Discussion Paper Series A No.750

# Estimating trend inflation in a regime-switching Phillips curve

Jouchi Nakajima
(Institute of Economic Research, Hitotsubashi University)

December 2023

Institute of Economic Research
Hitotsubashi University
Kunitachi, Tokyo, 186-8603 Japan

# Estimating trend inflation in a regime-switching Phillips curve*

Jouchi Nakajima†

December 2023

**Abstract**

This study develops a regime-switching Phillips curve model to estimate trend inflation. Extending the earlier work, we allow trend inflation, the slope of the Phillips curve, and the oil price pass-through rate to follow a regime-switching process. An empirical analysis using Japan's consumer price index illustrates that including the oil price and its time-varying pass-through rate improves the model's ability to forecast inflation. The empirical results also show that the obtained trend inflation highly correlates with firms' inflation expectations.

*JEL classification*: C22, E31, E42, E52, E58

*Keywords*: Inflation expectations, Oil prices, Phillips curve; Regime-switching model; Trend inflation

---

*The author is grateful to Mitsuhiro Osada and Esturo Shioji for valuable comments and discussions. Financial support from the Ministry of Education, Culture, Sports, Science, and Technology of the Japanese Government through a Grant-in-Aid for Scientific Research (No. 22K20157, 23H00050) and the Hitotsubashi Institute for Advanced Study is gratefully acknowledged.

†Institute of Economic Research, Hitotsubashi University (E-mail: nakajima-j@ier.hit-u.ac.jp)

# 1 Introduction

Trend inflation is a key variable driving a wide range of economic activities. It is usually defined as the level at which inflation is expected to converge without any future shock or prevail in the long run (e.g., Faust and Wright, 2013). This concept is closely related to inflation expectations and their term structure, because long-term inflation expectations are treated interchangeably with trend inflation. Ascari and Sbordone (2014) comprehensively summarize the concept and previous theoretical works.

Given that trend inflation is not observed directly, existing studies have attempted to develop a framework to estimate it (e.g., Nason and Smith, 2008). A popular approach employs multivariate time-series models such as vector autoregressive (VAR) models (e.g., Quah and Vahey, 1995; Claus, 1997; Mertens, 2016). Estimating trend inflation using the VAR model requires appropriate restrictions to extract trend inflation from a multivariate time series of macroeconomic variables. Several existing studies use survey data on inflation expectations to estimate trend inflation (e.g. Brissimis and Magginas, 2008; Kozicki and Tinsley, 2012).

Alternatively, an unobserved component model has become the standard approach for estimating trend inflation (e.g., Stock and Watson, 2007; Cecchetti et al., 2007; Kiley, 2008; Clark and Doh, 2014). These studies assume that the dynamics of actual inflation can be decomposed into trends and cycle components. In addition, Stock and Watson (2007) propose stochastic volatility (SV) for innovations in trend and cycle components, resulting in a flexible model for describing various changes in inflation dynamics.

From a different viewpoint, Kaihatsu and Nakajima (2018) propose a novel econometric framework for estimating trend inflation with regime-switching structure embedded into the Phillips curve. The unique and effective aspect of its modeling strategy is that the regimes are specified as being equally spaced. This equally spaced regime-switching model provides a parsimonious time-varying structure for both trend inflation and the slope of the Phillips curve.

One caveat of this previous study is that an input cost variable, such as oil price, is not considered in its model, although it has been one of the important drivers in inflation dynamics. Therefore, the current study extends the regime-switching model by explicitly incorporating the effect of oil price on inflation and assuming its pass-through rate to follow the equally spaced regime switching process. As an empirical analysis, using Japan's macroeconomic data, the time-varying nature of the Phillips curve, including the impact of oil prices, is investigated.

This study contributes to existing literature in two ways. The first is the estimation framework for trend inflation. This study specifies inflation dynamics based on an autoregressive process with a time-varying intercept as inflation converges to an implied endpoint that changes over time, following previous studies, such as Kozicki and Tinsley (2012), Mertens (2016), Chan et al. (2018). In the current study, we incorporate the Phillips curve components of the output gap and oil prices into the model as the important factors driving the inflation rates. Okimoto (2019) estimates the Japan's Phillips curve including trend inflation and other macroeconomic variables using a standard regime-switching model. The unique modeling strategy of the current study is that trend inflation, the slope of the Phillips curve, and the oil price pass-through rate follow an equally spaced regime-switching process.

The second is the oil price pass-through to inflation. In this context, fruitful discussions and empirical evidence are provided by De Gregorio et al. (2007), Chen (2009), Choi et al. (2018), and others. Choi et al. (2018) address changes in oil price pass-through rate over time, providing empirical findings that the rate differs depending on sub-sample periods. However, little evidence has been provided regarding the possibility of continuous or gradual changes in the oil price pass-through rate, which is the focus of this study. Shioji and Uchino (2011) empoy a time-varying parameter vector autoregressive (TVP-VAR) model to estimate possibly gradual changes in the oil-price pass-through rate using Japan's data (see also, Sekine, 2006; Yagi et al., 2022). This study estimates the changes based on regime switching and shows that the assumption of time variation in the pass-through rate is important in terms of the predictive ability of the trend inflation model.

The remainder of this paper is organized as follows. Section 2 explains the equally spaced regime-switching model and proposes a new model that includes oil prices. Section 3 describes the proposed model's estimation framework. Section 4 provides an empirical analysis of the Japan's data. Finally, Section 5 concludes.

# 2 The model

## 2.1 Trend inflation

We define $\pi_t$ as inflation rate and $\mu_t$ as trend inflation in period $t$. Following previous studies such as Kozicki and Tinsley (2012), Mertens (2016), Chan et al. (2018), we define trend inflation as the expected infinite-horizon forecast of $\pi_t$, given the information set in $t$. We formulate it as:

$$\mu_t = \lim_{j \to \infty} \mathrm{E}[\pi_{t+j} | \Omega_t], \tag{1}$$

where $\Omega_t$ denotes the information set at time $t$. We assume that the expected value of trend inflation equals the current value, that is, $\mu_t = \mathrm{E}_t \mu_{t+1}$. While previous studies assume that trend inflation follows a random walk process, this study assumes a regime-switching process, as described below.

To formulate the inflation rate dynamics, following Kozicki and Tinsley (2012) and Kaihatsu and Nakajima (2018), we specify

$$\begin{aligned} y_t &= \sum_{i=1}^{k} \alpha_i y_{t-i} + \left(1 - \sum_{i=1}^{k} \alpha_i\right) \mu_t + \beta_t x_t + \gamma_t z_t + \varepsilon_t, \\ \varepsilon_t &\sim N(0, \sigma_t^2), \end{aligned}$$

for $t = 1, \ldots, n$, where $x_t$ denotes the output gap and $z_t$ denotes an input cost variable, such as oil price. This study introduces the effect of the input cost variable on inflation using the term $\gamma_t z_t$. We assume that the coefficients $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_k)$ satisfy the stationarity condition $\left|\sum_{i=1}^{k} \alpha_i\right| \leq 1$, because trend inflation is the level at which the actual inflation rate is expected to prevail in the long run, when $x_t = z_t = 0$.

## 2.2 Equally-spaced regime-switching structure

A standard regime-switching model specifies multiple states of unknown discrete values for a parameter of interest and assumes that the parameter shifts from one state to another over time or remains in the same state with a certain probability (e.g., Kim and Nelson, 1999; Sims et al., 2008; Kim et al., 2014). In contrast, the equally spaced regime-switching model proposed by Kaihatsu and Nakajima (2018) assumes that the values of the states are pre-specified at equally spaced intervals. For example, we assume that trend inflation takes the value of zero, one, two percent, and so on, and that it shifts from one to another over time as in the standard regime-switching model. Given that the states are known, an inference of the state values is not required for the estimation, which generally decreases the estimation uncertainty of the model. Furthermore, the equally-spaced intervals often lead to intuitive implications for understanding estimation results. For instance, by tracking the regime probability of trend inflation of two percent, we can effectively address the degree of anchoring inflation at an inflation target of two percent in the current and historical developments of inflation rates.

Specifically, we assume $\mu_t \in \{\tilde{\mu}_1, \tilde{\mu}_2, \ldots, \tilde{\mu}_{N(\mu)}\}$ and $\beta_t \in \{\tilde{\beta}_1, \tilde{\beta}_2, \ldots, \tilde{\beta}_{N(\beta)}\}$, $\gamma_t \in \{\tilde{\gamma}_1, \tilde{\gamma}_2, \ldots, \tilde{\gamma}_{N(\gamma)}\}$, with all the states are equally spaced for each parameter, i.e., $\tilde{\lambda}_k - \tilde{\lambda}_{k-1} = \tilde{\lambda}_\ell - \tilde{\lambda}_{\ell-1}$ for any $k$ and $\ell$, where $\lambda \in \{\mu, \beta, \gamma\}$. Note that $N(\lambda)$ denotes the number of states, which can differ depending on parameter.

We formulate a first-order Markov process for the switching mechanism with a switch probability set smaller to a more distant regime. We first define the probability that the parameter remains in the same regime from $t-1$ to $t$, as $p_\mu$, $p_\beta$, and $p_\gamma$:

$$p_\lambda \equiv \Pr[\lambda_t = \tilde{\lambda}_i | \lambda_{t-1} = \tilde{\lambda}_i], \quad \text{for all } i \in \{1, \ldots, N(\lambda)\},$$

for $\lambda \in \{\mu, \beta, \gamma\}$. We assume the probability that the parameter shifts from one state to another as follows:

$$\Pr[\lambda_t = \tilde{\lambda}_i | \lambda_{t-1} = \tilde{\lambda}_j] = 2^{-|i-j|} q_{\lambda,j}, \quad \text{for } i, j \in \{1, \ldots, N(\lambda); i \neq j\},$$

where $q_{\lambda,j}$ is a function of $p_\lambda$ that satisfies:

$$\sum_{i=1}^{N(\lambda)} \Pr[\lambda_t = \tilde{\lambda}_i | \lambda_{t-1} = \tilde{\lambda}_j] = 1,$$

for all $j$. We do not consider an explicit correlation between the parameters $(\mu_t, \beta_t, \gamma_t)$ a priori, and the transition of each parameter is assumed to be independent.

