Essays on Aggregation in Macroeconomics

by

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Chapter 1

Introduction

Macroeconomics is a field of study that seeks to understand the behavior of the economy as a whole. Nonetheless, the economy is complex, with various goods, households, and firms exhibiting heterogeneity, making it difficult to capture a full picture of the economy. Consequently, the process of aggregating such heterogeneity is a fundamental aspect of macroeconomics as it enables a comprehensive analysis of the economy.

One challenge of heterogeneity in macroeconomics is the diversity of goods in the economy. Goods can vary in character, function, purpose, and other dimensions, making it difficult to aggregate their price and quantity into a single measure.

Recent advances in theoretical and empirical methods have led to new aggregation techniques that can capture the heterogeneity of goods. For instance, the seminal study by Diewert (1976) proposes the concept of a superlative index, which allows measuring the cost of living under a flexible utility function. This research is pivotal as it allows for the economically meaningful inclusion of substitution between heterogeneous goods in the price index. This study has also significantly impacted official statistics, leading to the adoption of the superlative index, specifically the Fisher index, in calculating the Gross Domestic Product (GDP) deflator in the US and Canada.

Another promising research field is the utilization of new data sources. For example, Broda and Weinstein (2010) utilize household scanner data to estimate the impact of product turnover on the cost of living index. This innovative approach is particularly important as it allows for the incorporation of new features into traditional aggregates, which help provide a more accurate and comprehensive picture of economic activity.

This thesis focuses on three studies on heterogeneity and aggregation, exploring how the aggregation of goods affects our understanding of key macroeconomic variables. By providing insights into the importance of incorporating heterogeneity in macroeconomic analysis, this thesis aims to contribute to the advancement of the field.

Chapter 2: The Effect of Aging on the Age-Wage Profile in Japan

Chapter 2 is based on a published study by Inoue (2022). In recent years, the Japanese economy has experienced a phenomenon known as the "flattening of age-wage profile," in which the wage gap between older and younger workers has dramatically shrunk. If such a phenomenon is interpreted by a simple model which assumes a competitive labor market and homogeneous labor, these changes in wage difference stem from a change in worker's productivity. This study seeks to provide a different interpretation of this change in age-wage profiles by challenging the assumption of homogeneous labor in such a simple model.

This study argues that the aging of the workforce is an important factor causing the flattening of the age-wage profile of the Japanese economy. Assuming imperfect substitution between older and younger workers, I derive the labor demand function from the firm's optimization problem. I estimate the elasticity of substitution between different aged workers, and quantify the effect of aging on the age-wage profile. I find that aging of the workforce can explain more than 80% of the change in the age-wage profile from 2000 to 2019.

While not explicitly discussed in Chapter 2, aging is linked to research topics about prices, which are examined in Chapters 3 and 4. As demonstrated by Aguiar and Hurst (2007), Unayama and Keida (2011), and Abe and Shiotani (2014), price levels exhibit heterogeneity across different age groups, suggesting that aging may influence the price level of the entire economy by changing the composition of the age group. Further evidence of heterogeneity has been documented by Diamond et al. (2020) for inflation rates and inflation expectations. As demographic aging becomes more pronounced in numerous countries, delving into the relationship between an aging population and prices will be an increasingly significant area of future research.

Chapter 3: Price Index Numbers under Large-Scale Demand Shocks: The Japanese Experience of the COVID-19 Pandemic

Chapter 3 is based on Abe et al. (2022), a joint work with Naohito Abe and Hideyasu Sato. The consumer price index is often designed to measure the cost of living. One of the problems in measuring the cost of living is that it is necessary to assume a constant utility function and no shocks to demand. To solve this problem, a recent development by Redding and Weinstein (2020) proposes a method to measure the cost of living index by allowing time-varying parameters of the utility function, but there is not enough empirical analysis using this method.

Using a recently developed index number formula that is exact for the constant elasticity of substitution utility function with variable preferences, we quantify the degree of demand shock caused by the pandemic. Specifically, we investigate the prices and quantities of face masks when the COVID-19 pandemic was particularly serious to understand the impact of demand shocks on the cost of living index (COLI). The empirical analysis revealed that shifts in preferences during the pandemic were so significant that the COLI with variable tastes became vastly different from the standard superlative indexes. While the prices of face masks decreased in the Fisher index in May 2020 by 0.76% per week, the COLI increased by 1.92% per week.

This increase in the COLI relative to the Fisher index is interpreted to reflect the actual change in the purchasing behavior of households for masks during the COVID-19 pandemic. Essentially, the COLI used in this research is assumed to decrease (increase) as the dispersion of preferences for each mask increases (decreases). The rise in COLI for May 2020 indicates a decrease in the dispersion of preferences. This is consistent with the actual economic situation. In fact, during the COVID-19 pandemic, households placed less importance on the brand or packaging of masks, prioritizing their functionality instead. Households purchased whatever masks were available in the stores. This change in mask preferences is captured by COLI, which showed a greater increase than the Fisher index.

Chapter 4: The Effect of Seasonality on Elementary Index

Chapter 4 is based on a working paper by Inoue (2023). To create annual indicators in real terms (e.g., real gross domestic product, real consumption, etc.), statistical authorities calculate the annual prices and quantities of each commodity for the inputs of an index formula. One of the challenges in creating commodity-level prices and quantities is dealing with seasonal goods. The annual price and quantity of seasonal goods are typically calculated using unit prices from the monthly quantity-weighted average prices (also known as the unit value price index), and a summation of monthly quantities. However, the studies of the index number theory have revealed that this aggregation technique is appropriate for homogeneous goods but not for heterogeneous goods (Silver, 2010). Therefore, this study seeks to investigate whether seasonal goods are consumed as homogeneous goods throughout the year or as heterogeneous goods in different seasons.

By estimating the Constant Elasticity of Substitution (CES) utility function with monthly commodity level data, I show that a significant number of fresh foods are non-homogeneous commodities throughout the year. This non-homogeneity can result in up to a 15% difference in the annual commodity-level aggregated quantity between the simple summation of the monthly quantity and the quantity aggregated using the CES utility function. Non-homogeneity results in significant long-term differences in quantity changes, with up to a 20% point difference between the relative simple summation quantity and the Fisher quantity index over a 40-year period. Additionally, my findings indicate that these differences at the commodity level affect higher-level aggregations, such as the annual aggregated price and quantity indicators for fresh foods and real consumption.

Non-homogeneity affects the aggregate results through two factors. First, the utility of consumption changes with seasons. The estimated results of the CES utility functions show that the preference weights are uneven for almost all goods. This means that the same amount of consumption in different seasons results in different levels of utility. Possible reasons for these uneven weights include changes in quality, such as taste and nutritional value, and in consumer value, influenced by seasonal traditions and customs. Another contributing factor is the imperfection of temporal substitution. The estimation results of the CES utility functions indicate that substitution between different months is imperfect for many goods. This implies that households prefer more balanced consumption and consider seasonal fluctuations inconvenient. This inconvenience of seasonal fluctuations lowers the aggregates of the CES utility function. These two factors create a discrepancy between the aggregates of the CES utility function and the simple summation.

Chapter 2

The Effect of Aging on the Age-Wage Profile in Japan[1]

2.1 Introduction

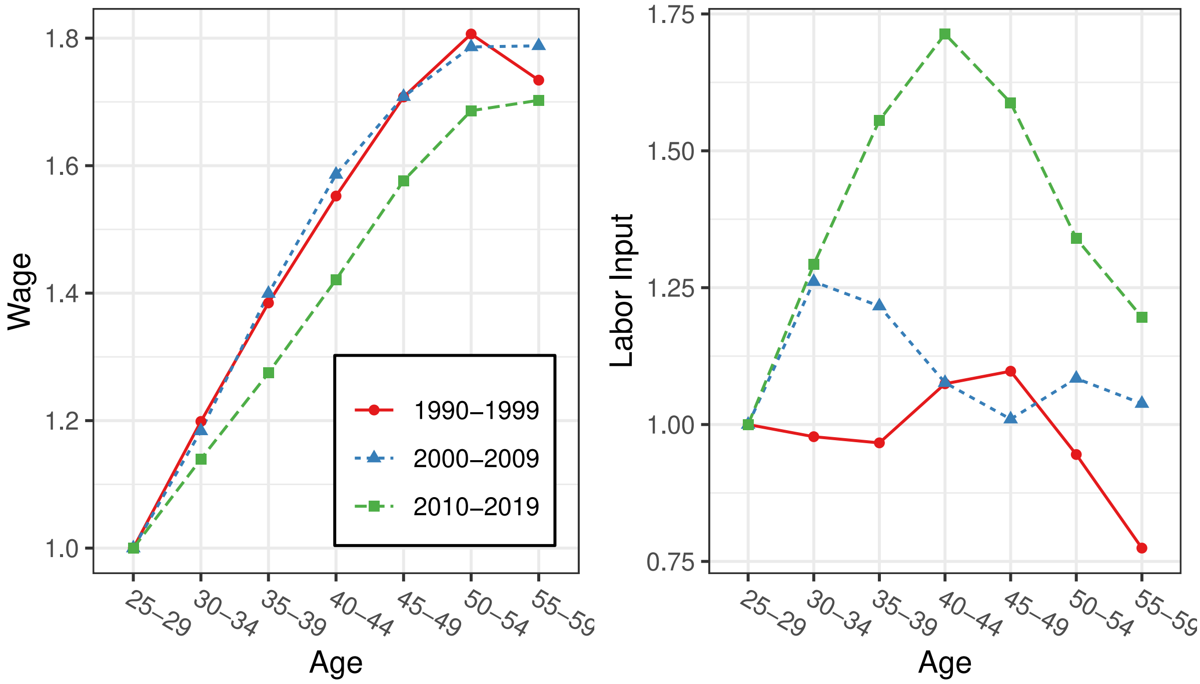
The wage gap between older and younger workers has shrunk dramatically over the past two decades in Japan. The left panel of Figure 2.1 shows that the wages of men aged 35–39 and 45–49 were approximately 1.4 and 1.7 times higher than those of men aged 25–29 in the 1990s and 2000s, but in the 2010s they decreased to 1.27 and 1.57 times higher, respectively.[2] In other words, the "age-wage profile" has flattened over the last two decades.[3] This is an interesting observation in the Japanese labor market; traditionally, both lifetime employment ("Shushin Koyo") and a seniority-based wage system ("Nenko Chingin") are considered to be signifying characteristics of the Japanese labor market, in which a worker typically enjoys steady and predictable seniority-based raises from a single employer (Mincer and Higuchi, 1988). However, the data suggest that this tradition may have been either weakened or changed; thus, these days, average workers do not enjoy seniority-based raises as much as previous generations.[4] Needless to say, the flattening of the age-wage profile is important for the welfare of individual workers. However, its influence on society extends beyond an individual worker's welfare. For example,

[1]This chapter is based on Inoue (2022).

[2]We only use data on male workers, because we are interested in the change in labor supply induced by demographic change. Labor force participation rate and employment rate of men have been relatively stable for many years in Japan. Thus, most of the change in the male workforce is attributed to demographic factors. If we include female workers in the analysis, we have to consider other factors such as the rise in the labor force participation or change in employment types (Kitao and Mikoshiba, 2020).

[3]The age-wage profile here is calculated by taking the hourly wage of each group and dividing it by the youngest group's hourly wage.

[4]Strictly speaking, the flattening of the cross-sectional age-wage profile shown in Figure 2.1 does not directly support the flattening of the individual age-wage profile. However, direct evidence is reported by other researchers. For example, Hamaaki et al. (2012) show that the recent cohorts have less seniority-rise in their real earning than the older cohorts.



Note: The right and left panel shows the relative value of hourly wage and working hours (person-hours) of age-group, respectively. The detail information of data construction is described in Section 2.3.
Source: The Statistical Survey of Actual Status for Salary in the Private Sector and the Labour Force Survey

Figure 2.1: Flattening of Age-wage Profile and Aging of Male Workforce in Japan

the age-wage profile is one of the key factors for income inequality (Kitao and Yamada, 2019). Also in terms of the government, Braun and Joines (2015), Imrohoroglu et al. (2016), and Kitao (2018) study the social security system in Japan and use the age-wage (or age-earning) profile to calibrate their model for simulation.

Another, and arguably the most important change in the Japanese labor market is aging. The right panel of Figure 2.1 plots labor inputs by different age groups, measured in hours worked. It shows that in the 2000s and earlier, firms hired roughly the same amount of labor services from workers aged 40–44 and workers aged 25–29, but in the 2010s, they hired 70% more labor services from workers aged 40–44 than workers aged 25–29.[5] Workers aged 40–44 have become more prominent, or possibly abundant, within firms and in the Japanese labor market. If workers are homogeneous, these demographic changes should not matter for firms or the economy. However, anecdotally, firms do not view workers that way. For example, Okunishi (2008) reports that approximately 80% of medium-sized firms have some form of age-restriction in hiring. A newspaper article also reported that 10 out of 16 firms that announced voluntary retirement have limited their offer to workers aged 45 or over.[6] Many voluntary retirement

[5]It is reasonable to assume that these changes were primarily driven by the demographic factor, because both the employment rate and relative working hour were stable in this period (Lise et al., 2014).

[6]See the Asahi shimbun, 25, May 2019.

offers are limited to middle-aged workers. This would be an unnecessary practice if workers were homogeneous.

We think that these two observations, i.e., the flattening of the age-wage profile and the aging of the workforce, are closely connected. Our hypothesis is that firms see workers of different ages as different inputs. They may be good at different tasks; for example, younger workers may be competent at using new technologies, like the information and communications technologies prevalent today, and older workers, with their experience, may be good at leading teams as managers. Perhaps firms operate most effectively when they form a good team that strikes the right balance between youth and experience. If this is the case, the demographic change in the labor market has an important consequence on the age-wage profile. When firms employ a lot of older workers, they have less incentive to hire additional older workers at the same wage as before, and they are willing to pay higher wages to hire scarce younger workers.

This chapter formalizes the aforementioned idea of workers as an economic model, and quantifies the impact of aging on the age-wage profile. We begin by modeling the production side of the economy. Importantly, we allow for imperfect substitution between different aged workers. The key parameter of our model is the elasticity of substitution between different aged workers, and we estimate it from the data. Following the literature (Card and Lemieux, 2001; Brunello, 2010), we first apply ordinary least square (OLS) estimation and instrumental variable methods to estimate the elasticity of substitution. However, we find that the results are not robust to the sample period in our data; both the point estimates and the confidence bands change wildly depending on the range of data. As we argue in the chapter, this lack of robustness may be due to the omitted variable bias caused by unobserved productivity change such as human capital accumulation in our data. To overcome this issue, we employ the identification strategy of Feenstra (1994), which is free from the omitted variable bias. With this method, we obtain a robust estimate that is not sensitive to the choice of sample period. The estimation result supports the imperfect substitution; the estimated elasticity of substitution is approximately 4.2, which implies that a 1% increase in labor inputs of an age group causes a decrease of 0.24% in their wage.

Our main finding is that aging is the single most important factor that explains the flattening of the age-wage profile. Our argument is based on the model with the estimated parameter. We use the actual labor inputs as the model input, and decompose the change in age-specific wage into the effect of demographics and other factors. We find that the demographic factor can explain a sizable amount of change in wage after 2000; it accounts for about 80% of the change in wages from 2000 to 2019. The demographic factor was less important before 2000; it accounts for only a third of the change in wages from 1980 to 2000. This result suggests that demographic change, or aging, has a significant impact on the age-wage profile since 2000.

This study contributes to the literature on the flattening of the age-wage profile in Japan. Yamada and Kawaguchi (2015) estimate a Mincer-type wage function to detect the flattening of the experience-wage profile. They argue that this flattening may be caused by the change in demand due to aging of the workforce, but do not show formal evidence or a structural model formalizing their argument. Our study supports their argument with a structural model and empirical evidence. Hamaaki et al. (2012) and Kimura et al. (2019) examine the change in human capital accumulation in Japan and analyze the factors causing it. Both papers capture the human capital accumulation process throughout the age-wage profile controlled by various variables. Our study provides a new perspective on the flattening of the age-wage profile, which enables us to extract changes in human capital accumulation more precisely and further develop this research field.

Our study is also related to the literature on the demographic effect on wage. The negative effect of cohort size on wage is reported by many studies. Welch (1979), Freeman (1979), and Berger (1985, 1988) examine a decline in wages for young people when the baby boomers started working in the U.S. Card and Lemieux (2001) studies the effect of changes in the relative supply of highly educated worker in U.S., Canada, and the U.K. Kawaguchi and Mori (2016) also investigate the impact of cohort size on the college wage premium in comparison between Japan and the United States. Brunello (2010), and Moffat and Roth (2016) specify the impact of an aging population in the EU. Several studies quantify the demographic effect on wages in Japan too, but the estimated effects in previous research are either small or not statistically different from zero (Okamura, 2000, 2001; Ohta, 2016). Our study finds that this is not the case in recent decades, that is, we also confirm that the cohort size effect on wages is small before 2000, but it has a large effect on wages after 2000. Regarding the elasticity of substitution between workers belonging to different age groups, Noro and Ohtake (2006) report a negative value of elasticity of substitution in each 5-year old group, but it is hard to interpret economically because it implies an upward sloping labor demand curve and a positive effect of cohort size on wage. In contrast, we estimate the elasticity of substitution parameter with an alternative identification strategy, and obtain a result that is both economically meaningful and statistically significant.

Our study is linked to the literature on the Japanese economy. Numerious empirical studies about the change in Japanese employment practices have been published in recent years, such as Kawaguchi and Ueno (2013) and Kambayashi and Kato (2017). One of the fact regarded as the evidence of the change in employment practice is flattening of the age-wage profile

(Hamaaki et al., 2012). In contrast, our study shows the new perspective on this literature that the flattening of age-wage profile can be explained by factors different from the change in employment practices. There are also numerous studies that examine the relationship between the extension of mandatory retirement (e.g., Clark and Ogawa, 1992; Mitani, 2003; Yamada, 2010; Kimura et al., 2019; Ueno, 2021). Our study contributes to this literature by showing the specific mechanism of the impact of extending retirement age on age-wage profile. In this regard, the closest work is Ueno (2021), who discusses the impact of changes in age composition caused by the extension of retirement age on the age-wage profile. However, they do not quantitatively assess the impact of change in the age composition of workers. In contrast, our study evaluates the quantitative importance of a specific channel causing the flattening of the age-wage profile by combining the economic model and the data.

The rest of this chapter is organized as follows. Section 2.2 formulates the model. Section 2.3 estimates the elasticity of substitution. Section 2.4 quantifies the effect of aging on the age-wage profile. Section 2.5 checks the robustness of our findings. Section 2.6 presents the conclusions.

2.2 Model Incorporating the Effect of Aging

This section presents the aggregate production function whose labor inputs are not perfect substitutes among age groups. First, we introduce the production function with prefect substitution, which is widely used in the macroeconomics literature. Then, we formulate the production function with imperfect substitution by generalizing the assumption of perfect substitution. From this production function, we derive the labor demand equation which, shows that the change in age composition of workers affects the relative wage.

2.2.1 Production Function with Perfect Substitution

Our starting point is the following production function:

$$Y_t = F(L_t, X_t),$$

where Y_t, L_t, and X_t are the output, aggregated labor input, and the other factors affecting the output, such as capital or technology, respectively. Here, we assume the separability between aggregated labor and the other factors. The aggregation of labor input in the standard macroeconomic model follows, such that[7]

$$L_t = \sum_{i=1}^{n} \beta_{it} L_{it}, \tag{2.1}$$

where β_{it} and L_{it} are the efficiency parameter and the labor inputs of group i at time t, respectively. Assuming that wage, W_{it}, is set in a competitive market, the age-wage profile based on $i = 1$ is given by

$$\frac{W_{it}}{W_{1t}} = \frac{\beta_{it}}{\beta_{1t}}.$$

Thus, age-wage profile depends only on the ratio of efficiency, β_{it}/β_{1t}, and the amount of labor service does not affect the relative wage.

2.2.2 Production Function with Imperfect Substitution

Now, to incorporate the mechanism in which the labor input share affects the wage, we introduce the imperfect substitution aggregator in the simplest manner, such that

$$L_t = \left(\sum_{i=1}^{n} \beta_{it} L_{it}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 0, \tag{2.2}$$

where ϵ denotes the elasticity of substitution among labor inputs. This constant elasticity of substitution (CES) type of aggregation is a generalized form of (2.1), because (2.2) reaches (2.1) when $\epsilon \to \infty$. From the first-order condition, the age-wage profile is given by

$$\frac{W_{it}}{W_{1t}} = \left(\frac{L_{it}}{L_{1t}} \right)^{-\frac{1}{\epsilon}} \frac{\beta_{it}}{\beta_{1t}}. \tag{2.3}$$

Taking the logarithm of (2.3) and using the notations $\log(W_{it}/W_{1t}) = w_{it}$, $\log(L_{it}/L_{1t}) = \ell_{it}$, and $\log(\beta_{it}/\beta_{1t}) = \tilde{\beta}_{it}$, the demand equation of labor can be written as

$$w_{it} = -\frac{1}{\epsilon}\ell_{it} + \tilde{\beta}_{it}. \tag{2.4}$$

Here, in addition to the relative efficiency, the age-wage profile is also affected by the relative labor inputs. A relatively high labor input in any age-group reduces their marginal productivity, and consequently, their wage. The magnitude of the effect of change in relative labor input on the age-wage profile depends on the value of elasticity of substitution, ϵ. In the next section,

[7] This type of labor aggregator is often used in a overlapping generation model. For example, Auerbach and Kotlikoff (1987) uses this type of function.

we will estimate this parameter.

Although this study uses the CES function to introduce imperfect substitution and assumes that substitution with any other age group is constant, it is possible to eliminate this assumption. For example, the Nested-CES function can provide different elasticities of substitution for close and distant ages. Another more flexible relationship can be expressed using the Trans-log function. However, a more flexible functional form requires more parameters to be estimated, which means that more data are needed to obtain plausible results. Owing to data limitations, this study utilized the CES function, which is the simplest and most straightforward extension. Extending the functional form is one of the remaining tasks in this study.

2.3 Estimating the Elasticity of Substitution

We now estimate the elasticity of substitution, which is an important parameter in analyzing the impact of changes in age composition of labor inputs on the age-wage profile. We estimate this parameter using the orthogonality condition of demand and supply shift. This is because, due to the characteristics of the data, simple panel data methods cannot guarantee robustness; the estimates vary widely by sample period. We begin this section with a description of the data used in the analysis. Then, we explain the estimation method. Finally, we summarize the estimation results.

2.3.1 Data

To examine the relationship between demographics and wages, the data that captures the broadest range of economic activity is desirable. From this perspective, we choose the following two data sources: the Statistical Survey of Actual Status for Salary in the Private Sector (*Minkan Kyuyo Jittai Tokei Chosa*, ASSPS) and the Labour Force Survey (*Roudouryoku Chosa*, LFS).[8]

The ASSPS is an annual survey of worker incomes at business establishments throughout Japan. The sample size of the ASSPS is approximately 18 thousand establishments and 243 thousand workers in each year. The survey items of ASSPS includes information about the establishment and employment status of workers.[9] From ASSPS, we use the annual income of male workers who have worked in the same workplace for an entire year to exclude the workers who have worked in more than one establishment during the same year. Unfortunately, the ASSPS does not include working hours, so we use data on working hours from the LFS. The

[8]The ASSPS is conducted by the National Tax Agency, and the LFS is conducted by the Statistics Bureau of the Ministry of Internal Affairs and Communications.

[9]The details of the ASSPS are described in Appendix 2.7.4.

Table 2.1: Relative Wage and Labor Input by Age and Years

	Age Range						
	25–29	30–34	35–39	40–44	45–49	50–54	55–59
Wage							
1980–1989	1.00	1.22	1.42	1.58	1.66	1.70	1.59
1990–1999	1.00	1.20	1.38	1.55	1.71	1.81	1.73
2000–2009	1.00	1.18	1.40	1.59	1.71	1.79	1.79
2010–2019	1.00	1.14	1.27	1.42	1.58	1.69	1.70
Labor Input							
1980–1989	1.00	1.21	1.28	1.15	1.01	0.81	0.57
1990–1999	1.00	0.98	0.97	1.07	1.10	0.95	0.77
2000–2009	1.00	1.26	1.22	1.08	1.01	1.08	1.04
2010–2019	1.00	1.29	1.56	1.71	1.59	1.34	1.20

Source: The Statistical Survey of Actual Status for Salary in the Private Sector and the Labour Force Survey.
Note: This table shows the relative values of wage and labor input of male workers averaged within the respective years. Each value of the wage and labor input is divided by the value of the base group, that is, workers aged 25–29.

LFS is a monthly survey of households which collects labor data that includes information regarding working hours. Combining these two sources, we obtain the hourly wage and labor inputs of male workers from 1980 to 2019.

Table 2.1 summarizes the relative values of wage and labor input compared with the 25–29-year-old group. We take the average value every 10 years and then divide them by the youngest group's one. The effect of aging on the age-wage profile is suggested between the 2000s and the 2010s. The relative labor inputs of all age groups in the 2010s are larger as compared to in the 2000s. The relative wage also decreased during this period. This negative correlation of relative labor input and wage supports the relationship in Equation (2.4) with the positive elasticity of substitution.

