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Independent Decisiveness, Dictatorship, and Inter-menu Consistency

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Abstract

This paper shows necessary and sufficient conditions for global dictatorship in the Arrovian social choice function approach. A well-known result, Sen's (1993) impossibility theorem in the choice function approach establishes that, given a set S of three or more alternatives, a social choice function defined on S satisfing Non-Emptiness, Weak Pareto, and Independent Decisiveness must be locally dictatorial on S. With novel inter-menu consistency conditions together with the above three axioms, we characterize globally dictatorial social choice functions in which a single dictator has rejection decisive power on all subsets of alternatives.

*This paper is born out of our scrutiny on Sen's choice functional impossibility theorem in the process of translating Sen's *Collective Choice and Social Welfare: Expanded Edition* into Japanese. This translation project was initiated by late Professor Kotaro Suzumura. Our research has been much inspired by lots of discussions that we had with Professor Suzumura. We truly respect his great spirit as a researcher.

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1 Introduction

Arrow's impossibility theorem is not only one of the greatest monuments in mathematical economics but a starting point in modern social choice theory. Its basic message is that, under the assumption of ordinal and interpersonally noncomparable individual utilities, any efficient and information parsimonious social ordering function in the sense of Weak Pareto and IIA must be a dictatorship (Arrow 1951/1963). In contrast, in the social choice function approach, it is easy to show that an impossibility theorem no longer holds as long as a simple independence condition is applied.[1] However, once some inter-menu consistency conditions are required, similar impossibility theorems are obtained even in the social choice function approach (Blair et al. 1976; Grether and Plott 1982).[2]

In contrast to the social choice function approach that explicitly imposes inter-menu consistency, Sen (1993) proposes Independent Decisiveness which is a weaker condition of Hansson's independence (Hannson 1969; Denicolo 1985; 1993), and proves that given one set S of alternatives, even if inter-menu consistency is not imposed, a social choice function on S that satisfies Non-Emptiness, Weak Pareto, and Independent Decisiveness must be a dictatorship.[3] Moreover, Deb (2011) shows that the three axioms are necessary and sufficient conditions for a dictatorship.

However, the combination of Non-Emptiness, Weak Pareto, and Independent Decisiveness implies nothing more than *local* dictatorship, as opposed to dictatorship in Arrow's sense. A local dictator has a rejection decisive power over *one* subset S of alternatives in the sense that he/she can always exclude any alternative that is less preferred to some other alternative in S from the choice set of S. But

[1] The choice functional version of Arrow's independence condition is as follows (Arrow 1951/1963; Grether and Plott 1982; Sen 1970/2017):

Arrow's Independence: For all $S \in \mathscr{S}$ and all $R_N, R'_N \in \mathscr{R}^N$, if $R_N|^S = R'_N|^S$, then $C(S, R_N) = C(S, R'_N)$.

Clearly, a social choice function that assigns the maximal elements of the Borda rule on S satisfies both weak Pareto and Arrow's independence.

[2] *Inter-menu consistency conditions* are usually called *internal consistency conditions*. We think that the former term reflects the contents better than the latter term.

[3] It is worth emphasizing that Hannson (1969) is the first to prove the functional version of the dictatorship theorem without imposing any inter-menu consistency conditions.

the local dictator does not necessarily have the same rejection decisive power over any other subset $S' \neq S$. In contrast, a dictator in Arrow's sense should have the rejection decisive power over *all* subsets of alternatives. We call such a dictator a *global* dictator.

In this paper, we introduce a novel inter-menu consistency condition, and establish that the above three axioms and the inter-menu consistency condition are necessary and sufficient for a social choice function to be a global dictatorship. Under the assumption of linear individual preference orderings, Deb (2011) also provides necessary and sufficient conditions for global dictatorship by adding the weak axiom of revealed preference to the three axioms. Since the assumption of linear orderings is rather restrictive, we assume the standard weak individual preference orderings. In fact, in the setting of weak orderings, the weak axiom of revealed preference is no longer a necessary condition for dictatorship.