As mentioned earlier, regime-switching models usually assume that regime values are unknown and are estimated. In contrast, the regimes in our model are pre-specified. It does not require the determination of the number of states, as in the standard regime-switching model. The range of equally spaced intervals is set, as they sufficiently covered the posterior distribution of the parameter. We will discuss this point in the empirical analysis below.

### 2.3 Stochastic volatility

For the error variance $\sigma_t^2$ in equation (1), we specify a standard stochastic volatility (SV) model by defining $h_t = \log \sigma_t^2$ and formulating the process as follows:

$$h_{t+1} = h_t + \eta_t, \quad \eta_t \sim N(0, v^2),$$

for $t = 1, \ldots, n-1$.

Existing studies measuring trend inflation employ the SV process for error variance (e.g. Stock and Watson, 2007; Chan et al., 2018). As Primiceri (2005) and Nakajima (2011) discuss, modeling the changes in the size of shocks or errors over time is relevant for estimating the time-varying state variables specified in other parts of the model. Using simulation data, Nakajima (2011) point out that a lack of time variation in the error variance can distort the estimates of time-varying state variables. Kaihatsu and Nakajima (2018) show that the predictive ability of the regime-switching trend inflation model is worsened when we assume a time-invariant error variance. Following these studies, we specify the SV process in the proposed model.

## 3 Estimation method

### 3.1 Bayesian estimation

Following previous studies on measuring trend inflation (Stock and Watson, 2007; Chan et al., 2018), we use a Bayesian estimation method with the Markov chain Monte Carlo (MCMC) algorithm because the model includes many state variables in a nonlinear form, and a maximum likelihood method is not feasible. Kaihatsu and Nakajima

(2018) develop an efficient MCMC algorithm to estimate the regime-switching trend inflation model. The current analysis extends it to the model with the oil price.

We generate samples of some of the parameters and state variables from their conditional posterior distributions, given the current values of the other parameters and state variables, and iterate conditional posterior sampling. To estimate the proposed model, we exploit existing sampling schemes for the regime-switching and SV models. To generate the samples efficiently, we employ key algorithms, the multi-move sampler for regime-switching models developed by Carter and Kohn (1994) and Chib (1996) and the one for the stochastic volatility model developed by Shephard and Pitt (1997) and Watanabe and Omori (2004). The details of the MCMC algorithm for the proposed model are presented in Appendix.

In the following empirical analysis, we generate 10,000 samples after discarding the initial 1,000 samples as a burn-in period. MCMC streams are clean and stable, with quickly decaying sample autocorrelations. The computation time is only a few minutes when using a generic laptop computer.

## 3.2 Data

For the empirical analysis, we use a year-on-year change in Japan's core inflation rate of the consumer price index that excludes fresh food for $y_t$ in the model. The effect of the increase in the consumption tax is adjusted. As a measure of Japan's output gap, the series provided by the Bank of Japan is used as $x_t$ in the model.

For the input cost variable $z_t$ in the model, we use oil prices. A year-on-year change in the spot price of West Texas Intermediate downloaded from the FRED database, where the price is in US dollars. The alternatives for the variable are exchange rates, import prices, and producer prices. The forecasting ability of the model is computed using these series as the input cost variables, in the same manner as in the forecasting exercise below. This pre-analysis shows that the model with oil prices yields the best forecasting performance among the candidates.

As the frequency of the output gap series is quarterly, we take a three-month average of the inflation rates and oil prices to adjust all variables to quarterly. The sample period is from 1981/Q1 to 2023/Q3. Figure 1 presents the time series of the data.

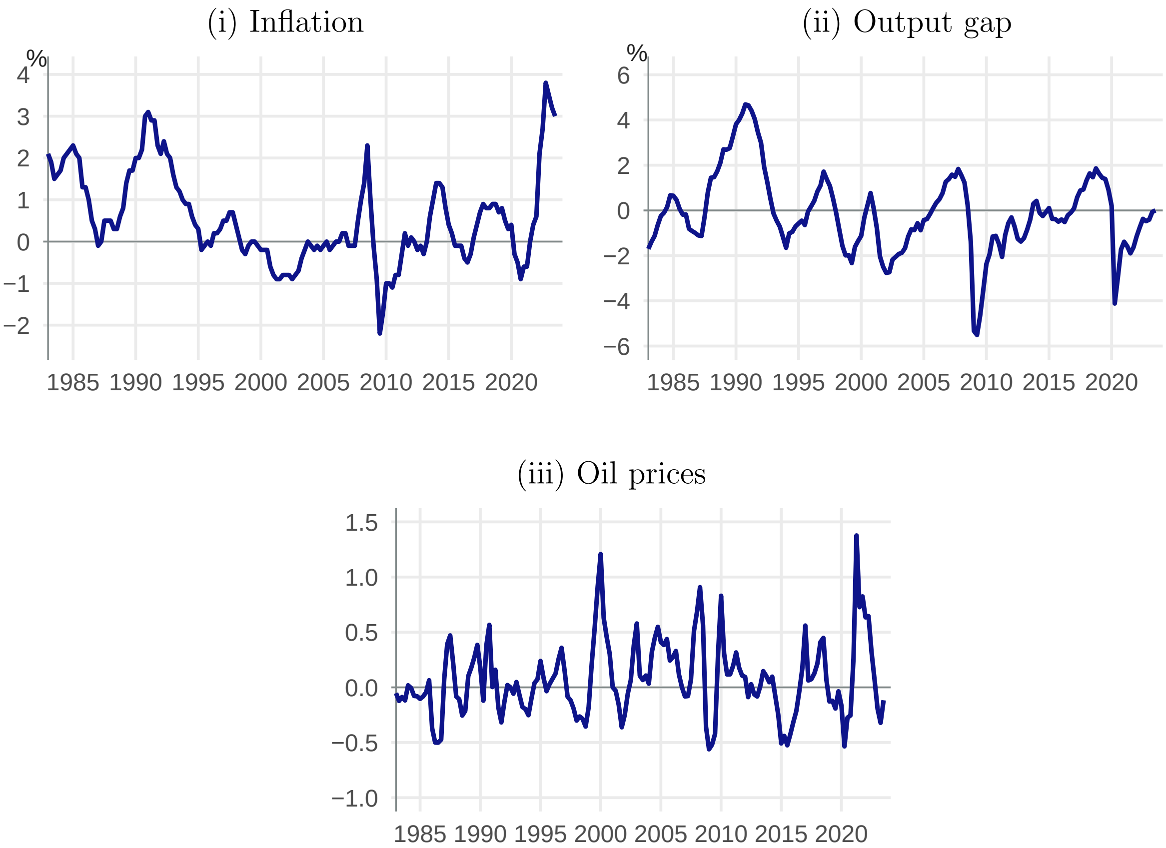


Figure 1: Japan's data. Inflation rates are a year-on-year change in the consumer price index excluding fresh food. Output gap is the series provided by the Bank of Japan. Oil price is a year-on-year change in the spot price of West Texas Intermediate. All the series are quarterly.

To find the best lag length for explanatory variables $x_t$ and $z_t$, we conduct the forecasting exercise explained below and measure the forecasting performance of each model with a different lag length. We examine the lag of the zero quarter (i.e., simultaneous), one quarter, and so on, up to four quarters for each of $x_t$ and $z_t$. The best lag length results in two quarters for the output gap ($x_t$) and zero quarters for the oil price ($z_t$), which we consider in the following analysis. The AR order $k$ is selected based on the same forecasting performance. Based on an exercise with up to $k = 12$, the model with $k = 8$ is found to be the best.

### 3.3 Priors

Following Kaihatsu and Nakajima (2018), we set the prior distributions:

$$\boldsymbol{\alpha} \sim TN_{\Theta}(\mathbf{0}_k, \boldsymbol{I}_k), \quad v^2 \sim IG(5, 0.2), \quad p_\lambda \sim B(990, 10), \text{ for } \lambda \in \{\mu, \beta, \gamma\},$$

where $TN_\Theta$ denotes the truncated multivariate normal distribution that has a positive density only in the domain $\Theta \equiv \{\boldsymbol{\alpha} | \left|\sum_{i=1}^{k} \alpha_i\right| \leq 1\}$ to satisfy the stationarity condition; and $IG$ and $B$ denote the Gamma and Beta distributions, respectively. Note that the specified priors for the regime-switching probability $p_\lambda$ imply a high probability of remaining in the same regime, which is an essential assumption of the regime-switching model. Examining the other values of hyperparameter for $p_\lambda$, we find that the result below does not change qualitatively as long as the alternative priors have a considerable mass within the range of 0.95–0.99. For the model with oil price but a time-invariant pass-through rate, we set a prior: $\gamma \sim N(0, 10)$.

We specify the trend inflation regimes as $\mu_t \in \{-2, -1, 0, 1, 2, 3\}$ on a percentage basis, covering the range of actual inflation rates during the sample period. Adding a 4 % regime might be an alternative, although we find that the result does not change significantly. Following Kaihatsu and Nakajima (2018), we set the regimes of the Phillips curve slope to $\beta_t \in \{0.00, 0.05, \ldots, 0.30\}$, which roughly covers 95% of the credible intervals of the slope in a model in which the slope is assumed to be constant over time.

