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The Product Cycle with Firm Heterogeneity

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The Product Cycle with Firm Heterogeneity

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March, 2008

Abstract

The present paper investigates a new product cycle in which each firm faces different productivity and endogenous organization. We show that the coexistence of different organizational forms gradually occurs according to a firm’s productivity level. In particular, the control of production shifts from integration to non-integration and the location of production shifts from a high-wage country to a low-wage country. These shifts occur first within low-productivity firms and then within high-productivity firms. It is also shown that the incompleteness of international contracts plays a significant role in this product cycle.

Keywords: product cycle; firm heterogeneity; incomplete contracts; organization of production

JEL Classification: D23; F12; F14; F21; F23; L22; L23

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1 Introduction

International trade and foreign direct investment (FDI) have been growing fast in intermediate inputs these days. It is often documented that international trade has grown faster in components than in final goods.\(^1\) Moreover, with the advance of computer-aided manufacturing, this growth of input trade has taken place in larger parts across firm boundaries, i.e., arm’s length trade.\(^2\) This feature shows that in the procurement of intermediate inputs, the organizational choice is an important factor for the firms’ strategy. In other words, firms have to simultaneously decide not only on their location of production of different parts but also on their control over these activities, i.e., integration or outsourcing.\(^3\)

In addition to the above phenomena, recent empirical data reveals that differences in productivity level play a key role in the firms’ decision. In contrast to conventional trade theory in which all firms can export everywhere, it has shown that only a small fraction of firms export. Furthermore, these exporters are larger and more productive than non-exporters.\(^4\) These findings suggest that successful theoretical frameworks for studying firms and their decisions to export should include productivity differences that lead only the most productive firms to engage in foreign trade.

To capture these empirical evidence, it is necessary to incorporate two crucial factors in firm-level heterogeneity: productivity difference and organizational form. As seen from the above evidence, these two kinds of heterogeneity are related in the sense that differences in productivity induce different choices for the organization of production. We investigate the role of firm heterogeneity in international trade by integrating elements of two recent papers.

Antràs (2005) develops a dynamic model of North-South trade in which the incompleteness of international contracts leads to the emergence of product cycles. Using the property-rights approach in Grossman and Hart (1986), Antràs introduces the boundaries of the firm to the classical product cycle model of Vernon (1966). He shows that the shift of manufacturing occurs first within firm boundaries through FDI and, at a later stage, through outsourcing to independent firms in the South, without addressing differences in productivity across firms.

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\(^1\)See, for example, Hummels et al. (2001) and Yeats (2001).

\(^2\)Feenstra (1998) illustrates this international specialization in intermediate inputs, taking an example of the production of the Barbie doll. Hanson et al. (2005) show that the growth of foreign outsourcing by U.S. firms might have outpaced the growth of their foreign intrafirm sourcing.

\(^3\)Following Helpman (2006), we define “integration” and “outsourcing” as follows: integration means the production of intermediate inputs within the boundaries of the firm, while outsourcing means the acquisition of intermediate inputs from an unaffiliated supplier.

\(^4\)Among valuable empirical evidence, see in particular Bernard et al. (2003). They report that exporters are in the minority (only 21% of U.S. plants) but they have, on average, a 33% advantage in labor productivity relative to non-exporters. The same results are confirmed in Helpman et al. (2004).
Antràs and Helpman (2004), on the other hand, follow Melitz (2003) to introduce differences in productivity across firms. They combine heterogeneous productivity of Melitz with firm boundaries in Antràs (2003) and realize equilibria featuring multiple organizational forms within industries. In equilibrium, each firm chooses the location of the production and the ownership structure according to its productivity level. However, they consider only a static model and do not explore how the manufacturing stage of production changes over time in the presence of firm heterogeneity.