As shown in the above mentioned papers, no condition of inter-menu consistency is necessary for local dictatorship to be established on a specific subset S where the cardinality of S is 3 or more. However, in order to establish global dictatorship, a certain inter-menu consistency condition is required. We introduce a new condition, which we call Extended Arrow's Axiom. It is an extension of Arrow's Axiom, a well-known necessary and sufficient condition for the existence of a full rationalization of a social choice function. The point in our conditions is to consider refinements of the original preference profile in which the indifference relations in the profile are replaced with some strict preference relations, and identify the set of all alternatives in each subset S that are chosen for some refinements. Let us call it the set of *potentially chosen* alternatives in S. Extended Arrow's Axiom states that if S_1 is a subset of S_2 and the choice set in S_2 has a non-empty intersection with S_1, then the intersection of S_1 and the set of potentially chosen alternatives in S_2 coincides with the set of potentially chosen alternatives in S_1. We also introduce two weaker conditions than Extended Arrow's Axiom, which are similar extensions of Chernoff's Axiom and Dual Chernoff's Axiom. These axioms focus on all the alternatives that are chosen for some refinements of the original individual preference profile. The operation of refining the original preferences is often useful for other applied areas such as voting theory, strategy-proofness, and matching theory, etc. in which there are contexts where indifference relations need to be replaced by strict preference relations. Our axioms may suggest some inter-menu consistency requirements of outcomes in these economic or political problems.

The organization of this paper is as follows. Section 2 introduces notation, definitions, and axioms in this paper. Section 3 proves and explains our main result. Section 4 summarizes the result of this paper.

2 Notation, Definitions, and Axioms

A finite set of individuals is given and denoted $N = \{1, \ldots, n\}$, where n is greater than 2. A finite set of alternatives is given and denoted X. We assume $|X| \geq 3$.[4] Let $\mathscr{S}$ be the set of all non-empty subsets of X.

Each individual $i \in N$ has an ordinal and interpersonally non-comparable preference relation R_i on X. We assume that each preference relation is reflexive, complete and transitive. Let $\mathscr{R}$ be the set of all preference relations. We denote P_i and I_i as the asymmetric and symmetric parts of $R_i \in \mathscr{R}$, respectively. For each $S \in \mathscr{S}$ and each $R_i \in \mathscr{R}$, let $M(S, R_N)$ and $G(S, R_N)$ be the set of maximal elements and the set of greatest elements in S for R_N, respectively, that is, $M(S, R_N) = \{x \in S \mid \text{There exists no } y \in S \text{ such that } y\ P_i\ x\}$ and $G(S, R_N) = \{x \in S \mid \forall y \in S, x\ R_i\ y\}$.

A list of preference relations of all individuals is called a *preference profile* and denoted $R_N = (R_1, \ldots, R_n)$. Let $\mathscr{R}^N$ be the set of all logically possible preference profiles.

For all $G \subseteq N$ and all $x, y \in X$, if $x\ P_i\ y$ for all $i \in G$, then we write $x\ P_G\ y$. For all $i \in N$ and all $R_i, R'_i \in \mathscr{R}$, if for all $x, y \in X$, $x\ R_i\ y \Rightarrow x\ R'_i\ y$ and $x\ P_i\ y \Rightarrow x\ P'_i\ y$, then we call R'_i a *refinement of* R_i. We say that R'_N is a *refinement of* R_N if for all $i \in N$, R'_i is a refinement of R_i. Let $r(R_i)$ denote the set of all refinements of R_i, and $r(R_N)$ the set of all refinements of R_N.

