Microeconomics of Declining Birthrate - Review of Existing Literature

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Abstract

This paper explores the implications of declining population on the economy. A simple model with household childbearing decision with macro-micro feedback rule is introduced to argue that population cannot continue to decrease forever. Then we review the innovation and growth literature to understand the possible implications.

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1 Introduction

There is general agreement that the declining birthrate in Japan is here to stay, at least in the short run. This is admittedly a belief since there seems to be no single identifiable cause of the decline and there has been no evidence of effectiveness of any of the policies implemented to address this problem.

While much of the debate has been on the cause and policy to counter the declining birthrate, perhaps now is the time to think about what will happen to the economy when the population decline continues. Akihiko Matsutani has taken this view (Matsutani (2004)). He has made several predictions about where the economy is going. His policy recommendations have been not how to stop the decline but what should be done to improve welfare given the decline continues. As result of population decline Matsutani predicts that,

- National income will have negative growth
- Per capita national income will not decline
- Firm profitability will not decline
- Changes will occur in
  - international specialization
  - domestic industrial structure
- Relative increase of imports will decrease the domestic-world price differential
- Becomes necessary to change and downsize social capital

In the next section, I introduce a simple framework with household childbearing decision. Individual decisions are related to macro variables. Then a feedback mechanism from labor market to childbearing decision is introduced to argue that neither population decrease or increase cannot continue forever. In the remaining sections, we turn to the innovation and economic growth literature (Grossman and Helpman (1991)) to understand the economic path with population decline. There seems to be no literature that analyzes the economic path when population (labor force) is declining. The economic development framework always assumes population is growing. There seems to be no literature that analyzes the economic path when population (labor force) is declining. The economic development framework always assumes population is growing. The innovation growth literature assumes innovation as the sole engine of growth. In particular population (both labor force and consumer) is assumed constant. We conduct comparative statics on the population.

2 Economics of Declining Birthrate

In this section we quickly review the economic foundation of birth decisions and population change.

Household

A consumer or a household lives 2 periods, 0 and 1. In period 0, there is income (I_0), and consumer decides how much to consume in each period (c_0 and c_2) and how many children to have, n. We normalize the price of consumption to 1. Price of a child, p, represents the cost of births and raising a child. There will be a return of r from each child. This return is pecuniary. For some households with a family business, this could be quite large. For instance, in a family of Kabuki actors, a successful child will mean prosperity of the family business. Cost of raising an heir in the Kabuki family will also be very costly, meaning a large p. On the other hand, for wage earners, r may be very low. Although not as expensive as Kabuki training, schooling costs can be quite high for wage earners too. This return does not include any "good feeling" from having a child. Such a factor should be in the utility function.

A consumer's utility is,

$$u(c_0, n) + \lambda u(c_1),$$

where λ is the discount factor. We make the usual regularity assumptions regarding derivatives of $u(\cdot)$.

A consumer's budget constraint is,

$$c_0 + pn + \lambda c_1 = I_0 + rn \quad \Leftrightarrow c_0 + +(p - r)n + \lambda c_1 = I_0.$$

Depending on the difference $p - r$, a child is a consumption or investment good. If one can draw substantial income from the family business so that $r > p$, then money should be spent on training a successful heir than putting money in the bank.

Births and Population

In this section we explain the relationship between individual childbearing decisions and change in population at the macro level.

Assume the difference $p - r$ is distributed (random variable μ) on $[-b, b]$. We denote the maximum marginal utility by $\bar{u}_n$. (This may be an additional assumption which is consistent with the regularity conditions on the derivatives.) This occurs at consumption 0 and all marginal utilities are less than $\bar{u}_n$. $\bar{n}$ denotes the maximum number of children that a household can physically have.

Optimal number of children will be,

$$n^*(p-r) = \begin{cases} 0 & p-r > \bar{u}_n \\ \hat{n}(p-r) & 0 < p-r \leq \bar{u} \\ \bar{n} & p-r \leq 0 \end{cases},$$

where $\hat{n}(p-r)$ is the interior solution to the optimization problem. In other words, it solve the first-order condition,

$$\frac{\partial u}{\partial n}(c_0, n) = p - r. \tag{1}$$

For the society as a whole, the total number of children born is

$$n^* = \int_{p-r<0} \bar{n} d\mu + \int_{0 \leq p-r < \bar{u}_n} \hat{n}(p-r) d\mu.$$

The new number of people, n^*, can be greater or less than the number of parents' generation. It depends on both cost of children (p) and return from children (r). Since children can be form of savings, generous pension could contribute to less children. This effect is the income effect which would be reflected in (1).

Feedback Relationship

The change in labor force follows

$$L_{t+1} - L_t = s_L \left(r_t - p_t \right) L_t,$$

where r_t and p_t are return and cost of having a child at time t. s_L is a constant that measure efficiency of child rearing, reflecting social capital.

We can make the following approximation regarding return from a child,

$$r_t = E_t\left[\frac{\partial f}{\partial L_{t+1}}\right], \quad p_t = \frac{\partial f}{\partial L_t}.$$

Return is future marginal product of labor and cost is the forgone wage , i.e., current marginal product of labor. When population declines over a period and labor supply declines, wages should become higher. This will increase the return of a child. If the population increases over a period, the wage and and in turn the return of a child will decline. Thus labor force should not continue to decrease or increase over long periods of time unless there is significant change in capital.

Labor productivity also depends on level of capital. Capital accumulation proceeds according to

$$K_{t+1} - K_t = s_K Y_t, \quad Y_t = f(K_t, L_t).$$

If $K_{t+1} > K_t$, it is reasonable to assume,

$$\frac{\partial f}{\partial L}(K_{t+1}, L_{t+1}) > \frac{\partial f}{\partial L}(K_t, L_t)$$

with $L_{t+1} - L_t > 0$. In this case, labor will increase for a prolonged length of time. If return to labor is perceived to decline in the future, it will be,

$$\frac{\partial f}{\partial L}(K_{t+1}, L_{t+1}) < \frac{\partial f}{\partial L}(K_t, L_t),$$

and labor will start to decrease. This will raise the marginal product of labor, and labor decline should not last forever.