For the newly introduced regimes of the oil price pass-through rate, we first estimate a model in which the pass-through rate is assumed to be constant over time with trend inflation and the slope of the Phillips curve following regime switching. Based on the posterior estimates of the time-invatiant pass-through rate i.e., $\gamma_t = \gamma$ for all $t$, we set the regimes to $\gamma_t \in \{0.0, 0.1, \ldots, 0.5\}$ to cover the 95% credible intervals of the time-invariant $\gamma$.

The final prior setting is the probability that the initial regime is $t = 1$. For trend inflation, as the actual inflation rate was positive and significantly different from zero around the early 1980s, we set $\Pr[\mu_1 = i] = 0$ for $i \leq 0$, and $\Pr[\mu_1 = j] = 1/3$ for $j \geq 1$. For the Phillips curve slope, previous literature on the Japan's Phillips curve points out that the slope is positive and apart from zero. Based on this evidence, we set $\Pr[\beta_1 = i] = 0$ for $i \in \{0.00, 0.05\}$ and $\Pr[\beta_1 = j] = 1/5$ for $j \geq 0.10$. Similarly, for the oil price pass-through rate, we assume $\Pr[\gamma_1 = 0.0] = 0$ and $\Pr[\beta_1 = j] = 1/5$ for $j \geq 0.1$. Posterior estimates for the state variables in the early sample period depend slightly on these prior assumptions regarding the initial regimes. However, those in the subsequent periods do not change significantly, regardless of the prior setting.

# 4 Empirical analysis

## 4.1 Estimation result

As a baseline model, we first estimate the original regime-switching trend inflation model of Kaihatsu and Nakajima (2018), where the oil price is not included. Their study use a sample period of up to 2014/Q4, so this estimation is an updated version of their analysis. Figure 2 shows the posterior regime probability of the trend inflation. In this figure, a pseudo-real-time (i.e., filtered) posterior mean of the probability is plotted following the idea discussed by Stock and Watson (2007). In our analysis, we estimate the filtered posterior mean of trend inflation by computing a quantity, $\mathrm{E}\left[\Pr[\mu_t = j|\mathbf{\Lambda}^{(m)}, y_1, \ldots, y_t]\right]$, at the $m$-th iteration in the MCMC algorithm, where $\mathbf{\Lambda}^{(m)}$ denotes the current values of all the other state variables and parameters at the $m$-th iteration. We call the plotted series as the "pseudo" real-time estimate because $\mathbf{\Lambda}^{(m)}$ is generated from the conditional posterior distribution given the data for all the sample period $(y_1, \ldots, y_n)$.

The estimated real-time regime probability shows that the trend inflation switches from one to zero percent around 1995. Since then, the zero-percent regime mostly dominated the distribution of trend inflation up to the end of the sample. Two notable breaks during this period are the probability of minus-one-percent regime hikes in 2000–2002, when the actual inflation rate fell to minus one percent. In 2013–2014, when the actual inflation rate increased significantly, the one-percent regime probability increased and reached a level similar to the zero-percent regime probability, which declined from approximately 80% to 40%. Presumably, this change in regime probability partly reflects the introduction of an inflation target of two percent in January 2013

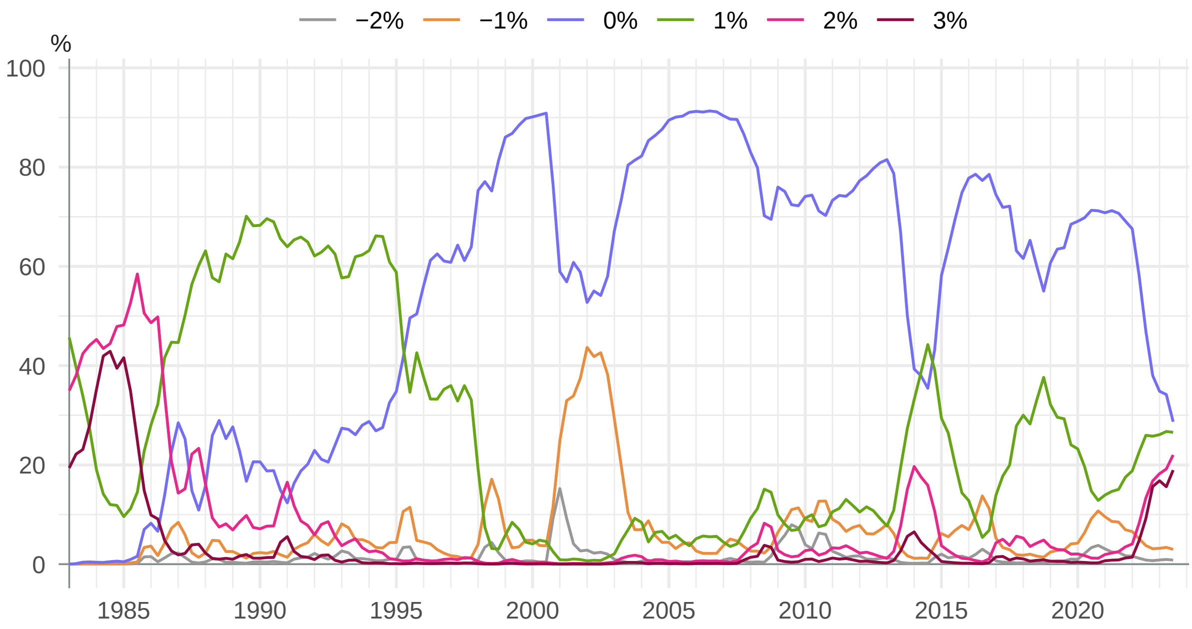


Figure 2: Posterior means of regime probability of trend inflation based on a pseudo real-time estimate in the baseline model.

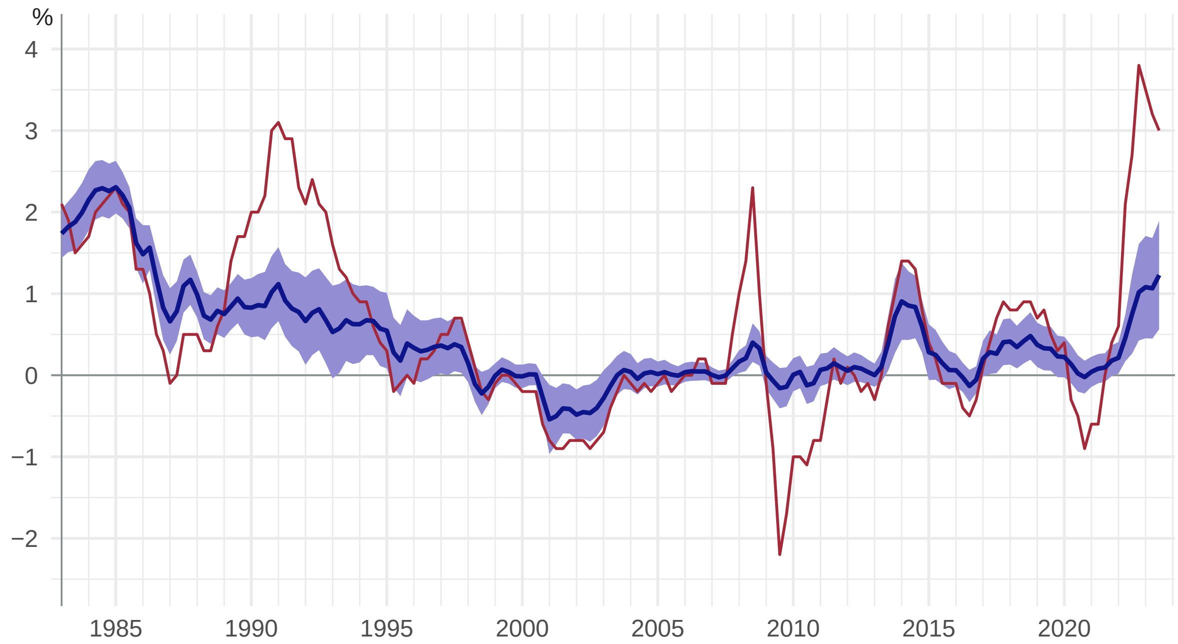


Figure 3: Posterior mean (bold line) and one-standard-deviation intervals (filled area) of trend inflation based on a pseudo real-time estimate in the baseline model. The solid line indicates the actual inflation rate.

and a new monetary policy framework of quantitative and qualitative monetary easing in April 2013, both implemented by the Bank of Japan. These findings are consistent with those in Kaihatsu and Nakajima (2018). After the hike in the one-percent regime probability, it peaked quickly and decreased to a low level in 2016, partly driven by a notable decline in oil prices. The probability increased in 2018–2019, but declined again in 2020 when the Covid-19 pandemic occurred.

A remarkable change occurred toward the end of the sample period. A significant increase in commodity prices and considerable wage pressure will push up inflation globally in 2022–2023. In Japan, while wage pressure is not as strong as in other developed countries, the inflation rate increases notably. The zero-percent regime probability decreases to approximately 30% and one-percent probability gradually increases. At the same time, the two- and three-percent regime probabilities rise to 15–20%, which has been historically high since the late 1980s.

Figure 3 plots the posterior means and one-standard-deviation credible intervals of the trend inflation. We computed them by taking the weighted average of the values in each regime based on pseudo real-time regime probabilities. The posterior mean shows notable changes throughout the sampling period, as described above. After the introduction of the inflation target and the quantitative and qualitative monetary easing in 2013, the trend inflation rose three times, but it has not reached the inflation target of two percent. Around the end of the sample, the trend inflation reaches at about one percent and the two-percent regime probability shown in Figure 2 does not dominate the other regimes.