For each $S \in \mathscr{S}$ and each $R_i \in \mathscr{R}$, the *restriction of* R_i *on* S is defined as $R_i|^S = \{(x, y) \in R_i \mid x, y \in S\}$. We write $R_N|^S = (R_1|^S, \ldots, R_n|^S)$. Let $\bar{\mathscr{S}} \subseteq \mathscr{S}$ be given. A *social choice function* C on $\bar{\mathscr{S}}$ is a set-valued function that associates with each $S \in \bar{\mathscr{S}}$ and each $R_N \in \mathscr{R}^N$, a subset $C(S, R_N) \subseteq S$.

[4]For each finite set A, $|A|$ denotes the number of elements in A.

Our goal is to characterize a social choice function that satisfies several properties or *axioms*, each of which is generally considered desirable. We consider the following four axioms. The first two are standard. First, a social choice function should assign a non-empty set of alternatives to any choice situation.

Non-Emptiness: For all $S \in \bar{\mathscr{S}}$ and all $R_N \in \mathscr{R}^N$, $C(S, R_N) \neq \emptyset$.

Second, if everyone prefers x to y, then a social choice function should not choose y in the presence of x.

Weak Pareto: For all $S \in \bar{\mathscr{S}}$, all $x, y \in S$ and all $R_N \in \mathscr{R}^N$, if $x \, P_N \, y$, then $y \notin C(S, R_N)$.

The next axiom is due to Sen (1993). It means that if a group G has "rejection decisive power over (x, y)" in some choice situation in the sense that $x \, P_G \, y$, and x is chosen but y is not, then the rejection power of G over (x, y) is not affected by any preference changes over alternatives involving $z \neq x, y$.

Independent Decisiveness: For all $G \subseteq N$, all $x, y \in X$, all $S \in \bar{\mathscr{S}}$ with $x, y \in S$, if for all $R_N \in \mathscr{R}^N$ with $x \, P_G \, y$, there exists $R'_N \in \mathscr{R}^N$ such that $R_N|^{\{x,y\}} = R'_N|^{\{x,y\}}$, $x \in C(S, R'_N)$ and $y \notin C(S, R'_N)$, then for all $R_N \in \mathscr{R}^N$ with $x \, P_G \, y$, $y \notin C(S, R_N)$.

Independent Decisiveness is a weaker version of similar independence conditions introduced by Hannson (1969) and Denicolo (1985).

Next, we define a condition of inter-menu consistency for social choice functions. This condition can be understood as an extension of Arrow's axiom (Arrow 1959), which is well-known as a necessary and sufficient condition for the existence of a full rationalization of a choice function. For each preference profile, consider refinements of the profile in which some indifference relations are replaced by strict preference relations. Now, suppose that a set S_1 is a subset of S_2, and that S_1 includes some alternatives chosen in S_2. In this case, our inter-menu consistency condition requires that if an alternative $x \in S_1$ is chosen in S_2 for some refinement of the original profile R_N, then x should be chosen in S_1 for some refinement of R_N, and vice versa.

Extended Arrow's Axiom: For all $S_1, S_2 \in \bar{\mathscr{S}}$ with $S_1 \subseteq$

S_2, and all $R_N \in \mathscr{R}^N$, if $\bigcup_{\bar{R}^N \in r(R_N)} C(S_2, \bar{R}^N) \cap S_1 \neq \emptyset$, then $\bigcup_{\bar{R}^N \in r(R_N)} C(S_2, \bar{R}^N) \cap S_1 = \bigcup_{\bar{R}^N \in r(R_N)} C(S_1, \bar{R}^N)$.

Note that this condition is equivalent to the following condition.

Extended Complement Arrow's Axiom: For all $S_1, S_2 \in \mathscr{S}$ with $S_1 \subseteq S_2$, and all $R_N \in \mathscr{R}^N$, if $\bigcup_{\bar{R}^N \in r(R_N)} C(S_2, \bar{R}^N) \cap S_1 \neq \emptyset$, then $\bigcap_{\bar{R}_N \in r(R_N)} [S_2 \setminus C(S_2, R_N)] \cap S_1 = \bigcap_{\bar{R}_N \in r(R_N)} [S_1 \setminus C(S_1, \bar{R}_N)]$.