In this paper, we extend Antràs (2005) by adding the following two elements from Antràs and Helpman (2004). First, we consider heterogeneous productivity that comes from marginal costs and “beachhead” costs. As mentioned above, Antràs (2005) assumes homogeneous productivity across firms and, as a result, the coexistence of organizational form never occurs in equilibrium. In practice, however, the different organizational forms can coexist, such as some firms procure intermediate inputs via FDI and others through foreign outsourcing. To realize this coexistence, we need to introduce intra-industry heterogeneity of the Melitz (2003) type. Second, we consider domestic incomplete contracts in Antràs (2005). For simplicity, Antràs puts the assumption that a foreign country has incomplete contracts, whereas a domestic country has complete contracts. This assumption is not entirely realistic, however, because contracts between parties could not specify all aspects in advance and thus become incomplete. Instead, we assume that the domestic country has also incomplete contracts but the quality of contracting institution is better in the domestic country than in the foreign country. Under these circumstances, our model shows that the coexistence of different organizational forms gradually occurs according to a firm’s productivity level. In particular, the control of production shifts from integration to non-integration and the location of production shifts from a high-wage country to a low-wage country. These shifts occur first within low-productivity firms and then within high-productivity firms.

The rest of the paper is organized as follows. Section 2 describes the closed-economy model and considers the dynamics under which each firm has heterogeneous productivity level. Section 3 extends the closed-economy model to the open-economy model in which the North can produce intermediate inputs in the South and studies the product cycle. Section 4 concludes. The proofs and discussions of the main results are provided in the Appendix.

5Several researchers give the rationale that incomplete contracts necessarily arise between the principal-agent relationship. For example, Hart (1995) emphasizes the fact that contracts are not “comprehensive” and are revised and renegotiated in every contingency.

6Recent empirical analysis illustrates that the degree of contract incompleteness has a large effect on international trade flow. Nunn (2007) and Levchenko (2007), for example, show that countries with better legal systems export goods that are more intensive in contract-dependent input. Acemoglu et al. (2007), on the other hand, propose a theoretical model in which better contracting institutions lead to the choice of technologies that are more sensitive to contractual frictions.
2 The Closed-Economy Model

In this section, we focus on a closed economy, where firms decide only their ownership structure. In the next section, we extend the model to an open economy, where firms decide not only on the extent of control but also on where to locate their production stages.

2.1 Setup

Consider an economy in which a single good $x$ is produced only with labor. We denote wage rate in this economy by $w$ and assume it is fixed. Consumer preferences are such that the unique producer of $x$ faces the following isoelastic demand function:

$$x = Ap^{-1/(1-\alpha)}, \quad 0 < \alpha < 1$$ (1)

where $p$ is the price of the good and $A$ is a parameter that the producer takes as given.\(^7\)

Firm behavior is similar to Melitz (2003). To start producing, a firm needs to bear a fixed cost of entry. Upon paying this fixed cost, a producer draws a productivity level $\theta$ from a known distribution $G(\theta)$. After observing this productivity level, the final-good producer decides whether to exit the market or start producing; in the latter case, an additional fixed cost of organizing production needs to be incurred. As discussed below, this additional fixed cost is a function of the structure of ownership.\(^8\)

Production of any final-good requires a combination of two inputs, $h$ and $m$, which we associate with headquarter services and manufactured components, respectively. Output of the final-good is Cobb-Douglas function of the inputs:

$$x = \theta \left( \frac{h}{\eta} \right)^\eta \left( \frac{m}{1-\eta} \right)^{1-\eta}, \quad 0 < \eta < 1$$ (2)

where $\eta$ measures represents headquarter intensity. The larger $\eta$ is, the more intensive the sector in headquarter services.

There are two types of agents engaged in production: final-good producers $H$ and component suppliers $M$. Upon paying the fixed cost of entry and observing the productivity level $\theta$, every $H$ receives a lump-sum transfer $T$ from $M$ and chooses whether to insource or outsource intermediate inputs.\(^9\) In addition to the fixed cost of entry, $H$ incurs another fixed cost that varies with organi-

\(^7\)Although we focus our attention on the partial-equilibrium approach in this paper, this demand function is derived from the Dixit-Stiglitz preferences under the general-equilibrium setup.