This simple analysis suggests

- Population decline cannot last forever.
- Policy variable other than p should be considered, such as social capital (s_L).

The proceeding simple model is a skeleton model and other variables should be considered. There are also likely to be a other relationships between the existing and yet to be added variable. As the first step towards a more comprehensive model, we review the innovation growth literature in the following sections.

3 Solow Model

We start with the classic Solow Model. The economy is,

$$Z = F(K, AL),$$

where Z is output, K is capital, A is productivity of labor and L is labor. It is assumed that the function $F(\cdot)$ is concave and linearly homogeneous.

We can write output per effective labor, z as function of capital per effective labor, k,

$$z = \frac{Z}{AL} = F(\frac{K}{AL}, 1) = f(k), \text{ where } k = \frac{K}{AL}.$$

The first equality can be interpreted to mean population decline results in increased per capita output (actually per effective capita). However Z also depends effective labor as well as capital. Denoting saving rate by s, capital accumulates according to

$$\dot{K} = sZ = sALf(k).$$

Capital per effective labor grows according to,

$$\dot{k} = sf(k) - (\frac{\dot{A}}{A} + \frac{\dot{L}}{L})k.$$

In traditional growth theory, technical advance $\frac{\dot{A}}{A}$ and population growth $\frac{\dot{L}}{L}$

are both assumed positive. On the other hand, $\frac{\dot{L}}{L} < 0$ will increase capital per effective labor *ceteris paribus*. The relationship also implies that decline in savings (which is often associated with aging society) will reduce capital accumulation *ceteris paribus*. Overall if per labor capital $\frac{K}{L} = Ak$ and $\frac{\dot{A}}{A}$ are not very large, then $\dot{k} < 0$. Since $z = f(k)$, this means output per effective labor will *decline*.

Getting back to outputs,

$$\dot{Z} = (\dot{AL})z + (AL)\dot{z}.$$

Growth may be negative even even if $\dot{z} > 0$. Per capita (not per effective capita),

$$(\dot{Az}) = \dot{A}z + A\dot{z},$$

may not be positive if $\dot{z} < 0$.

Last but not least, we note that without homogeneity of the function, $F(K, AL)$,

$$\dot{Z} = \frac{\partial F}{\partial K}\dot{K} + \frac{\partial F}{\partial L}\left(\dot{A}L + A\dot{L}\right).$$

The relationship between population change and output is eve more complex.

4 Grossman-Helpman Product Variety Model

In this section we consider an economy where consumers have preference for variety and innovation increases variety. Specifically, utility of representative household

$$U_t = \int_t^\infty exp^{\rho(\tau-t)} \log\left[\int_0^n (\tau)x_(j)^\alpha dj\right]^{\frac{1}{\alpha}} d\tau, \quad 0 < \alpha < 1,$$

where elasticity of substitution is $\epsilon = \frac{1}{1-\alpha} > 1$. Parameter n is measurement of product variety available. Additional variety is achieved by innovation which requires ℓ units of labor,

$$dn = \frac{\ell}{a} dt, \quad a > 0.$$

Each firm produces one variety and products are perfectly product differentiated. Thus manufacturer of brand j is a monopolist, and gets profit

$$\pi = \frac{1-\alpha}{n},$$

with price $p = \frac{w}{\alpha}$ where w is the wage.

Equilibrium in capital markets requires ,

$$\pi + \dot{v} = rv,$$

where v is the value of firm and r is the interest rate. The exogenous supply of labor is a constant L and demands are from the innovation and manufacturing sectors. The equilibrium condition in the labor market is,

$$a\dot{n} + \frac{1}{p} = L.$$

Denoting discount rate by ρ, we have $\rho = r$. Incorporating the capital markets equilibrium we have the no arbitrage condition,

$$\dot{v} = \rho v - \frac{1-\alpha}{n}.$$

This is the downward sloping curve in Figure 1a. Free entry and resource constraint implies,

$$\dot{n} = \begin{cases} \frac{L}{a} - \frac{\alpha}{v} & v > \bar{v} = \frac{\alpha a}{L} \\ 0 & v \leq \bar{v}. \end{cases} \tag{2}$$

This is the horizontal line in Figure 1a. The steady state, $(\bar{n}, \bar{v})$, is the intersection of the two curves, point E. More specifically, $\bar{v} = \frac{\alpha a}{L}$ and $\bar{n} =$

$\frac{(1-\alpha)L}{\alpha\rho a}$.

Now we consider what happens if labor decreases from L to L'. Only equation (2) is affected and it shifts upward. The steady state variety decreases to $\bar{n}' < \bar{n}$, and firm value increases to $\bar{v}' > \bar{v}$ (Figure 1b). However, this is only comparison of steady states. In the short run, as result of sudden reduction of labor implies economy is below equation (2). There is no innovation and v decreases. This suggests that the speed of adjustment to steady state after reduction is very important.

5 Product Variety Model with Public Knowledge Capital

In this model, innovation results in accumulation of knowledge capital which can be shared by everyone in society. Knowledge capital in turn increases product variety, each product again comprising a single market (firms are monopolists). Knowledge capital K_n (n varieties) is a public good and contributes in the following way to increasing product variety,

$$\dot{n} = \frac{LK_n}{a},$$

where L is the total labor of the economy. We can normalize so that $K_n = n$. Labor is used for manufacturing and innovation. The labor market equilibrium is.

$$\frac{a\dot{n}}{n} + \frac{1}{p} = L. \tag{3}$$

The following substitutions are made for convenience,

$$V = \frac{1}{nv}, \quad g = \frac{\dot{n}}{n}.$$

Then the free entry condition with (3) implies,

$$g = \begin{cases} \frac{L}{a} - \alpha V & V < \frac{L}{\alpha a} \\ 0 & V \geq \frac{L}{\alpha a} \end{cases}. \tag{4}$$

Higher rate of innovation leaves less labor for manufacturing. This leads to higher higher price and higher profit for those firms that can produce. Graphically, this the downward sloping line and the vertical axis above its vertical intercept in Figure 2a. Finally the no arbitrage condition is,

$$\frac{\dot{V}}{V} = (1-\alpha)V - g - \rho.$$

This is the upward sloping line in Figure 2a.