Contrary to the trend since 2000, relative labor inputs and wages were positively correlated from the 1980s to the 1990s. The relative labor inputs and wages of the group over 45 years old increased from the 1980s to the 1990s. Because of these trends, the coefficient of the regression of wages on labor input using data from this period is expected to be positive, and hence the estimated elasticity of substitution is negative. However, we suspect that these trends are explained by change in productivity, such as human capital accumulation, and that simple OLS estimates have bias due to lack of control variables for productivity. The details of this point will be discussed in the later sections.

The positive correlation between labor input and wage before 2000 considerably affects the estimates of the simple panel data methods such as OLS estimation on the demand curve in Equation (2.4). Please see Appendix 2.7.1 for details of the estimation. The OLS estimate of the elasticity of substitution in demand Equation (2.4) is 23.8 in the period 1980–2019. But, if we exclude the first 10 years from the sample, the elasticity decreases to 7.2. Moreover, the estimate in the period 1980–2009 is -11.7, which is hard to interpret economically because negative elasticity implies an upward labor demand curve. It is difficult to determine which value of the parameter is suitable for our model in a situation where the estimate changes as the sample period of the data changes. Thus, we select another estimation method to ensure robustness for the choice of the sample period.

It should be noted that the wage data used in this study are based on all workers, without distinguishing between internal and external labor markets. The effects of aging examined in this study are likely to be more pronounced in the external labor market, where wages are determined by marginal productivity. Therefore, the effects of aging may be reflected more in the job transition and wages of short-term employees than in wages for long-term employment. Although there is a possibility of more plausible data, this study uses wages for all workers because of the limitations of the available data.

2.3.2 Estimation Method

To avoid the endogeneity problem, we use the identification strategy developed by Feenstra (1994). Our identification originates from the orthogonality between the demand and supply shocks in the simultaneous equations. Using the panel feature of the data, we can estimate the system of equations without exogenous variables.

We first formulate the supply side of labor market, such that,

$$\rho \log W_{it} = \log\left(\frac{L_{it}}{N_{it}}\right) + \lambda_i + \kappa_t + u_{it}, \tag{2.5}$$

where W_{it}, L_{it}, and N_{it} are wage, person-hour labor supply, and population of age group i at time t, respectively. λ_i and κ_t are fixed effects, and u_{it} is the error term. ρ shows the elasticity of labor supply. By taking the difference to age group 1, we can obtain the following expression of labor supply function,

$$\rho w_{it} = \ell_{it} - n_{it} + \tilde{u}_{it}, \tag{2.6}$$

where we use notations such that $\log(W_{it}/W_{1t}) = w_{it},\ \log(L_{it}/L_{1t}) = \ell_{it},\ \log(N_{it}/N_{1t}) = n_{it}$, and $u_{it} - u_{1t} + \lambda_i - \lambda_1 = \tilde{u}_{it}$, Our estimation model consists of a simultaneous equation system of labor demand and supply functions for each age group as follows:

$$w_{it} = -\frac{1}{\epsilon}\ell_{it} + \tilde{\beta}_{it} \tag{2.7}$$

$$\rho w_{it} = \ell_{it} - n_{it} + \tilde{u}_{it}. \tag{2.8}$$

The demand Equation (2.7) is the same as Equation (2.4). The relative efficiency, $\tilde{\beta}_{it}$, is interpreted as the sum of fixed effect and demand shock. The Equation (2.8) represents the supply of labor service. $\tilde{u}_{it}$ also contains the supply shocks and fixed effect of age groups. The time effect is already controlled because all variables are divided by the base group, $i = 1$.

To estimate the system of equations (2.7) and (2.8), we take the first difference of each equations such that

$$\Delta w_{it} = -\frac{1}{\epsilon}\Delta\ell_{it} + \Delta\tilde{\beta}_{it} \tag{2.9}$$

$$\rho\Delta w_{it} = \Delta\ell_{it} - \Delta n_{it} + \Delta\tilde{u}_{it}, \tag{2.10}$$

where Δ means first difference. By taking the first difference, we can control the fixed effects on both demand and supply shocks. Then, using the panel feature, we set the $n-1$ identification conditions, such that

$$\mathbf{Cor}_i[\Delta\tilde{\beta}_{it}\Delta\tilde{u}_{it}] = 0 \quad (\text{for } i = 2, \cdots, n). \tag{2.11}$$

Here, we assume that the correlations of $\Delta\tilde{\beta}_{it}$ and $\Delta\tilde{u}_{it}$ are equal to zero overtime for every age group. The identification condition is economically reasonable because this condition implies that, on average, demand and supply shocks to individual groups are not correlated. The shocks that have the same effect on all groups, such as macro shocks, are controlled for by taking the difference to group 1. Moreover, the simultaneous shocks on both supply and demand are averaged out and do not violate the condition unless such shocks occur frequently. This identification strategy was originally developed by Feenstra (1994) and is used in multiple fields, such as international trade and consumption (e.g., Broda and Weinstein, 2006, 2010; Arkolakis et al., 2018).[10]

[10] The original identification condition of Feenstra (1994) is expressed as $\mathbf{E}[\Delta\tilde{\beta}_{it}\Delta\tilde{u}_{it}]$. This is done to reduce computational complexity by avoiding the non-linear GMM. By using the correlation as the moment condition, we can control the size of variance in each age group.

Table 2.2: Result of Estimation on the Elasticity of Substitution

Sample Period	1980–2019	1990–2019	1980–2009
ϵ	4.22	6.44	4.40
	(1.34)	(2.34)	(1.29)
ρ	1.00	0.88	1.04
	(0.65)	(0.66)	(0.61)
J	1.97	1.35	3.63
	[0.74]	[0.85]	[0.46]

Note: The standard errors are reported in parentheses and p-values are reported in brackets. The standard errors are calculated by the delta method. J is the J-statistics of the overidentification test of GMM

2.3.3 Results

Table 2.2 shows the estimation results of ϵ and ρ, using the generalized method of moments (GMM) estimation. We use the efficient GMM estimator, which sequentially updates the weight until the estimated value and weight converge. The details of the estimation method are described in Appendix 2.7.2. The first column of the table shows the estimated results using the data for the entire period. The elasticity of substitution estimated by the full sample is 4.22. Using the demand Equation (2.4), this estimate implies that a 1% increase in relative labor supply causes a decrease of 0.236% in relative wages.

The estimates are robust despite changes in the sample period. The second and third columns of Table 2.2 show the results of estimation using the data from 1990–2019 and 1980–2009. The estimated result for the data after 1990 is 6.44, which is similar to the estimate of the entire period, considering that the standard error of the entire period is 2.13. Moreover, the elasticity estimated in the data before 2009 is 4.40, which is almost the same as the estimates from 1980 to 2019. This robustness for the sample period is surprising because other methods, such as the OLS estimation, produce both negative and positive values of the elasticity of substitution depending on sample period.[11]

We check the robustness of the estimation under different identification conditions. We conduct the robustness check using two different moments: product of demand and supply shocks and their covariance. All identification conditions imply the zero-correlation of demand and supply shocks, but they differ in taking deviations from mean and adjusting of size of variance. In Appendix 2.7.3, we introduce the robustness check in detail and show the estimation result under different identification conditions. The estimates of the elasticity of substitution are mostly stable under different identification conditions. Therefore, our estimates are not

[11] Please see the Appendix 2.7.1 for detail.

sensitive to the form of identification condition.

2.4 Quantifying the Effect of Aging

2.4.1 Decomposition of Change in Age-wage Profile

From the demand Equation (2.4), we can decompose relative wage into two parts, relative labor input and efficiency, such that

$$\log \frac{W_{it}}{W_{1t}} = -\frac{1}{\epsilon} \log \frac{L_{it}}{L_{1t}} + \log \frac{\beta_{it}}{\beta_{1t}}.$$

From this equation, the change in the age-wage profile can be decomposed, such that,

$$\log \frac{W_{it}}{W_{1t}} - \log \frac{W_{it'}}{W_{1t'}} = -\frac{1}{\epsilon} \left(\log \frac{L_{it}}{L_{1t}} - \log \frac{L_{it'}}{L_{1t'}} \right) + \left(\log \frac{\beta_{it}}{\beta_{1t}} - \log \frac{\beta_{it'}}{\beta_{1t'}} \right). \tag{2.12}$$

$\log \frac{W_{it}}{W_{1t}} - \log \frac{W_{it'}}{W_{1t'}}$ denotes the change in age-profile, $-\frac{1}{\epsilon} \left(\log \frac{L_{it}}{L_{1t}} - \log \frac{L_{it'}}{L_{1t'}} \right)$ is the effect of change in labor inputs on the age-wage profile, and $\left(\log \frac{\beta_{it}}{\beta_{1t}} - \log \frac{\beta_{it'}}{\beta_{1t'}} \right)$ shows the change in efficiencies.

By using Equation (2.12), we decompose the change in age-wage profile during two periods: from 1980 to 2000 and from 2000 to 2019. Figure 2.2 shows the decomposition of change in the age-wage profile in the two periods. In the figure, the black lines show the actual change in the relative wage, the white-filled bars show the effect of the change in labor inputs, and the gray-filled bars show the effect of the change on efficiencies. As the figure shows, the change in the wage profile from 1980 to 2000 is mostly due to changes in efficiency; on the other hand, the change from 2000 to 2019 is largely accounted for by the changes in labor input. The white area accounts for 33% of the total area of the white and gray bars in the left panel, while it accounts for 84% in the right panel. In the decomposition of the Figure 2.4 in the appendix, there is no change in the tendency of the share of changes in labor input to increase in recent years.

It is important to note that the wages we analyze are for the same age range but from different years and therefore come from different groups of people. Therefore, changes in efficiency can be caused by replacing cohorts with different productivity levels. Examples of this replacement effect include differences in productivity between the war-torn generation and baby boomers and between the cohort affected and unaffected by the entry condition of the recession in the late 1990s. The following discussion also addresses this point, as the change in productivity among middle-aged and older workers from 1980 to 2000 can be explained by cohort-specific factors.

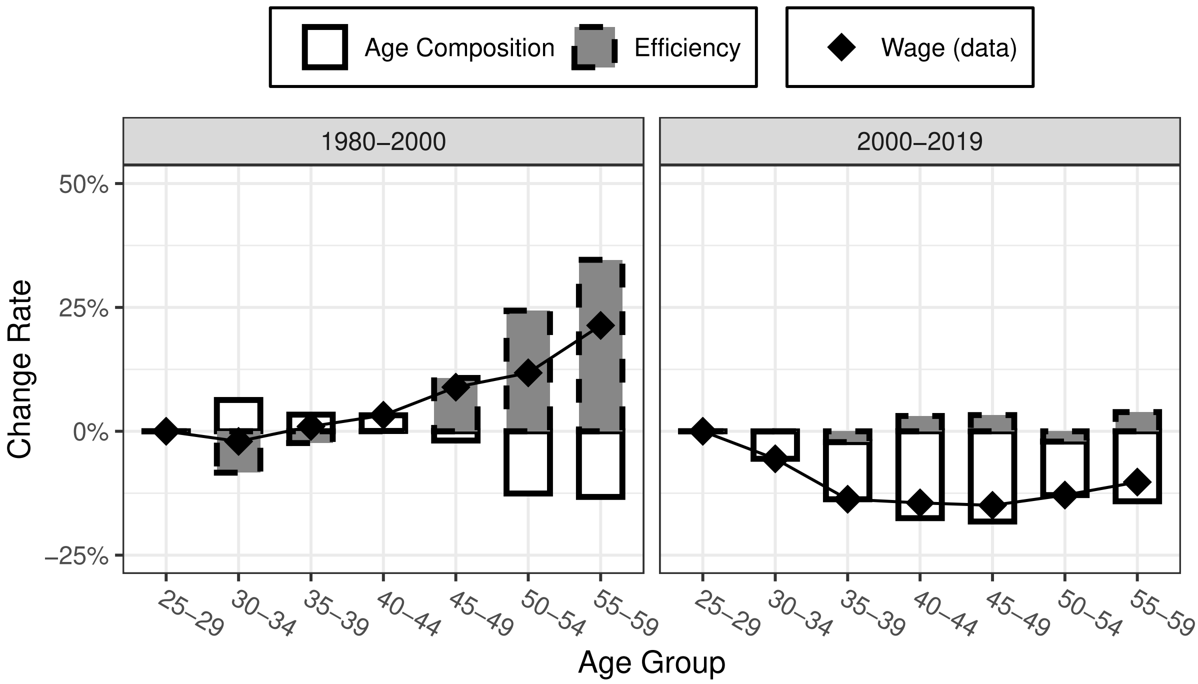


Figure 2.2: Decomposition of the Change in Age-wage Profile

2.4.2 Discussion

Source of Change in Labor Input

We found that changes in labor input largely explain the flattening of the age-wage profile. This change in labor inputs can be explained by demographics, i.e., the aging of the population. To show this, we decomposed the labor inputs into three factors: working hours, employment rate, and population. The decomposition is conducted by taking the difference in log of the following equation from 2000 to 2019.

$$(\text{Labor input}) = (\text{Employment rate}) \times (\text{Working hour}) \times (\text{Population})$$

In addition, to compare against the youngest group, we calculated the relative value of change rates based on the youngest age group. The results of this decomposition are shown in Table 2.3. In the table, labor inputs and population look highly correlated; the decrease in the labor input of the youngest group can be explained by changes in the population by more than 4 times, than by change in the working hours. For the 40s age group, the change in labor input is relatively low, and the population also does not change significantly. The change in working hours is also large. However, in terms of relative change, the change in working hours is a small factor in explaining the differences between the groups, as the trend is similar for all groups.

Table 2.3: Decomposition of Change in Labor Inputs

Age Group	Labor Input	Employment Rate	Hour	Population
Changes from 2000 to 2019 (%)				
25–29	-57.7	0.3	-12.3	-45.8
30–34	-37.9	-1.3	-11.9	-24.7
35–39	-17.2	-1.4	-9.9	-5.8
40–44	3.6	-0.8	-8.5	12.9
45–49	1.8	-0.8	-6.5	9.1
50–54	-24.2	0.0	-5.5	-18.7
55–59	-15.5	1.3	-5.7	-11.1
Relative changes from 2000 to 2019 (%)				
25–29	(base)	–	–	–
30–34	19.9	-1.6	0.4	21.1
35–39	40.6	-1.8	2.3	40.0
40–44	61.3	-1.1	3.7	58.7
45–49	59.5	-1.1	5.7	54.9
50–54	33.5	-0.3	6.7	27.1
55–59	42.3	0.9	6.6	34.7

On the other hand, the relative change in population is still huge. Even for the least affected group, changes in population are four times more influential than changes in working hours. Therefore, we consider that much of the effect of labor input on the age-wage profile that we have shown in Figure 2.2 is attributable to demographic changes.

The Change in Productivity Before 2000

There are two implications of the increase in the productivity of the elderly in the decomposition from 1980 to 2000 in Figure 2. The first is that this change in productivity is consistent with previous studies. The 45–59 year olds in 1980 correspond to those born between 1921 and 1935, and there are significant differences between them and the generation born 20 years later. Since the end of the war in 1945, Japan has experienced rapid economic growth, which has led to a steeper age-wage profile due to various factors such as an increase in highly educated workforce (Kawaguchi, 2011) and the establishment of Japanese employment practices (Genda and Kambayashi, 2002). In addition, the war shortened the number of years of education and led to the drafting of soldiers, which likely hindered the accumulation of human capital. From 1980 to 2000, this wartime generation was replaced by the postwar generation in the middle-aged group, and hence the productivity increased within the 45–59 age group.

Since this increase in productivity occurred in the postwar generation, which is the first baby boomer generation, a bias can be generated in estimating the demand curve by using methods such as OLS. In Equation (2.4), the error term represents the change in productivity. Thus if there is a positive correlation between labor inputs and productivity, the estimated coefficient will have an upward bias. The elasticity of substitution corresponds to the negative value of the inverse of the coefficient, which means that the positive bias of the coefficients makes the estimates of elasticity of substitution large or possibly even negative. This is the case in Appendix 2.7.1, where the elasticity estimates for the data up to 2010 are negative. The negative estimates of the elasticity are not a unique problem to our data. Noro and Ohtake (2006) report negative elasticity of substitution among different aged workers for each five year age group by using the Basic Survey on Wage Structure from 1976 to 2001.

Changes in the Age-wage Profile due to Another Factor

Figure 2.2 indicates that the change in the age-wage profile from 2000 to 2019 can be explained by the change in age composition. It also shows that the relative labor productivity by age group has remained unchanged or slightly increased during this period. However, the question is whether there are any other factors that affect the age-wage profile. For example, the increase in non-regular employment is supposed to make the age-wage profile flat. We discuss why the increase in non-regular employment seems to have a small effect on the age-wage profile from 2000 to 2019.

In recent years, non-regular employment has been increasing in Japan. According to the LFS, the share of non-regular workers has increased from 26% in 2000 to 38% in 2019. One of the characteristics of non-regular employment is a flat age-wage profile (Esteban-Pretel and Fujimoto, 2021). Therefore, the increase in the number of non-regular employed workers is expected to have a significant impact on the age-wage profile.

As the increase in non-regular employment is primarily for younger workers, the impact on the age-wage profile is considered limited for now. The increase in the share of non-regular employment concentrates on the younger male workers. Esteban-Pretel and Fujimoto (2021) show with data that the profile of the share of regular employees is decreasing, particularly for younger age groups. As the increase in non-regular employment spreads to the middle-aged and older workers, the impact on age-wage profile might become more significant.

Relationship with the recent structural changes in the Japanese labor market

Next, we discuss how this study relates to the rise of non-regular employment and the female workforce, which are not directly examined in this study. The Japanese economy has recently undergone major structural changes, with a rise in non-regular employment and the female workforce. However, this study targets only male employees who have worked at the same workplace for at least one year; the two structural changes are not considered in the analysis. We discuss how this study is positioned with respect to these two structural changes and whether potential biases exist due to the exclusion of these markets from the analysis.

First, we review the recent increase in non-regular employment. According to Kambayashi (2017), the increase in non-regular employment among men has been accompanied by a decline in self-employment rather than in regular employment, which accounts for the majority of the working population. As an exception, he finds that the regular employment of young male workers is replaced by non-regular employment with a permanent contract[12] but that the share of regular employment is stable after the entry stage. He concludes that regular employment remains the dominant employment type for male workers in Japan.

The decline in self-employment, accompanied with replacement by non-regular workers, slightly expands the share of the male workforce included in our data. Since our data are based on male workers employed by the same firm for one year, the majority of regular workers and parts of non-regular workers are included, but self-employed workers are not included.

Therefore, the workers that move from self-employment to non-regular employment are newly included in the survey if they work at the same firm for one year. However, this impact of the replacement on data is quantitatively unclear. We can get a few hints from official statistics. According to the 2017 Employment Status Survey (ESS), 77% of non-regular workers have worked for more than one year in the same firm. So, at least some portion of workers moving from self-employment to non-regular employment is likely included in this data. However, please note that this number does not necessarily equal the percentage of working for the same firm for at least one year in the workers moving from self-employment to non-regular employment. To infer whether the inclusion rate of workers moving from self-employment to non-regular employment is higher or lower than the non-regular workers' average, we need a more detailed micro-data analysis. In summary, the replacement of self-employment with informal employment has increased the share of surveyed workers in the total workforce, although the magnitude of this effect is unclear.

The entry-level replacement of regular employment with non-regular employment potentially overestimates the aging of workers in our data. In the 2017 ESS, 63% of non-regular workers who were employed on permanent contract, had been employed for at least one year, which is lower than 78% of regular workers. This shift in employment status at the entry level may accelerate the aging of the population in our data beyond the aging of the entire workforce.

[12] Kambayashi (2017) uses the definition of non-regular employment by job title.

Another potential bias stems from the decrease in self-employment and the assumption of separability between self-employed and employed workers. We assume that the intermediate inputs produced by self-employed workers are separable from the labor inputs of male employees.

If the decline in self-employment impacts the marginal rate of substitution between different aged male employees,

our model's specification would fail, and it would require a more flexible substitution structure. Consequently, to ensure the validity of our analysis, further research is essential to confirm whether age-related heterogeneity exists in the substitution between self-employed and employed workers.

Next, we summarize the changes in female labor participation in recent years. According to the LFS, women's labor participation rate increased from 48% to 53% between 1980 and 2019. The female labor entrants are highly skewed towards the healthcare service sector. Kawaguchi and Mori (2019) show that women's employment in the healthcare service industry increased by 2.7 million, while overall female employment rose by 3.5 million, between 2002 and 2018. As suggested by this skewed entry, occupational gender segregation still exists, but has narrowed within each industry. Uchikoshi and Mugiyama (2020) show that occupational gender separation decreased within the industry between 1980 and 2005, which offsets the widening effect of skewed entry by the female workforce on occupational gender segregation. Another noteworthy phenomenon is the change in the employment status of women. As pointed out by Kambayashi (2017), regular and permanent non-regular employment has increased among women under the age of 40 years.

In the data we used, the age structure of female workers shows aging, but the age profile of women is steeper from 2000 to 2019, which suggests an important mechanism other than imperfect substitution. There are multiple hypotheses to explain this difference in changes in the wage profiles of women and men. If women have a higher elasticity of substitution than men, aging has a weaker flattening effect on age-wage profile. Another hypothesis is that the increase in regular employment among women in their early careers has led to a faster accumulation of human capital and a steeper age-wage profile. Additional analysis is promising not only for estimating the elasticity of substitution, but also for quantifying the effect on regular employment for those early in their career.

A potential bias in our analysis could arise from the assumption that female labor inputs are separable from those of males. From the shrinking segregation within industry by Uchikoshi and Mugiyama (2020) and the increase in regular employment among young people in Kambayashi (2017), it is possible that women employment can substitute for regular male employment, especially in younger age groups. Ignoring the labor supply of women that can substitute for men in the younger age group may overestimate the aging of the labor force.

2.5 Robustness Check

2.5.1 Comparison of the ASSPS with Other Data

In this subsection, we compare the ASSPS with the Basic Survey on Wage Structure (BSWS), which is often used in empirical studies of the labor market in the Japanese economy.[13] Both ASSPS and BSWS are annual surveys for establishments regarding employment and wage, but they differ significantly in the sample selection of establishments.

Before comparing the two statistics, we briefly introduce the BSWS. The BSWS is an annual survey of business establishments to examine the employment conditions and wages of their workers. The sample establishment of the BSWS is extracted from the population list of establishments. The population list is created by the enumeration survey of establishments before 2015 and the population database of establishments in 2015 or later. The sample establishments randomly select their workers at a specific rate and answer their questions. The survey items of the BSWS include age, education, monthly salary, annual bonus, and working hours. In 2019, the BSWS surveyed approximately 780,000 establishments and 1.63 million workers.

The advantage of the ASSPS is the broad coverage of establishments. The sample of the ASSPS is constructed to represent all business establishments, including small businesses. In 2019's ASSPS, 6% of male workers are employed at establishments with 1–4 employees, and 8% of male workers are employed at establishments with 5–9 employees. Contrary to the ASSPS, the BSWS does not take samples from establishments with 1–4 employees and extracts a sample of the establishments with 5–9 employees only from firms with 5–9 employees. Furthermore, using the microdata of the ASSPS, Kawaguchi and Toriyabe (2022) reveal that the demographics of workers in the establishments with 1–4 employees, which is dropped from the BSAWS's coverage, are statistically different from those of the other group.

Therefore, ASSPS is superior as its sample design covers small businesses with less than 10 employees, which represents the economy as a whole.

An additional advantage of ASSPS is that the population list of establishments is updated annually. The ASSPS uses tax data to create the business establishment population list for each year. Therefore, the population of the ASSPS reflects current economic activity. In contrast, the population list of the BSWS was dependent on the enumeration survey for establishments before

[13]The BSWS is conducted by the Ministry of Health, Labour and Welfare.

2015. Creating a population list of business establishments requires a major effort. In Japan, the enumeration survey for establishments was conducted once every few years. Probably due to this sampling design, the BSWS's data seemingly have some large changes when the population list is updated.[14] Therefore, the ASSPS's systemically updating the population list is another advantage.[15]

The advantage of the BSWS is the variety of survey items. Notably, the BSWS surveys hours worked as well as compensation to workers, which allows simultaneous measurement of the hourly wage and labor supply for the same worker. Also, the BSWS surveys the educational background of workers and the number of employees by firm. Therefore, the BSWS enables the study of aging in the population by controlling the education level and heterogeneity of firm size. We also use the BSWS as a robustness check to compliment the findings derived from the ASSPS data.

2.5.2 Estimation with the BSWS Data

In this subsection, we estimate the elasticity of substitution using the BSWS data and compare the results with those of the ASSPS. The estimation result by the BSWS still shows that the aging workforce flattens the age-wage profile. The elasticity of substitution estimated from the BSWS is smaller than that of the ASSPS, which suggests that the effect of worker aging on flattening the wage profile is larger. Notably, the difference in the estimates is not statistically significant; however, the different estimation results may be due to differences in the characteristics of data.