Figure 4 displays the posterior means and one-standard-deviation credible intervals of the slope of the Phillips curve and the stochastic volatility. Note that this figure shows smoothed estimates rather than filtered estimates. We find that the slope of the Phillips curve gradually declined from the 1980s to the late 1990s and remained at almost the same level until the end of the sample period, which indicates a flattening of the Phillips curve (e.g., De Veirman, 2009). Okimoto (2019) provides a similar empirical result that the trend inflation regime switched from above one percent to about zero percent, using a smooth transition regime-switching model. Kaihatsu et al. (2023) point out that the trend inflation and the Phillips correlation decreased since the 1980s. Regarding the stochastic volatility, the figure shows its notable increase in

2008–2010, when the global financial crisis (GFC) significantly dampened the inflation rate, and in 2020–2022, when the Covid-19 pandemic had a significant impact on the economy.

Next, we estimate the regime-switching trend-inflation model with oil prices but a time-invariant pass-through rate, assuming $\gamma_t = \gamma$ for all $t$. An estimation result indicates that the posterior mean of $\gamma$ is 0.285, and the 95% credible intervals are [0.137, 0.446]. As the credible intervals do not include zero, the results indicate that the oil price pass-through is relevant to Japan's core inflation rates, which is consistent with existing studies (e.g., Shioji and Uchino, 2011; Yagi et al., 2022).

Finally, we estimate the model using the oil price and regime-switching pass-through rate, which is the full specification proposed in this study. Figure 5 plots the pseudo-real-time posterior mean of the regime probability. The results show several key differences from the baseline model. The most distinct difference from the baseline model is the evolution of the regime probabilities after 2013. In the full model, one-percent regime increases more persistently and reaches 50% in 2015. It exhibits a one-off drop in 2016, but quickly returned to 50% in 2017. Figure 6 plots the posterior distribution of trend inflation, which shows that the trend inflation reaches one percent in 2014 and gradually declined afterwards, but it did not fall to zero in 2016. Figure 7 compares the two posterior estimates of the trend inflation from the baseline and full models. The full model captures an impact of the oil price dropping significantly in 2015–2016 and provides an estimate of higher trend inflation than the baseline model during this period. After 2020, the increases in the regime probabilities of one, two, and three percent are less pronounced in the full model. The baseline model allocates a greater fraction of the large increase in actual inflation after 2020 to an increase in trend inflation than does the full model.

Figure 8 plots the posterior estimates of the Phillips curve slope, oil price pass-through rate, and stochastic volatility, estimated using the full model. The time evolution of the Phillips curve slope did not change significantly from that of the baseline model. It is evident that the posterior mean of stochastic volatility in the full model is significantly lower than that in the baseline model because the oil price and its time-varying pass-through rate explain a fraction of the unexplained component left in the baseline model. This result indicates that incorporating the oil price compo-

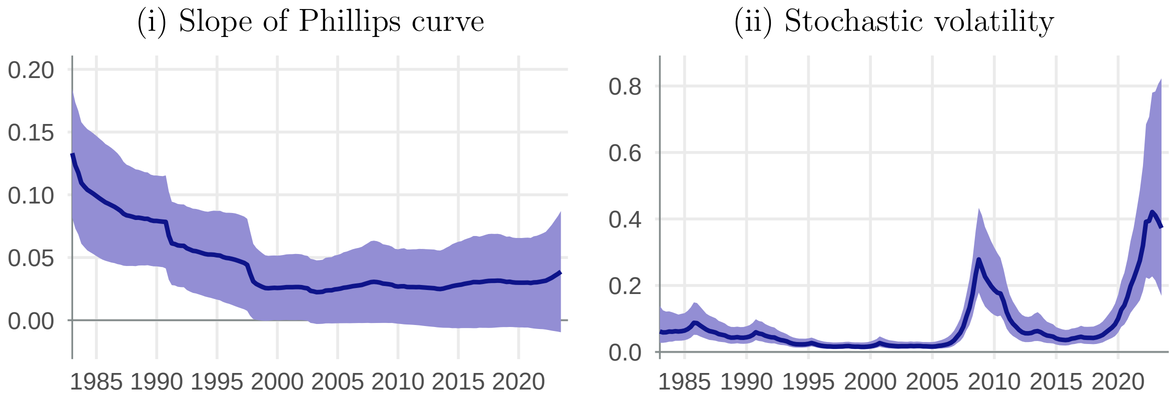


Figure 4: Posterior mean and one-standard-deviation intervals of (i) the slope of Phillips curve ($\beta_t$, left panel) and (ii) the stochastic volatility ($\sigma_t^2$, right panel) based on smoothed estimates in the baseline model.

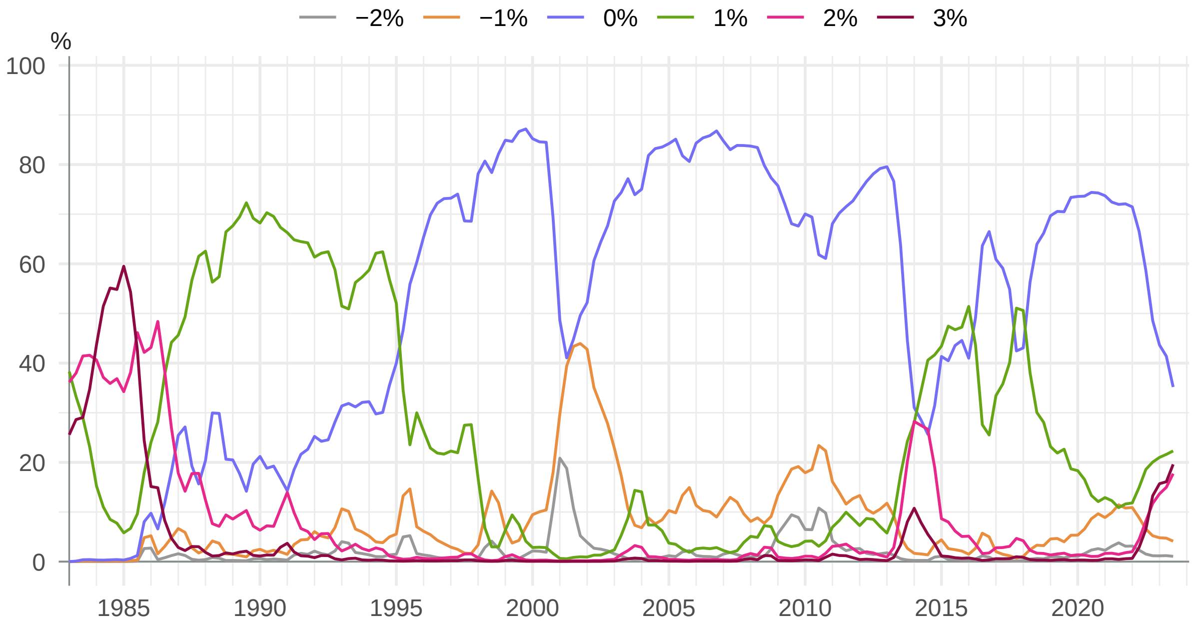


Figure 5: Posterior means of regime probability of trend inflation based on a pseudo real-time estimate in the full model.

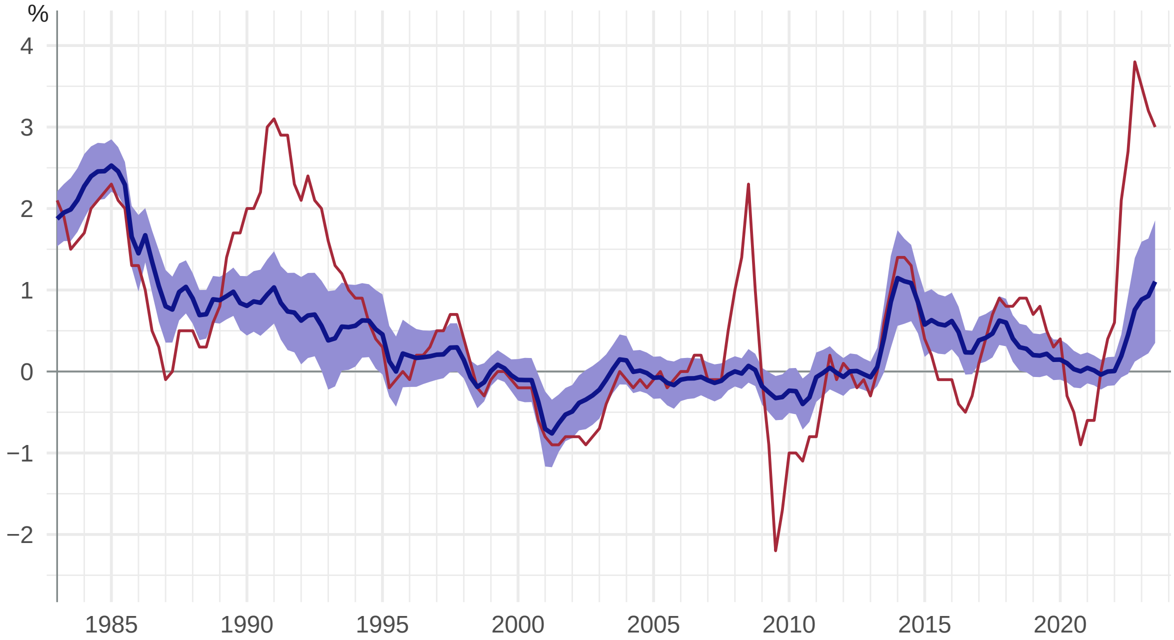


Figure 6: Posterior mean (bold line) and one-standard-deviation intervals (filled area) of trend inflation based on a pseudo real-time estimate in the full model. The solid line indicates the actual inflation rate.