\(^8\)In an open economy, this fixed cost consists of the location of production as well as the ownership structure.

\(^9\)As in many other property-rights literature, this transfer $T$ enables final-good producers to choose their organizational form.
zational form. Following Antràs and Helpman (2004), we term all these costs fixed organizational costs. Since an organizational form consists of the ownership structure alone in the closed economy, we denote the fixed costs by $f_k$ and assume

$$f_V > f_O.$$  \hspace{1cm} (3)

This assumption reflects that there are more managerial overloads such as the supervision of the production under vertical integration than under outsourcing. As Antràs and Helpman (2004) note, this inequality may not necessarily hold in a particular case.\(^{10}\) However, we believe this assumption to be appropriate, and therefore we maintain it in the main analysis.

From the demand function (1) and the production function (2), a revenue function is given by

$$R(h, m) = A^{1-\alpha} \theta^\alpha \left( \frac{h}{\eta} \right)^{\alpha \eta} \left( \frac{m}{1-\eta} \right)^{\alpha (1-\eta)}.$$  \hspace{1cm} (4)

At the bargaining stage, the revenue $R(h, m)$ is divided by Symmetric Nash Bargaining, in which the final-good producer obtains half of the ex post gains from the relationship.\(^{11}\) Note that the distribution of revenue also depends on organizational form, which determines every party’s outside option.

When outsourcing takes place, the outside options at the bargaining stage are zero for both parties, because one party owns $h$ and the other owns $m$, and both inputs have been customized so that they have no value outside the relationship. As a result, $H$ and $M$ share the revenue equally.

On the other hand, when it comes to integration, both $h$ and $m$ belong to $H$ because $M$ is $H$’s employee. Following the property rights in Grossman and Hart (1986), however, if the bargaining fails and $M$ does not cooperate, $H$ is able to produce only a fraction of $\delta \in (0, 1)$ of the output in (2). In this case, the outside option of $M$ is zero, while the outside option of $H$ is a fraction $\delta^\alpha$ of the revenue (4). As a result, in the bargaining stage $H$ receives a fraction

$$\beta_V = \delta^\alpha + \frac{1}{2} (1 - \delta^\alpha)$$

$$= \frac{1}{2} (1 + \delta^\alpha) > \beta_O = \beta = \frac{1}{2}$$  \hspace{1cm} (5)

of the revenue, and $M$ receives a fraction $1 - \beta_V$; $H$ can increase the bargaining power by employing $M$. As in Grossman and Hart (1986), this affects ex ante investment incentives for the both parties.

This completes the description of the model. The timing of events is summarized in Figure 1.

\(^{10}\)Grossman et al. (2005) construct a model in which (3) does not hold.

\(^{11}\)The following results hold even under Generalized Nash Bargaining, i.e., $\beta \neq 1/2$. However, the analysis becomes rather complicated in this case. See footnotes 14 and 21.
Choice of ownership
Ex ante transfer $T$

Components $m$ produced
Symmetric Nash bargaining
Final good produced and sold

**Figure 1**: Timing of events

### 2.2 Equilibrium

Since the delivery of the inputs $h$ and $m$ is not contractible ex ante, the parties choose their quantities non-cooperatively; every supplier maximizes its own payoff. In particular, $H$ provides an amount of headquarter services so that

$$\max_h \beta_k R(h, m) - wh,$$

whereas $M$ provides an amount of components so that

$$\max_m (1 - \beta_k) R(h, m) - wm.$$

Combining the first-order conditions of these two parties, the optimal price and the total operating profits are expressed as

$$p_k = \left( \frac{1}{\theta \alpha} \right) \left( \frac{w}{\beta_k} \right)^{\eta} \left( \frac{w}{1 - \beta_k} \right)^{1-\eta},$$

and

$$\pi_k = A \theta^{\alpha/(1-\alpha)} \psi_k(\eta) - f_k,$$

where

$$\psi_k(\eta) = \frac{1 - \alpha [\beta_k \eta + (1 - \beta_k)(1 - \eta)]}{(p_k \theta)^{\alpha/(1-\alpha)}}. \quad (7)$$