The steady state equilibrium is the intersection E. In steady state, variety continues to increase ($g > 0$) but firm value is constant. Division of labor between innovation and manufacturing also remains constant in steady equilibrium.

Now we consider what happens if labor decreases from L to L'. This means equation (4) moves downward (Figure 1b). The new steady state equilibrium must be E'.[1] The rate of variety increase declines. The value of the firm remains constant at a lower level.

6 Human Capital Investment

We consider a model where there is both product and labor heterogeneity. High-technology products require high-skilled labor but resources can be used

[1]Other expectations on return on firm value are not consistent. If expectation on return on v is greater than at E', then variety stops at finite number ($g = 0$) and V goes to infinity or v goes to 0. But this is not possible since firm profit remains strictly positive with finite number of variety. If expectation on return is less than E', then v goes to infinity while variety grows at maximum rate. Increasing number of variety means there is an upperbound of v. Again, not possible.

to improve quality of labor. Specifically, there is unskilled labor (L) and high skilled labor (H). Individuals invest in human capital (take time for schooling) to become high skilled labor. Innovation requires high skilled labor but manufacturing uses low skilled labor. The rate of innovation is γ.

There are three types of manufactured goods: traditional (Z), high-tech (Y) and intermediate input (X). Manufacturing of the traditional manufactured good requires only low-skilled labor. The high-tech product uses high-skilled labor and the intermediate good. The intermediate good is directly influenced by innovation. Each good has fixed coefficients production process.

Steady state equilibrium can be characterized by the following three relationships. First, the no-arbitrage condition is,

$$\begin{aligned} & \frac{(1-\delta)p_X X}{c_\gamma(w_L, w_H)} = \rho + \gamma \\ \Leftrightarrow \quad & B_X c_X - B_Z c_Z = \frac{(1-\delta)\sigma}{\rho+\gamma}, \end{aligned} \tag{Π}$$

where c_Z, c_Y, and c_X are unit costs of production, $c_\gamma = c_\gamma(w_L, w_H)$ is the unit cost of R&D, w_H and w_L are wages, and p_X is the price of the traditional good.

Using the notation convention a_{LX} as input coefficient of L labor for producing X, the labor market clearing condition, one for each type, are,

$$a_{L\gamma}\gamma + \frac{a_{LX}\sigma\delta}{c_X} + \frac{a_{LZ}(1-\sigma)}{c_Z} = L, \tag{L}$$

$$a_{H\gamma}\gamma + \frac{a_{HX}\sigma\delta}{c_X} + \frac{a_{HZ}(1-\sigma)}{c_Z} = H. \tag{H}$$

The three equations are depicted in Figure 3a. The common intersection of all three lines, E, is the steady state equilibrium.

First let us consider the reduction of both types of labor. This shifts both labor curves (H) and (L) downward. Then the rate of innovation γ

must be reduced to move the (Π) curve downward. (The new intersection A in Figure 3b.) This makes the labor curves shift upward until all three lines again intersect at the same point. (Intersection E in the same Figure.) This is the new steady state with a *lower* γ.

Now let us consider the reduction of L only. This time only L moves downward. Accordingly now Π must move upward to the new intersection A in Figure 3c. This increases γ and both labor curves move downward. (The new equilibrium E). As result of reduction of low skilled labor, rate of innovation *increases.*

The result of reduction on only high-skilled labor is summarized in Figure 3d. The rate of innovation is reduced as result. The proceeding analysis shows that there are different effect of labor reduction on rate of innovation. We have shown that it is not possible to categorically claim effect of population reduction on pace of innovation.

7 Innovation and Trade

Japan is an open economy and trade is an essential part of the economy. We consider a model with two countries, A and B. Each country produces two goods (innovative and traditional). There is only one type of labor and both goods require it. Labor also is used to improve the the innovative good.

Each country has labor market and no-arbitrage conditions as in the previous section. In addition, there is a market clearing condition.

In equilibrium, only Country A innovates if A has sufficiently large stock of innovation. The production of the traditional good may be specialized. Specifically, both countries produce traditional good if,

$$\frac{L^B}{L^A+\rho} \leq \frac{1-\sigma}{\sigma}. \tag{5}$$

Only Country B produces traditional good if,

$$\frac{L^B}{L^A + \rho} \geq \frac{1 - \sigma}{\sigma}. \tag{6}$$

In equilibrium, initially wages will be equal in both countries, meaning consumers in both countries are equally well-off. But as only country A continues to innovate, it must provide increasingly many high-technology products. Eventually, labor becomes very scarce in country A and $w^A > w^B$.

A decrease in population of country A may change the world regime from (5) to (6). Then Country A will only produce high-tech goods which Country B will only produce traditional good. As result of population decrease in one country, trade increases and specialization becomes more extreme. Eventually, the high demand for labor in one country leads to wage differentials.

References

Gene M. Grossman and Elhanan Helpman (1991): *Innovation and Growth in the Global Economy*, The MIT Press.

Akihiko Matsutani (2004): "Declining Births and Population Decline (Shyoushika to Jinkou Genshyou) (1)," *Nihon Keizai Shinbun*, August 3, 25.