This condition requires that the set of alternatives in S_1 that are never chosen in S_2 for any refinements of an original profile is equal to the set of alternatives that are never chosen in S_1 for any refinements. It is clear from the definition that if all individual preferences are linear orders, then the above two axioms are equivalent to the original Arrow's axiom.

Two weaker conditions than Extended Arrow's Axiom may be defined as follows. These conditions can be seen as extensions of Chernoff's Axioms and Dual Chernoff's Axiom, which are also well-known.

Extended Chernoff's Axiom: For all $S_1, S_2 \in \mathscr{S}$ with $S_1 \subseteq S_2$, and all $R_N \in \mathscr{R}^N$, if $\bigcup_{\bar{R}^N \in r(R_N)} C(S_2, \bar{R}^N) \cap S_1 \neq \emptyset$, then $\bigcup_{\bar{R}^N \in r(R_N)} C(S_2, \bar{R}^N) \cap S_1 \subseteq \bigcup_{\bar{R}^N \in r(R_N)} C(S_1, \bar{R}^N)$.

Extended Dual Chernoff's Axiom: For all $S_1, S_2 \in \mathscr{S}$ with $S_1 \subseteq S_2$, and all $R_N \in \mathscr{R}^N$, if $\bigcup_{\bar{R}^N \in r(R_N)} C(S_2, \bar{R}^N) \cap S_1 \neq \emptyset$, then $\bigcup_{\bar{R}^N \in r(R_N)} C(S_1, \bar{R}^N) \subseteq \bigcup_{\bar{R}^N \in r(R_N)} C(S_2, \bar{R}^N) \cap S_1$.

Finally, we define functional versions of local dictatorship and global dictatorship.

Local Dictatorship: A social choice function C is *locally dictatorial on* $\bar{\mathscr{S}}$ if and only if for each $S \in \bar{\mathscr{S}}$, there exists $d_S \in N$ such that for all $R_N \in \mathscr{R}^N$, $\emptyset \neq C(S, R_N) \subseteq M(S, R_{d_S})$.

Global Dictatorship: A social choice function C is *globally dictatorial on* $\bar{\mathscr{S}}$ if and only if there exists $d \in N$ such that for all $S \in \bar{\mathscr{S}}$ and all $R_N \in \mathscr{R}^N$, $\emptyset \neq C(S, R_N) \subseteq M(S, R_d)$.

Local dictatorship requires that, for a fixed subset S, a social choice function assigns a non-empty subset of a local dictator's maximal elements irrespective of preferences of all the other individuals. Note that since the local dictator has decisive power only on the set S, the same individual does not necessarily have decisive power over different sets. In fact, it is debatable whether local dictatorship should be considered undesirable or anti-democratic. Consider two subsets S_1 and S_2 such that the elements of S_1 represent various feasible options of public health policies, and the elements of S_2 are feasible alternatives of transportation policies, given all other things equal. In this case, having the best expert on public health policies choose one in S_1 and having the best expert on transportation policies select one in S_2 are quite reasonable solutions for social choice, and indeed we often use this kind of delegation rules. Moreover, if every member of society has decisive power over some subset of alternatives, then the distribution of decisive power can be quite equal. In that situation, local dictatorship may not be anti-democratic at all.[5] In contrast, global dictatorship is clearly anti-democratic: *only one* dictator has decisive power over *all* subsets of alternatives.

Notice also that both local dictatorship and global dictatorship do not specify which alternative is chosen when there are two or more maximal elements of the dictator. In other words, these dictatorships say nothing about a tie-breaking method that specifies what should be chosen among maximal elements for which the dictator is indifferent. They are compatible with all tie-breaking methods and consist of the broadest class of dictatorships in the social choice function approach.