Figure 2 illustrates the relationship between (7). Simple calculation yields

$$\frac{\partial \psi_V(\eta)}{\partial \eta} > 0, \quad \frac{\partial \psi_O(\eta)}{\partial \eta} = 0, \quad \psi_V(1) > \psi_O(1), \quad \text{and} \quad \psi_V(0) < \psi_O(0). \quad (6)$$

$\psi_k(\eta)$ is concerned with the **holdup problem** in the following sense. When the parties engage in relationship-specific investment under incomplete contracting, it is well known that the level of investment is short of efficiency.\(^{13}\) Under the circumstance, Grossman and Hart (1986) propose that the ownership should be given to the party whose investment is important, because this can mitigate

\(^{12}\)See Appendix A.1 for the proof of upward-sloping of $\psi_V(\eta)$.

\(^{13}\)See, for example, Grossman and Hart (1986), Hart and Moore (1990), and Hart (1995).
the holdup problem. In our model, when \( \eta \) is close to one, \( H \)'s investment is more important in production. Following Grossman and Hart (1986), it is better to give the ownership to \( H \) (\( \psi_V(\eta) > \psi_O(\eta) \)). When \( \eta \) is close to zero, however, it is no longer optimal to maintain integration; since \( M \)'s investment is more important, outsourcing generates higher joint surplus (\( \psi_O(\eta) > \psi_V(\eta) \)). As a result, \( \psi_V(\eta) \) and \( \psi_O(\eta) \) intersect at only one point \( \tilde{\eta} \) as in the figure. Then, it is easy to see that

\[
\eta \geq \tilde{\eta} \iff \psi_V(\eta) \geq \psi_O(\eta).
\]

(8)

Summarizing the above observation, the following lemma is obtained:\(^{14}\)

**Lemma 1.** There exists a unique headquarter intensity threshold \( \tilde{\eta} \in (\frac{1}{2}, 1) \) such that for \( \eta > \tilde{\eta} \) (resp. \( \eta < \tilde{\eta} \)), all firms earn higher variable profits by integration (resp. outsourcing).\(^{15}\)

\(^{14}\)Under Generalized Nash Bargaining, \( \psi_O(\eta) \) is no longer flat. That is,

\[
\beta \geq \frac{1}{2} \iff \frac{\partial \psi_O(\eta)}{\partial \eta} \geq 0.
\]

In this case, however, we can see that \( \psi_V(1) > \psi_O(1) \) and \( \psi_V(0) < \psi_O(0) \) and thus the relationship in equation (8) still holds. Therefore, the following results can be obtained even under Generalized Nash Bargaining.

\(^{15}\)This threshold \( \tilde{\eta} \) is similar to that in Antràs (2003). We can prove that, however, \( \tilde{\eta} \) is necessarily greater than one-half under Symmetric Nash Bargaining. See Appendix A.3 in detail.

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**Figure 2:** The relationship between \( \psi_k(\eta) \)
It is important to note that $\tilde{\eta}$ does not always determine the optimal ownership structure $k^*$ under condition (3). That is, $\tilde{\eta}$ concerns only variable profits (7); the optimal ownership structure is determined so that total profits (6) are maximized. To investigate the ownership structure in equilibrium, we have to take account of the effect of the fixed organizational cost.

Figure 3 illustrates the profit function (6). The variable $\theta^{\alpha/(1-\alpha)} \equiv \Theta$ is measured along the horizontal axis and operating profits measured along the vertical axis. It is evident that the operating profit function $\pi_k(\cdot)$ is linear in $\Theta$, and it has the slope $\psi_k(\eta)$ and the intercept $-f_k$. Then, it follows from (8) that integration and outsourcing can coexist when $\eta > \tilde{\eta}$: firms with high-productivity choose to integrate, whereas firms with low-productivity choose to outsource. This coexistence arises because only high-productivity firms can earn positive profits from integration after paying higher fixed costs. Due to this fixed cost, it is more profitable for low-productivity firms to choose to outsource than to integrate. In other words, the tradeoff between (5) and (3) leads to the coexistence.