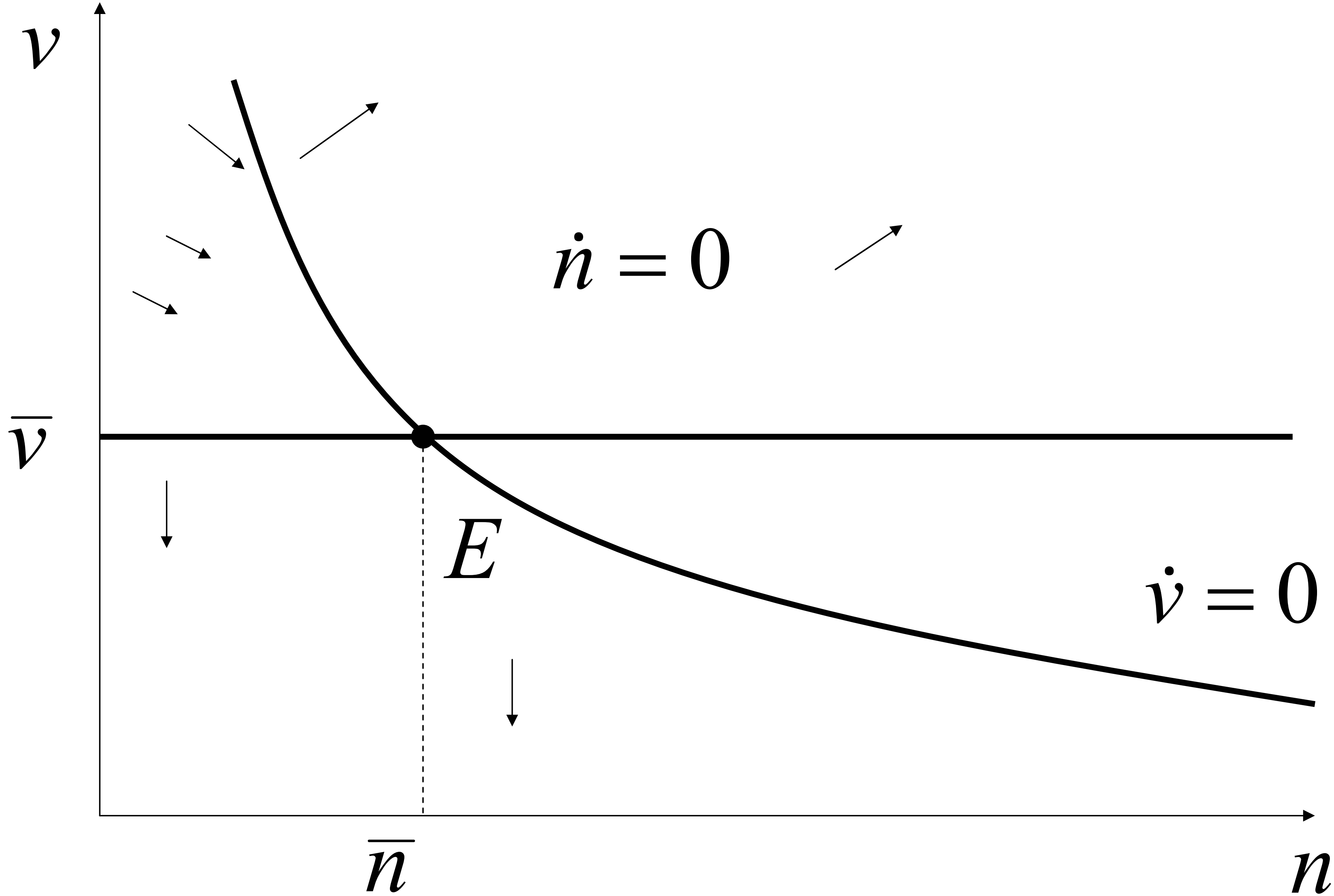

Figure 1a: Innovation in Product Variety

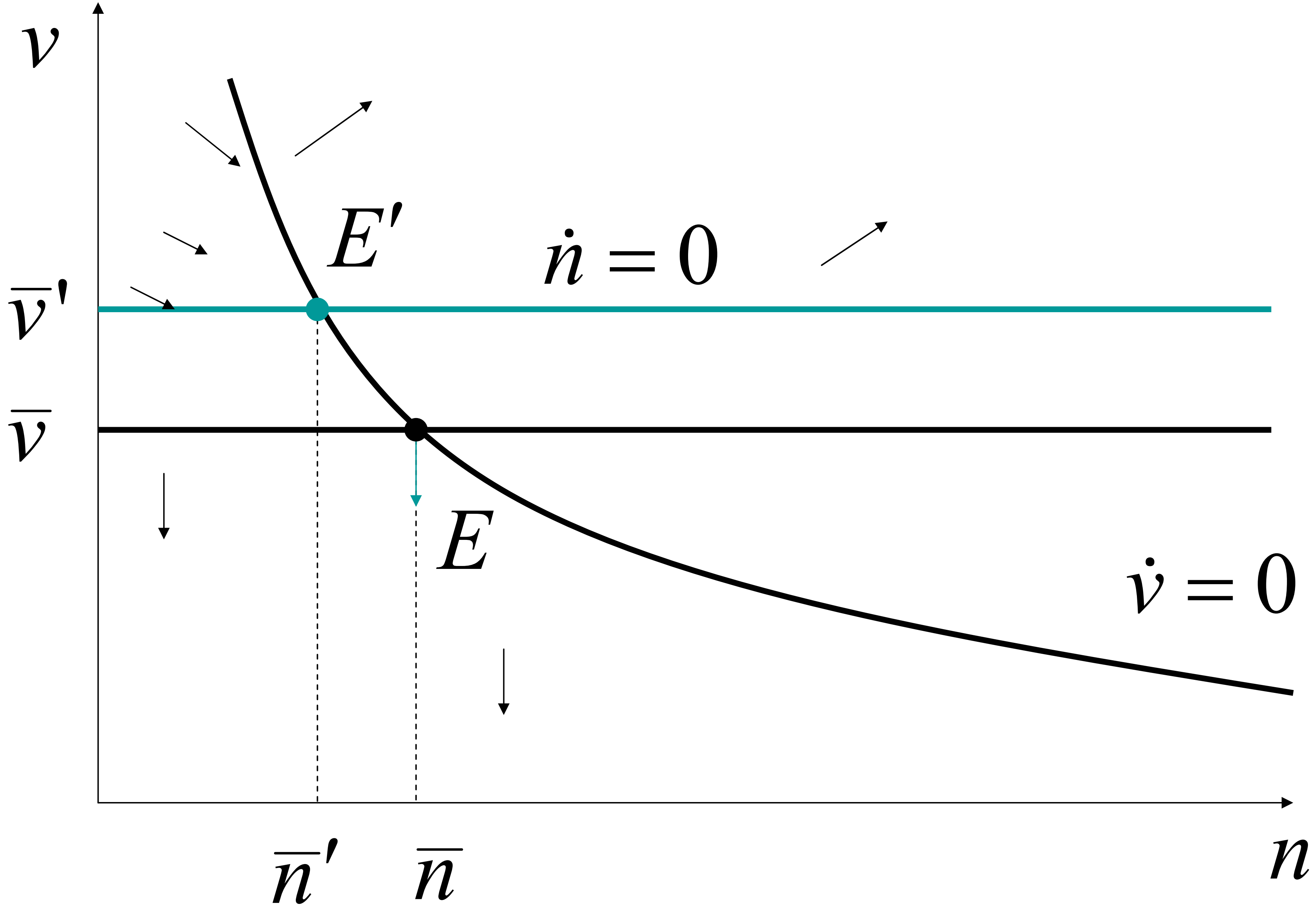