The data used from the BSWS is close to that of the ASSPS in terms of time period and attributes. We use the BSWS from 1981 to 2019 for male regular workers aged 25–59 in establishments with 10 or more employees. The BSWS from 1981 to 2015 is provided by the Japan Institute for Labour Policy and Training. We connect this to the data from 2015 to 2019, available on the website of the statistical bureau. For wages, we use the hourly wage calculated from the monthly salary, annual bonus, and monthly hours worked. We also use the population from the LFS, same as in the estimation by the ASSPS. Using the above data, we conduct GMM estimation of the simultaneous equations (2.9) and (2.10).

Table 2.4 compares the elasticity of substitution of labor between different ages estimated by the BSWS and ASSPS. The details of the estimation results are provided in Table 2.9 in the

[14] For example, a total number of employees in firms with 10 or more employees changes from 19.5 million to 23.4 million between 2011 and 2012, when the population list is updated.

[15] Another potential problem of the BSWS is identified by Shinozaki (2008), who argues that the BSWS is disconnected before 2004 and after 2005 due to changes in the definition of part-time workers and the addition of additional surveyed occupations.

Table 2.4: Comparison of the Estimated Elasticity of Substitution

	1981–2019	1990–2019	1981–2009
BSWS	2.49	2.04	3.12
	(0.68)	(0.53)	(1.04)
ASSPS	3.84	6.44	4.14
	(1.41)	(2.34)	(1.30)

Note: The standard errors are reported in parentheses. The details of the estimation results for the BSWS data are reported in Table 2.9

appendix. The estimates show incomplete substitution regardless of sample periods or data. For all sample periods, the estimates of the BSWS are lower than those of ASSPS, which signifies larger impact of aging on the age-wage profile.

There are numerous possible reasons why the BSWS estimates are lower than those of the ASSPS. The first reason considered is statistical error. The 95% confidence interval of estimated elasticity using the ASSPS from 1981 to 2019 is [1.60, 6.84]. This interval overlaps the estimates of BSWS. Therefore, both estimates may be close if the sample period is sufficiently large. Next, we also discuss the possibility that differences in the data affect the estimates. The characteristic differences between the two datasets may cause the results to vary. Since the BSWS does not survey small firms, if small firms can more easily substitute the labor of different ages than big firms, the estimated elasticity with the ASSPS may be higher. Also, using regular worker's data for the BSWS may restrict the workers who have jobs that are more difficult to replace.

2.5.3 Heterogeneity in Firm Size

In Section 2.2, our model assumes representative firms, which ignores the heterogeneity of firms. However, the studies of age-wage profiles in Japan, such as Noro and Ohtake (2006) or Hamaaki et al. (2012), often focus on age-wage profiles by firm size. Following these studies, we relax the assumption of a representative firm and examine the effect of aging on age-wage profile by firm size.

We estimate the elasticity of substitution using data separately aggregated by firm size. Specifically, we use the number of regular employees of the BSWS as firm size. The firm size is divided into three categories: 10 to 99 employees, 100 to 999 employees, and 1,000 or more employees. Table 2.5 shows the estimation result of data by firm size. The estimated results for the elasticity of substitution between different aged labor, ϵ, do not differ significantly by firm size.

Table 2.5: Result of estimation on the elasticity of substitution by firm size

firm size	10-99	100-999	1000-
ϵ	5.11	4.09	4.28
	(3.83)	(3.32)	(1.80)
ρ	1.08	1.64	1.07
	(0.93)	(1.40)	(0.53)
J	1.16	0.96	0.86
	[0.88]	[0.92]	[0.93]

Note: The first row shows the firm size regarding the number of employees. The standard errors are reported in parentheses and p-values are reported in brackets. The standard error of ϵ is calculated by the delta method. J is the J-statistics of the overidentification test of GMM

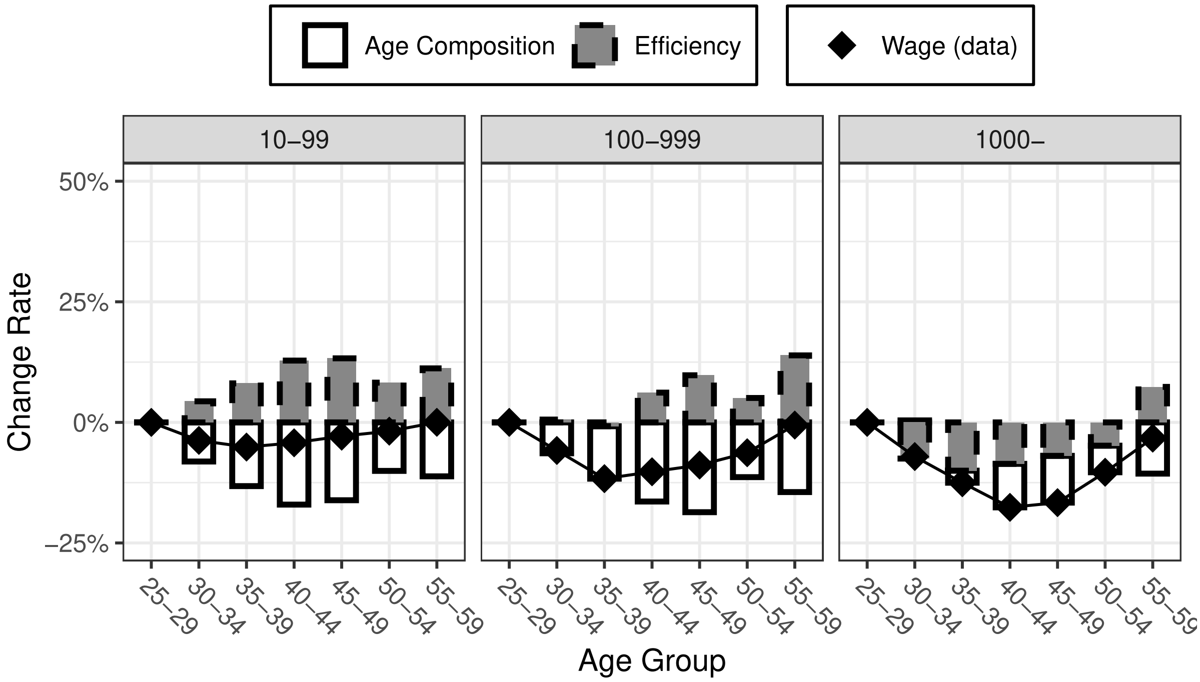


Figure 2.3: Decomposition of the Change in Age-wage Profile by Firm Size

Figure 2.3 shows the results of decomposition of the change in age-wage profile from 2000 to 2019 using the equation (2.12). For firms with 10 to 99 regular employees, the effect of change in age composition is large but is offset by changes in productivity; thus, the change in age-wage profile in this group is smaller than in other groups. The age-wage profiles of firms with 100 to 999 regular employees are explained by changes in the change in age composition. Firms with more than 1,000 employees are less affected by changes in age composition than other firm sizes, but their age-wage profile flattens due to the change in efficiency. These decomposition results are also uncertain because of the large standard error of the elasticity of substitution.

2.5.4 Robustness Check for Identification Condition

This subsection provides the estimates that consider potential violations of the identification condition. Specifically, we consider the following two possibilities. First, we consider the possibility that the extension of mandatory retirement age affects both demand and supply. Second, we consider the effect of change in college enrollment.

The Extension of Mandatory Retirement Age

Extending the mandatory retirement age affects both supply and demand in the labor market. On the supply side, the labor supply of elderly increases. Kondo (2016) and Kondo and Shigeoka (2017) report the increase in labor supply of older workers induced by the extension of the retirement age. On the demand side, several effects of extending the retirement age are considered. One is the effect of hiring older workers who had previously retired. By being employed continually in the same firms, some older workers will remain in their skilled jobs, and their productivity may rise. In another effect, the human capital investment process may decline, and then the productivity profile may flatten, as shown by Kimura et al. (2019). Thus, there is a possibility that demand and supply shocks may co-move by extending the retirement age.

A series of policies to extend the retirement age may violate the identifying condition. The identifying condition, in our estimation, is that the demand and supply shocks are not correlated in the long run. However, it is possible that the demand and supply shocks are temporarily correlated due to the extension of the retirement age. In addition, the mandatory retirement age has been gradually extended over a long period; hence, it will not be a temporary shock and may violate the identification condition.

Therefore, we conduct a robustness check by removing the older workers, who are primarily affected by extending the retirement age. Table 2.6 shows the results of the elasticity estimation without older workers. The estimate of the elasticity of labor by age is 2.83 when workers aged 55 and older are removed, which is lower than the estimate of 4.22 that includes workers up to age 59. An even smaller estimate is obtained when workers aged over 50 are excluded. The main primary conclusion that the labor inputs are imperfect substitution is robust, but there is a possibility of underestimating the impact of aging on age-wage profile.

Change in Worker's Educational Background

The change in educational background of workers may affect both demand and supply in the labor market. For the demand side, the change in composition of the educational background

Table 2.6: Robustness Check for Identification Condition

	Age < 55	*Age* < 50	*Education*
ϵ	2.83	3.09	2.85
	(1.46)	(1.53)	(0.82)
ρ	2.01	2.01	2.22
	(1.86)	(1.74)	(0.44)
J	1.52	0.56	7.38
	[0.68]	[0.75]	[0.69]

Note: The first and second columns show the estimated result without older workers. The third column shows the estimated result of the model in which the labor inputs are divided by educational groups. The standard errors are reported in parentheses and p-values are reported in brackets. The standard errors are calculated by the delta method. J is the J-statistics of the overidentification test of GMM

causes a change in productivity, which is regarded as a demand shock in the present model. Further, the change in composition of the educational background causes a change in productivity, which is regarded as a demand shock in this model. On the supply side, if education alters the supply behavior of workers, the change in educational background also has an impact. Therefore, it is likely that demand and supply shocks are correlated when the composition of workers' educational backgrounds is shifting.

We, therefore, extend our model to control the demand shock caused by the change in educational background. We divide the labor input by the educational background of workers. Specifically, we consider the following production function,

$$Y_t = F(L_{1t}, L_{2t}, X_t),$$

where L_{1t} and L_{2t} are aggregated labor inputs for high school and college graduates, and X_t denotes other factors of production. The aggregation of labor inputs is given as follows,

$$L_{kt} = \left(\sum_{i=1}^{n} \beta_{kit} L_{kit}^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \quad (k = 1, 2)$$

where L_{1it} and L_{2it} are labor inputs of high school and college graduates of age group i.

Then, as in the previous estimation, the supply function is defined as

$$\rho \log W_{kit} = \log \left(\frac{L_{kit}}{N_{it}} \right) + \lambda_{ki} + \kappa_{kt} + u_{kit}, \tag{2.13}$$

where λ_{ki} and κ_{kt} are the fixed effect, and u_{kit} is a supply shock. Note that the supply shock varies with the increase in the college enrollment rate.[16]

We estimate this model with the GMM. As before, the identification condition assumes no correlation between demand and supply shocks. That is, for this model, an increase in college enrollment should have no effect on productivity.

The estimation results are shown in the third column of Table 2.6. The elasticity of substitution between different aged labor inputs still shows incomplete substitution. Therefore, the main results are robust when labor input is separated by education.

2.6 Conclusion

This study empirically evaluates the effect of aging on the age-wage profile in Japan. We estimate the elasticity of substitution among0 different age groups. We find that the flattening of the age-wage profile is well explained by the change in the age composition of labor input after 2000.

Our study has a number of potential applications. In macroeconomics, overlapping generation models usually assume the perfect substitution between different ages, and for this reason it is difficult to reproduce the flattening of age-wage profiles in recent years. Sakuragawa and Makino (2007) is an exception for using an incomplete substitution labor aggregator, and our study can help with using and calibrating such a model. In terms of income inequality, the implication of our model is that a generation with a larger population size than the surrounding generations lowers their wages. Our findings suggest that the negative cohort effect on income of the ice age generation, which is also known as the second baby boomer generation, is due to their cohort size.

One of the remaining tasks is to extend the model to imperfect competition. In the present study, we adopt the competitive labor market in which wages and marginal productivities are always identical. Although the competitive market is a natural benchmark for investigating our hypothesis, it lacks various important factors such as unemployment or dynamic contracts with asymmetric information. Moreover, the effect of demographic changes cannot fully explain the observed change in the age-wage profile. We suspect that a model without a competitive market may be helpful in explaining these gaps, which will be our next task.

[16] If we replace the population of the age group, N_{it}, with the population of the age-education group, the increase in college-enrollment rate does not affect the error term. Unfortunately, we could not find publicly available annual data on the population by education and by each 5-year age group.

2.7 Appendix

2.7.1 Reduced Form Estimation of the Elasticity of Substitution

In this section, we estimate the elasticity of substitution using the reduced form estimation. We estimate the following model:

$$w_{it} = \gamma \ell_{it} + u_i + e_{it} \tag{2.14}$$

where $w_{it}(= \log(W_{it}/W_{1t}))$ and $\ell_{it}(= \log(L_{it}/L_{1t}))$ are the log of relative value of wage and labor inputs, which are the similar notation in Section 2.2. u_i is a fixed effect, and e_{it} represents any demand shocks such as a change in productivity of workers. The coefficient, γ, is equal to the negative value of the inverse of elasticity of substitution. In addition to OLS estimation, we use instrument variable (IV) method to estimate Equation (2.14). As instruments variables, we use population, which provides exogenous variation for labor inputs.

We also conduct simple estimation on the elasticity of substitution by following the two-way fixed effect (FE) model,

$$\log W_{it} = \alpha \log L_{it} + u_i + g_t + e_{it}.$$

Table 2.7 shows the estimated result. The OLS and IV estimation using data from 1980 to 2019 shows the elasticity of substitution as close to 20. However, if the sample period is changed to post-1990, the elasticity of substitution is approximately 1/3 of the value of the whole period. Negative elasticities are reported if the sample is restricted to 2009. Regarding the estimation by the two-way FE model, both negative and positive values are reported if the sample period is changed.

Table 2.7: Result of Estimation on the Elasticity of Substitution

	1980–2019	1990–2019	1980–2009
A. OLS			
coefficients	-0.042	-0.14	0.085
	(0.019)	(0.012)	(0.016)
EoS	23.8	7.2	-11.7
B. IV			
coefficients	-0.059	-0.152	0.045
	(0.020)	(0.010)	(0.019)
EoS	17.0	6.6	-22.4
C. two-way FE			
coefficients	0.054	-0.083	0.12
	(0.018)	(0.016)	(0.012)
EoS	-18.5	12.1	-8.3

Note: The robust standard errors are reported in parentheses. EoS shows the point estimates of elasticity of substitution, which equals to negative value of inverse of coefficients.

2.7.2 Details of Estimation Algorithm

In this appendix, we describe the details of our estimation method. We use the following simultaneous equation and moment condition to perform the estimation,

$$
\begin{aligned}
&\text{simultaneous equation:} && w_{it} = -\frac{1}{\epsilon}\ell_{it} + \tilde{\beta}_{it} \\
& && \rho w_{it} = \ell_{it} - n_{it} + \tilde{u}_{it} \\
&\text{moment condition:} && \mathbf{Cor}[\Delta\tilde{\beta}_{it}\Delta u_{it}] = 0 \quad (\text{for } i = 2, \cdots, n).
\end{aligned}
$$

We set function $g_t(\theta)$ such that

$$
g_t(\theta) = \begin{pmatrix} \frac{(\tilde{\beta}_{2t}-\overline{\beta}_2)(\tilde{u}_{2t}-\overline{u}_2)}{\sqrt{T^{-1}\sum_{t=1}^{T}(\tilde{\beta}_{2t}-\overline{\beta}_2)^2}\sqrt{T^{-1}\sum_{t=1}^{T}(\tilde{u}_{2t}-\overline{u}_2)^2}} \\ \frac{(\tilde{\beta}_{3t}-\overline{\beta}_3)(\tilde{u}_{3t}-\overline{u}_3)}{\sqrt{T^{-1}\sum_{t=1}^{T}(\tilde{\beta}_{3t}-\overline{\beta}_3)^2}\sqrt{T^{-1}\sum_{t=1}^{T}(\tilde{u}_{3t}-\overline{u}_3)^2}} \\ \vdots \\ \frac{(\tilde{\beta}_{nt}-\overline{\beta}_n)(\tilde{u}_{nt}-\overline{u}_n)}{\sqrt{T^{-1}\sum_{t=1}^{T}(\tilde{\beta}_{nt}-\overline{\beta}_n)^2}\sqrt{T^{-1}\sum_{t=1}^{T}(\tilde{u}_{nt}-\overline{u}_n)^2}} \end{pmatrix} \quad \left(\text{where } \overline{\beta}_n = T^{-1}\sum_{t=1}^{T}\tilde{\beta}_{nt}, \overline{u}_n = T^{-1}\sum_{t=1}^{T}\tilde{u}_{nt}\right).
$$

A GMM estimator minimizes a quadratic form in $\sum_{t=1}^{T} g_t(\theta)$:

$$
\min_{\theta} \left[\sum_{t=1}^{T} g_t(\theta)\right]' \hat{\Xi} \left[\sum_{t=1}^{T} g_t(\theta)\right]
$$

We start weight matrix, $\hat{\Xi}$, from identity matrix and update weight matrix by following equation,

$$
\hat{\Xi} = T^{-1}\sum_{t=1}^{T} g_t(\hat{\theta})g_t(\hat{\theta})'.
$$

We repeat the estimation and updating of weights until the estimates and weights converge.

2.7.3 Robustness Check of GMM Estimation

In this appendix, we check the robustness of the GMM estimation. We conduct the GMM estimation using two identification conditions: the expected value of the product of demand and supply shocks is zero, and their covariance is zero. The estimation results are shown in Table 2.8. The estimates of the 1980–2019 sample are similar to the 4.22 shown in Section 2.3. The estimates using the 1980–2009 sample also do not change much. The results for 1990–2019, especially for the case using the expected value of the product, deviate from the original estimates, but the standard errors are also estimated to be larger. Therefore, the different identification conditions may change the efficiency of the estimation or some characteristics in small samples. In general, the estimates do not change significantly in most cases, which indicates that our estimates do not depend much on the form of the identification condition.

Table 2.8: Result of Estimation on the Elasticity of Substitution

	1980–2019	1990–2019	1980–2009
A. $\mathbf{E}[\Delta\tilde{\beta}_{it}\Delta\tilde{u}_{it}] = 0$			
ϵ	4.11	11.13	4.75
	(1.53)	(6.62)	(1.78)
ρ	1.14	0.38	1.12
	(0.72)	(0.62)	(0.66)
J	0.97	1.74	2.40
	[0.92]	[0.78]	[0.66]
B. $\mathbf{Cov}[\Delta\tilde{\beta}_{it}, \Delta\tilde{u}_{it}] = 0$			
ϵ	4.83	7.22	5.72
	(1.75)	(3.01)	(2.41)
ρ	0.78	0.70	0.69
	(0.64)	(0.66)	(0.70)
J	1.94	1.30	3.79
	[0.75]	[0.86]	[0.43]

Note: The standard errors are reported in parentheses and p-values are reported in brackets. The standard errors are calculated by the delta method. J is the J-statistics of the overidentification test of GMM.

2.7.4 Details of the ASSPS

The Statistical Survey of Actual Status for Salary in the Private Sector (*Minkan Kyuyo Jittai Tokei Chosa*, ASSPS) is one of Japan's Fundamental Statistics. The ASSPS is intended to reveal the actual annual salaries in the private sector and to provide basic data for estimating tax revenues, examining the tax burden, and managing tax administration. The ASSPS was initiated in 1949 and has been conducted annually since then.

The sampling procedure of the ASSPS is conducted in two stages. In the first stage, private establishments are selected based on a population list of establishments. The population list of establishments is created using the tax withholding records. In 2019, there are 3.5 million establishments in the surveyed population, of which 27,320 are sampled, and 18,529 establishments respond. In the second step, the surveyed establishments select sample employees based on a certain selection rate specified by firm size. In the 2019 survey, 243,018 employees are extracted.

The ASSPS survey items are divided into two parts, one for establishments and the other for employees. For establishments, the survey items include firm name, address, type of business, capital, number of salaried employees, total annual salaries paid, etc. For employees, the survey items include name, gender, age, working years, number of months of salary received during the year, amount of salary, amount of taxes, etc.

2.7.5 Decade-by-decade Decomposition of Age-wage Profile

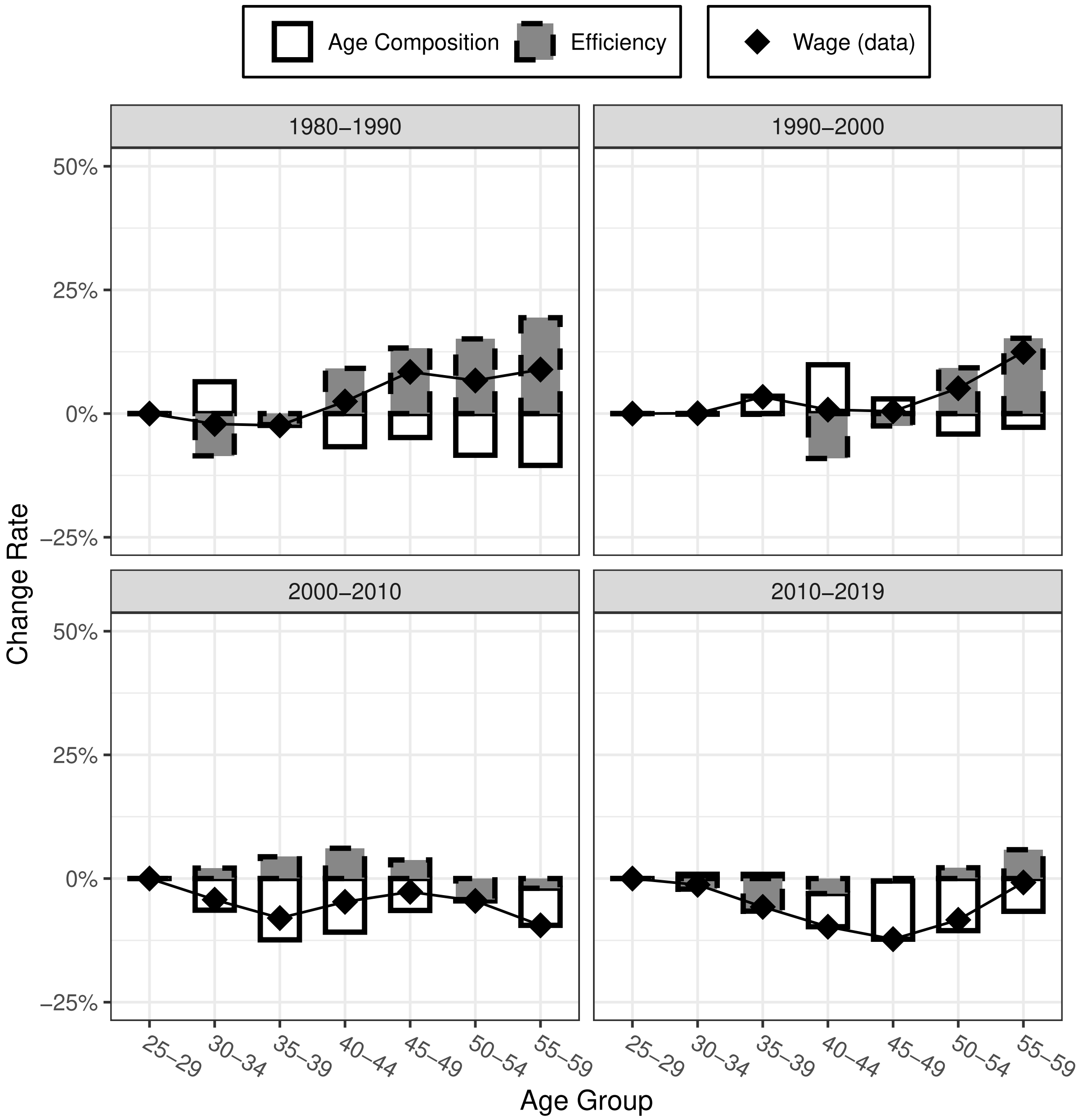


Figure 2.4: Decomposition of the Change in Age-wage Profile

2.7.6 Estimation Result by Using the BSWS Data

Table 2.9: Result of Estimation on the Elasticity of Substitution by Using the BSWS

Sample Period	1981–2019	1990–2019	1981–2009
ϵ	2.49	2.04	3.12
	(0.68)	(0.50)	(1.04)
ρ	1.34	2.03	1.26
	(0.62)	(0.63)	(0.51)
J	1.50	2.52	1.12
	[0.83]	[0.64]	[0.89]

Note: The standard errors are reported in parentheses and p-values are reported in brackets. The standard errors are calculated by the delta method. J is the J-statistics of the overidentification test of GMM.

Chapter 3

Price Index Numbers under Large-Scale Demand Shocks: The Japanese Experience of the COVID-19 Pandemic[1]

3.1 Introduction

The coronavirus 2019 (COVID-19) pandemic promoted massive stockpiling behavior among consumers worldwide, and Japan was no exception. The first case of COVID-19 in Japan was reported in mid-January, 2020. Immediately after the news was reported, the demand for face masks and sanitizers surged. On March 23, 2020, the Governor of Tokyo warned that a lockdown might be imposed, causing several people to flock to supermarkets and grocery stores to purchase food and other necessary items.