Notice that there exists another threshold, namely $\hat{\eta}$, such that only vertical integration is chosen in equilibrium. Clearly, the headquarter intensity $\tilde{\eta}$ identifies the profit line which passes through the cutoff point $\Theta$. Then, the figure shows that when $\eta > \tilde{\eta}$ only integration is pervasive, whereas when $\eta > \eta > \tilde{\eta}$ integration and outsourcing coexist in equilibrium. Combining the thresholds $\tilde{\eta}$ and $\hat{\eta}$, we have the following proposition:

\[ \text{Figure 3: Equilibrium in the closed economy} \]

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16 Since $\frac{\partial \psi_V(\cdot)}{\partial \delta^{\alpha/(1-\alpha)}} |_{\eta=1} > 0$ and $\frac{\partial \psi_V(\cdot)}{\partial \delta^{\alpha/(1-\alpha)}} |_{\eta=0} < 0$, the larger outside option $\delta^{\alpha}$ leads to the steeper slope of $\psi_V(\eta)$. This means that if $\delta^{\alpha}$ is relatively small, $\tilde{\eta}$ may not exist.
Proposition 1. Assume that there are the fixed organizational costs satisfied with (3). Then, if final-good producers’ outside option $\delta^\alpha$ is sufficiently high, there exist two thresholds $\tilde{\eta}$ and $\hat{\eta}$ with $\hat{\eta} > \tilde{\eta}$ such that: (i) if $\hat{\eta} < \eta < 1$, all firms choose to insource; (ii) if $\tilde{\eta} < \eta < \hat{\eta}$, high-productivity firms choose to insource and low-productivity firms choose to outsource; and (iii) if $0 < \eta < \tilde{\eta}$, all firms choose to outsource.

The immediate implication of Proposition 1 is that high-tech intensive products ($\hat{\eta} < \eta < 1$) are manufactured within the firm, whereas low-tech intensive products ($0 < \eta < \tilde{\eta}$) are procured in the market. The production of neutral products ($\tilde{\eta} < \eta < \hat{\eta}$), on the other hand, depends upon a firm’s productivity level.

2.3 Dynamics

Now, following Antrás (2005), we consider the simple dynamic extension of the static model developed above. The objective in this subsection is to show that as a good matures the ownership structure gradually changes according to a firm’s productivity level.

The setting is as follows. Time is continuous, indexed by $t$, with $t \in [0, \infty)$. Consumers are lived infinitely and, at any $t \in [0, \infty)$, their preferences for good $x$ are captured by the demand function (1). In addition, we assume that the output elasticity of the headquarter services decreases through time. In particular, this elasticity is a function of time $\eta = g(t)$ with

$$g'(t) < 0, \quad g(0) = 1, \quad \text{and} \quad \lim_{t \to \infty} g(t) = 0. \tag{9}$$

This assumption is meant to capture the idea that most goods require a lot of R&D and product development in early stages, while the assembling or manufacturing becomes a much more significant input in production as the good matures. We will take these dynamics as given.

Figure 4 depicts the above dynamics. In the figure, the relationship between $\psi_k(\eta)$ that is derived from Figure 3 is given in the second quadrant, while the locus of $g(t)$ that satisfies (9) is given in the first quadrant. For simplicity, we assume that the outside option $\delta^\alpha$ is time-invariant so that $\tilde{\eta}$ is not affected by time and determined uniquely.\(^{17}\) Under the circumstance, $\hat{\eta}$ is determined at the point where $\psi_V(\eta)$ and $\psi_O(\eta)$ intersect. Figure 4 shows that given the threshold $\tilde{\eta}$ in the second quadrant, $\tilde{t}$ is uniquely fixed in the first quadrant.\(^{18}\) It can be said that $\tilde{t}$ is a time threshold at which all firms can earn the same variable profits from integration and outsourcing in the dynamics.