Figure 1b: Innovation in Product Variety

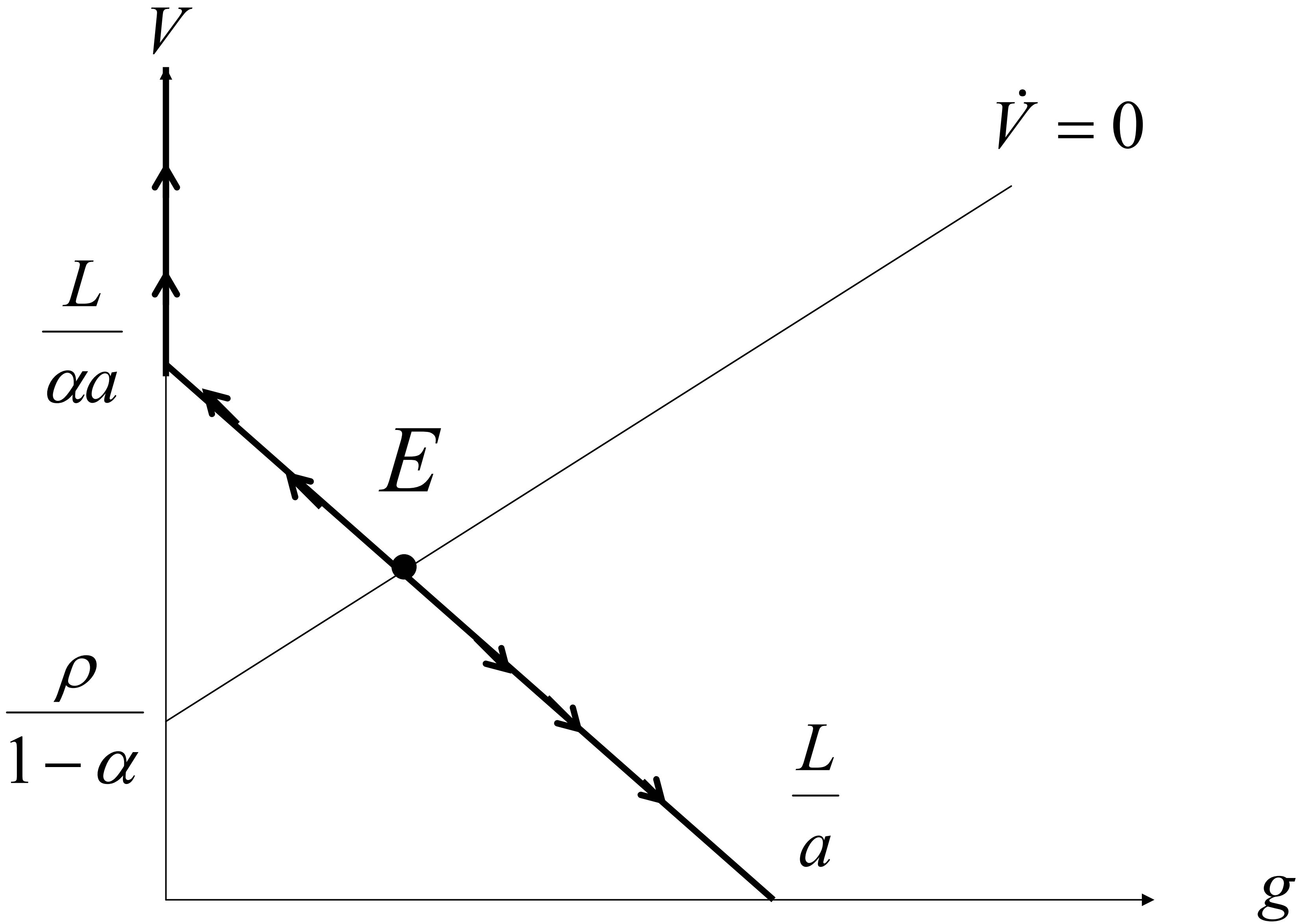

Figure 2a: Innovation with Public Knowledge