Due to the COVID-19 threat, people have changed their consumption behaviors to a large extent. Figure 3.1 shows the weekly rates of change of the chained Laspeyres and Paasche indexes of face masks in Japan based on scanner data.[2] As Ivancic et al. (2011) show, weekly scanner data often exhibit large discrepancies between Laspeyres and Paasche indexes, which can be observed in the figure. In the middle of January 2020, both indexes increased to a great extent; then, the Paasche index overtook the Laspeyres index.[3] According to the Bortkiewicz decomposition of the Laspeyres-Paasche (L-P) gap, the negative L-P gap implies that the cor-

[1]This chapter is based on Abe et al. (2022).

[2]Section 3.4 examines detailed information on the dataset we use in this study.

[3]See Diewert (2022) for detailed discussion on the chain drift observed when using scanner data.

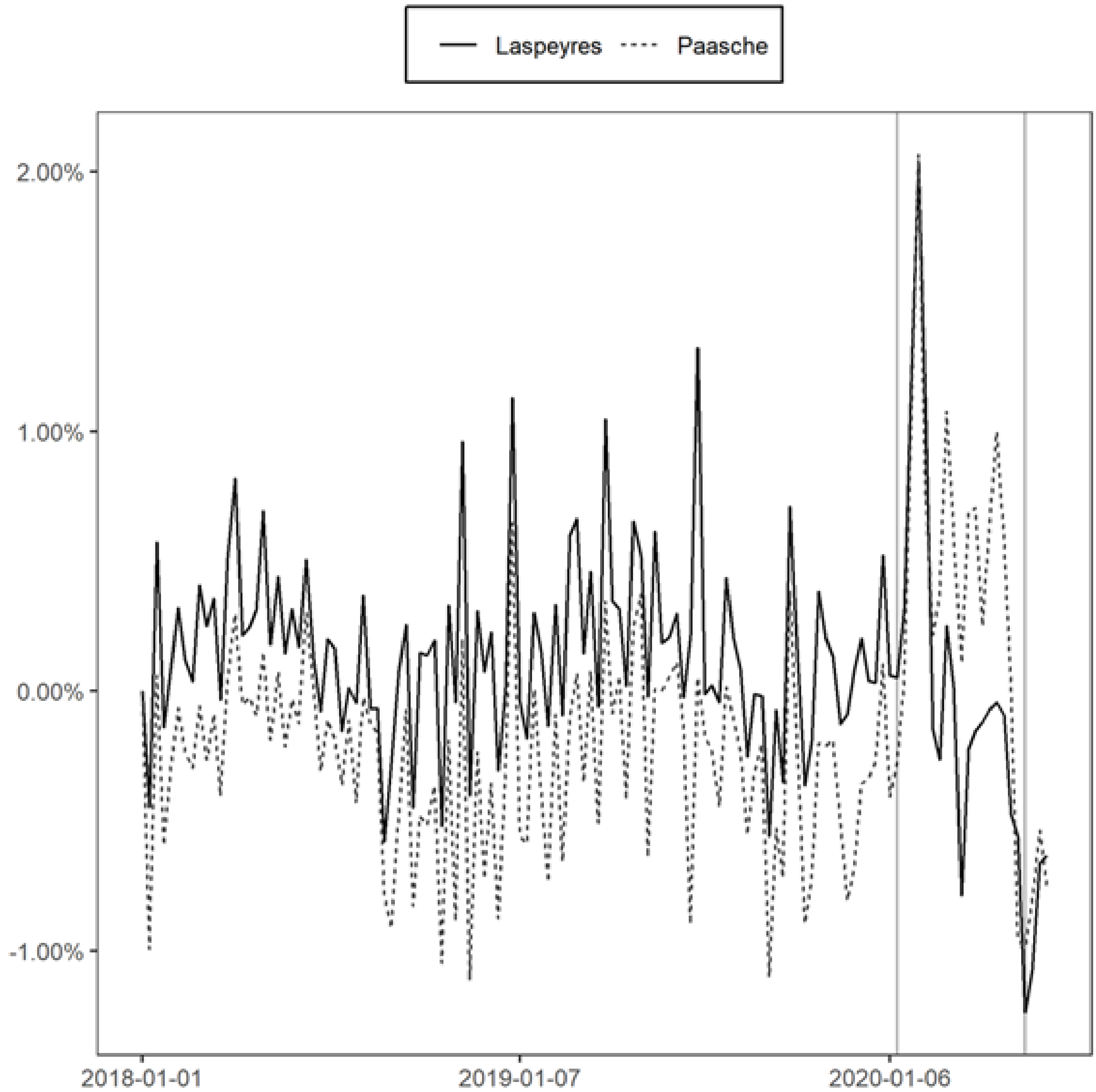
relation between quantities and prices is positive. Although, theoretically, a positive correlation between quantities and prices is not impossible, it is quite unlikely. A natural interpretation is that during this period, large-scale demand shocks occurred, which shifted prices and quantities along an upward-sloping supply curve.

In this study, using Japanese scanner data for face masks, we investigated the impacts of demand shocks on the COLI caused by the COVID-19 in 2020. The traditional theory of COLI assumes that the preference is unchanged.[4] Therefore, changes in demands or preferences are not captured in the COLI. In a recent path-breaking paper, Redding and Weinstein (2020) propose the constant elasticity of substitution (CES) unified price index (CUPI). The CUPI has several important characteristics. First, CUPI is exact for CES COLI without any restrictions on the relationship between quantities and prices. Second, CUPI can be decomposed into two effects; price and taste effects. Price effects can take various forms. In this study, following Redding and Weinstein (2020), we adopt the Sato-Vartia index as the price effect. Taste effects capture changes in demand or preferences. Redding and Weinstein (2020) call this effect "bias" in the traditional COLI such as the Fisher or Tornqvist indexes.

Our empirical analyses based on Japanese weekly scanner data of face masks revealed that the 2020 COVID-19 caused large-scale demand shocks. This increased the discrepancies between the traditional COLI such as the Fisher and Tornqvist indexes and CUPI to a great extent. That is, the demand shocks caused by the pandemic caused a significant change in the COLI. More specifically, while the prices of face masks decreased in the Jevons and Fisher indexes in May 2020 by 0.06% and 0.76% per week, respectively, the COLI increased by 1.92% per week. The magnitude of the changes caused by the demand shock is so substantial that traditional index numbers may carry incorrect information on the cost of living among consumers.

The chapter is organized as follows. Section 3.2 presents a brief history of the COVID-19 pandemic in Japan. Section 3.3 introduces the index number formula by Redding and Weinstein (2020) and discusses the measures of demand shocks. Section 3.4 describes the dataset. Section 3.5 presents the empirical results. Section 3.6 concludes.

[4] One notable exception is Fisher and Shell (1971) that proposes calculating the difference between two cost of living indices (COLIs), one using the old preferences, and one using the new preference. Although this carries information on the effects of having different preferences on the COLI, it does not provide us with information on how the cost of living changes when preferences vary. Phlips (1974) criticizes Fisher and Shell (1971) and proposes a cardinal COLI that compares the minimum expenditures between two time periods assuming two different utility levels are comparable, that is, the utility function is cardinal. Balk (1989) proposes a COLI based on ordinal utility functions. He introduces the reference vector. The minimum expenditure is arrived at which the utility level at the reference vector are assured. Martin (2020) provides us with a brief survey on the cost of living index with variable preferences.

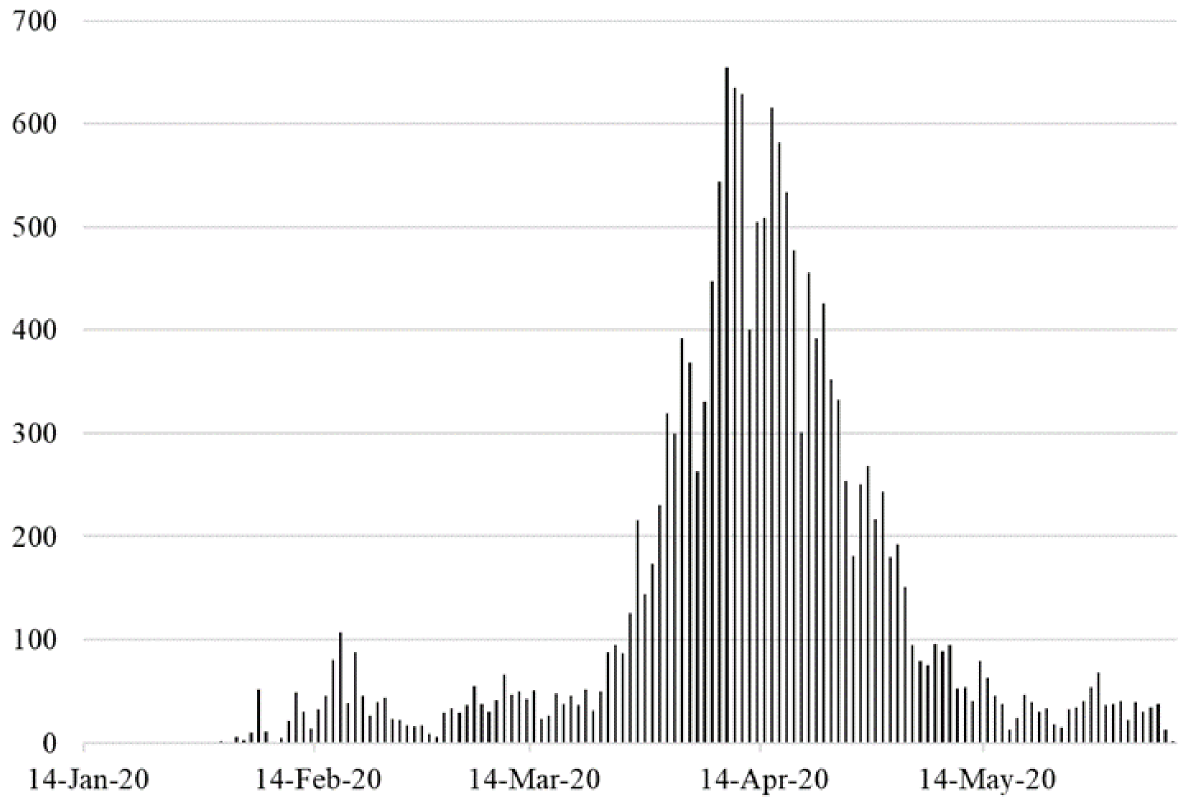


Note: Based on weekly Japanese scanner data. See Section 3.5 for the detail of the dataset.

Figure 3.1: Chained Laspeyres and Paasche Indexes of Face Masks.

3.2 The 2020 COVID-19 Pandemic in Japan and Face Masks

The first COVID-19 infection was reported in Japan on January 16, 2020. From Figure 3.2, which depicts the change in the number of infected persons by reported date, it can be seen that the number of COVID-19 cases started to increase significantly from February 2020. In response to this pandemic, the Japanese government announced its first emergency plan on February 13. The government also requested manufacturers to increase the production of masks, which had already started to run short, and prefectures to allocate stockpiles to medical institutions. On March 10, the second emergency plan was formulated, and the resale of masks, still in short



Source: The National Institute of Infectious Diseases, Japan.

Figure 3.2: Number of COVID-19 Infections in Japan.

supply, was legally prohibited. In addition, the government decided to purchase 20 million reusable cloth masks in bulk to be distributed to nursing homes and nursery schools. They also decided to secure 15 million masks for medical institutions as mask imports augmented, and manufacturers were requested to increase their production.

The infection continued to spread, and the Governor of Tokyo mentioned the possibility of a lockdown at a press conference on March 23, resulting in a temporary increase in consumer demand for food and daily necessities. On April 7, a one-month state of emergency was declared in seven prefectures, including Tokyo and Osaka, and residents were requested to avoid leaving their prefectures as much as possible. The declaration was extended to all prefectures on April 16, and the period was extended to May 31. However, as the number of infected persons began to decrease in May, the declaration was lifted in 39 out of 43 prefectures on May 14, followed by three more prefectures on May 21. In response to chronic mask shortages, two reusable cloth masks were distributed during the state of emergency to each child, student, faculty, and staff member attending or working at schools, in addition to two masks to each household nationwide. These distributions were completed on June 20.

3.3 The Price and Cost of Living Index with Taste Shocks

The CUPI by Redding and Weinstein (2020) (RW) consists of the two price indexes. The first is the CES common variety (CCV) price index, and the second is the Redding-Weinstein (RW) index, which includes the effects of changing product variety. The CCV between time and is defined as

$$\ln CCV(p_s, q_s, p_t, q_t) = \sum_{i=1}^{N} \omega_{ist}^{*}(\ln p_{it} - \ln p_{is}) + \sum_{i=1}^{N} \omega_{ist}^{*}(\ln \varphi_{is} - \ln \varphi_{it}), \tag{3.1}$$

$$\omega_{ist}^{*} = \frac{w_{it} - w_{is}}{\ln w_{it} - \ln w_{is}} \Bigg/ \left(\sum_{i=1}^{N} \frac{w_{it} - w_{is}}{\ln w_{it} - \ln w_{is}} \right), \tag{3.2}$$

$$w_{it} = p_{it} q_{it} \Bigg/ \left(\sum_{i=1}^{N} p_{it} q_{it} \right), \tag{3.3}$$

where φ_{it} and q_{it} are the taste parameter and the quantity of a commodity i at time t, respectively. We denote the vector of prices, quantities, and taste parameters at time t as follows.

$$p_t = (p_{1t}, p_{2t}, ..., p_{Nt}), \;\; q_t = (q_{1t}, q_{2t}, ..., q_{Nt}), \;\; \varphi_t = (\varphi_{1t}, \varphi_{2t}, ..., \varphi_{Nt}).$$

The taste parameter φ_{it} is also a function of prices and quantities as follows,[5]

$$\varphi_{it} = \varphi \left(\frac{p_{it}}{p_{1t}} \right) \left(\frac{w_{it}}{w_{1t}} \right)^{\frac{1}{\sigma-1}} \left[\prod_{k=2}^{N} \left(\left(\frac{p_{it}}{p_{1t}} \right) \left(\frac{w_{it}}{w_{1t}} \right)^{\frac{1}{\sigma-1}} \right)^{-\frac{1}{N}} \right], \tag{3.4}$$

where φ is a positive constant.

The first term on the right-hand side of the Equation (3.1) is the Sato-Vartia (SV) index. The second term, the taste term, captures the changes in the COLI caused by the changes in the preference parameters, φ_{it}, over time. RW shows that the CCV defined in (3.1) and (3.4) is the COLI for the following utility function and the normalization condition.

$$U_t(q_t; \varphi_t, \sigma) = \left(\sum_{i=1}^{N} (\varphi_{it} q_{it})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{3.5}$$

$$\prod_{i=1}^{N} \varphi_{it}^{\frac{1}{N}} = \varphi, \tag{3.6}$$

where $\sigma > 1$ is the elasticity of substitution and N is the number of commodities. Since the above utility function is linearly homogeneous with respect to the quantities, the minimum expenditure function can be written as the product of the unit expenditure function, $C(p_t; \varphi_t)$

[5] Please see Appendix A for the derivation of Equation (3.4).

and utility level.

$$E(p_t, U_t; \varphi_t) = C(p_t; \varphi_t) \times U_t.$$

Here the unit expenditure function takes the following functional form.

$$C(p_t; \varphi_t) = \left(\sum_{i=1}^{N} \left(\frac{p_i t}{\varphi_i t} \right)^{1-\delta} \right)^{\frac{1}{1-\delta}}$$

A notable feature of the utility function in Equation (3.5) is that the taste parameter, φ_{it} , can vary over time. The COLI corresponding to (3.1) is given by,

$$\begin{aligned} COLI(s,t) = & \frac{E\left(p_t, U_t = U, \varphi_t\right)}{E\left(p_s, U_s = U; \varphi_s\right)} \\ = & \frac{C\left(p_t; \varphi_t\right) \times U}{C\left(p_s; \varphi_s\right) \times U} \\ = & \frac{C\left(p_t; \varphi_t\right)}{C\left(p_s; \varphi_s\right)} \\ = & \left(\frac{\sum_{i=1}^{N} \left(\frac{p_{it}}{\varphi_{it}} \right)^{1-\sigma}}{\sum_{i=1}^{N} \left(\frac{p_{is}}{\varphi_{is}} \right)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} . \end{aligned} \tag{3.7}$$

The important characteristics of the COLI in (3.7) are: (i) its concavity with respect to the taste-adjusted prices, p_{it}/φ_{it} , and (ii) its symmetric treatment of the taste-adjusted prices. The concavity of COLI comes from cost minimization. The symmetric treatment stems from our assumption that the differences among quantities in the utility function are represented by the taste parameter, φ_{it} . These two characteristics of the COLI imply that if the taste-adjusted prices become more heterogeneous, that is, if the taste-adjusted prices are more dispersed, the consumers benefit from the diversity, which enables them to attain the given utility level by smaller expenses. In other words, if the taste-adjusted prices become more dispersed, the COLI becomes smaller. Please note that this result comes not from the assumption of the CES preferences but from the equal treatments of the taste-adjusted prices.[6]

While the CCV by RW enables us to construct the COLI with variable tastes, there are several issues to be considered. First, we could not identify one of the taste parameters, φ_t, from data. For example, assume that we multiply all the taste parameters φ_t by a constant, say $\kappa > 0$, while φ_s is unchanged, since such a change is a monotonic transformation of the utility function at time t, we obtain identical demand functions at time t while the demand

[6] As Redding and Weinstein (2020) found, it is not difficult to generalize the CCV so that the utility functions can take the form of the translog function with variable taste parameters.

functions at time s are unchanged. However, the COLI will take a different value because it is a decreasing function of the taste parameters. This identification problem may seem severe because the choice of the normalization condition affects the index number.[7] Recently, Abe and Rao (2020) investigated the axiomatic properties of normalization conditions using RW. They found that the normalization condition in form 3.6 is the necessary and sufficient condition for the CCV; first to pass the commensurability test, the index number must be free from the measurement units of price and quantities, and second, to treat all the quantities equally in the normalization conditions. For example, instead of 3.6, if we adopt the arithmetic mean, such as $(1/N)\sum_{i=1}^{N}\varphi_{it} = \varphi$ the CCV becomes sensitive to the choice of the measurement units of commodities such as pound or kilogram.[8] Therefore, in this study, we also use Equation (3.6) as the normalization condition.

The second issue is the interpretation of the taste term, the second term in 3.1. Martin (2020) argues that the magnitude of the taste term can be so large that the contribution of the price changes is swamped. Martin (2020) concludes that pure taste change effects are arguably out of the scope of a consumer price index and further clarifies the difference between the average price changes and the cost of living index. The CCV captures the effects of differences in taste, in addition to the effects of price changes. The taste term in the CCV reflects a strict concavity in the expenditure function. If the expenditure function is linear, that is, if δ is infinite, the taste term disappears from 3.1. As RW shows, it is not difficult to generalize the assumption of CES preferences to a more general class of utility functions, such as Translog preferences. That is, the CCV can be regarded as the simplest case of the COLI with demand shocks, which provides us with information on the impact of demand shocks on economic welfare.

A demand shock for commodity occurs at time when the taste parameter changes, that is,

$$\varphi_{it} \neq \varphi_{it-1}.$$

Using equations (3.4) and (3.6), we can estimate the taste parameter, φ_{it} , from the data of the expenditure shares and prices at time t. That is, we can observe changes in taste parameters over time. A natural measure of the degree of the demand shock at time t is the root mean

[7]Comparing various different normalization condition, Kurtzon (2020) argues that an arbitrary choice of normalization can yield any desired CCV.

[8]Abe and Rao (2020) show that the CCV passes the transitivity test as well as the monotonicity test but fails the identity test.

square deviation (RMSD), such as,[9]

$$RMSD_t = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\ln\varphi_{it} - \ln\varphi_{it-1})^2}. \tag{3.8}$$

If the RMSD increases, then the departure between the SV and COLI is expected to be greater. The actual effects of the demand shock on the price index can be captured by the taste term in 3.1,

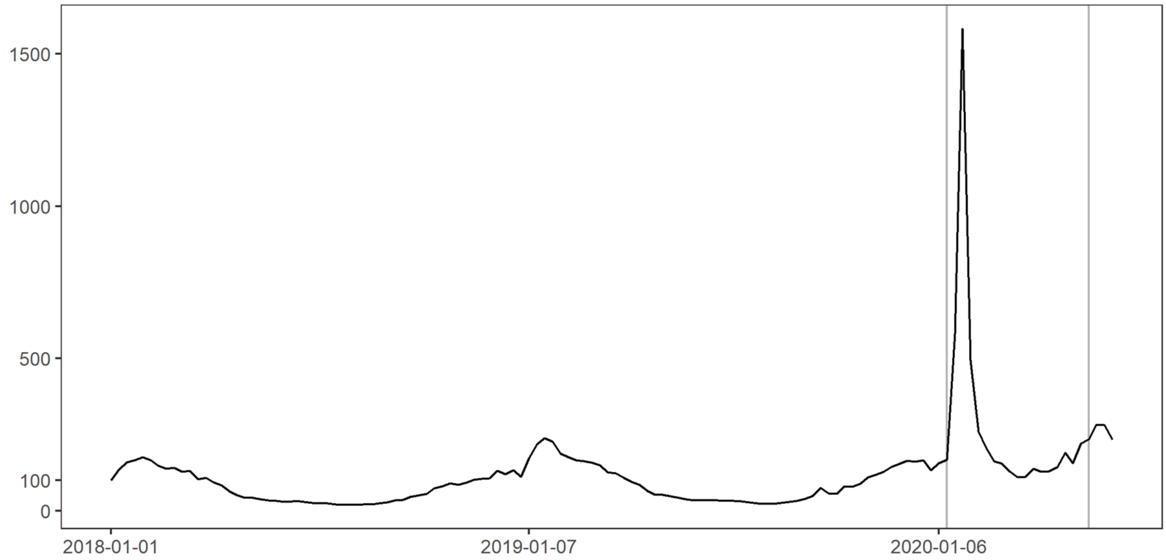
$$\sum_{i=1}^{N}\omega_{ist}^{*}\left(\ln\varphi_{is} - \ln\varphi_{it}\right).$$

It is worth noting that to show the equivalence between the Sato-Vartia index and the COLI for the CES utility function, the taste parameters must be fixed over time. That is, the Sato-Vartia index or other superlative indexes become COLI only when the preferences are constant over time. In other words, the observed prices and quantities must always be on the time-invariant demand function, which is a strong assumption, particularly for the COVID-19 pandemic period in 2020. Using the CCV, we can construct the cost of living index without assuming constant demand curves.

3.4 Data

In this study, we used the scanner data of face masks provided by Intage Holdings Inc. The dataset contains barcode level weekly sales and quantity information from nationwide retail stores in Japan. The scanner data provided by Intage is the largest point of sales data in Japan collected from more than 3,000 retail stores such as general merchandise stores, supermarkets, convenience stores, and drug stores all over Japan. Moreover, the retailers were chosen to get a nationally representative sample. We chose data for face masks between the week starting January 1, 2018, and the week starting June 8, 2020. Price information is obtained by dividing the weekly total sales at each store and the barcode by the quantities sold. When constructing such unit values, we must choose the data frequency. RW uses quarterly unit values while Diewert (2022) adopts a monthly frequency. Comparing price index numbers based on the unit values at various frequencies, Bradley (2005) found that a monthly unit value would lead to an upward bias in the cereal price index because of the aggregation of transactions at different prices. To mitigate the problems of using the unit values, we chose the weekly store-barcode

[9] Note that due to the normalization condition, the simple geometric average of the taste parameters is always constant.



Note: The total sales in the first week of 2018 are normalized as 100.
Source: Scanner data provided by Intage.

Figure 3.3: Movements of the Total Sales of Face Masks.

Table 3.1: Descriptive statistics of face masks in Japan.

	Mean	Std. Dev.	Min	P5	P25	P50	P75	P95	Max
Total Sales (million yen)	101	131	16.4	19.3	31.7	86.5	130	200	1330
Mean $\Delta(\ln \text{Price})$	-0.03	0.26	-0.49	-0.43	-0.19	-0.05	0.12	0.39	1.00
Std. Dev. $\Delta(\ln \text{Price})$	0.07	0.01	0.05	0.06	0.07	0.08	0.08	0.08	0.09
Mean $\Delta(\ln \text{Share})$	0.77	6.48	-41.3	-6.09	-1.87	0.44	3.09	10.7	21.6
Std. Dev $\Delta(\ln \text{Share})$	0.80	0.16	0.69	0.70	0.72	0.74	0.75	1.18	1.29

Note: Scanner data of face masks between the week starting January 1, 2018, and the week starting June 8, 2020. Data is provided by Intage, covering approximately 3,000 retail stores all over Japan.

level unit value, which were the finest data available.[10]

The general movements of the total sales of face masks are shown in Figure 3.3. In the week starting January 13, 2020, the demand for face masks surged. The period of the 2020 COVID-19 pandemic was set as the period between the week starting January 13, 2020, and the week starting May 18, 2020. This period is illustrated as the interval between the two vertical gray lines in Figure 3.3. The impact of the pandemic on the sales of face masks is clear in the figure. A surge in sales appeared in the week starting from January 13.