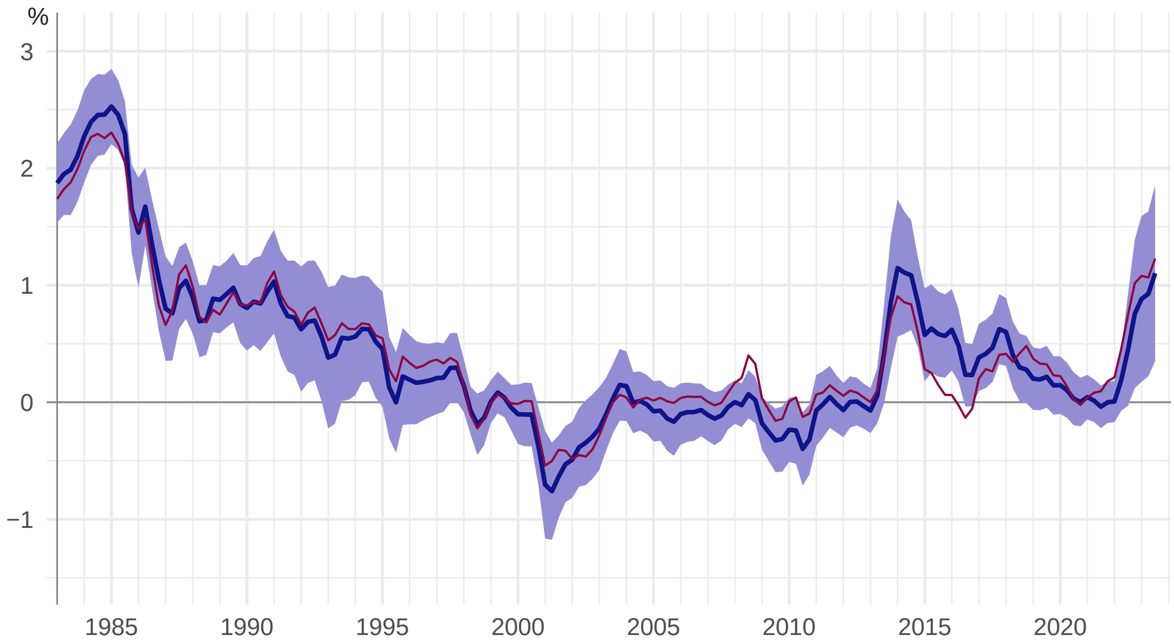


Figure 7: Comparison of the estimates of trend inflation: Posterior mean (bold line) and one-standard-deviation intervals (filled area) based on a pseudo real-time estimate in the full model, and posterior mean (solid line) in the baseline model.

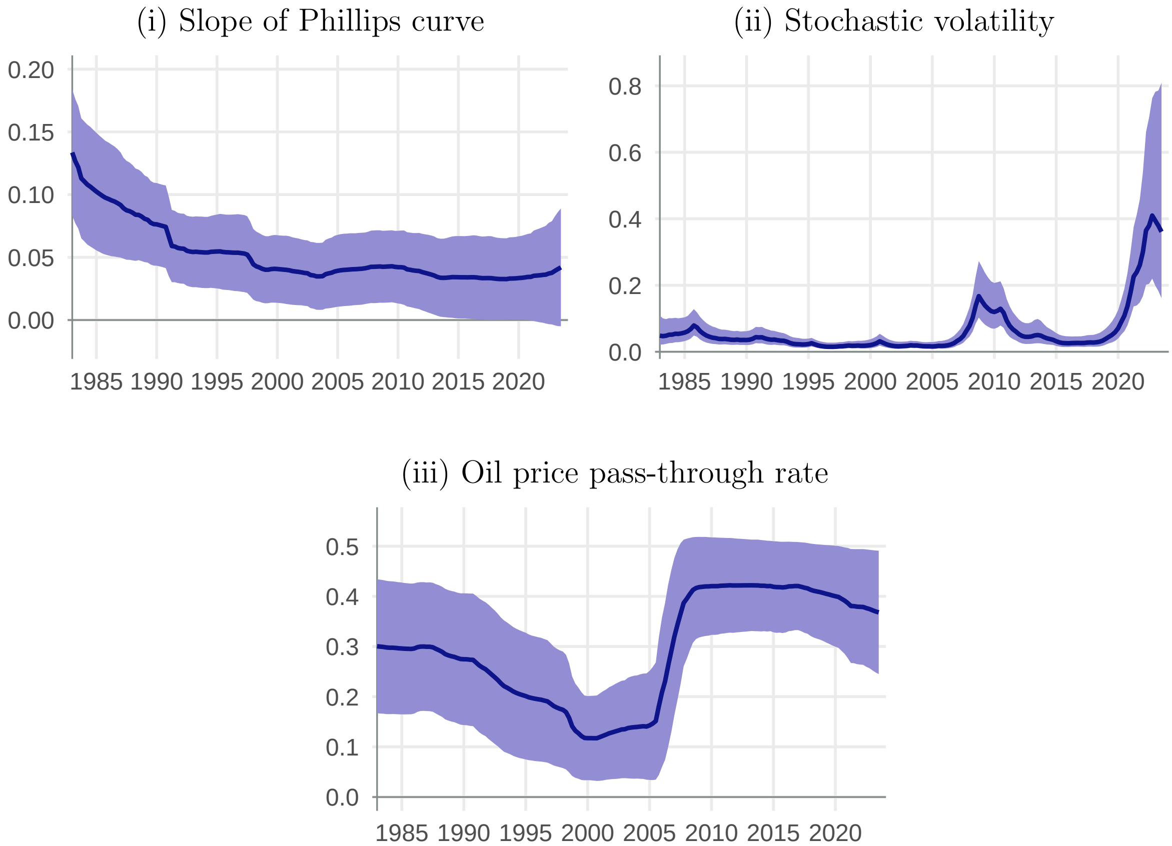


Figure 8: Posterior mean and one-standard-deviation intervals of (i) the slope of Phillips curve ($\beta_t$, top left panel) and (ii) the stochastic volatility ($\sigma_t^2$, top right panel), and (iii) the oil price pass-through rate ($\gamma_t$, bottom panel), based on smoothed estimates in the full model.

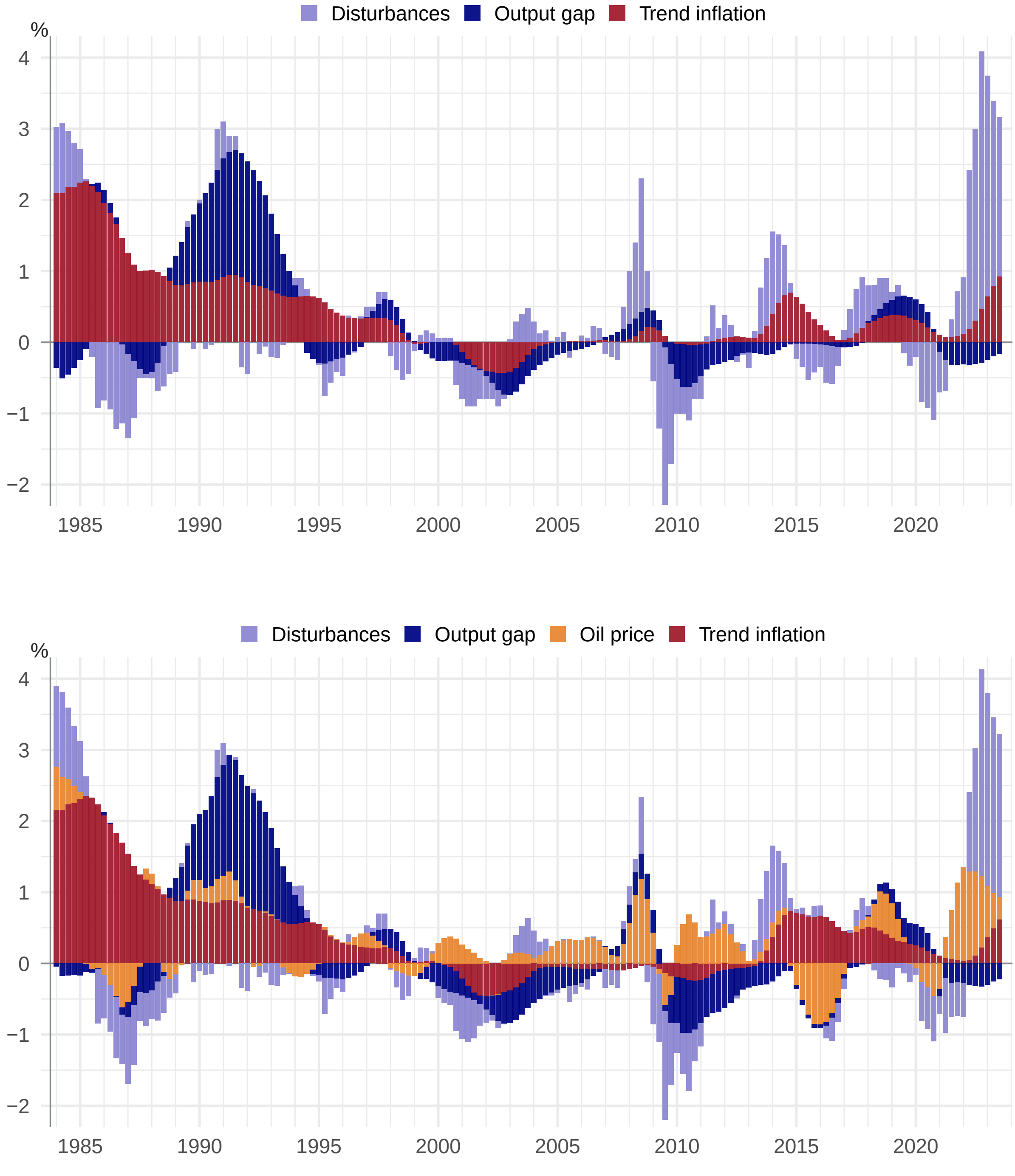


Figure 9: Historical decomposition: Contributions of model components based on their posterior mean in the baseline model (top) and the full model (bottom).

nent clearly improves the in-sample fit. Figure 9 shows the historical decomposition of actual inflation based on the posterior means of the parameters and state variables. The contribution of the oil price component partly explains the surges in inflation in 2005–2008 and 2020–2021 and the large decline in 2015–2016.