In the next section, we will examine necessary and sufficient conditions for the two different dictatorships.

3 Main Results

We first give Sen and Deb's theorem. Let $\bar{\mathscr{S}}_{\geq 3} = \{S \in \mathscr{S} \mid |S| \geq 3\}$.

Theorem 1 (Sen 1993; Deb 2011). *A social choice function C on*

[5]Needless to say, if a distribution of decisive powers among individuals is significantly biased, the situation can be said to be anti-democratic. However, local dictatorship is only a matter of decisive power on a single subset, and the distribution of decisive power over all subsets is not included in its scope. Furthermore, it can be said that local oligarchy is closer to the reality of a committee of experts than local dictatorship. For the related local oligarchy theorems and local veto theorems, see Denicolo (1993), Deb (2011), and Cato (2015).

$\bar{\mathscr{S}}_{\geq 3}$ *satisfies Non-Emptiness, Weak Pareto, and Independent Decisiveness if and only if it is locally dictatoriral on* $\bar{\mathscr{S}}_{\geq 3}$.

For the proof of sufficiency of the above axioms for local dictatorship, see Sen (1993). Basically, he proves this theorem through functional versions of the famous two lemmas—the field expansion lemma and the group contraction lemma—developed by Sen (1970/2017) in the proof of Arrow ' s impossibility theorem. Deb (2011) shows that these axioms are also necessary conditions for local dictatorship.

Some remarks about Sen and Deb's theorem should be made here. First, it is required that there are three or more alternatives in S. In fact, if $|S| = 2$, then the simple majority decision rule satisfies Non-Emptiness, Weak Pareto, and Independent Decisiveness. That is, a local dictatorship is not entailed by the three axioms when $|S| = 2$. Second, a locally dictatorial choice function can assign any set of maximal elements of the local dictator for various choice situations. In other words, all possible tie-breaking methods among the local dictator's maximal elements are acceptable for the choice function. Finally, local dictators may be different for different subsets. Therefore, an additional condition is needed for the local dictators on different subsets to be the same individual.

We now state our main theorems.

Theorem 2 *If a social choice function* C *on* $\bar{\mathscr{S}}_{\geq 3}$ *satisfies Non-Emptiness, Weak Pareto, Independent Decisiveness, and Extended Chernoff's Axiom, then* C *is globally dictatorial on* $\bar{\mathscr{S}}_{\geq 3}$.

Proof.
Assume that C satisfies the above four axioms. It follows from Non-Emptiness, Weak Pareto, Independent Rejection Decisiveness, and the theorems of Sen's Theorem that for every $S \subseteq X$ with $|S| \geq 3$, there exists a local dictator $d_S \in N$ such that for every $R_N \in \mathscr{R}^N$, $C(S, R_N) \subseteq M(S, R_{d_S})$.

Let $S \subseteq X$ with $S \neq X$ and $|S| \geq 3$. Let $x, y \in S$ with $x \neq y$. Consider $R_N \in \mathscr{R}^N$ such that
(i) R_{d_X} is a linear order with $M(X, R_{d_X}) = \{x\}$, and
(ii) for all $i \neq d_X$, R_i is a linear order with $M(X, R_i) = \{y\}$.
Suppose, to the contrary, that $d_S \neq d_X$. Then, by Non-Emptiness and local dictatorship, $C(X, R_N) = M(X, R_{d_X}) = \{x\}$ and $C(S, R_N) = M(S, R_{d_S}) = \{y\}$. Hence, $\bigcup_{\bar{R}^N \in r(R_N)} C(X, \bar{R}^N) \cap S = C(X, R_N) \cap S = \{x\}$ but $\bigcup_{\bar{R}^N \in r(R_N)} C(S, \bar{R}^N) = C(S, R_N) = \{y\}$. This means that C

violates Extended Chernoff's Axiom, which is a contradiction. Thus, $d_S = d_X$. This holds for every $S \subseteq X$. Therefore, C is globally dictatorial. ■

It is clear from the above proof that Extended Chernoff's Axiom can be replaced with Extended Dual Chernoff's Axiom or Extended Arrow's Axiom.