\(^{17}\)This time-invariant outside option is not crucial to the following result. Indeed, Appendix A.4 shows that if the outside option decreases over time, the same result can be obtained.

\(^{18}\)Strictly speaking, $\tilde{t}$ is derived from the inverse function of $\psi_k(\eta)$, i.e., $\tilde{t} = \psi_k^{-1}(\tilde{\eta})$. 

9
With this simplified, dynamic setup, the following lemma is a straightforward application of Lemma 1 to the dynamic model:

**Lemma 2.** There exists a unique time threshold $\tilde{t} \in [0, \infty)$ such that when $t < \tilde{t}$ (resp. $t > \tilde{t}$), all firms earn higher variable profits by integration (resp. outsourcing).

Recall that under (3) there is another threshold $\hat{\eta}$ such that only integration is pervasive. We denote this time threshold by $\hat{t}$. Furthermore, Proposition 1 indicates that only integration is pervasive if $H$’s outside option $\delta^s$ is sufficiently high. To realize this outcome, we assume that this outside option is initially large enough so that

$$\psi_V(1) > \psi_V(\hat{\eta}). \quad (10)$$

Then, if all firms incur the fixed cost whenever they produce, the ownership structure in dynamic equilibrium is given as follows:

$$\hat{\eta} < \eta < 1 \iff 0 \leq t < \tilde{t} \iff \text{integration;}$$

$$\hat{\eta} < \eta < \hat{\eta} \iff \hat{t} < t < \tilde{t} \iff \text{coexistence;}$$

$$0 < \eta < \hat{\eta} \iff \hat{t} < t < \infty \iff \text{outsourcing.}$$

---

\[19\] If the outside option $\delta^s$ is also decreasing in time, (10) can be abandoned. See Appendix A.4 in detail.

\[20\] For simplicity, we assume that all firms enter at $t = 0$ without late entry. For justification of this assumption, see Ederington and McCalman’s (2007, 2008) technology adoption model.
We show these implications below in the framework of Figure 3. First, note that the difference in the static model is that $\eta$ is affected by time. Since $\psi_O(\eta)$ is independent of $\eta$, we can ignore the effect of time to the slope of $\pi_O$: 

$$\frac{\partial \psi_O(\eta)}{\partial \eta} \cdot \frac{dg(t)}{dt} = 0.$$ 

On the other hand, $\psi_V(\eta)$ is an increasing function of $\eta$ and, from (9), $\eta = g(t)$ is a decreasing function of $t$. Combining these effects, we have 

$$\frac{\partial \psi_V(\eta)}{\partial \eta} \cdot \frac{dg(t)}{dt} < 0,$$ 

which means that the slope of $\pi_V$ becomes smaller over time.

Figure 5 depicts the dynamics in the closed economy. The figure shows that in an early stage with $t < \hat{t}$, only integration appears in equilibrium. Since the slope of $\pi_V$ declines over time, two ownership structures begin to coexist in $\hat{t} < t < \tilde{t}$: integration with relatively high-productivity firms and outsourcing with relatively low-productivity ones. When enough time satisfied with $t > \tilde{t}$ has passed to standardize the good, the whole $\pi_V$ locates under $\pi_O$. As a result, integrated firms exit the industry and outsourcing becomes the only option of organizational form for profit-maximizing firms.\textsuperscript{21} We summarize this result in the following proposition:

\textsuperscript{21}Since the least productive firms (whose productivity is less than $\Theta$ in Figure 5) earn negative profits, they must exit the industry. Furthermore, this threshold becomes greater over time; as the industry matures, the number of firms tends to drop. (Under Generalized Nash Bargaining satisfied with $\beta > 1/2$, the number of exiting firms becomes much larger because $\psi_O(\eta)$ is also decreasing in time.) The author thanks Stephen Yeaple for pointing out this “shakeout” phenomenon and for suggesting Ederington and McCalman (2007) for further remarks on industrial evolution.