Table 3.1 reports the descriptive statistics for each weekly aggregated variable. As shown

[10] Although Japanese article number (JAN) code is supposed to be the unique identifier of products, sometimes, manufactures keep the identical JAN codes when they change the contents of the products. To deal with this problem, Intage creates an additional code, sequential code, to identify the difference of the commodities with the identical JAN codes if there are any differences. In this chapter, as the commodity identifier, we use the combination of both JAN and sequential codes. The total number of commodities stores is about 47,000.

Table 3.2: Changes in variables during the COVID-19.

Dependent variable	Constant	Changes after the outbreak
ln(Sales)	4.90	0.615
	[4.69, 5.10]	[0.137, 1.09]
Mean $\ln \Delta p_{it}$ (%)	-0.0794	0.113
	[-0.183, 0.0245]	[-0.090, 0.315]
Std. Dev. $\ln \Delta p_{it}$	0.0708	-0.0164
	[0.0563, 0.0853]	[-0.0238, -0.0090]
Std. Dev. $\ln \Delta w_{it}$	0.630	0.371
	[0.502, 0.758]	[0.302, 0.440]
L-P gap (%)	0.530	-0.772
	[0.290, 0.77]	[-1.11, -0.433]

Note: This table shows the results of regression analysis that measures the change in statistics during the COVID-19 outbreak. The seasonality and trend changes were controlled using monthly dummies and a trend term. In brackets, we report 95% confidence intervals estimated by heteroskedasticity and autocorrelation consistent standard errors. The first column shows the statistics used as dependent variables. L-P Gap is the difference between the logarithms of the chained Laspeyres and Paasche indexes. The second column shows the estimated value of the constant term in January. The third column shows the estimated coefficients of the dummy variables after January 13, 2020, that is after the outbreak.

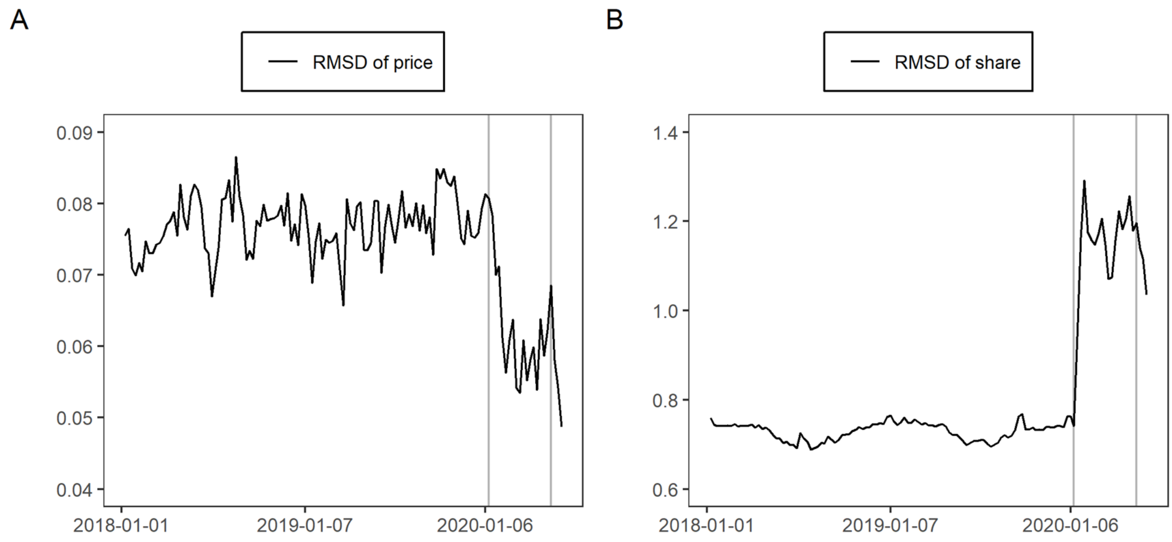
in the first row, the maximum value of total sales is enormous compared to the 95th percentile point. This distortion of sales distribution mainly appeared in the week when the demand for masks increased sharply in January 2020. In the second to the fifth row, we report the mean and standard deviation of the log change in prices and expenditure share in the common product calculated per week. The table shows that changes in expenditure shares are more volatile than changes in prices.

To see the changes in variables during the COVID-19 pandemic, we conducted the following regression.

$$y_t = \alpha + \beta D_t + \sum_{i=2}^{12} \lambda_i M_t^i + \delta t.$$

The second term on the right-hand side, D_t, is a dummy variable that is set to unity during the 2020 COVID-19. The third term controls seasonality by the dummy variables, M_t^i, which represents the months from February to December. The fourth term represents the trend term. We use multiple variables as dependent variables to examine their changes during the COVID-19.

The results of the regression analysis are shown in Table 3.2. The sales of face masks increased by 61.5% during the 2020 COVID-19. The arithmetic mean of price changes did not show a statistically significant change and the standard deviation of price changes fell. Contrary to price changes, the standard deviation of the change in expenditure share increased during



Source: Scanner data provided by Intage.

Figure 3.4: Movements of the RMSD of prices and shares of face masks in Japan.

the disaster. As shown in Figure 3.2, the L-P gap decreased significantly.

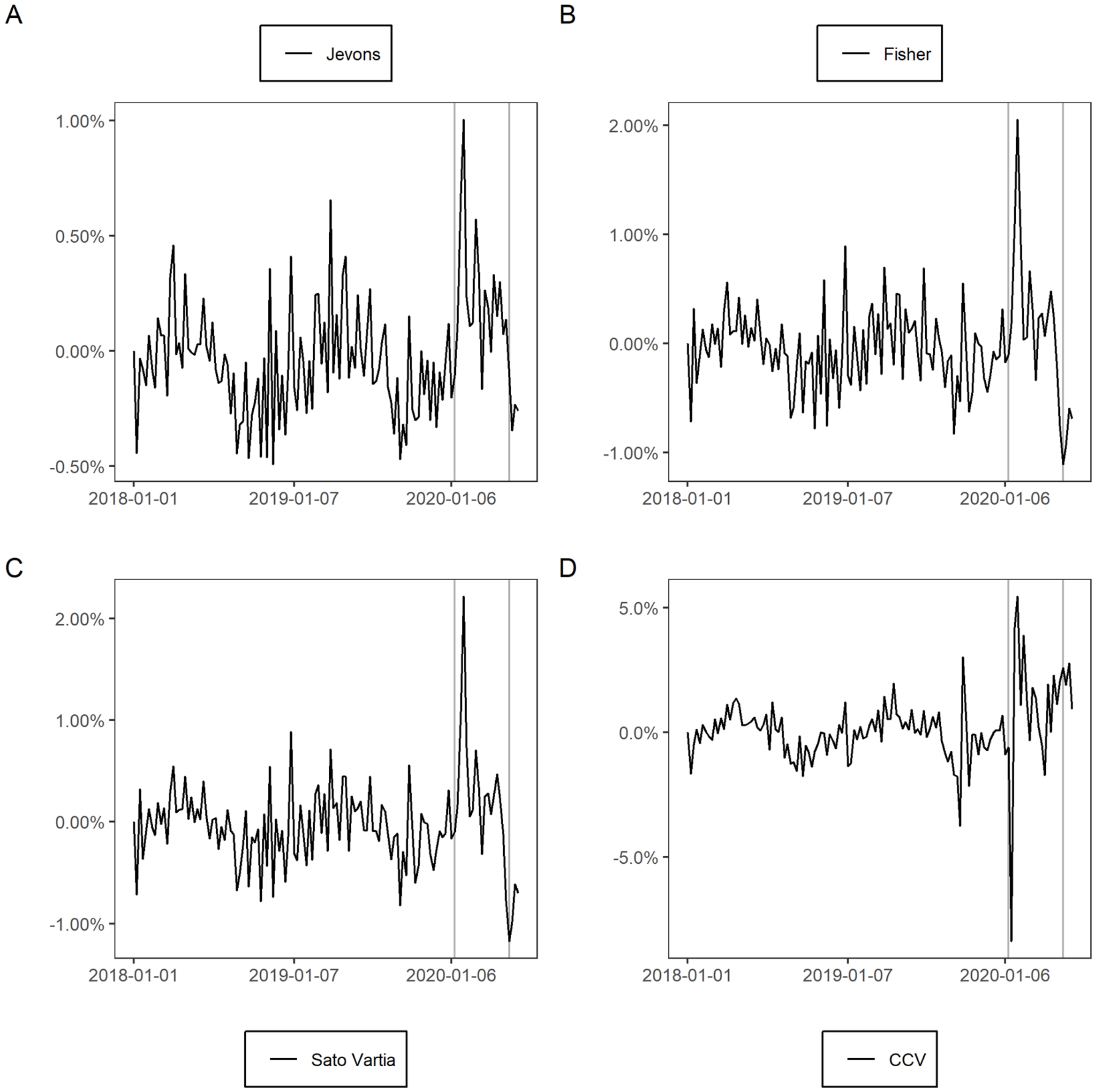
Figure 3.4 reports the RMSD of the changes in logged prices and logged expenditure shares. During the first week of the 2020 COVID-19 pandemic, prices become less volatile while the fluctuation of market shares surged. If the demand curve is stable, smaller volatility in prices should come with stable market shares. Thus, Figure 3.4 suggests that the demand curve changes during the first week of the COVID-19 pandemic.

3.5 Empirical Results

Figure 3.5 shows the weekly change rates of several chained price indexes including the CCV. Panel A shows the movements of the simple geometric average price, the Jevons index, which is known to be free from chain drift. Panels B and C report the movements of the Fisher and SV indexes, respectively. The Fisher and SV indexes were close to each other.[11] The Jevons, Fisher, and SV indexes exhibited a sharp increase in the first week of the COVID-19 period. However, when we consider changes in preferences, the movements of prices become very different. The CCV shows a sharp drop in prices during the first week of the COVID-19 pandemic, which then increased substantially.[12] Table 3.3 reports the movements of the indexes including changes

[11] Although not depicted in the figure, the Tornqvist index is also very close to the Fisher index.

[12] The point estimate of the elasticity of substation is 5.87, which is between the 25th and 50th percentiles reported by Redding and Weinstein (2020). We adopt the methods developed by Feenstra (1994) to estimate the elasticity of substitution using balanced data during 2018-2019. We chose the periods because if we include observations during 2020, the estimates become unstable. The constant elasticity over time is surely a restrictive



Note: Weekly change rates of chained indexes. CCV stands for CES common variety price index defined in Equation (3.1).

Figure 3.5: Weekly change rates of several price indexes of face masks.

in sales during January–February, 2020.[13] In the week starting from January 20, the sales of face masks surged, and an over 50% increase from the previous week was recorded. However, traditional price indexes, such as the Fisher price index changed little from the previous week. The CCV dropped by 8.37% in the week when the sales of face masks surged.

Figure 3.6 shows the movement of the RMSD of the taste parameters (Panel A) and the taste shock defined as $\sum_{i=1}^{N} \omega_{ist}^{*} \left(\ln \varphi_{is} - \ln \varphi_{it}\right)$ (Panel B). First, from Panel A, we can observe that

assumption. However, the estimation methods by Feenstra (1994) and Redding and Weinstein (2020) as well as the CCV critically depend on the assumption that elasticity of substitution is constant over time. The considerations of variable elasticity will be our future tasks.

[13] Appendix Table reports the index numbers of the entire sample periods.

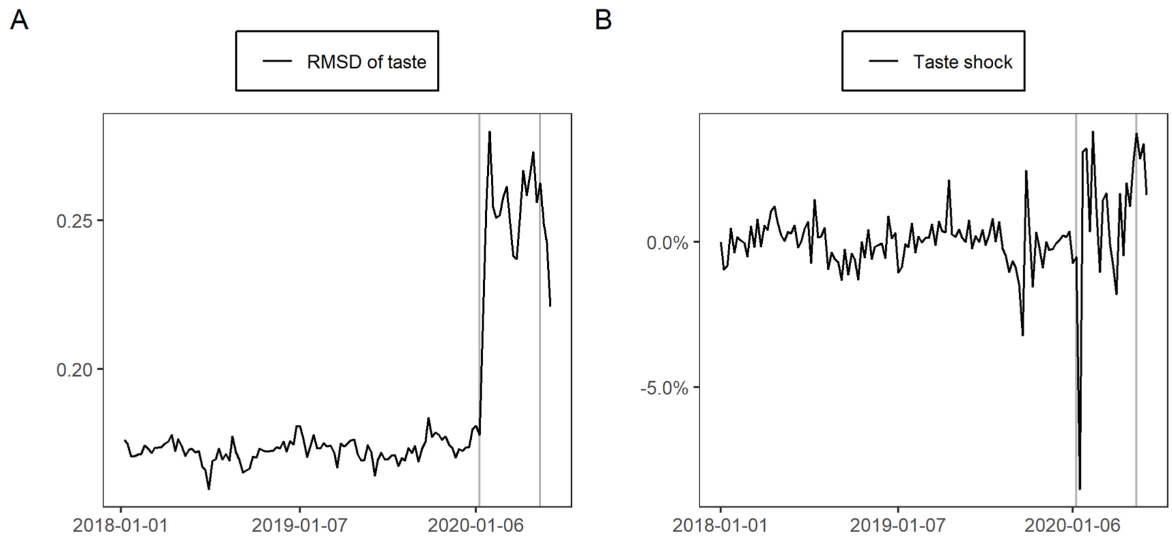
Table 3.3: Comparisons of price indexes.

	Sales	Jevons	Fisher	Sato Vartia	CCV
2020/1/13	2.97	-0.11	-0.11	-0.10	-0.60
2020/1/20	53.92	0.12	0.18	0.17	-8.37
2020/1/27	43.46	0.68	1.09	1.08	4.21
2020/2/3	-50.59	1.00	2.05	2.22	5.45
2020/2/10	-27.88	0.23	0.91	0.73	1.10
2020/2/17	-9.90	0.11	0.03	0.05	3.87
2020/2/24	-10.38	0.12	0.06	0.12	1.27
Average. May, 2018	-2.93	0.00	0.09	0.10	0.33
May, 2019	-3.52	0.06	0.13	0.13	0.33
May, 2020	4.29	-0.06	-0.76	-0.76	1.92

Note: The weekly rates of change (%) of the chained indexes. More comprehensive numbers are reported in the Appendix Table.

the RMSD of the taste parameters are always positive even before the COVID-19 period, which indicates that the assumption of the constant taste parameters is violated. From the figure, we can also observe that in January 2020, large negative taste shocks with a surge in the RMSD occurred, which makes the CCV drop to a great extent. The intuition behind the decline the CCV is as follows. As Equation (3.7) indicates, the CCV is a concave and symmetric function of the taste-adjusted prices, p_{it}/φ_{it} . This implies that people can obtain greater utility when taste-adjusted prices are more dispersed. Before the pandemic, some face masks were more popular than others, regardless of the price, which was represented as variations in the taste parameters among commodities. When people realized that face masks are effective in avoiding COVID-19 infections, their evaluation of each face mask changed to a great extent, which led to greater variations in the taste-adjusted prices than before the pandemic. The greater the dispersion, the more CCV dropped as discussed in Section 4. Opposite effects were observed in May 2020. As Table 3.3 shows, the CCV became positive while the standard price indexes were negative. At that time, the RMSD of tastes began to decrease, which led to smaller dispersions in the taste-adjusted prices, thus, resulting in positive taste effects.

Finally, Figure 3.7 shows the levels of several price indexes. In the figure, the chained Fisher and SV indexes do not depart from the Jevons index much, which suggests that chain drifts of the face mask are not serious. The deep trough of the CCV in January 2020 disappeared within a few weeks. The Jevons, Fisher, and SV indexes became smaller after the pandemic, while the CCV continued to increase. In early June 2020, the Jevons, Fisher, and SV indexes were around 97, while the CCV was more than 115. In other words, the magnitude of the cumulative effects of taste shocks was large.



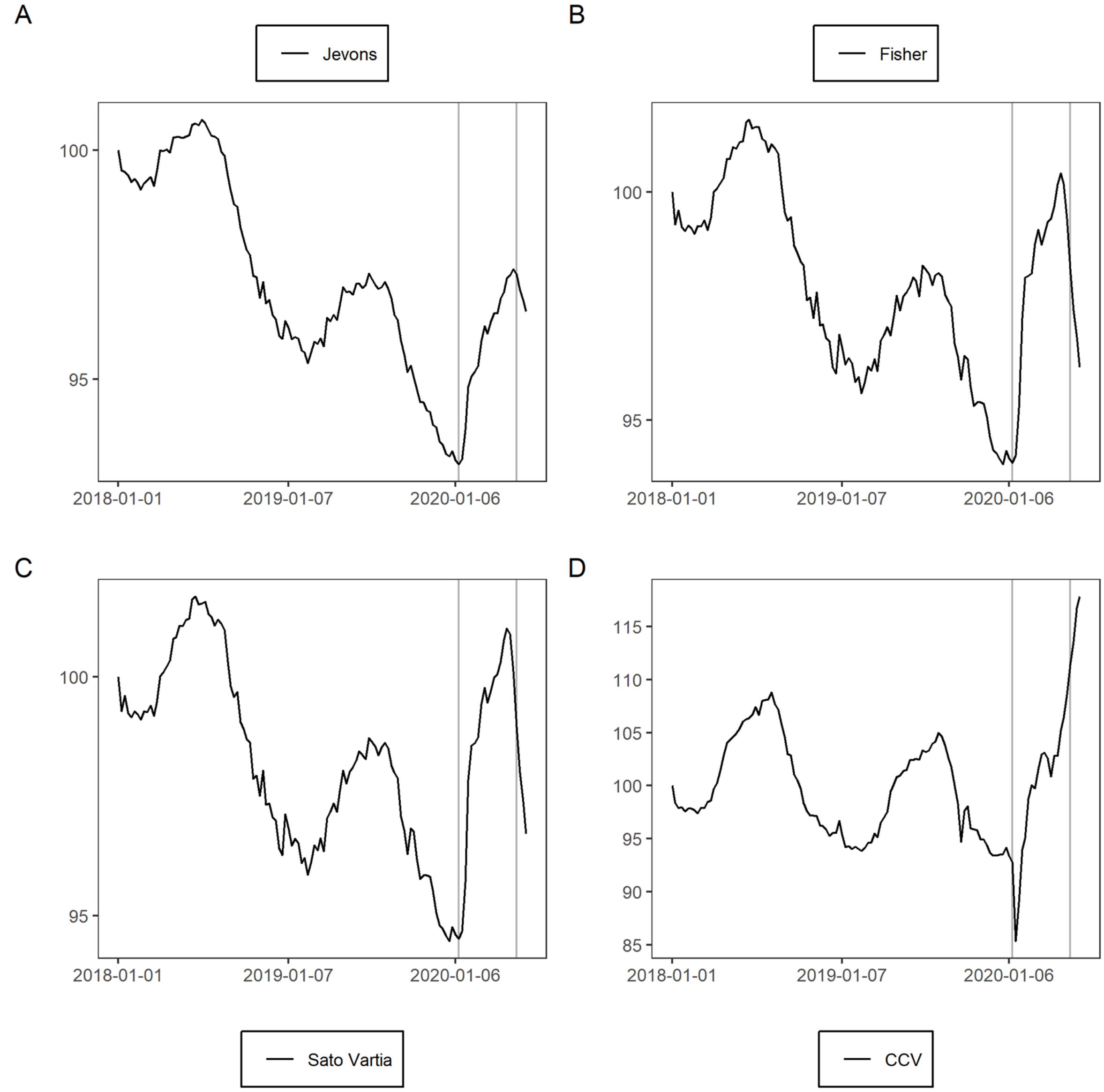
Note: The RMSD of taste is defined in (3.8), while the taste shock is defined as the second term of the R.H.S. of (3.1)

Figure 3.6: The RMSD of Taste Parameters and the Taste Shock.

3.6 Conclusion

By investigating the prices and quantities of face masks in Japan during the serious threat of the COVID-19 pandemic in 2020, we considered the impact of demand shocks on the COLI. We found that the demand shock that occurred during this period was large, which makes the Laspeyres index to be smaller than the Paasche index. The demand shocks measured by the changes in the taste parameters for the CES utility function created large taste effects that are not captured in the Sato-Vartia or superlative indexes such as the Fisher index. While the prices of face masks decreased in the Jevons and Fisher indexes in May 2020 by 0.06% and 0.76% per week, respectively, the COLI increased by 1.92% per week.

This study has several limitations. When the demand for face masks surged, face masks were rationed, which complicates the construction of the COLI. If we could identify a product that was not rationed during the sample period, it could be possible to adopt the method developed by Tobie and Houthakker (1950) and Neary and Roberts (1980) to construct the cost of living under rationing. However, as long as we use scanner data, the existence of rationing cannot be identified. If rationing occurs, COLI tends to be greater than the index without rationing. Therefore, our estimates of the cost of living in this study should be regarded as a lower bound. Second, although the CCV allows for variable taste parameters, we need to assume that the elasticity of substitution is constant over time, which is a restrictive assumption when a strong demand shock occurred. Although we could assume some form of stochastic processes



Note: The levels were obtained by taking the cumulative logged weekly changes in the chained indexes. The indexes were normalized to 100 in the first week of 2018.

Figure 3.7: Level of chained price indexes.

for the elasticity of substitution and conduct estimations, we have not been able to obtain stable and robust estimates. Finally, we did not discuss the variety of effects developed by Feenstra (1994) and Redding and Weinstein (2020) on COLI. Appendix B reports some results of the various effects; however, we have obtained unreasonably large negative variety effects on the COLI. Investigations of the effects of rationing, variable elasticities over time, and the effects of changing variety will be our next task.

In addition, this study has a potential bias due to supply constraints caused by the COVID-19 pandemic, which leads to missing price data. Diewert and Fox (2022a,b) point out that price information that becomes unobservable during the pandemic causes bias in COLI unless it is implemented by reservation prices, which are high enough to diminish demand. As this study excludes commodities that miss price information from the analysis, the calculated index may get bias.

Beyond the scope of this study, the missing price, and more broadly, entry and exit of goods is a major issue in the estimation of COLI. When consumers have a love of variety, the goods' entry and exit in the market affect consumer welfare and change COLI. In this approach, the pioneering work of Feenstra (1994) uses a CES utility function to estimate the effect of a new variety of imported goods on the cost-of-living index. This study presents the entry and exit using his method, as shown in Figure 3.8. However, Boldsen (2022) points out that the COLI capturing entry and exit effects is not always appropriate for use in the CPI. One reason is that the estimated effect of the entry and exit of goods varies greatly depending on the choice of utility function (Diewert and Feenstra, 2022). Another reason is that the objective of CPI is to measure the inflation rate of a good as a market signal, not a sign of consumer's welfare.

3.7 Appendix

3.7.1 Derivation of 3.4

The demand function generated by the Equation (3.5) can be written in terms of the expenditure share as follows,

$$\ln w_{it} = (\sigma - 1)\left(\ln P_t + \ln \varphi_{it} - \ln p_{it}\right), \tag{3.9}$$

where $\ln P_t = \ln C\left(p_t; \varphi_t\right)$ (A1) can be rewritten as

$$\ln \varphi_{it} = \frac{1}{\sigma - 1} \ln\left(\frac{w_{it}}{w_{1t}}\right) + \ln\left(\frac{p_{it}}{p_{1t}}\right) + \ln \varphi_{1t}. \tag{3.10}$$

Combined with the normalization condition in Equation (3.6), we can obtain Equation (3.4).

3.7.2 The Variety Effects

During the COVID-19 pandemic, due to the increasing demand for face masks, the variety of masks changed over time. One method of quantifying the effects of the changes in the product variety on the price index is provided by Feenstra (1994). Redding and Weinstein (2020) also consider a case wherein the variety of commodities changes over time. This is the second CUPI in their paper. The RW index, which is the COLI when the product variety changes, is defined as follows,

$$\ln RW\left(p_s, q_s, p_t, q_t\right) = \ln CCV\left(p_s, q_s, p_t, q_t\right) + \frac{1}{\sigma - 1}\left(\ln\left(\lambda_t^s\right) - \ln\left(\lambda_s^t\right)\right). \tag{3.11}$$

Here, λ_t^s is the ratio of the expenditure share of common products in the periods t and s to the total expenditure at time t,

$$\lambda_t^s = \frac{\sum_{i \in C_{t,s}} p_{i,r} x_{i,r}}{\sum_{i \in L_{t,r}} p_{i,r} x_{i,r}} \tag{3.12}$$

I_t: Set of all commodities at time t.

C_t: Set of common commodities at time t and s.

The second term on the right-hand side of Equation (3.11) is called the log λ ratio. Note that if we replace $\ln RW$ in Equation (3.11) with SV, then $\ln RW$ becomes the price index by Feenstra (1994). The movements of λ, the log λ ratio, Feenstra's index, and RW index are reported in the Figure 3.8. As is clear from the figure, the magnitudes of the various effects during the COVID-19 period are extremely large. We suspect that this occurs because of the product turnover of the identical products. Suppose face mask A was sold at store X. Then, the next week, mask A did not appear in the store because of the huge demand for the mask. Two weeks later, mask A returned to store X. Although, this turnover is not related to the introduction of the new product, the log ratio is interpreted as the introduction of new products, thus affecting the cost of living index.