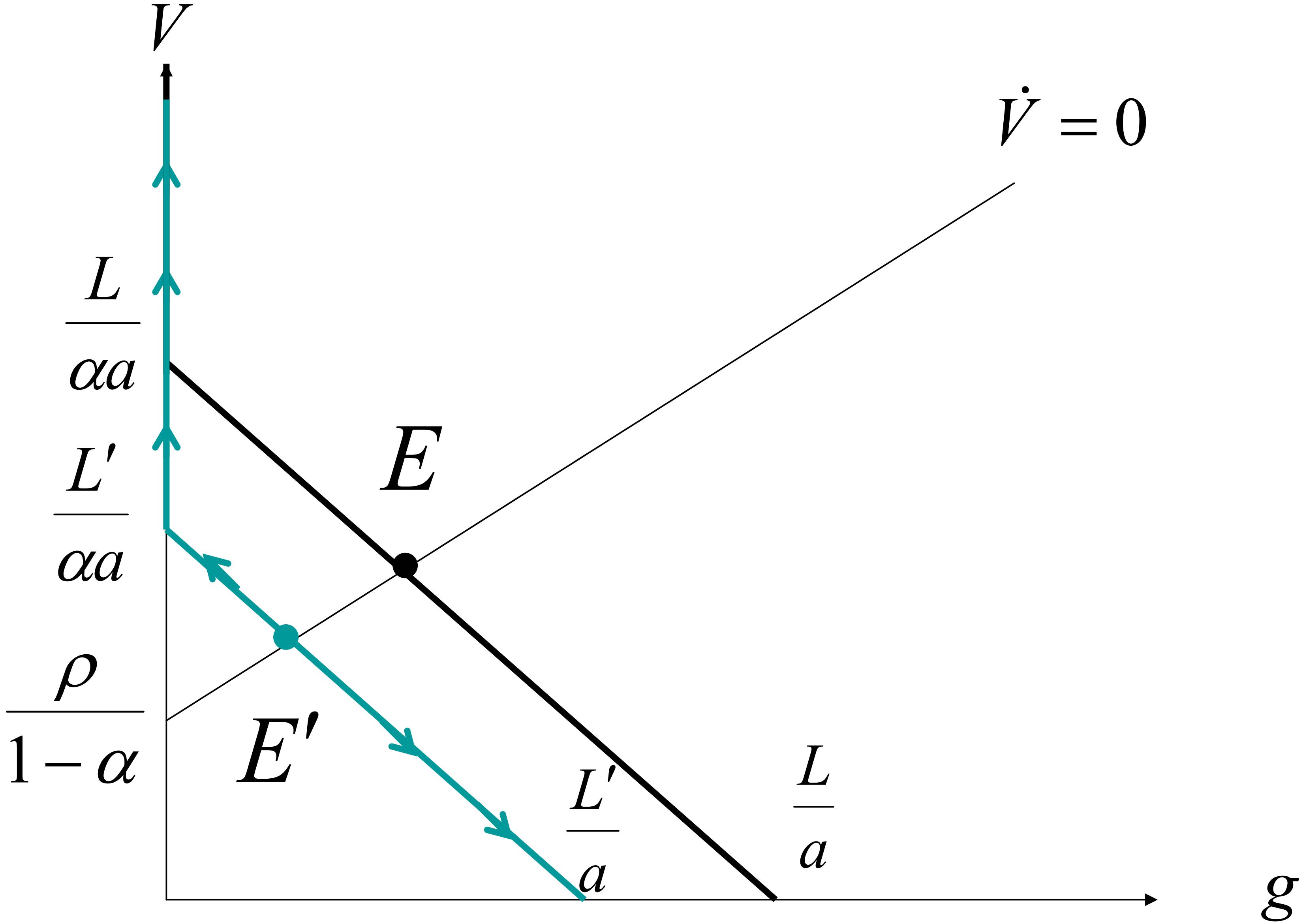

Figure 2b: Innovation with Public Knowledge

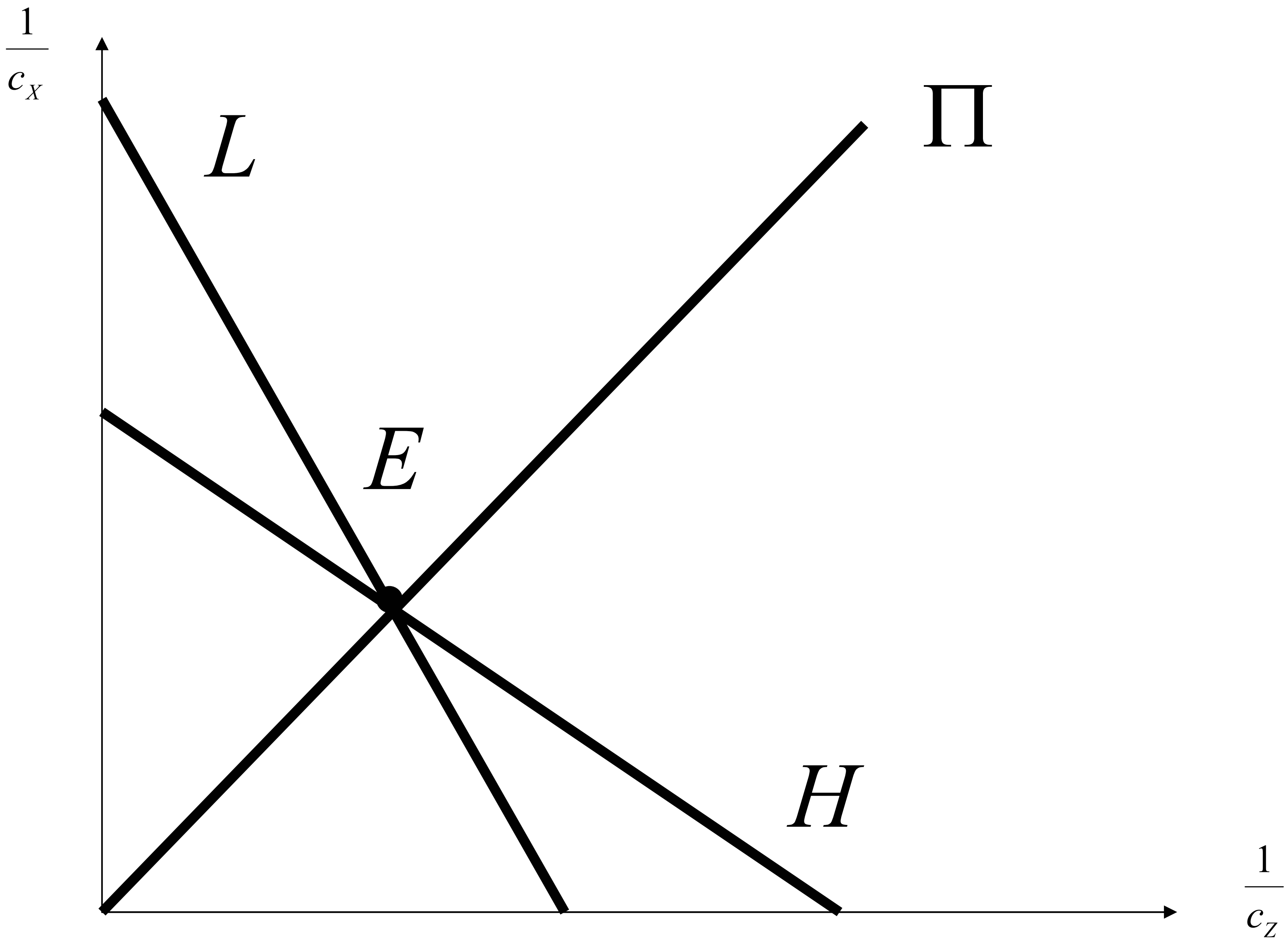