Figure 5: Dynamics in the closed economy
Proposition 2. When the good is new, i.e., $0 \leq t < \hat{t}$, the manufacturing stage of production takes place only within integrated firms initially. When the good starts to be mature, i.e., $\hat{t} < t < \tilde{t}$, firms with low-productivity give their manufacturing to non-integrated firms, while firms with high-productivity maintain to integrate their supplier. When the good is sufficiently mature or standardized, i.e., $\hat{t} < t < \infty$, manufacturing is undertaken only within non-integrated firms. In every stage, the least productive firms exit the industry.\textsuperscript{22}

The key of this proposition is that the shift of organization is related to productivity. The intuition why the shift occurs first within low-productivity firms is as follows. As Figure 4 shows, the revenue from integration gradually decreases over time. When $\hat{t} < t < \tilde{t}$, $\psi_V(\eta) > \psi_O(\eta)$ still holds, which means that all firms have an incentive to maintain integration in this stage. Since the difference between $\psi_V(\eta)$ and $\psi_O(\eta)$ becomes much smaller, however, condition (3) induces low-productivity firms to change their organization; although they can earn the higher revenue from integration, they cannot cover the fixed cost of integration. By contrast, high-productivity firms can still earn higher profits and thus maintain integration. As a result, when $\hat{t} < t < \tilde{t}$, two organizations come to coexist in equilibrium, thereby letting low-productivity firms switch their manufacturing first.

3 The Open-Economy Model

In this section, we extend the closed-economy model developed in the previous section to an open-economy model. In an open economy, all firms have to decide on the extent of their control over their activity and also on where to locate the production of different parts of their value chains simultaneously. In our model, intermediate inputs can be produced domestically or in a low-wage country. Firm heterogeneity plays a key role in this section, too. International trade with firm heterogeneity leads to new insights which are not seen in the previous product-cycle literature.

3.1 Setup

Consider a world with two countries, the North and the South, whose unique factor of production is labor. The setting is similar to that in the previous section, except for several points that are noted below.

\textsuperscript{22}Two points should be stressed in comparison with Antràs (2005). First, since Antràs assumes domestic complete contracts, firm boundaries in the domestic country are not considered. Second, since Antràs assumes homogeneous firms, two organizational forms never coexist in the same industry. These extensions will play a crucial role in the open economy model in the next section.
First of all, we assume that the wage rate differs between two countries. We denote by $w^\ell$ the wage rate in country $\ell \in \{N, S\}$. These wage rates are fixed and the northern wage is higher than the southern wage:

$$w^N > w^S. \quad (11)$$

Second, only the North knows how to produce headquarter services $h$ and final-good varieties $x$, while intermediate inputs $m$ can be produced in the North and in the South. Thus, the problem that all final-good producers face is where to produce $m$; $H$ has to choose to transact with $M$ in the North or the South.

Third, the fixed organizational costs vary not only by the ownership structure but also by the location of $M$. We denote them by $f^\ell_k$ and assume they are ranked as follows:

$$f^S_V > f^S_O > f^N_V > f^N_O. \quad (12)$$

Finally, the outside option of $H$ is assumed to be larger in the North than in the South:

$$(\delta^N)^\alpha > (\delta^S)^\alpha. \quad (25)$$

As a result, $H$’s ex post fraction of revenue $\beta^\ell_k$ under ownership structure $k$ and locational choice $\ell$ is given by

$$\beta^N_V = \frac{1}{2}[1 + (\delta^N)^\alpha] > \beta^S_V$$

$$= \frac{1}{2}[1 + (\delta^S)^\alpha] > \beta^N_O = \beta^S_O = \beta = \frac{1}{2}. \quad (13)$$

3.2 Equilibrium

Incomplete contracts in both countries means that inputs $h$ and $m$ cannot be specified ex ante, and thus $H$ and $M$ choose their quantities non-cooperatively. As in the closed economy, this brings about underinvestment for both parties. In particular, the profit maximization for $H$ is given by

$$\max_h \beta^\ell_k R(h, m) - w^N h,$$

---

23 In this paper, we restrict our attention to the partial equilibrium model in which these wage rates are exogenously given. Antràs (2005) extends this model to a general equilibrium model and shows that the equilibrium wage in the North is necessarily higher than that in the South.