Theorem 3 *Every globally dictatorial social choice function on $\bar{\mathscr{S}}_{\geq 3}$ satisfies Non-Emptiness, Weak Pareto, Independent Decisiveness, and Extended Arrow's Axiom.*

Proof.
Let a social choice function C be a globally dictatorial social choice function. By definition, C satisfies Non-Emptiness. It is trivial that C satisfies Weak Pareto. By Deb (2011), C satisfies Independent Decisiveness. It remains to show that C satisfies Extended Arrow's Axiom. There exists $d \in N$ such that for all $S \subseteq X$, all $R_N \in \mathscr{R}^N$, $C(S, R_N) \subseteq M(S, R_d)$. Assume that $S_1, S_2 \subseteq X$ and $R_N \in \mathscr{R}^N$ satisfy $S_1 \subseteq S_2$, $S_1 \neq S_2$, and $C(S_2, R_N) \cap S_1 \neq \emptyset$.

Let $x \in M(S_1, R_d)$. Let $\bar{R}_N \in r(R_N)$ be a profile such that
(i) $\bar{R}_d \in r(R_d)$ is a linear order with $M(S_1, \bar{R}_d) = \{x\}$, and
(ii) for all $i \neq d$. $\bar{R}_i = R_i$.
It follows from Non-Emptiness and $C(S_1, \bar{R}_N) \subseteq M(S_1, \bar{R}_d)$ that $\{x\} = C(S_1, \bar{R}_N)$. Hence, $x \in \bigcup_{\bar{R}_N \in r(R_N)} C(S_1, \bar{R}_N)$. This holds for every $x \in M(S_1, R_d)$. Thus, $\bigcup_{\bar{R}_N \in r(R_N)} C(S_1, \bar{R}_N) = M(S_1, R_d)$. Similarly, we have $\bigcup_{\bar{R}_N \in r(R_N)} C(S_2, \bar{R}_N) = M(S_2, R_d)$. Because R_d is complete, $M(S_k, R_d) = G(S_k, R_d)$ for each $k = 1, 2$. Obviously, $G(S_2, R_d) \cap S_1 = G(S_1, R_d)$. All together, we have $\bigcup_{\bar{R}_N \in r(R_N)} C(S_2, \bar{R}_N) \cap S_1 = \bigcup_{\bar{R}_N \in r(R_N)} C(S_1, \bar{R}_N)$. ■

Because Extended Arrow's Axiom implies Extended Chernoff's Axiom and Extended Dual Chernoff's Axiom, every globally dictatorial social choice function satisfies these three axioms.

By Theorems 2 and 3 together, we have shown that a globally dictatorial social chioce function is characterized by Non-Emptiness, Weak Pareto, Independent Decisiveness, and one of the three axioms, namely, Extended Arrow's Axiom, Extended Chernoff's Axiom and Extended Dual Chernoff's Axiom.

Sen and Deb's theorem requires that there be at least three alternatives in S to establish local dictatorship for S. In our next theorem, thanks to Extended Dual Chernoff's Axiom, global dictatorship can be extended to all subsets with two alternatives *except* the set of the worst and the second worst alternatives for the dictator. The next result clarifies this.

Let $\bar{\mathscr{S}}_{\geq 2} = \{S \in \mathscr{S} \mid |S| \geq 2\}$.