Table 3.4: Index numbers.

	Sales	Jevons	Fisher	Sato Vartia	CCV
2019/1/7	0.190	-0.002	-0.003	-0.003	-0.014
2019/1/14	0.100	-0.003	-0.004	-0.004	-0.012
2019/1/21	0.039	0.001	0.002	0.002	0.001
2019/1/28	-0.021	0.000	-0.001	-0.001	-0.003
2019/2/4	-0.084	-0.003	-0.004	-0.004	0.002
2019/2/11	-0.026	0.000	0.001	0.001	-0.002
2019/2/18	-0.026	-0.003	-0.004	-0.004	-0.002
2019/2/25	-0.007	0.002	0.002	0.003	0.003
2019/3/4	-0.015	0.003	0.004	0.004	0.005
2019/3/11	-0.022	-0.001	-0.001	-0.001	0.000

Table 3.4: Index numbers.

	Sales	Jevons	Fisher	Sato Vartia	CCV
2019/3/18	-0.072	0.001	0.003	0.003	0.009
2019/3/25	-0.014	-0.002	-0.003	-0.003	-0.004
2019/4/1	-0.055	0.007	0.007	0.007	0.014
2019/4/8	-0.058	-0.001	0.001	0.001	0.005
2019/4/15	-0.041	0.002	0.002	0.002	0.005
2019/4/22	-0.123	-0.001	-0.002	-0.002	0.020
2019/4/29	-0.087	0.003	0.005	0.004	0.007
2019/5/6	-0.003	0.004	0.004	0.004	0.006
2019/5/13	-0.052	-0.001	-0.003	-0.003	0.002
2019/5/20	-0.048	0.000	0.003	0.003	0.004
2019/5/27	-0.038	-0.001	0.001	0.001	0.001
2019/6/3	-0.043	0.002	0.001	0.001	0.009
2019/6/10	0.016	0.000	0.002	0.002	0.000
2019/6/17	-0.011	-0.001	-0.001	-0.001	0.001
2019/6/24	-0.017	0.001	-0.003	-0.001	-0.001
2019/7/1	-0.011	0.003	0.007	0.004	0.009
2019/7/8	-0.003	-0.001	-0.001	-0.001	-0.002
2019/7/15	-0.025	-0.001	-0.001	-0.001	0.002
2019/7/22	-0.055	-0.001	-0.002	-0.002	0.006
2019/7/29	-0.037	0.000	0.002	0.002	0.002
2019/8/5	-0.038	0.001	0.000	0.001	0.008
2019/8/12	-0.005	-0.002	-0.001	-0.001	-0.003
2019/8/19	0.059	-0.002	-0.004	-0.004	-0.008
2019/8/26	0.055	-0.004	-0.002	-0.002	-0.012
2019/9/2	0.054	-0.001	-0.001	-0.001	-0.008
2019/9/9	0.062	-0.005	-0.008	-0.008	-0.017
2019/9/16	0.108	-0.003	-0.003	-0.003	-0.018
2019/9/23	0.184	-0.004	-0.005	-0.005	-0.037
2019/9/30	-0.123	0.002	0.005	0.006	0.030
2019/10/7	-0.008	-0.003	-0.001	-0.001	0.005
2019/10/14	0.157	-0.003	-0.006	-0.006	-0.021

Table 3.4: Index numbers.

	Sales	Jevons	Fisher	Sato Vartia	CCV
2019/10/21	-0.009	-0.003	-0.005	-0.004	-0.001
2019/10/28	0.055	0.000	0.001	0.001	-0.001
2019/11/4	0.091	-0.002	0.000	0.000	-0.009
2019/11/11	0.033	0.000	0.000	0.000	0.000
2019/11/18	0.033	-0.003	-0.003	-0.003	-0.006
2019/11/25	0.051	0.000	-0.004	-0.005	-0.007
2019/12/2	0.028	-0.003	-0.003	-0.003	-0.003
2019/12/9	0.028	-0.001	-0.001	-0.001	0.000
2019/12/16	-0.009	-0.002	-0.001	-0.002	0.001
2019/12/23	0.012	-0.001	-0.001	-0.001	0.001
2019/12/30	-0.096	0.001	0.003	0.003	0.007
2020/1/6	0.073	-0.002	-0.002	-0.002	-0.009
2020/1/13	0.030	-0.001	-0.001	-0.001	-0.006
2020/1/20	0.539	0.001	0.002	0.002	-0.084
2020/1/27	0.435	0.007	0.011	0.011	0.042
2020/2/3	-0.506	0.010	0.020	0.022	0.054
2020/2/10	-0.279	0.002	0.009	0.007	0.011
2020/2/17	-0.099	0.001	0.000	0.001	0.039
2020/2/24	-0.104	0.001	0.001	0.001	0.013
2020/3/2	-0.023	0.006	0.007	0.007	-0.003
2020/3/9	-0.082	0.003	0.003	0.003	0.018
2020/3/16	-0.066	-0.002	-0.003	-0.003	0.014
2020/3/23	0.009	0.003	0.002	0.002	0.002
2020/3/30	0.091	0.002	0.003	0.003	-0.005
2020/4/6	-0.031	0.000	0.001	0.001	-0.017
2020/4/13	0.006	0.003	0.003	0.003	0.019
2020/4/20	0.038	0.002	0.005	0.005	0.000
2020/4/27	0.126	0.003	0.002	0.002	0.023
2020/5/4	-0.087	0.001	-0.002	-0.001	0.011
2020/5/11	0.151	0.001	-0.008	-0.008	0.020
2020/5/18	0.029	-0.001	-0.011	-0.012	0.026

Table 3.4: Index numbers.

	Sales	Jevons	Fisher	Sato Vartia	CCV
2020/5/25	0.079	-0.003	-0.009	-0.010	0.019
2020/6/1	-0.001	-0.002	-0.006	-0.006	0.028
2020/6/8	-0.082	-0.003	-0.007	-0.007	0.009

Note: The weekly rates of change (%) of the chained indexes.

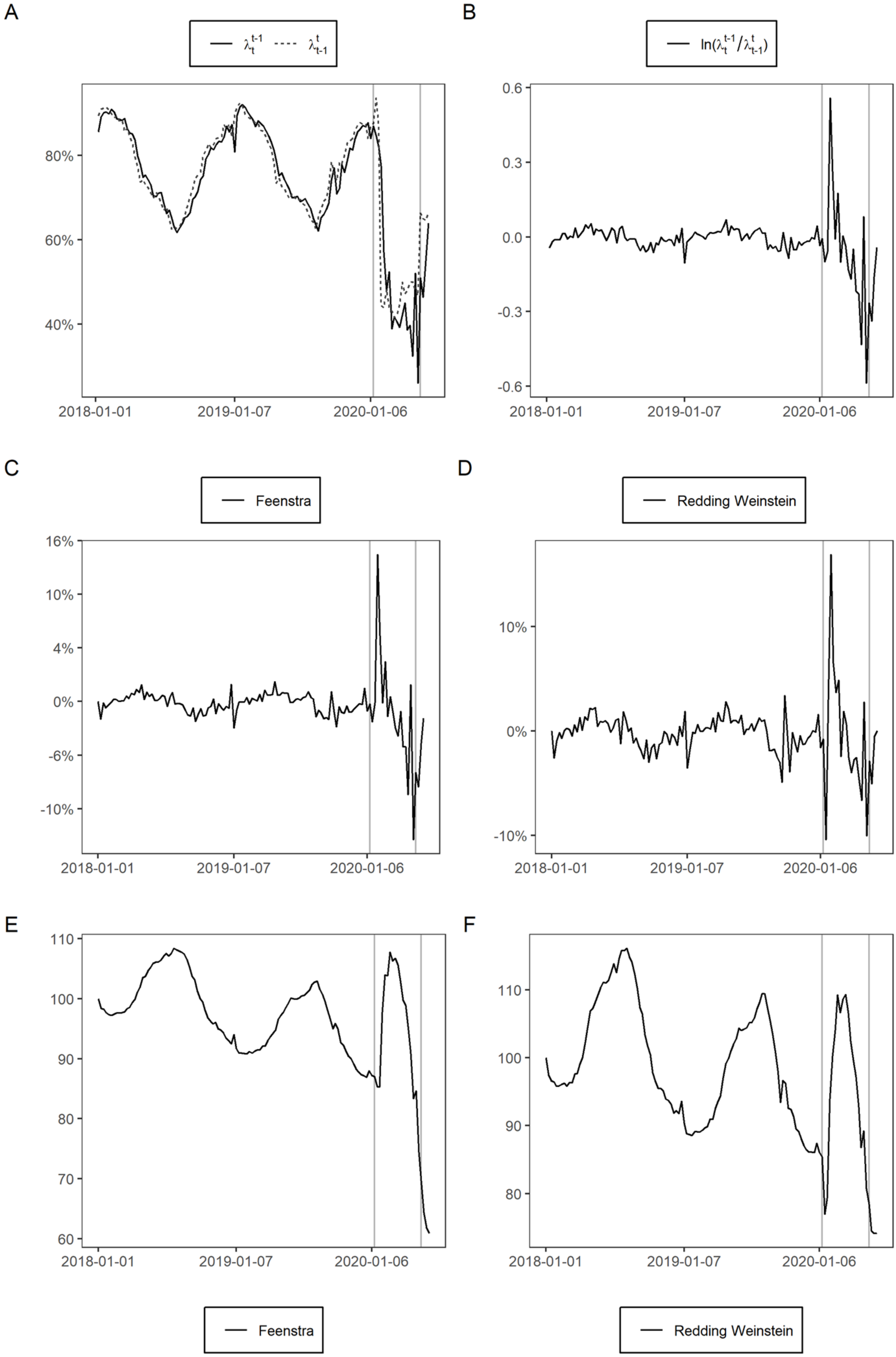


Figure 3.8: Other indexes.

Chapter 4

The Effect of Seasonality on Elementary Index[1]

4.1 Introduction

Seasonal changes significantly impact economic activities such as production, consumption, and trade, resulting in cyclical variations in price and quantity throughout the year. Importantly, seasonal fluctuations influence many commodities. For example, to calculate the seasonally adjusted Consumer Price Index (CPI), the Bureau of Labor Statistics makes seasonal adjustments to 45 of the 81 price series.[2][3] These fluctuations can be substantial, as illustrated by the significant and cyclical monthly variations in the price and consumption of tomatoes in Japan in Figure 4.1. Thus, it is crucial to consider the impact of seasonality to conduct a more accurate and precise economic analysis.

Seasonal fluctuations in price and quantity pose challenges in accurately measuring economic activity. Adjusting for seasonality in quarterly and short-term indicators is necessary to extract the signal, and even annual indicators can be affected by seasonality. To measure the annual performance of an economic activity in real terms (e.g., real gross domestic product (GDP), real aggregated consumption, etc.), statistical authorities calculate the annual prices and quantities of each commodity for the elementary prices and quantities. However, seasonal goods pose a challenge because different calculation methods can produce significantly different results due to seasonal fluctuations. For example, the annual price of tomatoes in Figure 4.1 and its growth rate can vary largely depending on the calculation method. The simple monthly price averages

[1]This chapter is based on Inoue (2023).

[2]Bureau of Labor Statistics, "Consumer Price Index: Components for Seasonal Aggregation to all items," https://www.bls.gov/cpi/seasonal-adjustment/questions-and-answers.htm, accessed January 30, 2023

[3]In the Japanese CPI, seasonally adjusted indices are created by making seasonal adjustments to the aggregated indices, and individual seasonally adjusted price series are not published.

Note: This figure shows the monthly price and quantity of tomatoes purchased in Japan. The price data is taken directly from the Japanese CPI, and the quantity is calculated by dividing the expenditure from the Family Income and Expenditure Survey (FIES) by the price.

Figure 4.1: Price and quantity of tomatoes purchased by households in Japan

were 110 yen in 2016 and 100 yen in 2017, with a growth rate of -8.9%. However, the quantity-weighted averages indicated monthly prices of 105 yen in 2016 and 97 yen in 2017, with a growth rate of -7.9%. The price level diverges by 3–5% and the growth rates differ by 1% point. Thus, this example highlights the importance of considering seasonality when calculating annual indices.

The conventional method of aggregating seasonal goods has certain limitations. There are two common methods of annual aggregation. One is to use the unweighted average of monthly prices.[4] In general, it is desirable to include the expenditure or quantity share in the calculation of the elemental index in order to reflect the economic importance (CPI manual, 2020). In particular, since sales of seasonal products vary significantly by season, it is problematic to equalize the weights for any period. The annual price and quantity of seasonal goods are also calculated using unit prices from the monthly quantity-weighted average prices (also known as the unit value price index), and a summation of monthly quantities.[5]

However, this method of aggregation is only suitable for homogeneous goods and fails to account for heterogeneity (Silver, 2010). Therefore, the latter conventional method is inappropriate and potentially biased when applied to seasonal products that are heterogeneous in different seasons.

[4]For exmaple, the U.K. uses this method for calcurating the average annual price in CPI.

[5]The aggregation method of price and quantity is usually determined in pairs so that their product is equal to the annual expenditure.

Two types of consumer behavior may deviate from the consumption of homogeneous goods when it comes to seasonal products. Firstly, a product with high value during a particular season, such as turkeys during the Christmas season in the U.S., would not be considered a homogeneous good, as consumers would receive different utility by consuming it throughout the year. For reflecting this consumer's valuation, monthly weights are good to aggregate such seasonal products. Secondly, seasonal goods in a particular season may not be perfect substitutes for those in another season. Households would not necessarily consider that only the annual quantity of consumption matters, disregarding their monthly consumption. Instead, maintaining a balanced monthly consumption can be beneficial to households. Assuming homogeneity, which set a perfect substitution between seasons, we ignore the potential benefits of this consumption smoothing. Our guess is that the above-mentioned heterogeneity of seasonal goods is not negligible for accurate measurement of consumption.

This study quantifies the neglected effect of seasonality on elemental-level price and quantity. To identify this hidden effect, we analyze consumption aggregation using the economic approach to index number theory. In such an approach, the aggregated quantity is defined by the utility of consumers and the aggregated price by the minimum expenditure given at a certain utility level. Importantly, to be justified under this approach, the standard elemental-level temporal aggregation method requires that an annual utility is a simple summation of monthly consumption, which is a severe restriction.

We propose two alternative methods that allow more flexible utility functions. The first method uses a Constant Elasticity of Substitution (CES) utility function, which is a generalized form of simple summation utility that allows unequal weights and imperfect substitution. For using the CES utility function, we estimate the monthly weights and elasticities of substitution for each good. These estimated parameters test whether the simple summation form and the assumption of homogeneity are supported by the data. From the estimated parameters, we can also calculate the annual quantity and hidden seasonal effects by comparing them with the simple summation quantity.

The second method uses the Fisher quantity index, a class of superlative indices, which allows the aggregation of quantities using a flexible utility function (Diewert, 1976).[6] A major advantage of this method is that it does not require a parameter estimation. Note that while the Fisher quantity index can calculate the rate of change of the aggregate quantities, it cannot quantify the level. Therefore, this method allows us to quantify the hidden effects of seasonal

[6]Other superlative indexes can also be used. In this study, the Fisher index is used because it is one of the most prominent superlative indexes and has various good properties in terms of the test approach to index number theory.

changes, but not the full extent of those effects.

Our analysis reveals that overlooked seasonality significantly affects elemental-level and aggregated prices and quantities. We use the aforementioned methods to analyze the price and expenditure data for fresh foods in Japan's CPI. Focusing our analysis on fresh food products, which exhibit large seasonal fluctuations and long-term change in the fluctuations, we quantitatively evaluate our proposed method. The empirical analysis, conducted using the CES framework, reveals the neglected impact of seasonality. The results of the CES parameters suggest that seasonal goods in different seasons are heterogeneous for almost all fresh foods. The CES function's annual aggregated quantities differ from the simple calculation by as much as -15% to +15%. This quantity deviation exhibits significant variation over time, with the CES-aggregated quantity falling below the simple calculation by the late 1980s. However, after 1990, when seasonal fluctuations decreased, the CES-aggregated quantity generally exceeded the simple calculation. A Fisher-type elemental quantity index indicates similar long-term changes, with a higher growth rate than that of the simple calculation index.

Our study, like Diewert et al. (2022), proposes methods for calculating annual indices that consider the heterogeneity caused by seasonality. The main difference between these studies is the calculation procedure for the index. While they directly aggregates monthly-commodity-level goods to an annual price index, our study conducts commodity-level aggregation of monthly data to commodity-level annual prices and quantities and then aggregates these annual prices and quantities to higher-level indices. Our research offers a methodology that can be easily incorporate into the current deflators in System of National Accounts (SNA) by refining the elementary aggregation, whereas the their method requires a more comprehensive revision of the implementation. However, our method requires an additional assumption of separability on the consumer's utility function compared to their method. Therefore, our research and theirs complement each other by having different advantages that contribute to the accurate measurement of the hidden effect of seasonality.

Our study offers two unique contributions to the existing literature on index number theory, focusing on the unit value price index, which is calculated as the change in average price per unit. While various studies have explored the advantages and disadvantages of the unit value index (for example Párniczky, 1974; Holmes, 1973; Diewert, 1995; Balk, 1998; Bradley, 2005; Silver, 2009a,b, 2010; Diewert and von der Lippe, 2010), Silver (2010) concludes that the unit value index should be used for homogeneous goods but not for heterogeneous goods, based on the perspective of price index theory. Thus, the classification of heterogeneous and homogeneous goods is a crucial problem for the application of the unit value index. Our study's first contribution is the application of statistical methods to identify the homogeneity of goods, which is a novel approach in current literature. Secondly, we focus on the seasonal heterogeneity of household consumption goods, which distinguishes our research from previous studies that mainly focus on trade statistics. By examining the seasonal effect overlooked by unit value price indices in household consumption, our study broadens the scope of the problems highlighted in previous literature.

The remainder of this chapter is organized as follows. Section 2 describes the elemental-level aggregation conducted by the statistical authorities. Section 3 presents a method to capture the effect of seasonality overlooked by the conventional method. Further, Section 4 describes the data, and Section 5 shows the empirical results. Lastly, the conclusions are presented in Section 6.

4.2 The elemental-level temporal aggregation in the conventional method

This section explains the actual procedure of temporal aggregation by statistical authorities to create commodity-level indices. Computing an index number typically involves two stages: the creation of elemental-level prices and quantities, and the aggregation of them to form an index number. Creating elemental-level data frequently incorporates temporal aggregation, particularly when constructing annual indices. Unfortunately, the conventional methods used for elemental-level temporal aggregation needs strong assumption to be justified when applied to seasonal goods.

The process of creating price and quantity indices from raw data is divided into two stages. We focus on the first stage, which involves temporal aggregation of seasonal goods. Here, the raw data is aggregated to create elementary indices and commodity-level price and quantity (or price and expenditure) datasets. Depending on available data, type of index, and commodity property, statistical authorities conduct various types of elemental-level aggregation. This includes temporal aggregation from monthly data to annual prices and quantities. In the second stage, the commodity-level price quantity is aggregated into a higher-level index. This two-step aggregation has been adopted to create several indices, such as GDP, CPI, and Producer Price Index.[7] Therefore, accurately measuring economic activity in higher-level indices depends heavily on the temporal aggregation of seasonal goods.

Elemental-level temporal aggregation is exemplified in the initial stage of computing annual

[7] These index procedures are confirmed by manuals such as Bureau of Economic Analysis (2021), International Labour Office et al. (2020), and International Monetary Fund (2004)

indices like GDP and real consumption. The first stages of these indices involve aggregating monthly (or shorter period) data to form annual price and quantity (or price and expenditure) data, which is then aggregated in the second stage.[8]

Index formulas typically used for elemental-level aggregation have some limitations when applied to temporal aggregation of seasonal goods. There are generally two types of element-level aggregation procedures. One type calculates the aggregated price from price information and subsequently calculates the aggregated quantity by dividing the annual expenditure by this price. To apply this method for elemental-level temporal aggregation, the arithmetic or geometric mean of monthly prices is used to obtain an annual price. This approach is unsuitable for temporal aggregation of seasonal goods because it assigns equal weights to peak and off-peak seasons.

The second type uses price and quantity (or price and expenditure) data, such as the unit value price index. For temporal aggregation, the aggregation process by unit value price index is straightforward: the aggregated quantity is determined by summing monthly quantities, and the aggregated price is determined by calculating the average price per quantity.[9] This method of elemental-level temporal aggregation treats goods in different months as homogeneous, which is a crucial feature. In the next section, we will discuss the economic conditions under which this feature is justified.

4.3 Model for measuring the effect of seasonality

This section presents the model and method to quantify the hidden impacts of seasonality using the economic approach to index number theory. Specifically, we aim to design a consumer price index to measure the cost of living and a real consumption index to measure the utility of households. We model the utility function of households to capture annual consumption from an economic perspective. The utility function incorporates a two-step aggregation process, following the conventional method of statistical authorities. Using this model, we derive the desirable elemental-level temporal aggregation. Additionally, we present two methods to quantify the impact of seasonality in lower-level aggregation for quantitative analysis.

[8] A similar example uses a temporally averaged value as the reference price and quantity. The CPI manual recommends using the annual average price as a reference because the monthly prices of some commodities may not be completely accessible and can exhibit relatively high or low variations (International Labour Office et al., 2020).

[9] The aggregation method for price and quantity is chosen to ensure that the total expenditure is equal to the product of the aggregated price and quantity.

4.3.1 General framework

We construct a model of a representative household that maximizes its annual utility by making monthly consumption decisions at the commodity level. This household has the following nested utility function:

$$
\begin{aligned}
&U(q_1, q_2, ..., q_n)\\
&q_i = u_i(q_{i1}, q_{i2}, ..., q_{i12}) \quad (i = 1, 2, ..., n),
\end{aligned}
$$

where q_i represents the aggregated annual consumption of commodity i, and q_{im} represents the monthly consumption of commodity i in month m.[10] $U(\cdot)$ represents the higher-level utility function denoting the annual utility, and $u_i(\cdot)$ represents the lower-level utility function for commodity i. The household optimizes its consumption to maximize utility, within a given budget M:

$$
\begin{aligned}
\max_{\{q_{im}\}_{1\leq i\leq n, 1\leq m\leq 12}} \quad & U(q_1, q_2, ..., q_n)\\
\text{s.t.} \quad & q_i = u_i(q_{i1}, q_{i2}, ..., q_{i12}) \quad (i = 1, 2, ..., n)\\
& \sum_{i=1}^{n} \sum_{m=1}^{12} p_{im} q_{im} \leq M,
\end{aligned}
$$

where p_{im} represents a price of commodity i in month m. Following the economic approach to index number theory, we aim to measure the cost function at a given level of utility as the aggregated price and to measure utility as the aggregated quantity.

We can formalize the above utility maximization problem as one with a non-nested utility function by using appropriate elementary prices. Considering two-stage budgeting, we can convert the maximization problem to a higher-level budgeting problem incorporating optimal consumption of lower-level commodities as follows:

$$
\begin{aligned}
\max_{\{e_i\}_{1\leq i\leq n}} \quad & U(q_1, q_2, ..., q_i)\\
\text{s.t.} \quad & q_i = v_i(p_{i1}, p_{i2}, ..., p_{i12}, e_i) \quad (i = 1, 2, ..., n)\\
& \sum_{i=1}^{n} e_i \leq M,
\end{aligned}
$$

[10]Our model focuses on annual utility, as the goal of this study is to measure annual consumption. Therefore, we assume separability between consumptions in different years.

where $v_i(\cdot)$ is the indirect utility function such that,

$$v_i(p_{i1}, p_{i2}, ..., p_{i12}, e_i) = \underset{\{q_{im}\}_{1 \le m \le 12}}{\text{argmax}} \; u_i(q_{i1}, q_{i2}, ..., q_{i12}) \quad \left(\text{s.t.} \sum_{m=1}^{12} p_{im} q_{im} \le e_i\right).$$

If all of $u_i(\cdot)$ are homothetic functions, then the problem becomes simpler,

$$\begin{aligned} &\max_{\{q_i\}_{1 \le i \le n}} \; U(q_1, q_2, ..., q_i) \\ &\text{s.t.} \quad \sum_{i=1}^{n} q_i p_i \le M, \end{aligned}$$

where p_i is defined as

$$p_i = C(p_{i1}, p_{i2}, ..., p_{i12}, 1),$$

where $C(\cdot)$ denotes the cost function. We can convert the original problem with a nested function into the standard utility maximization problem if p_i and q_i are measured accurately. Therefore, accurately identifying $u_i(\cdot)$ ensures the correct measurement of the lower-level aggregation, p_i and q_i, which contributes to the accurate measurement of the annual aggregate price and quantity.

Note that to justify using the unit value index for lower-level aggregation, $u_i(\cdot)$ must take a restrictive functional form. In this aggregation method, the aggregated quantity at the elementary level is expressed as a summation of the monthly quantity, meaning $u_i(\cdot)$ is specified as follows:

$$u_i(q_{i1}, q_{i2}, ..., q_{i12}) = \sum_{m=1}^{12} q_{im}. \tag{4.1}$$

This utility function assumes that all $q_{i,m}$ are homogeneous. Therefore, the household's demand for goods does not increase in any season.[11] Moreover, assuming that any pair of monthly quantities are perfect substitutes eliminates a household's incentive to smooth consumption. In the subsequent subsection, we introduce the use of more flexible functions to improve the measurement of lower-level aggregation.