Interestingly, the oil prices pass-through rate plotted in Figure 8 appears to have gradually declined from the 1980s to 1990s and quickly increased in the 2000s. The formar evolution of the pass-through rate is found by several studies. Sekine (2006) and Chen (2009) show empirical results indicating a decline in Japan's input-cost pass-through rate from the 1980s to the 2000s. The decline reflects an increase in import penetration in the 1980s and 1990s as pointed out by Yagi et al. (2022) and other studies.

The latter evolution is also found by recent studies. Shioji and Uchino (2011) argue that when the oil price increased in the first half of the 2000s, the share of oil in overall production cost gets larger than before, and then retail prices of products can be more sensitive to oil prices. It could have happened, while it is interesting that the pass-through rate estimated by our model stays at a higher level for the rest of the sample, up to the 2020s than the previous years. Yagi et al. (2022) point out that the pass-through rate increases after the 2000s, providing empirical evidence based on the TVP-VAR model. The study discusses that the supply chain length between industries got shorter after the 2000s, which could make price negotiation quicker, and therefore the pass-through rate rises (see also, Hara et al., 2015). Sasaki et al. (2022) provide empirical evidence showing that the exchange rate pass-through rate to the CPI inflation also increases after the GFC.

## 4.2 Forecasting performance

We examine the predictive ability of the proposed model to address how the additional component of the oil price pass-through works in forecasting future inflation. As a benchmark model, we use the model without the oil price component (labelled Model 1, the baseline model we labeled earlier), which is equivalent to that proposed by Kaihatsu and Nakajima (2018). As extended models, we investigat the forecasting performance of the models with oil prices. We consider the model with a constant pass-through rate, which is formulated by $\gamma_t = \gamma$ (Model 2), and the model with a regime-switching oil price pass-through (Model 3, the full model labeled earlier). We assume that in all the candidate models, the trend inflation and the slope of the Phillips curve are regime-switching, and the error variances follow stochastic volatility, because Kaihatsu and Nakajima (2018) show that these components contribute to improving forecasting performance compared to a model with a constant trend inflation, slope of the Phillips curve, and volatility.

A recursive real-time forecasting exercise is conducted to implement realistic forecasting exercises. We begin with a dataset up to 1991/Q1 and compute one- to four-quarters-ahead forecasts of the core inflation rates using each model. In other words, we forecast the inflation from 1991/Q2 to 1992/Q1. Next, we update the data set to 1992/Q1 and obtain forecasts for up to four quarters ahead (1993/Q1). Thus, we update the dataset and forecast inflation sequentially until we update the dataset to 2023/Q1. Note that the historical time series of inflation rates and output gaps have been revised several times, owing to changes in the reference year and measurement method. As vintages of the time series are not available, we use the latest version of the data for the forecasting exercise.

In forecasting inflation, the future values of the output gap and oil prices should be computed. As assessing the dynamics of the output gap and oil prices and their interaction with inflation is beyond the scope of this study, we treat them as exogenous and calculate future values using a simple time series model. Specifically, we fit AR(2) and AR(1) models to the output gap and oil price, respectively, using the real-time dataset and compute their recursive forecasts up to four quarters ahead. We assume that these variables converge to zero without any additional shocks, and specify autoregressive models without an intercept. The lag lengths of these autoregressive models are determined by searching for the best forecasting performance among the candidate models with a lag length of up to four quarters.

Table 1(a) shows the root mean squared forecast error (RMSFE) for each forecasting horizon and the model from 1991/Q2 to 2023/Q1 as the full forecasting period. For all horizons from one quarter to four quarters, the RMSFE of Model 3 is lower than those of Models 1 and 2, which indicates that the proposed model with the oil price and its regime-switching pass-through rate has a better predictive ability than the baseline

| | $z_t$ | $\gamma_t$ | 1-quarter | 2-quarter | 3-quarter | 4-quarter |
|---|---|---|---|---|---|---|
| (a) From 1991/Q2 to 2023/Q1 | | | | | | |
| Model 1 | Excluded | – | 0.362 | 0.569 | 0.815 | 1.025 |
| Model 2 | Included | constant | 0.418 | 0.682 | 0.924 | 1.099 |
| Model 3 | Included | regime-switching | 0.306 | 0.516 | 0.774 | 0.967 |
| (b) From 1991/Q2 to 2020/Q1 | | | | | | |
| Model 1 | Excluded | – | 0.358 | 0.493 | 0.702 | 0.775 |
| Model 2 | Included | constant | 0.373 | 0.531 | 0.746 | 0.792 |
| Model 3 | Included | regime-switching | 0.307 | 0.447 | 0.678 | 0.748 |

Table 1: Forecasting performance: Root mean squared forecast error (RMSFE). The forecasting period is (a) from 1991/Q2 to 2023/Q1 and (b) from 1991/Q2 to 2020/Q1.

model. Interestingly, the RMSFE of Model 2 is larger than that of Model 1, which implies that incorporating the oil price deteriorates the predictive ability if its pass-through rate is assumed to be time-invariant. Table 1(b) reports the RMSFE from 1991/Q2 to 2020/Q1 as a forecasting period excluding the Covid-19 pandemic time. The ranking of the competing models is the same as that of the full forecasting period, indicating that the results are robust.

This result uncovers a crucial aspect of the model: as the other components of the model are assumed to be time-varying, if the oil price component is assumed to be time-invariant, its possible time variation can be inappropriately absorbed by the time-varying components. This mechanism is discussed in Sims (2001) and Primiceri (2005) in the context of time-varying parameter-vector autoregression models. The results presented here highlight the importance of incorporating the oil-price component into the regime-switching pass-through rate.

## 4.3 Term structure of inflation forecast

Using the forecasting method described above, we can compute the term structure of inflation forecasts. In macroeconomics, the term structure of inflation expectations has been well discussed, and empirical assessments provide useful information for understanding macroeconomic developments and their roles of expectations in them (e.g.

Kozicki and Tinsley, 2012; Crump et al., 2016; Aruoba, 2020; Maruyama and Suganuma, 2020). The existing literature on inflation expectations (e.g., Coibion et al., 2022; Weber et al., 2022) shows that economic agents' inflation expectations can include biases in the sense that inflation expectations are often biased against actual inflation rates. Given that the current study does not employ any variable for inflation expectations, the resulting term structure of the inflation forecast is based purely on the actual inflation rate and time series model. Therefore, the term structure obtained is free from biases in agents' inflation expectations. While the biases themselves have important implications investigated by existing studies, the term structure implied by the time-series model and realized inflation rate is considered a meaningful reference for macroeconomic analysis.

Figure 10 shows the real-time inflation forecasts at one-year to ten-year ahead obtained from the full model at each time point. Compared to the term structure of firms' inflation expectations in Japan estimated by Nakajima (2023), as shown in Figure 11, the model-based inflation forecast converges to the trend inflation more slowly. This finding indicates a considerably persistent model structure that reflects the persistent dynamics of the inflation rate in Japan (Bank of Japan, 2021).

Figure 11 shows the model-based forecasts and firms' expectations at the horizons of five years ahead, where we take two-quarters backward and forward moving averages for the model-based forecasts to compare the two series. Notably, the changes in forecasts and expectations are similar, with an almost constant deviation between them, which is averagely about 0.8%. This deviation could correspond to the biases contained in firms' expectations.

# 5 Conclusion

This study proposes a regime-switching model to estimate trend inflation in the context of modeling the Phillips curve by considering the oil price and its time-varying pass-through rate. An empirical analysis using Japan's data demonstrates the usefulness of the models.[1] First, trend inflation is appropriately estimated, with its regime

[1]The estimates of the full model proposed in this paper are available and updated at the author's website, https://sites.google.com/site/jnakajimaweb/trend.