Theorem 4 *Suppose that a social choice function C on $\bar{\mathscr{S}}_{\geq 2}$ satisfies Non-Emptiness, Weak Pareto, Independent Decisiveness, and Extended Dual Chernoff's Axiom. Then, there exists a global dictator $d \in N$ on $\bar{\mathscr{S}}_{\geq 3}$ and moreover, for every $R_N \in \mathscr{R}^N$ and every $S \in \bar{\mathscr{S}}_{\geq 2}$ with $|S| = 2$, if there exist $x \in S$ and $y \in X \setminus S$ such that $x\, R_d\, y$,*[6] *then $C(S, R_N) \subseteq M(S, R_d)$.*

Proof. Let $R_N \in \mathscr{R}^N$ be given. Let $S \in \mathscr{S}$ with $|S| = 2$. Suppose that there exist $x \in S$ and $y \in X \setminus S$ such that $x\, R_d\, y$. Define $S' = S \cup \{y\}$. By the supposition, $G(S', R_d) \cap S \neq \emptyset$, and $G(S', R_d) \cap S = G(S, R_d)$. Because $|S'| = 3$, it follows from Theorem 2 that $C(S', R_N) \subseteq M(S', R_d)$. By the same reasoning as in the proof for Theorem 3, we can show $\bigcup_{\bar{R}_N \in r(R_N)} C(S', \bar{R}_N) = M(S', R_d) = G(S', R_d)$. It follows from Extended Dual Chernoff's Axiom that $\bigcup_{\bar{R}_N \in r(R_N)} C(S, \bar{R}_N) \subseteq \bigcup_{\bar{R}_N \in r(R_N)} C(S', \bar{R}_N) \cap S$. Obviously, $C(S, R_N) \subseteq \bigcup_{\bar{R}_N \in r(R_N)} C(S, \bar{R}_N)$. All together, we conclude $C(S, R_N) \subseteq G(S, R_d) = M(S, R_d)$. ■

In the above theorem, Extended Dual Chernoff's Axiom can be replaced with Extended Arrow's Axiom.

Two remarks are worth making. First, Deb (2011) shows that the axioms of Non-Emptiness, Weak Pareto, Independent Decisiveness, and a weak version of the Weak Axiom of Revealed Preference[7] characterize a globally dictatorial social choice function. Our results replace the weak version of the Weak Axiom of Revealed Preference with novel inter-menu consistency conditions, which we think are simple and natural.

[6]Note that $[\exists x \in S, \exists y \in X \setminus S,\ x\, R_d\, y] \Leftrightarrow \neg[\forall x \in S, \forall y \in X \setminus S,\ y\, P_d\, x]$, that is, S is *not* the set of the two alternatives that are strictly worse than any other alternatives for the dictator.

[7]Deb (2011) defines the axiom as follows. Pareto Restricted Weak Axiom of Revealed Preference: for all $x, y \in X$, all $S_1, S_2 \in \mathscr{S}_{\geq 3}$, and all linear preference profiles $R_N \in \mathscr{R}^N$ such that for all $z \in X$ and all $i \in N$, xP_iz and yP_iz, if $x \in C(S_1, R_N)$ and $y \in S_1 \setminus C(S_1, R_N)$, then $x \in S_2$ implies $y \notin C(S_2, R_N)$.

Second, in our definition of a globally dictatorial social choice function, like Sen's locally dictatorial social choice function, no tie-breaking methods among the best alternatives for the dictator are specified. Hence, we actually characterize the class of dictatorial social choice functions with all possible tie-breaking methods. In order to characterize this broadest class of dictatorial social choice functions, we need to extend inter-menu consistency conditions.

4 Conclusion

This paper has reexamined the implications of Sen's Independent Decisiveness. In his celebrated paper, Sen (1993) emphasizes that dictatorship is inevitable without imposing any conditions of inter-menu consistency in the social choice function approach. However, this dictatorship is only local dictatorship (or simple dictatorship on a fixed agenda), and not global dictatorship on all subsets of alternatives. The present study has shown that in order for dictatorship in Arrow's sense to be established, some kind of inter-menu consistency is required. The new consistency conditions that we have introduced may be useful in various contexts such as voting, strategy-proofness, and matching where indifference relations often need to be replaced by strict preference relations.

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