Figure 3a: Human Capital

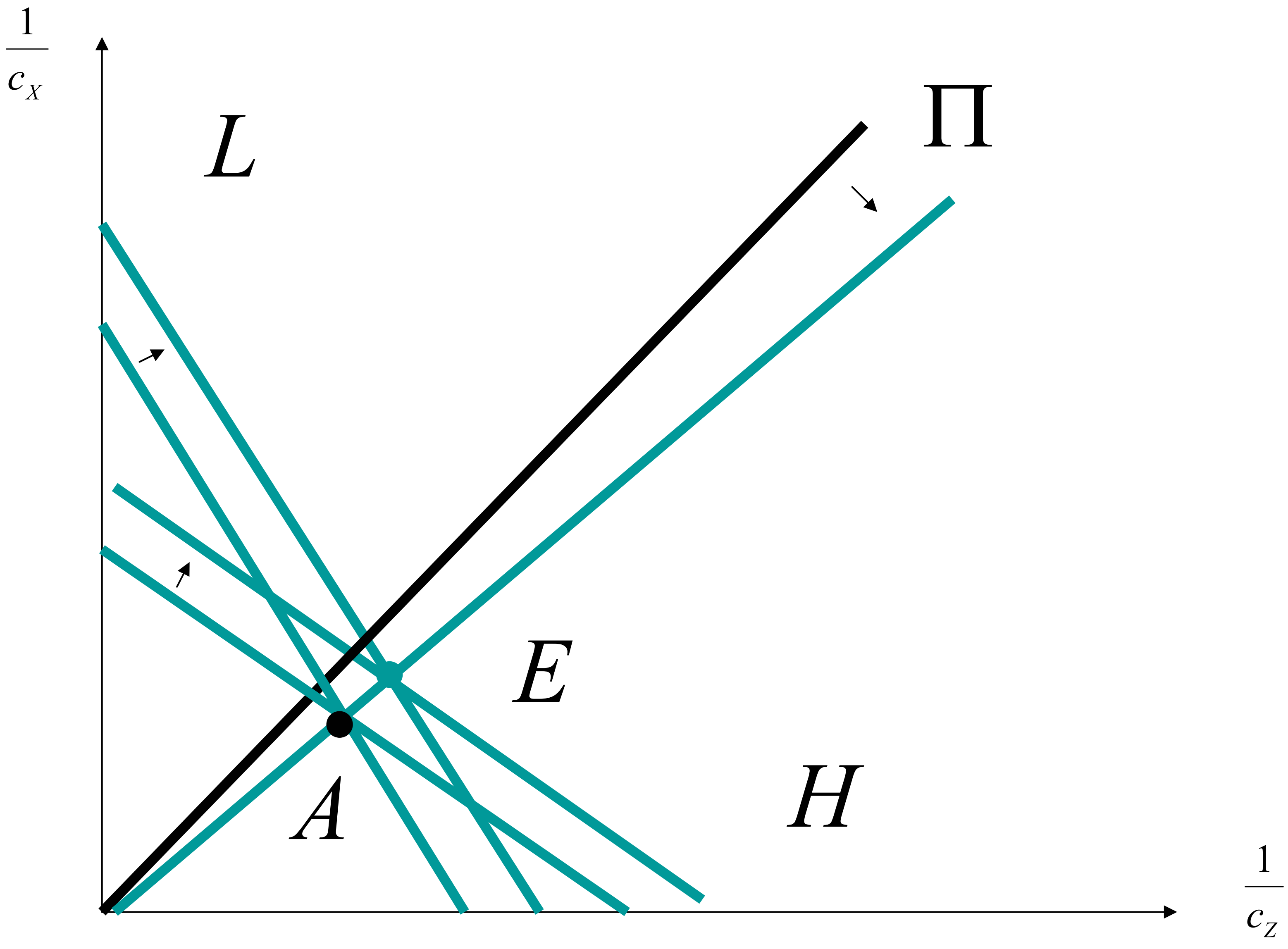

Figure 3b: Human Capital