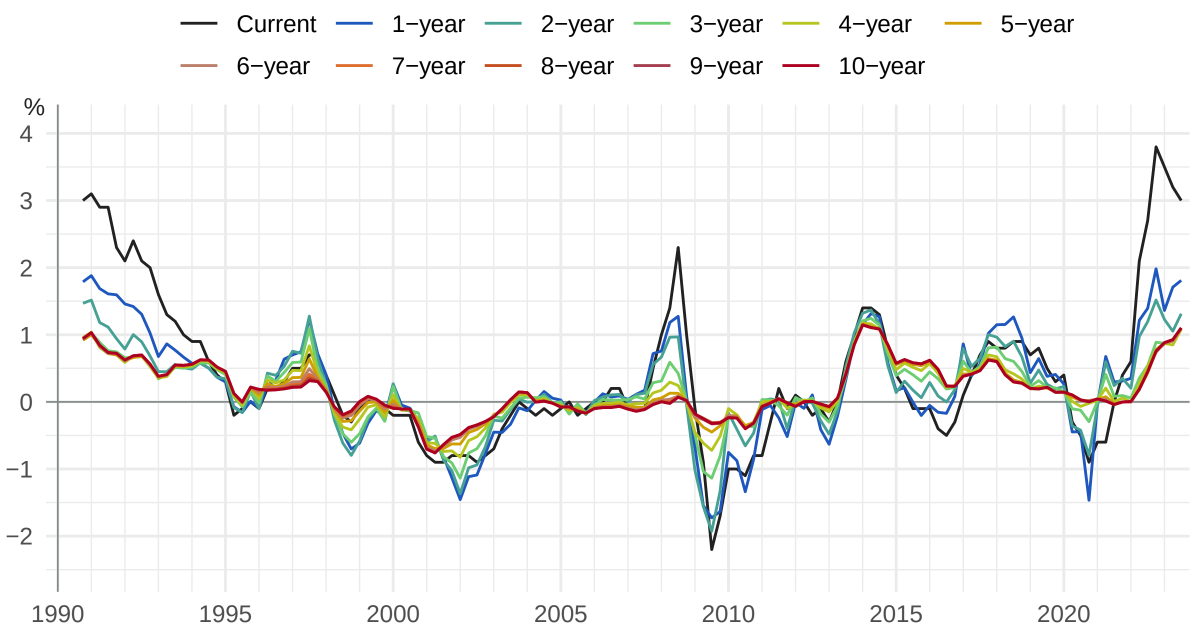


Figure 10: Term structure of real-time inflation forecasts estimated in the full model.

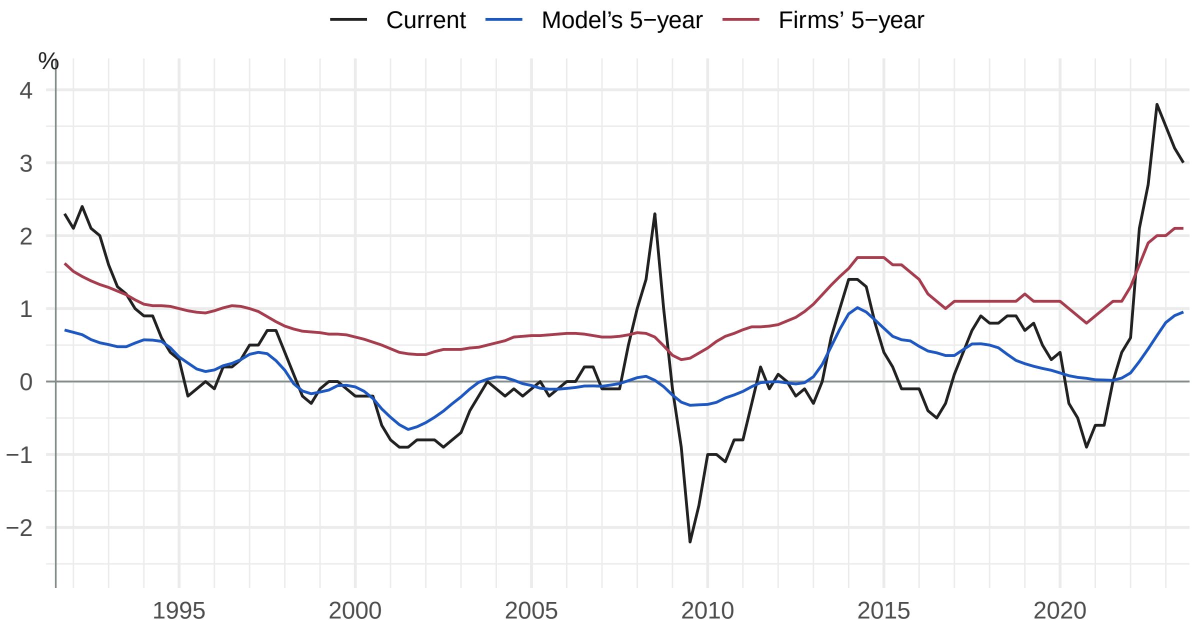


Figure 11: Real-time inflation forecasts in the full model and firms' inflation expectations at five-year horizons. Two-quarters backward and forward moving averages are taken for the model-based forecasts.

probability uncovering its dynamics of trend inflation. Second, incorporating the oil price and its regime-switching pass-through rate improved the predictive ability of the trend inflation model in forecasting the actual inflation rate. Third, we find that the obtained trend in inflation is highly correlated with firms' inflation expectations. One caveat of the analysis in this study is that we do not identify the factors driving oil prices. As the pass-through of oil prices to inflation may depend on the driving factors (e.g. Kilian, 2009), introducing identified shocks of oil prices into the estimation framework in this study is of interest, which is left for future work.

## Appendix. Estimation method of the equally-spaced regime-switching model

In this appendix, we describe the estimation method for an equally spaced regime-switching model. Define $\boldsymbol{y} = (y_1, \dots, y_n)$, $\boldsymbol{\mu} = (\mu_1, \dots, \mu_n)$, $\boldsymbol{\beta} = (\beta_1, \dots, \beta_n)$, $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_n)$, and $\boldsymbol{h} = (h_1, \dots, h_n)$. We specify the prior distributions for the set of model parameters $\boldsymbol{\theta} = \{\boldsymbol{\alpha}, \boldsymbol{p}, v\}$, where $\boldsymbol{p} = \{p_\mu, p_\beta, p_\gamma\}$, and generate samples from the full joint posterior distribution $p(\boldsymbol{\theta}, \boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{h}|\boldsymbol{y})$. The MCMC algorithm for the proposed model is summarized as follows:

1. Generate $\boldsymbol{\mu} \,|\, \boldsymbol{\alpha}, p_\mu, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{h}, \boldsymbol{y}$.
2. Generate $\boldsymbol{\beta} \,|\, \boldsymbol{\alpha}, p_\beta, \boldsymbol{\mu}, \boldsymbol{\gamma}, \boldsymbol{h}, \boldsymbol{y}$.
3. Generate $\boldsymbol{\gamma} \,|\, \boldsymbol{\alpha}, p_\gamma, \boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{h}, \boldsymbol{y}$.
4. Generate $\boldsymbol{h} \,|\, \boldsymbol{\alpha}, \boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{y}$.
5. Generate $\boldsymbol{\alpha} \,|\, \boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{h}, \boldsymbol{y}$.
6. Generate $\boldsymbol{p} \,|\, \boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\gamma}$.
7. Generate $v \,|\, \boldsymbol{h}$.

The details of the generation at each step are described below.

### A.1 Sampling regime-switching state variables (Steps 1–3)

In Step 1, we use an efficient sampling algorithm for the Markov-switching model (e.g. Carter and Kohn, 1994; Chib, 1996) to generate a sample of $(\mu_1, \dots, \mu_n)$ from the joint conditional posterior distribution throughout the sample period. We define a state variable $s_t \in \{1, \dots, L\}$ such that $s_t = i$ if $\mu_t = \tilde{\mu}_i$ and further define $\boldsymbol{\vartheta} \equiv (\boldsymbol{\alpha}, p_\mu, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{h})$ and $\boldsymbol{Y}_t = \{y_\ell\}_{\ell=1}^t$. We recursively compute the following two densities

$$\begin{aligned}
&\text{Prediction} : p(s_t|\boldsymbol{Y}_{t-1}, \boldsymbol{\vartheta}) = \sum_{i=1}^{L} p(s_t|s_{t-1} = i, \boldsymbol{\vartheta})\, p(s_{t-1} = i|\boldsymbol{Y}_{t-1}, \boldsymbol{\vartheta}), \\
&\text{Filtering} : p(s_t|\boldsymbol{Y}_t, \boldsymbol{\vartheta}) \propto p(s_t|\boldsymbol{Y}_{t-1}, \boldsymbol{\vartheta})\, f(y_t|s_t, \boldsymbol{\vartheta}),
\end{aligned}$$

where

$$f(y_t|s_t, \boldsymbol{\vartheta}) \propto \exp\left\{ -\frac{(y_t - \boldsymbol{\alpha y}_{t-1:t-k} - \bar{\alpha}\mu_{s_t} - \beta_t x_t - \gamma_t z_t)^2}{2\sigma_t^2} \right\},$$

with $\boldsymbol{y}_{t-1:t-k} = (y_{t-1}, \dots, y_{t-k})'$, and $\bar{\alpha} = 1 - \sum_{i=1}^k \alpha_i$, for $t = 1, \dots, n$. We generate $s_n$ from $p(s_n|Y_n, \boldsymbol{\vartheta})$ and then recursively generate $s_t$ for $t = n-1, \dots, 1$, following the probability $p(s_t|Y_t, \boldsymbol{\vartheta}) \times p(s_{t+1}|s_t, \boldsymbol{\vartheta})$.

We apply the same sampling algorithm to $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$.

### A.2 Sampling stochastic volatility process (Step 4)

We define $y_t^* = y_t - \boldsymbol{\alpha y}_{t-1:t-k} - \bar{\alpha}\mu_t - \beta_t x_t - \gamma_t z_t$. Conditional on other state variables and parameters, the stochastic volatility is associated with the following nonlinear state-space model:

$$\begin{aligned}
y_t^* &= \exp(h_t/2) e_t, \\
h_t &= h_{t-1} + \eta_t, \\
\begin{pmatrix} e_t \\ \eta_t \end{pmatrix} &\sim N\left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & v^2 \end{pmatrix} \right).
\end{aligned}$$

We employ the so-called multi-move sampler for a standard stochastic volatility model without leverage developed by Shephard and Pitt (1997) and Watanabe and Omori (2004). See Appendix A.2 in Nakajima (2011) for further details.

