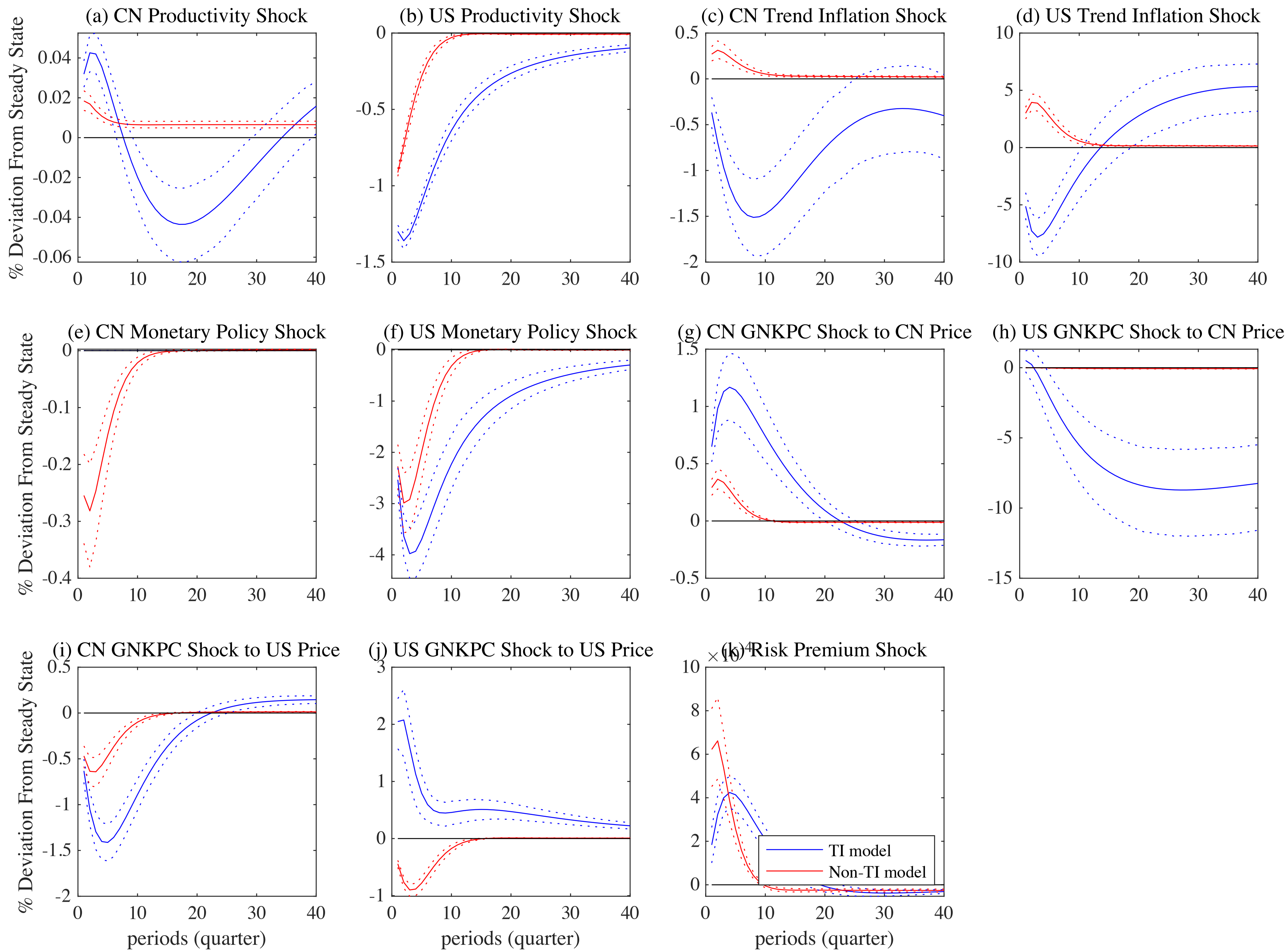

Figure 8: IRFs of the Foreign Hours Worked to Structural Shocks