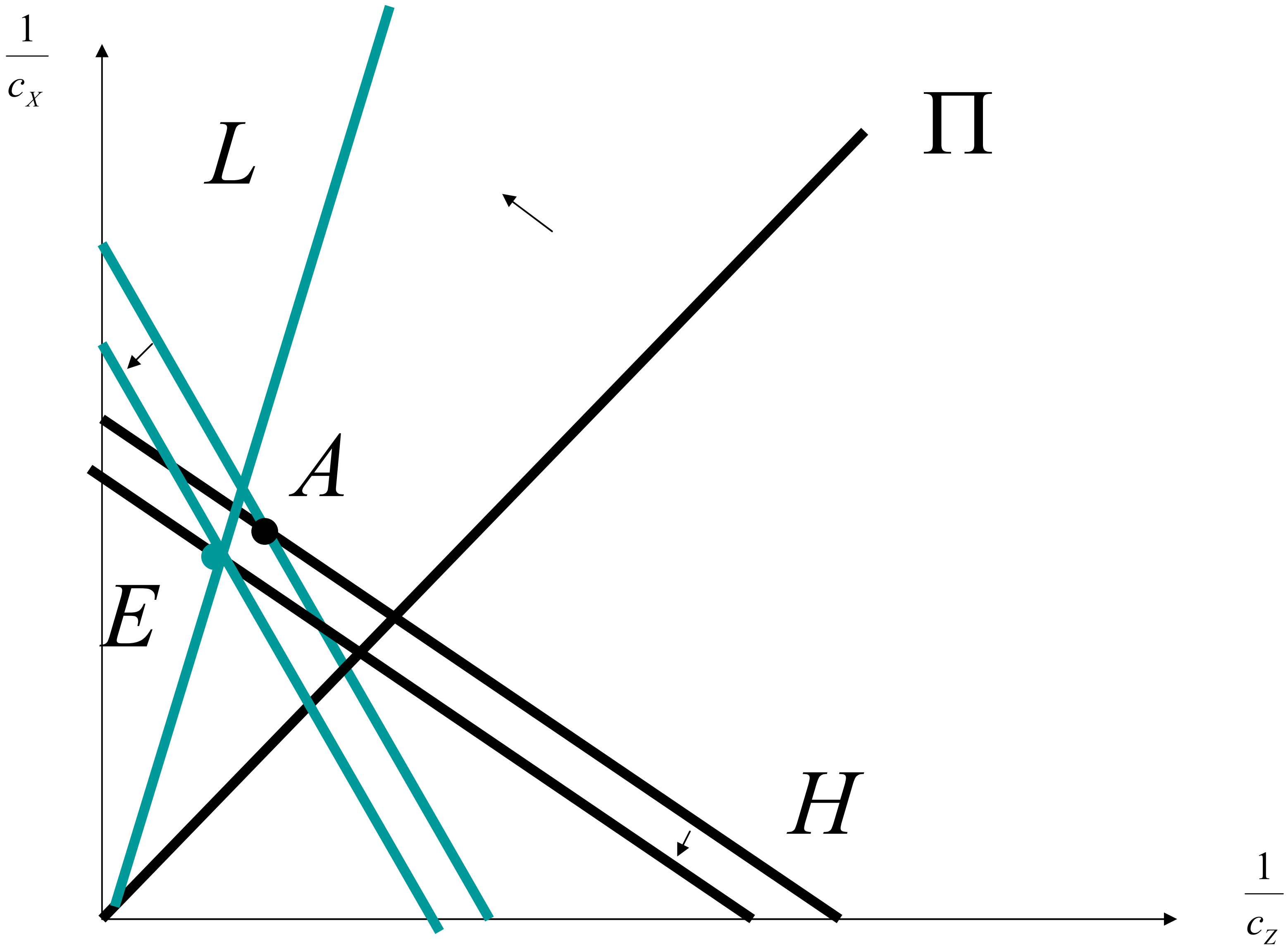

Figure 3c: Human Capital

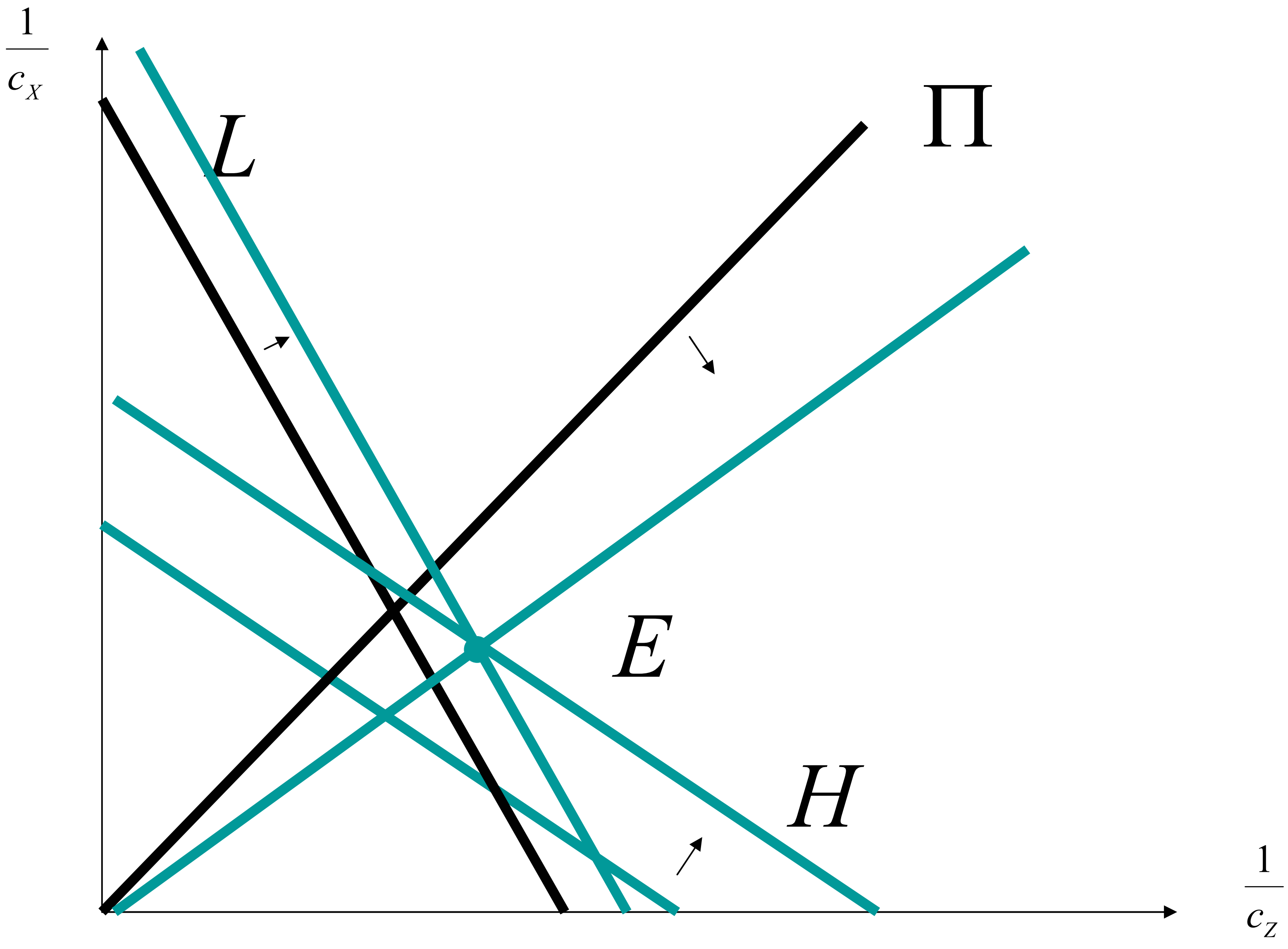

Figure 3d: Human Capital