### A.3 Sampling autoregressive parameters (Step 5)

We specify a prior $\boldsymbol{\alpha} \sim TN_\Omega(\alpha_0, \Sigma_0)$, where $TN_\Omega$ denotes a truncated multivariate normal distribution with positive density only in the domain $\Omega \equiv \{\boldsymbol{\alpha} | \left|\sum_{i=1}^k \alpha_i\right| \leq 1\}$. This prior is conjugate and the conditional posterior distribution is $\boldsymbol{\alpha} \,|\, \boldsymbol{\mu}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{h}, \boldsymbol{y} \sim TN_\Omega(\hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\Sigma}})$, where $\hat{\boldsymbol{\Sigma}} = (\boldsymbol{\Sigma}_0^{-1} + \boldsymbol{W}'\boldsymbol{W})^{-1}$, $\hat{\boldsymbol{\alpha}} = \hat{\boldsymbol{\Sigma}}(\boldsymbol{\Sigma}_0^{-1}\boldsymbol{\alpha}_0 + \boldsymbol{W}'\boldsymbol{u})$,

$$\boldsymbol{W} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix}, \quad \text{and} \quad \boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix},$$

with $w_t = [(y_{t-1} - \mu_t)/\sigma_t, \ldots, (y_{t-k} - \mu_t)/\sigma_t]$, and $u_t = (y_t - \mu_t - \beta_t x_t - \gamma_t z_t)/\sigma_t$.

### A.4 Sampling switching probability parameters (Step 6)

We set the prior $p_\mu \sim B(a_0, b_0)$, where $B$ denotes the beta distribution. Then, the conditional posterior distribution is given by $p_\mu \,|\, \mu \sim B(\hat{a}, \hat{b})$, where $\hat{a} = a_0 + n_c$, $\hat{b} = b_0 + n - n_c - 1$, and $n_c$ is the count of set $\{t \,|\, 1 < t < n-1, \mu_{t+1} = \mu_t\}$. The generations of $p_\beta$ and $p_\gamma$ is separately implemented in the same manner as $p_\mu$.

### A.5 Sampling variance of stochastic volatility process (Step 7)

We assume a prior $v^2 \sim IG(n_0/2, S_0/2)$, where $IG$ denotes the inverse gamma distribution. A sample is generated from the conditional posterior distribution $v^2 \,|\, \boldsymbol{h} \sim IG(\hat{n}/2, \hat{S}/2)$, where $\hat{n} = n_0 + n - 1$ and $\hat{S} = S_0 + \sum_{t=1}^{n-1}(h_{t+1} - h_t)^2$.

# References

Aruoba, S. B. (2020). Term structures of inflation expectations and real interest rates. *Journal of Business & Economic Statistics 38*, 542–553.

Ascari, G. and A. M. Sbordone (2014). The macroeconomics of trend inflation. *Journal of Economic Literature 52*, 679–739.

Bank of Japan (2021). Assessment for further effective and sustainable monetary easing (the background). March 19, 2021.

Brissimis, S. N. and N. S. Magginas (2008). Inflation forecasts and the New Keynesian Phillips curve. *International Journal of Central Banking 4*, 1–22.

Carter, C. K. and R. Kohn (1994). On Gibbs sampling for state space models. *Biometrika 81*, 541–553.

Cecchetti, S. G., P. Hooper, B. C. Kasman, K. L. Schoenholtz, and M. W. Watson (2007). Understanding the evolving inflation process. *US Monetary Policy Forum*, Volume 8.

Chan, J. C., T. E. Clark, and G. Koop (2018). A new model of inflation, trend inflation, and long-run inflation expectations. *Journal of Money, Credit and Banking 50*(1), 5–53.

Chen, S. S. (2009). Oil price pass-through into inflation. *Energy Economics 31*, 126–133.

Chib, S. (1996). Calculating posterior distributions and modal estimates in Markov mixture models. *Journal of Econometrics 75*, 79–97.

Choi, S., D. Furceri, P. Loungani, S. Mishra, and M. Poplawski-Ribeiro (2018). Oil prices and inflation dynamics: Evidence from advanced and developing economies. *Journal of International Money and Finance 82*, 71–96.

Clark, T. E. and T. Doh (2014). Evaluating alternative models of trend inflation. *International Journal of Forecasting 30*, 426–448.

Claus, I. (1997). A measure of underlying inflation in the United States. Bank of Canada Working Paper, 97-20.

Coibion, O., Y. Gorodnichenko, S. Kumar, and M. Pedemonte (2022). Inflation expectations as a policy tool? *Journal of International Economics 124*, 103297.

Crump, R. K., S. S. Eusepi, and E. Moench (2016). The term structure of expectations and bond yields. FRB of NY Staff Report No. 775.

De Gregorio, J., O. Landerretche, C. Neilson, C. Broda, and R. Rigobon (2007). Another pass-through bites the dust? Oil prices and inflation. *Economia 7*(2), 155–208.

De Veirman, E. (2009). What makes the output-inflation trade-off change? The absence of accelerating deflation in Japan. *Journal of Money, Credit and Banking 41*, 1117–1140.

Faust, J. and J. H. Wright (2013). Forecasting inflation. In G. Elliott and A. Timmermann (Eds.), *Handbook of Economic Forecasting*, Volume 2, pp. 3–56. Amsterdam: North-Holland.

Hara, N., K. Hiraki, and Y. Ichise (2015). Changing exchange rate pass-through in Japan: Does it indicate changing pricing behavior? Bank of Japan Working Paper Series, No. 15-E-4.

Kaihatsu, S., M. Katagiri, and N. Shiraki (2023). Phillips correlation and price-change distributions under declining trend inflation. *Journal of Money, Credit and Banking 55*(5), 1271–1305.

Kaihatsu, S. and J. Nakajima (2018). Has trend inflation shifted?: An empirical analysis with an equally-spaced regime-switching model. *Economic Analysis and Policy 59*, 69–83.

Kiley, M. T. (2008). Estimating the common trend rate of inflation for consumer prices and consumer prices excluding food and energy prices. Finance and Economics Discussion Series, 2008-38, Federal Reserve Board.

Kilian, L. (2009). Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market. *American Economic Review 99*(3), 1053–1069.

Kim, C.-J., P. Manopimoke, and C. R. Nelson (2014). Trend inflation and the nature of structural breaks in the New Keynesian Phillips curve. *Journal of Money, Credit and Banking 46*, 253–266.

Kim, C.-J. and C. R. Nelson (1999). *State-Space Models with Regime Switching.* Cambridge, MA: MIT press.

Kozicki, S. and P. A. Tinsley (2012). Effective use of survey information in estimating the evolution of expected inflation. *Journal of Money, Credit and Banking 44*, 145–169.

Maruyama, T. and K. Suganuma (2020). Inflation expectations curve in Japan. *Japanese Journal of Monetary and Financial Economics 8*, 1–28.

Mertens, E. (2016). Measuring the level and uncertainty of trend inflation. *Review of Economics and Statistics 98*, 950–967.

Nakajima, J. (2011). Time-varying parameter VAR model with stochastic volatility: An overview of methodology and empirical applications. *Monetary and Economic Studies 29*, 107–142.

Nakajima, J. (2023). Estimation of firms' inflation expectations using the survey DI. Discussion Paper Series A.749, Institute of Economic Research, Hitotsubashi University.

Nason, J. M. and G. W. Smith (2008). The New Keynesian Phillips curve: Lessons from single-equation econometric estimation. *FRB Richmond Economic Quarterly 94*, 361–395.

Okimoto, T. (2019). Trend inflation and monetary policy regimes in Japan. *Journal of International Money and Finance 92*, 137–152.

Primiceri, G. E. (2005). Time varying structural vector autoregressions and monetary policy. *Review of Economic Studies 72*, 821–852.

Quah, D. and S. P. Vahey (1995). Measuring core inflation. *Economic Journal 105*, 1130–1144.

Sasaki, Y., Y. Yoshida, and P. K. Otsubo (2022). Exchange rate pass-through to Japanese prices: Import prices, producer prices, and the core CPI. *Journal of International Money and Finance 123*, 102599.

Sekine, T. (2006). Time-varying exchange rate pass-through: Experiences of some industrial countries. BIS Working Papers, No. 202.

Shephard, N. and M. Pitt (1997). Likelihood analysis of non-Gaussian measurement time series. *Biometrika 84*, 653–667.

Shioji, E. and T. Uchino (2011). Pass-through of oil prices to Japanese domestic prices. In T. Ito and A. K. Rose (Eds.), *Commodity Prices and Markets*, pp. 155–189. University of Chicago Press.

Sims, C. A. (2001). Comment on Sargent and Cogley's 'Evolving post World War II U.S. inflation dynamics'. *NBER Macroeconomics Annual 16*, 373–379.

Sims, C. A., D. F. Waggoner, and T. Zha (2008). Methods for inference in large multiple-equation Markov-switching models. *Journal of Econometrics 146*, 255–274.

Stock, J. H. and M. W. Watson (2007). Why has US inflation become harder to forecast? *Journal of Money, Credit and Banking 39*, 3–33.

Watanabe, T. and Y. Omori (2004). A multi-move sampler for estimating non-Gaussian time series models: Comments on Shephard and Pitt (1997). *Biometrika 91*, 246–248.

Weber, M., F. D'Acunto, Y. Gorodnichenko, and O. Coibion (2022). The subjective inflation expectations of households and firms: Measurement, determinants, and implications. *Journal of Economic Perspectives 36*(3), 157–184.

Yagi, T., Y. Kurachi, M. Takahashi, K. Yamada, and H. Kawata (2022). Pass-through of cost-push pressures to consumer prices. Bank of Japan Working Paper Series, No. 22-E-17.