Although the length of temporal aggregation of our model is 12 months, it is changeable. For example, if we consider quarterly estimates of economic statistics, it is reasonable to set three months to correspond to the statistics. In this case, we would model the elemental-level

[11] While the demand remains constant, the quantity demanded can vary significantly. This is because, under this functional form, it is optimal to consume only in the month with the lowest price.

aggregation for the three time periods using the CES function. Similarly, it is possible to model longer periods, such as 18 or 24 months. By considering such a long-term model, control variables such as fixed effects can be more flexibly incorporated into the regression model. However, considering such a long aggregation period might raise the concern of no statistics corresponding to it. Thus, the relevance of empirical work to economic statistics will be weak. For this reason, we construct a model with a 12-month aggregation so that it does not deviate from the framework of annual statistics.

4.3.2 Empirical strategy

We used two different approaches for lower-level aggregation. The first approach uses the CES utility function that allows for unequal weights and imperfect substitution but requires parameter estimation. The second approach uses a superlative index that captures the effect of the change in seasonality without parameter estimation.

Implementation I: CES function

As the first implementation, we introduce the CES utility function for lower-level aggregation:

$$u_i(q_{i1}, q_{i2}, ..., q_{i12}) = 12 \left(\sum_{m=1}^{12} \alpha_{im} q_{im}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}} \quad \left(\varepsilon_i, > 0, \alpha_{im} \geq 0, \sum_{m=1}^{12} \alpha_{im} = 1 \right),$$

where ε_i represents the elasticity of substitution and α_{im} are the weighting parameters. These parameters are required to estimate.

The CES utility function specification has two advantages. Firstly, it includes the simple summation utility in Equation (4.1) as a special case when $\varepsilon_i \to \infty$ and $\alpha_{im} = 1/12$ (for all m). Therefore, it serves as a generalized form of the standard aggregation method.

Secondly, by utilizing the CES utility function, we can derive a simple demand curve form. Taking the first-order condition and the difference between the two months m and l, the demand curve is as follows:

$$\log p_{im} - \log p_{il} = -\frac{1}{\varepsilon_i}(\log q_{im} - \log q_{il}) + (\log \alpha_{im} - \log \alpha_{il}).$$

We estimate this demand curve and use the estimates to recover the parameters. We denote the base period as month b and define the base-adjusted variable as $\tilde{x}_m = \log x_m - \log x_b$. The regression model can then be written as

$$\tilde{p}_{mt} = b\tilde{q}_{mt} + \tilde{\alpha}_m + w_{mt}, \text{ (for } m \neq b) \tag{4.2}$$

where w_{mt} represents the newly added error term. Here, we drop the commodity index i for simplicity and add the year index t. We assume that the weighting parameter α_{im} is constant for the entire period. Based on this assumption, we estimate the weighting parameters from the monthly dummy variables. From these estimates, we can retrieve the parameters of the utility function, and using the null hypothesis we can test for equal weight $H_0 : \alpha_m = 0$ (for all m), for perfect substitution, $H_0 : b = 0$, and for simple summation utility function $H_0 : \alpha_m = 0$ (for all m) and $b = 0$.

A limitation of the CES utility function is that it does not consider change in seasonality from the demand side. Parameter α_{im} represents the fixed effect of seasonal demand, but is assumed to be constant over time. This means that seasonal demand changes are assumed to occur in the same manner every year. Importantly, our model generates changes in seasonality by supply-side factors.

Implementation II: superlative index

The second implementation uses a superlative index for elemental-level temporal aggregation. A superlative index is an index number based on a function capable of approximating twice-differentiable functions to a second-order level (Diewert, 1976). The functional form behind the superlative index is more flexible than that of simple summation utility. Therefore, the superlative index is expected to capture the hidden effects of seasonality.

Among the various candidates for superlative indices, we chose the Fisher index because of its popularity, such as the US SNA. Using the Fisher Price index and Fisher Quantity index, the elementary price and quantity relatives are defined as follows:

$$\begin{aligned}
\frac{p_i^{t+1}}{p_i^t} &= \sqrt{\frac{\sum_{m=1}^{12} p_{im}^{t+1} q_{im}^t}{\sum_{m=1}^{12} p_{im}^t q_{im}^t} \frac{\sum_{m=1}^{12} p_{im}^{t+1} q_{im}^{t+1}}{\sum_{m=1}^{12} p_{im}^t q_{im}^{t+1}}} \\
\frac{q_i^{t+1}}{q_i^t} &= \sqrt{\frac{\sum_{m=1}^{12} p_{im}^t q_{im}^{t+1}}{\sum_{m=1}^{12} p_{im}^t q_{im}^t} \frac{\sum_{m=1}^{12} p_{im}^{t+1} q_{im}^{t+1}}{\sum_{m=1}^{12} p_{im}^{t+1} q_{im}^t}}.
\end{aligned}$$

Diewert (1976) states that the Fisher index underlies the following utility function,[12]

$$u_i(q_{i1}, ..., q_{i12}) = \left(\sum_{j=1}^{12} \sum_{k=1}^{12} q_{ik} a_{jk} q_{ij} \right)^{\frac{1}{2}} \quad (a_{jk} = a_{kj}),$$

where a_{jk} represents the parameter of the utility function that does not require estimation. In contrast, CES implementation requires estimating the utility function's parameters.

The superlative index cannot identify the impact of seasonality on the price (quantity) levels, as it only observes the change in the price (quantity). Therefore, this implementation can only measure the hidden impacts of seasonal changes.

4.4 Data

This section covers the data used to quantify the impact of seasonality, which consists of the price and expenditure of Japanese representative households. This data is also used for Japan's official CPI. We mainly focus on fresh food data in the dataset as it exhibits significant seasonal fluctuations. Additionally, we provide some descriptive statistics for the data.

4.4.1 Price and expenditure data of a representative household

We use monthly commodity-level prices, provided by the Japanese official CPI, as price data for the empirical analysis. The advantage of CPI data is that it is produced through a consistent methodology over an extended period. The CPI's nationwide commodity price is an expenditure-weighted average of municipal-level price data obtained by rigorous procedural sampling. These expenditure weights were derived from the Family Income and Expenditure Survey (FIES), which we also use in this study.

In addition to the price data, we use the FIES as our expenditure data. The FIES is a comprehensive survey of household expenditures that regularly replaces its sample and maintains a sample size of approximately 9,000 households. The surveyed households are required to maintain detailed daily expenditure records, which are subsequently classified by the statistical authority to generate commodity-based expenditure data. We use data from households with two or more family members because it is available over the long term.

To create price expenditure data, we match these two datasets by commodity names. The matching is conducted monthly from 1975 to 2020, and only includes commodities that have not been added or dropped after 1975. Furthermore, we exclude data for any month that has

[12] Following Diewert (1976), this statement was first proven by Konüs and Byushgens (1926).

Table 4.1: Length of the observed month of fresh foods

Mean	SD	Min	Max	#month=12	N
11.1	2.6	3	12	87.3%	63

been added or omitted in the initial survey for each commodity. We also exclude category-level matching (e.g., both data contain "other processed seafood products") and single vs. multi-matching cases (e.g., CPI has "apple A" and "apple B" but FIES has only "apple"). From the matched data, we calculated the quantity by dividing the expenditure by the price. For the statistical analysis, we restricted our focus to fresh foods, which are highly affected by seasonality. It is important to note that the official CPI provides annual prices for fresh food as a quantity-weighted average of monthly prices. Therefore, we use the quantity-weighted average price (unit value price index) and simple summation as the benchmark for annual price and quantity in our elemental-level temporal aggregation.

Note that we eliminate data that miss either price or expenditure. For example, for a product whose price is only observed in the summer season and whose expenditure is observed every month, we only use the data of this product in the summer period. A product that misses price or quantity data by seasonality is called a strongly seasonal product. On the other hand, a product with seasonal fluctuations but no missing data is called a weakly seasonal product. The main focus of this study is the weakly seasonal product. We also use data from strongly seasonal products but do not extensively analyze their seasonality in terms of disappearance from the market.

4.4.2 Summary statistics

In this subsection, we present descriptive statistics of the data.

Most commodities in our dataset of fresh foods are observed throughout the year; however, a few exhibits are missing data during certain months. Table 4.1 summarizes the availability of price and expenditure data for each commodity within a year. While 87.3% of all commodities had completed data, some fresh food items were observed for only three months of the year. Due to data construction, the observed month of each commodity is fixed throughout the period.

Expenditure share on selected fresh food data has decreased over time, from approximately 10% in 1975 to half in the most recent data. Table 4.2 summarizes the share of fresh foods in total expenditures for a given year. The expenditure on fresh foods in our dataset accounted for 9.50% of total expenditure in 1975. However, this number decreased significantly to 5.48% in 1995 and further to 5.07% in 2015. Additionally, the mean, median, and maximum values of

Table 4.2: Percentage of the total expenditure of fresh foods

Year	Total	Mean	Min	Median	Max
1975	9.50	0.15	0.01	0.09	0.97
1995	5.48	0.09	0.02	0.07	0.61
2015	5.07	0.08	0.01	0.05	0.45

Table 4.3: Summary statistic of standard deviation of fluctuation within a year

data	year	Mean	Min	q25	Median	q75	Max
1. price	1975	12.77	0.10	1.28	5.62	20.51	55.57
	1995	7.74	0.20	0.58	3.91	13.63	33.47
	2015	7.21	0.22	0.97	3.14	11.19	37.24
2. expenditure	1975	35.63	8.95	17.76	30.64	39.22	101.18
	1995	25.32	6.04	15.53	22.30	30.54	76.27
	2015	24.58	4.10	13.65	21.53	30.60	92.02
3. quantity	1975	39.82	6.42	17.88	28.41	45.14	132.92
	1995	27.39	4.02	14.63	22.87	32.73	83.58
	2015	26.88	3.02	13.02	23.31	31.58	88.75

each fresh food item's share have also declined. Two primary factors underlie the decrease in share. The first is the decline in food expenditure share due to Japan's economic growth. The country's rapid economic development and high growth rate from the post-war era to around 1990 resulted in a decrease in food expenditure share. The second factor is the expansion of the FIES and CPI surveys in terms of coverage and temporal scope, which this study does not utilize. It is important to note that this study has a limitation in that it does not fully account for changes in seasonality expanding the observable period.

Seasonal fluctuations of fresh foods declined throughout our dataset's period: Table 4.3 illustrates the intra-annual variation calculated by the following formula for each commodity's price, expenditure, and quantity.

$$\sqrt{\frac{1}{\# M_i} \sum_{m \in M_i} (x_{itm} - \overline{x}_{it})^2},$$

where x denotes the data; i, t, and m represent commodity, year, and month, respectively. $\overline{x}_{it}$ denotes the average of x for commodity i in year t, M_i represents the set of observable month for commodity i, and $\# M_i$ represents the length of the observable month. The table illustrates that the mean and median of the seasonal variations in price, expenditure, and quantity have decreased over time.

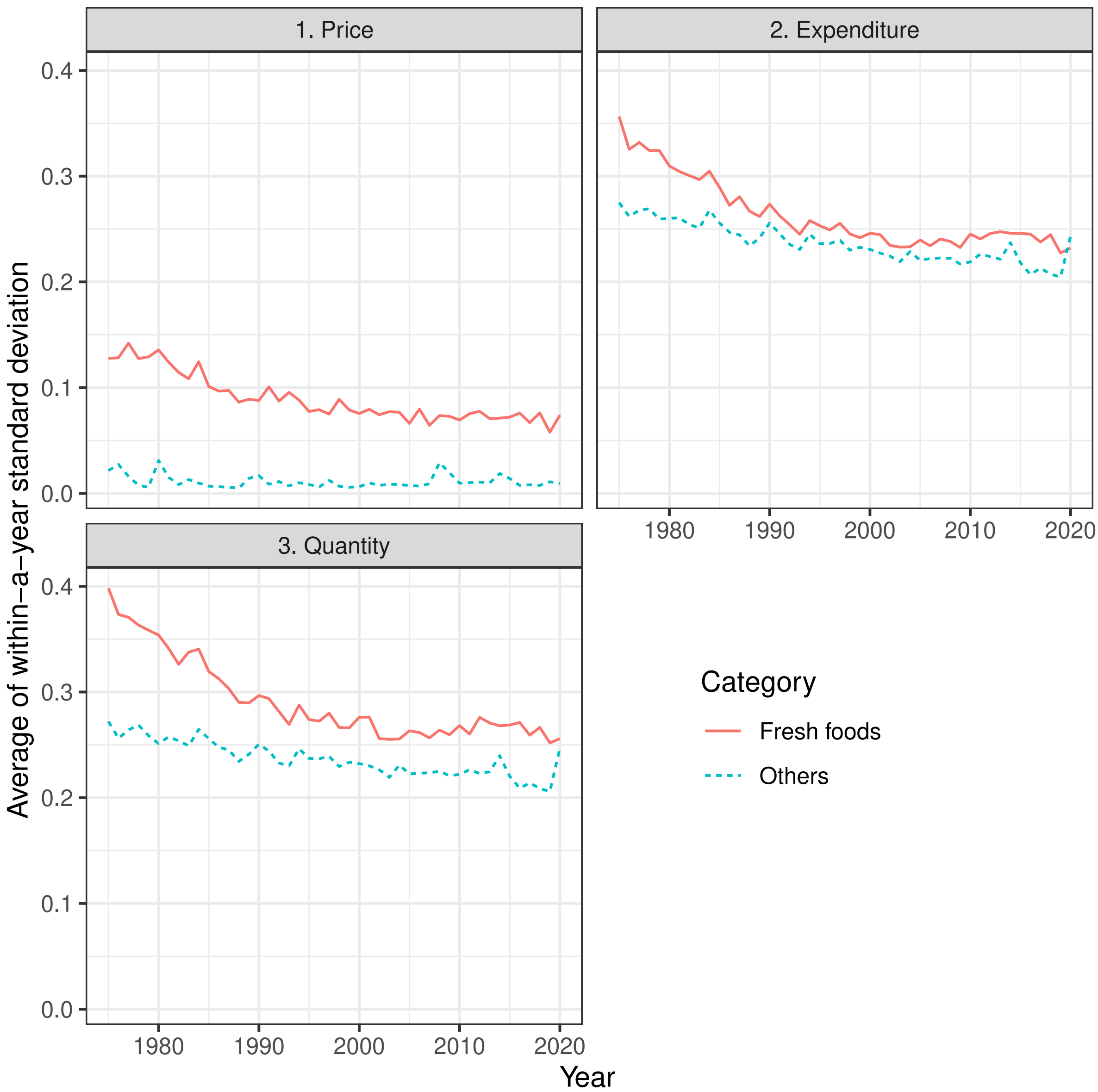


Figure 4.2: Averages of within year standard deviations

Fresh foods, particularly their prices, experience larger seasonal fluctuations compared to other commodities. Figure 4.2 compares the seasonal fluctuations of fresh foods to other commodities, which are calculated in a similar manner. Even though seasonal price fluctuations of fresh foods gradually decreased over time, they are still more substantial than other commodities. Similarly, seasonal expenditure fluctuations gradually decreased and are at nearly identical levels. The quantity fluctuations discrepancy has been gradually diminishing in recent years, yet it still exhibits a certain degree of difference.

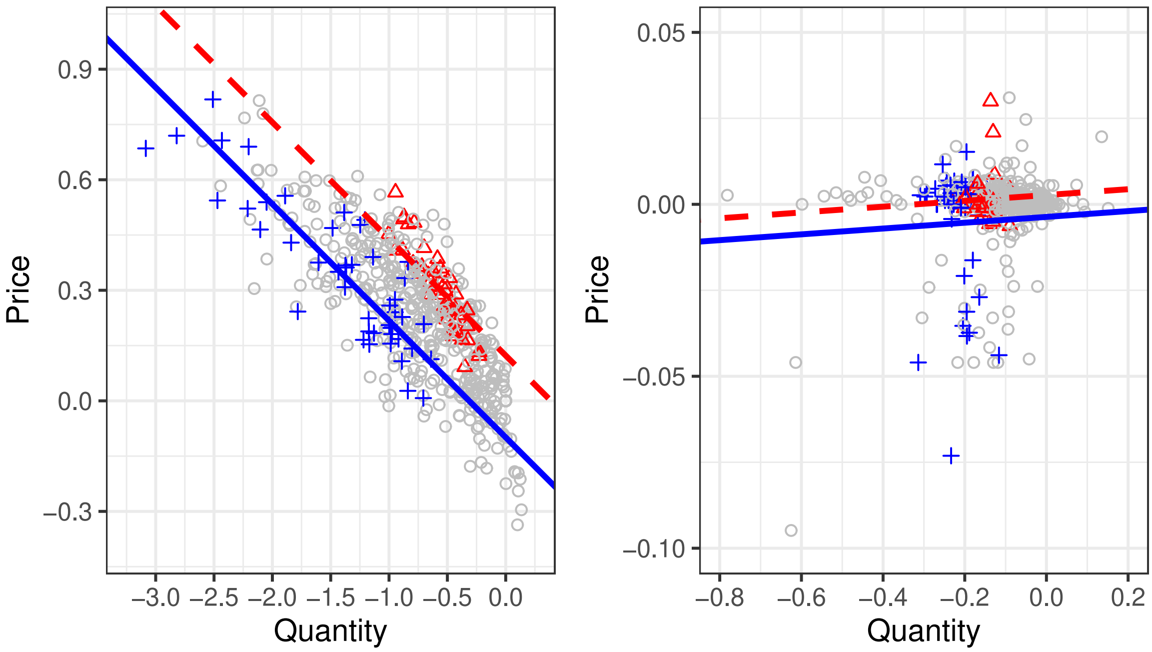


Figure 4.3: Examples of estimation results (left: tomato, right: milk)

4.5 Results

This section presents our findings on the impact of seasonality on elemental-level temporal aggregation, using two methods. The empirical results show the effects of seasonality that are not captured by summation-based quantity aggregation. Notably, long-term decreasing seasonal fluctuations increased the aggregated quantity at both the elemental and aggregated levels.

4.5.1 The result of implementation by CES utility function

To obtain the parameters of the CES utility function, we estimate the demand curve shown in Equation (4.2). We select the month with the highest average expenditure share for each commodity as a base month. We use the ordinary least squares method to estimate this equation.[13] We compute the robust standard error using the method proposed by White (1980). Additionally, to test the null hypothesis on multiple parameters, we use the likelihood-ratio test.

To illustrate the estimation results of the CES utility function, we present examples of commodities with and without significant hidden seasonal effects: for tomatoes and milk, respectively. Figure 4.3 displays the estimated demand curves, while Table 4.4 presents the estimated

[13]The identification condition is that the idiosyncratic shock to the demand function is uncorrelated with the monthly price variation. Featurely, if there is a positive correlation between the price and quantity supplied on the supply side, this condition may not hold. However, our analysis focuses solely on fresh food, making this scenario unlikely to occur since production levels are challenging to alter in the short term.

Table 4.4: Examples of estimation of CES utility

Commodity	Slope	Monthly Fixed Effect	
		Maximum	Minimum
tomato	-0.317 (0.016)	0.123	-0.099
milk	0.008 (0.020)	0.0027	-0.0037

Note: The robust standard errors are reported in parentheses.

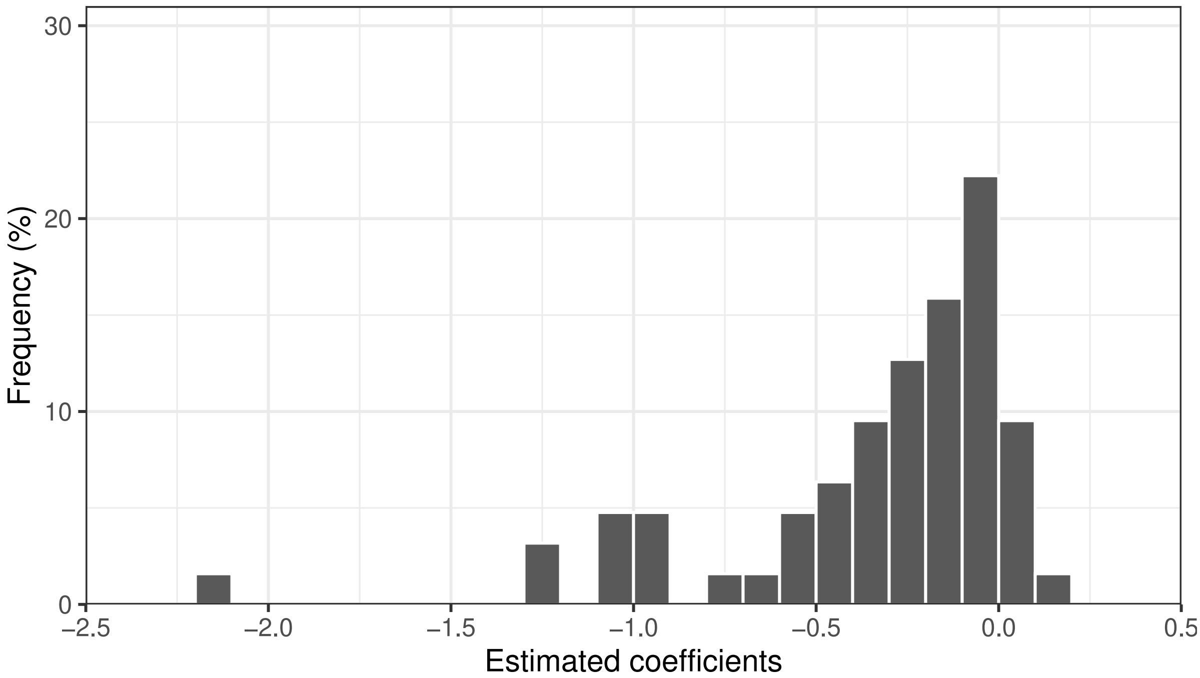


Figure 4.4: The histogram of the estimated slopes

parameters of the CES utility functions for each commodity. The left panel of Figure 4.3 shows that the estimated slope for tomatoes is negative, indicating imperfect substitution between different seasonal consumptions. Moreover, the estimated demand curve shifts monthly, indicating that utility obtained from the same quantity of consumption varies each month. In contrast, the demand curve for milk is estimated to have an almost horizontal slope that does not reject the null hypothesis of perfect substitution. Furthermore, the difference between the maximum and minimum estimated preference weights is smaller than that of tomatoes, indicating that the utility obtained is relatively consistent regardless of the consumption month.

As shown in the above examples, the estimation results vary among commodities. To present a general tendency, we summarize the estimated slopes of the demand function in Figure 4.4. The estimated slopes are predominantly negative, indicating that a significant proportion of fresh foods are imperfect substitutes. Table 4.5 shows that 83% of commodities have statistically significant negative estimates. However, this aspect of imperfect substitution is often overlooked

Table 4.5: Summary of the estiamted slopes

Description	Count	Share (%)
Total	63	100
Negative slope	56	88.9
Negative slope & Rejected horizontal	52	82.5
Rejected equal weight	57	90.5
Rejected summation utility	63	100

Note: The significance level is 5%.

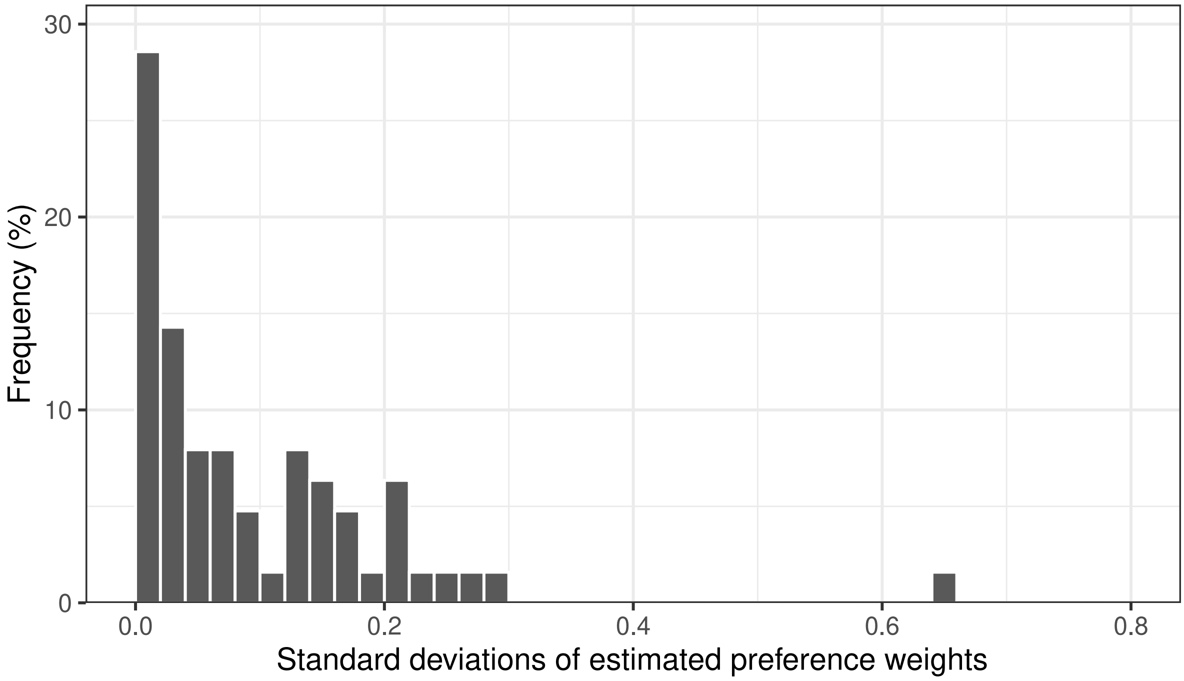


Figure 4.5: The histogram of the standard deviations of the estimated preference weights

by the standard method, which underlies the simple summation utility function.

Next, we present a summary of estimated utility weights, along with a histogram in Figure 4.5 that illustrates the standard deviation of the estimated preference weights for each commodity. A standard deviation of 0 indicates that all preference weights are identical, resulting in no variation in the demand curve. Conversely, a standard deviation greater than 0 indicates a temporal shift in the demand curve, and that preference weights are not identical. Figure 4.5 indicates that the distribution peaks around 0; however, a significant number of commodities have values that deviate from 0. Specifically, Table 4.5 shows that 90 % of commodities reject the null hypotheses of identical weights. This suggests that many products experience seasonal demand fluctuations and that equal-weighted aggregation frequently misrepresents actual consumer preferences.

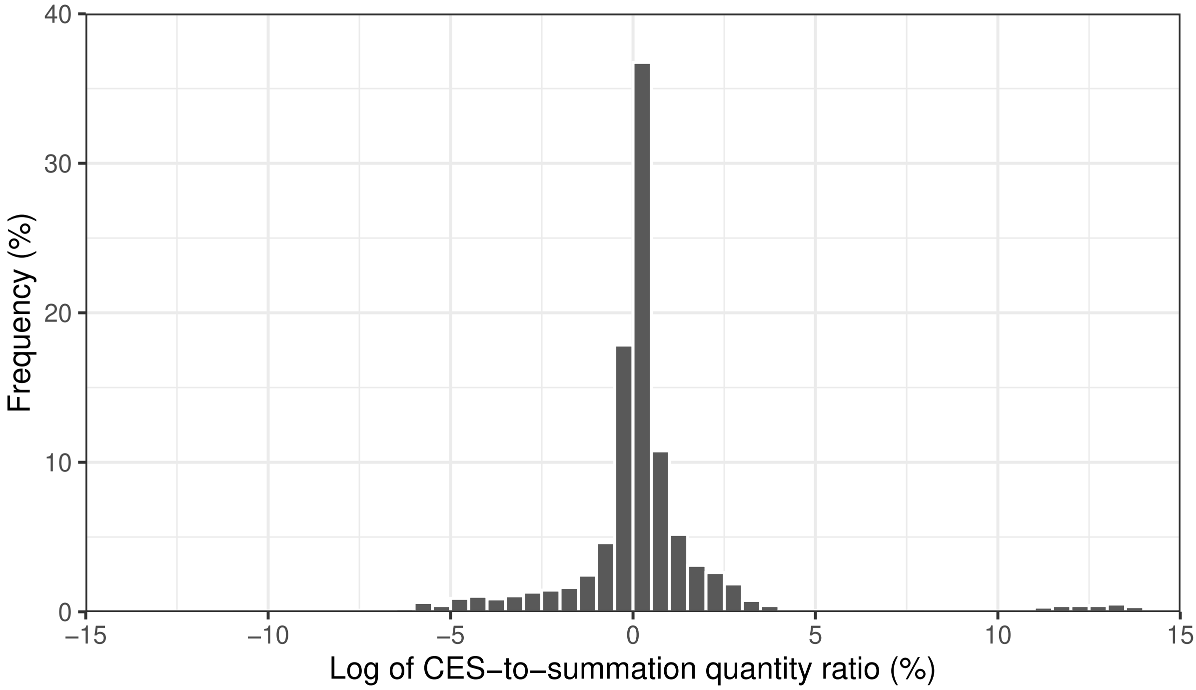


Figure 4.6: The histogram log differences of CES-to-summation quantity

Table 4.6: Summary statistics of the log of CES-to-summation quantity ratio

Statistics	Value (%)
Mean	0.30
Mean absolute value	1.29
Minimum	-14.24
25% quantile	-0.07
Median	0.09
75% quantile	0.72
Maximum	14.91

Now, we compare the elemental-level aggregate quantity derived from the CES utility function with the simple summation quantity. We use the estimates of the slope of the demand curve for the elasticities, but if the estimated slope is non-negative, we assume infinite elasticity (i.e., perfect substitution). Figure 4.6 is a histogram of the logarithmic differences of the quantities (i.e., CES quantity minus simple summation quantity) by year and commodity, while Table 4.6 presents a summary of these quantity differences. The differences between the two aggregation techniques are confirmed, with a mean absolute difference of 1.29%, and the maximum and minimum values exceeding 14% in absolute value. However, on average, these discrepancies are evenly distributed. The mean is 0.5%, and the median is 0.09%.

We decompose the difference between the two aggregates into two components: incomplete

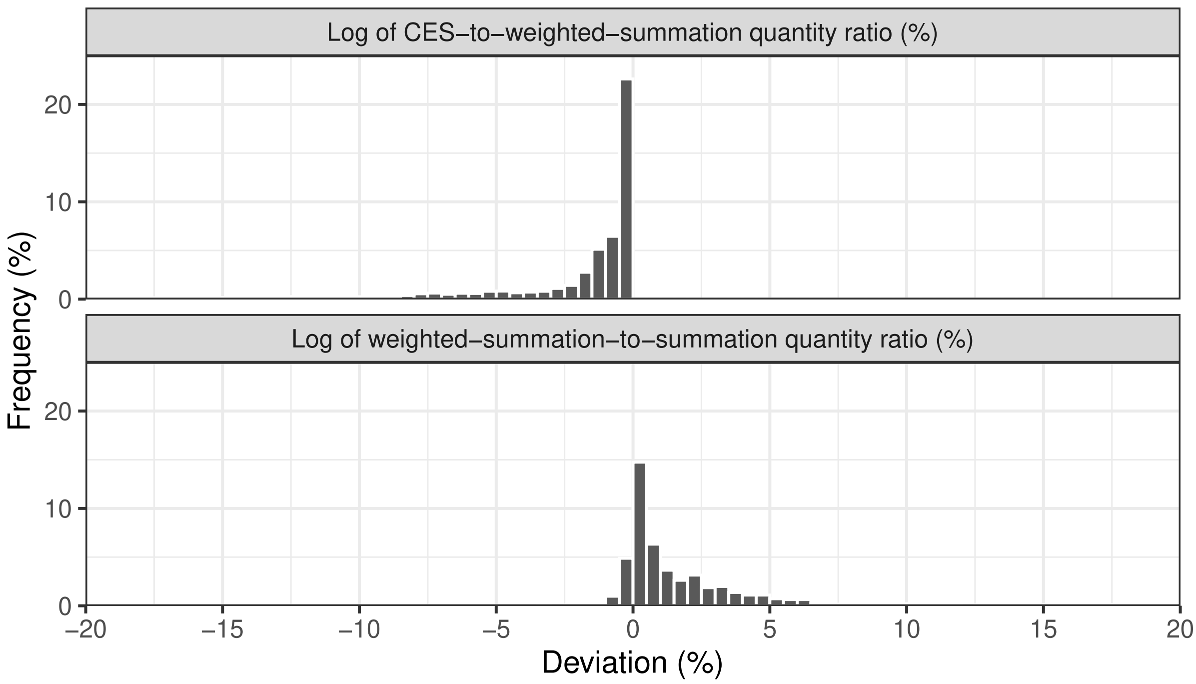


Figure 4.7: Decomposition of the log differences of the two aggregates (CES minus simple summation)

substitution and unequal weights. We use the following equations

$$S = \log\left[\#M_i\left(\sum_{m\in M_i}\hat{\alpha}_{mi}q_{mi}^{\frac{\hat{\varepsilon}_i-1}{\hat{\varepsilon}_i}}\right)^{\frac{\hat{\varepsilon}_i}{\hat{\varepsilon}_i-1}}\right] - \log\left(\#M_i\sum_{m\in M_i}\hat{\alpha}_{mi}q_{mi}\right)$$

$$W = \log\left(\#M_i\sum_{m\in M_i}\hat{\alpha}_{mi}q_{mi}\right) - \log\left(\sum_{m\in M_i}q_{mi}\right).$$

where S represents the difference between imperfect and perfect substitution utility function, indicating the seasonality effect overlooked by perfect substitution assumption. W represents the difference between unequal and equal preference weights, which captures the seasonality effect overlooked by the equal-weight assumption. These two differences are computed by year and commodity and are summarized in Figure 4.7. This figure shows that the CES utility function's aggregated quantities are reduced by imperfect substitution and increased by unequal weights.

These elemental-level differences' average shifted significantly from year to year. Figure 4.8 illustrates that the expenditure-weighted average logarithmic difference of the element-level quantities starts with negative values and continues to increase until 2000. This long-term shift suggests that the seasonal effect overlooked by simple summation could significantly impact both

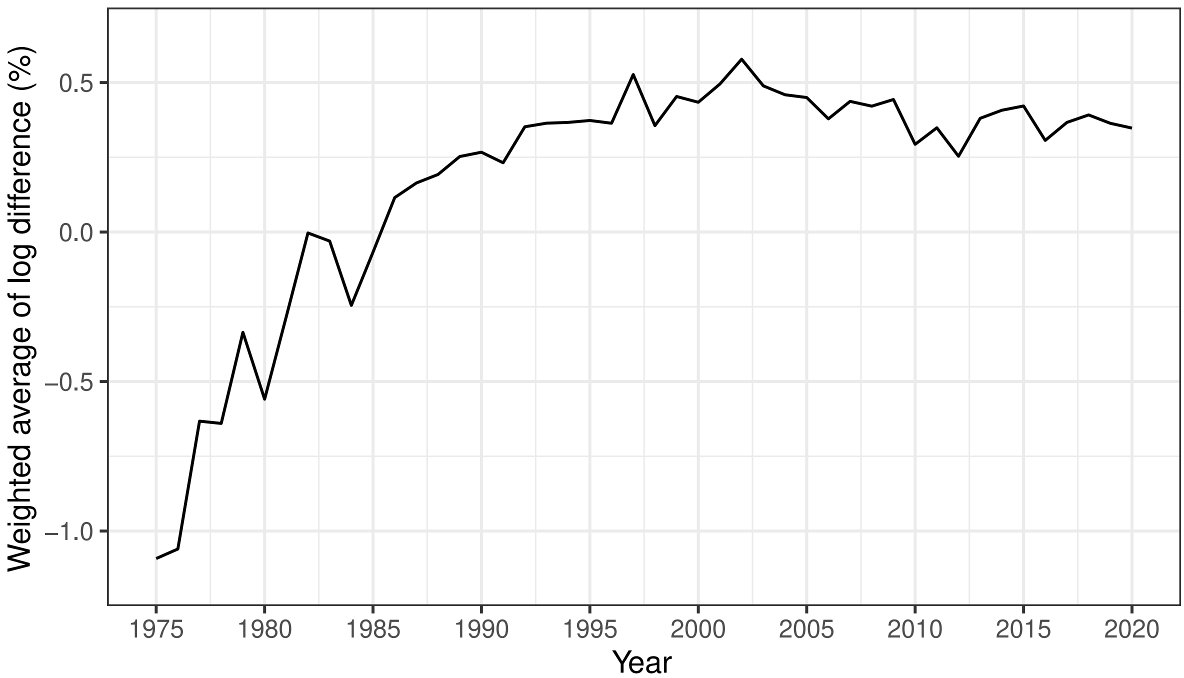


Figure 4.8: Weight average of log-differences by year

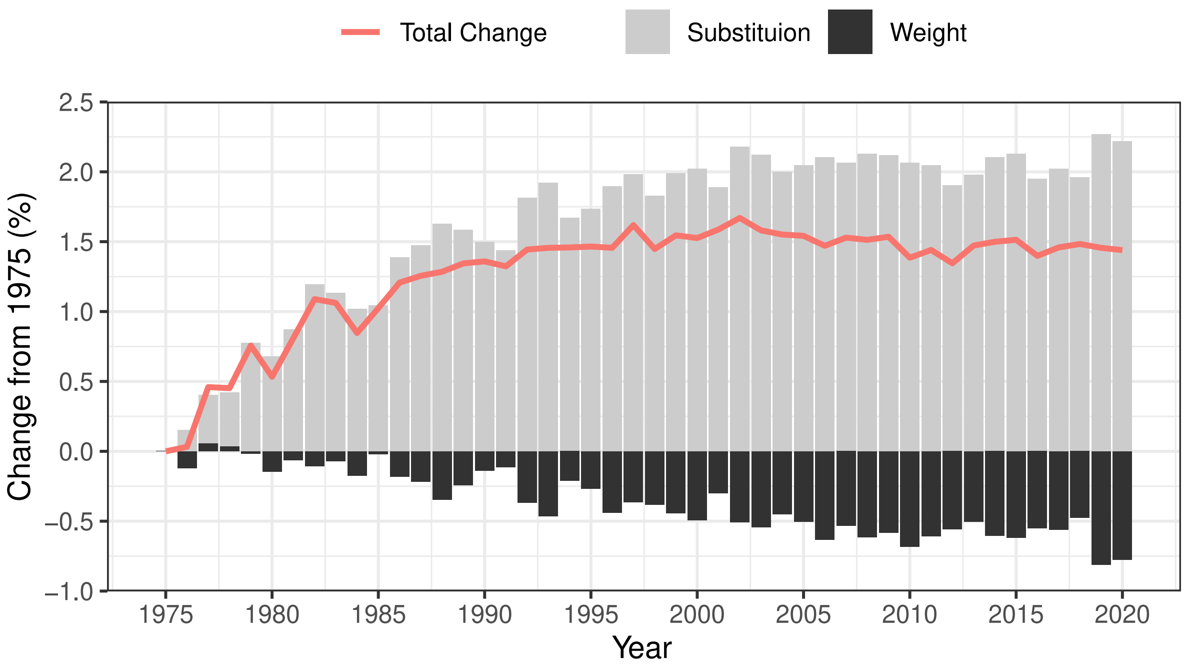


Figure 4.9: Decomposition of the weight average of log-differences by year

the level and long-term growth of the higher-level indices. What are the contributing factors to this shift? Figure 4.9 represents the decomposition of the average quantities difference into the impacts of imperfect substitution and unequal weights. The gray bar represents the effect of the imperfect substitution, effectively illustrating the overall change represented by the solid

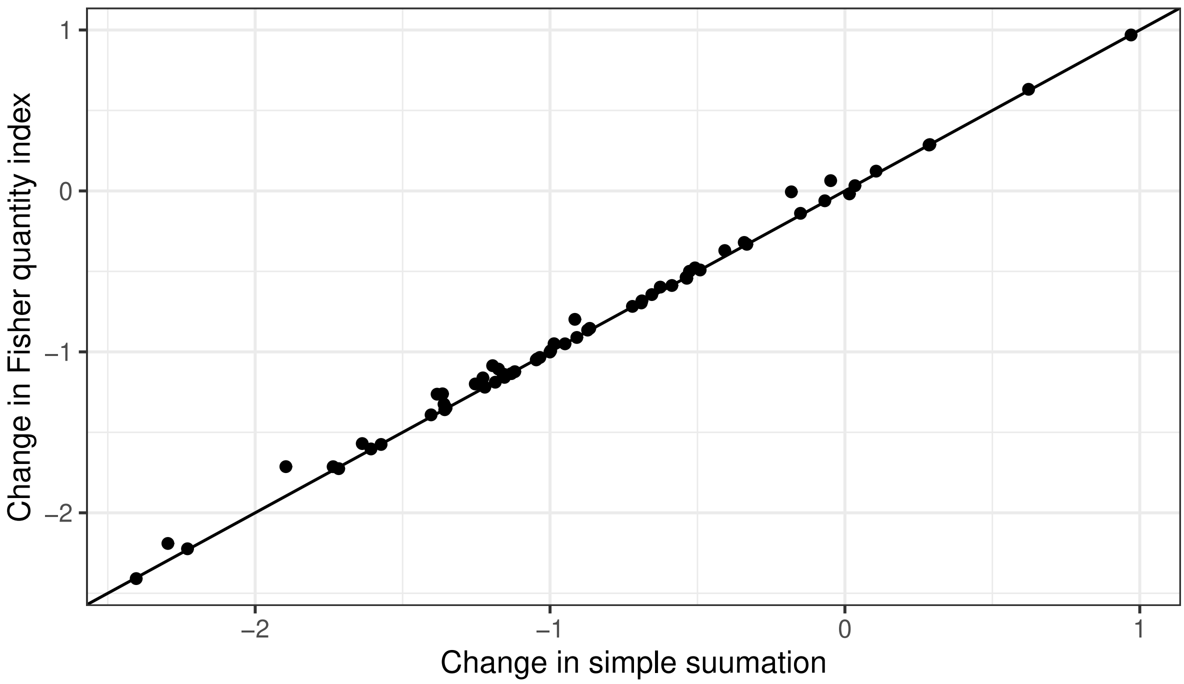


Figure 4.10: Scatter plot of elemental-level quantity index (x: summation-based index, y: Fisher quantity index)

line. The weighted average's temporal shift corresponds to the decline in seasonal fluctuation, as depicted in Figure 4.2. Therefore, our CES implementation captures the hidden effect of the decrease in seasonal fluctuations, increasing household utility.

Figure 4.9 also reveals that reduced seasonality does not necessarily reduce the bias caused by homogeneity. In our framework, if all monthly consumptions have the same value, the CES-type aggregate equals the simple summation for any parameter. Although one might think that the bias decreases when seasonal fluctuation decreases, our empirical analysis suggests that this is not always the case. Figure 4.2 shows a decrease in seasonal variation of quantity from 1975 to 2000. The average bias depicted in Figure 4.9 decreased from 1975 to 1985 but increased from 1985 to 2000.

4.5.2 Implementation by Fisher quantity index

In this subsection, we compare the elemental-level aggregation using the Fisher index and simple summation. Unlike aggregation through the estimated CES utility function, the Fisher index does not require parameter estimation, which is an advantage. However, it should be noted that it only measures the rate of change and not the level.

We aggregate monthly consumption at the elementary level from 1975 to 2020 using the chained Fisher index and compared it to aggregation using simple summation. Figure 4.10

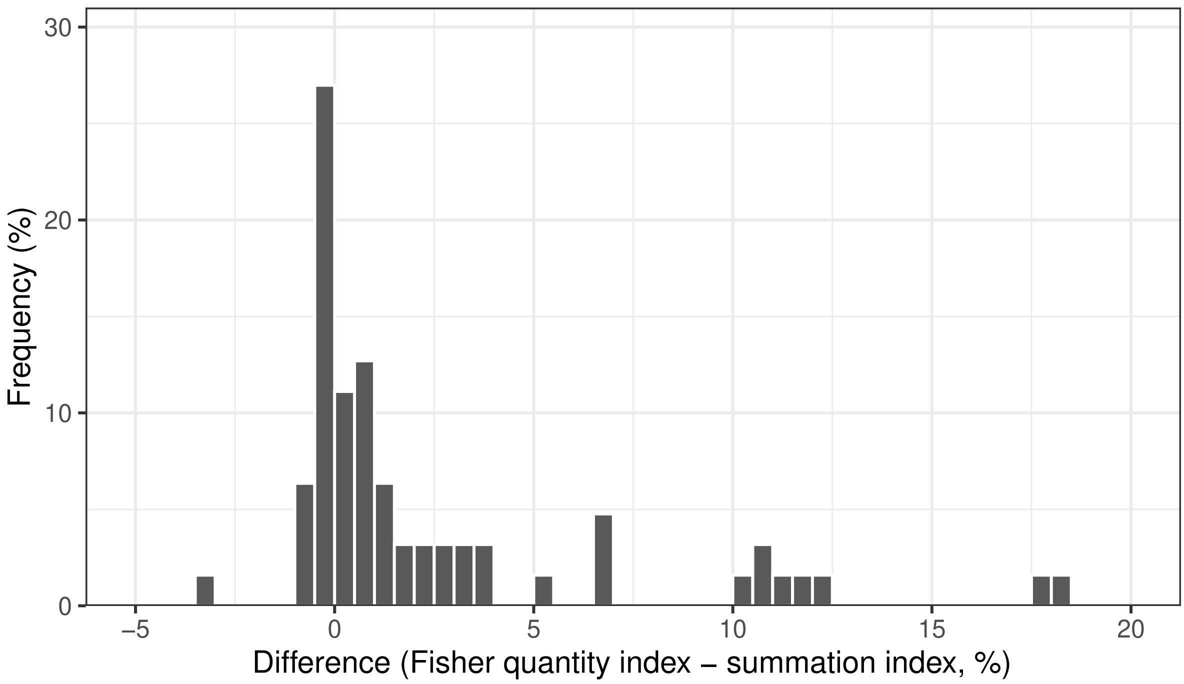


Figure 4.11: Log difference of the elemental-level indices

shows a scatter plot for each commodity, with the rate of change in the simple summation on the horizontal axis and the rate of change in the Fisher quantity index on the vertical axis. Numerous points fall above or near the 45-degree line, indicating that the Fisher quantity index shows a greater increase than the simple summation quantity. To further examine this difference, we have presented a histogram of the results in Figure 4.11. The histogram shows a rightward skew, clearly indicating that the differences in quantities are mostly above 0 and that the Fisher index has a higher rate of change.

Then, we quantify the impact of elementary index differences on higher-level indices. For the higher-level index formula, we use the Fisher-chained quantity index. Figure 4.12 displays the log difference between the higher-level quantity indices of fresh food using different elemental indices. The difference increased from 1975 to 2000 and then stabilized. This pattern is similar to the change in the average difference between CES utility, and simple summation is shown in Figure 4.8. Thus, the Fisher index implementation also captures the overlooked effect of the decline in seasonal variation.

4.6 Conclusion

We measure the impact of seasonality on the temporal elemental aggregation that is overlooked by the conventional method of statistical authorities. The newly observed impact of change

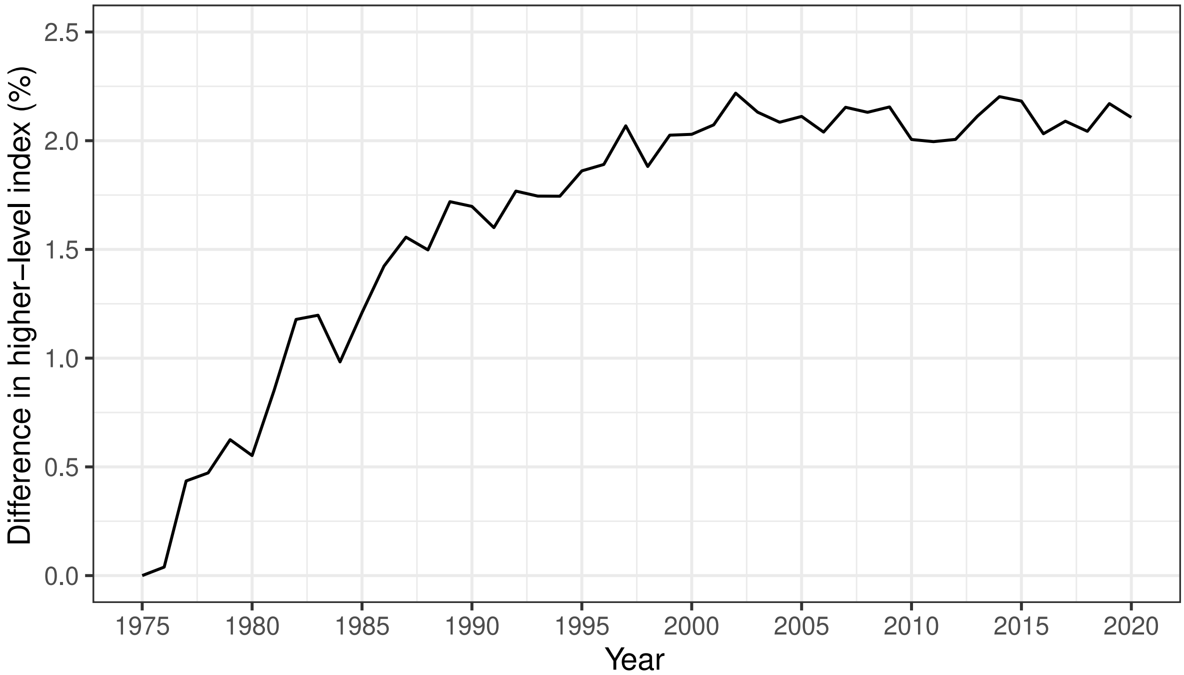


Figure 4.12: Log difference of the aggregated fresh foods quantity index (Fisher index based - summation based)

in seasonality in Japan's fresh foods does increase the quantity index through consumption smoothing. Such an overlooked seasonality effect to the quantity index (and simultaneously in the price index) is referred to as a "substitution bias" by Boskin et al. (1996). Therefore, it should be considered when accurately measuring utility and cost of living.

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