24 This ranking is the same as that in Antràs and Helpman (2004).

25 This reflects that the North has stronger legal protection than the South. Note that our analysis requires the strong inequality; the following results do not hold under the equal legal protection (see footnote 35). On the other hand, Antràs and Helpman (2004) permit the weak inequality and their results hold even in $\delta^N = \delta^S$. This is an interesting difference from Antràs and Helpman (2004).
whereas for $M$ it is
\[ \max_m (1 - \beta^M_k)R(h, m) - w^M m. \]
Combining the first-order conditions of these two programs, the optimal price and the total operating profits are expressed as
\[ p^\ell_k = \left( \frac{1}{\theta^\alpha} \right) \left( \frac{w^N}{\beta^\ell_k} \right)^\eta \left( \frac{w^\ell}{1 - \beta^\ell_k} \right)^{1-\eta}, \]
and
\[ \pi^\ell_k = A\theta^\alpha/(1-\alpha)p^\ell_k - f^\ell_k, \tag{14} \]
where
\[ \psi^\ell_k(\eta) = \frac{1 - \alpha\beta^\ell_k\eta + (1 - \beta^\ell_k)(1 - \eta)}{(p^\ell_k\theta)^\alpha/(1-\alpha)}. \tag{15} \]

Figure 6 illustrates the relationship between (15).\(^{26}\) It is clear that $\psi^N_k(\eta)$ is the same as $\psi_k(\eta)$ in the closed economy. On the other hand, when the wage differential across countries is satisfied with
\[ \frac{\beta^S_k}{1 - \beta^S_k} < \frac{w^N}{w^S}, \tag{27} \]
the relationship between $\psi^S_k(\eta)$ becomes
\[ \frac{\partial \psi^S_k(\eta)}{\partial \eta} < 0, \quad \frac{\partial \psi^S_k(\eta)}{\partial \eta} < 0, \quad \psi^S_k(1) > \psi^S_O(1), \text{ and } \psi^S_k(0) < \psi^S_O(0). \]

The intuition for the downward-sloping of $\psi^S_k(\eta)$ is as follows. In low $\eta$, components $m$ are important in production. Since firms can obtain components $m$ with the lower marginal cost via FDI and foreign outsourcing, revenues in the South are higher than those of the North in lower $\eta$ ($\psi^S_k(\eta) > \psi^N_k(\eta)$). As a result, $\psi^S_k(\eta)$ becomes downward-sloping.

Under these circumstances, we can see that
\[ \max_{k \in \{V, O\}, \ell \in \{N, S\}} \psi^\ell_k(\eta) = \begin{cases} \psi^N_k(\eta) & \text{if } \eta < \eta < 1 \\ \psi^S_k(\eta) & \text{if } \eta < \eta < \tilde{\eta} \\ \psi^S_O(\eta) & \text{if } 0 < \eta < \eta \end{cases}. \]

The following lemma follows directly from this observation:\(^{28}\)

---

\(^{26}\)As in the closed economy, the optimal organizational form $(k^*, \ell^*)$ is to maximize the total profits (14).

\(^{27}\)To be more precise, this condition ensures that $\psi^S_k(\eta)$ becomes downward-sloping. In other words, if the wage differential is relatively small, $\psi^S_k(\eta)$ can be upward-sloping. Since we consider vertical FDI here, however, we can assume that the wage differential is sufficiently large so that $\psi^S_k(\eta)$ is also downward-sloping. See Appendix A.2 for details.

\(^{28}\)Appendix A.3 shows that the threshold in the closed economy, $\hat{\eta}$, is necessarily between the thresholds in the open economy, $\eta$ and $\tilde{\eta}$; that is, $\eta < \hat{\eta} < \tilde{\eta}$. 

14
Lemma 3. There exist two thresholds $\bar{\eta}$ and $\tilde{\eta}$ with $\bar{\eta} > \eta$ such that: (i) if $\bar{\eta} < \eta < 1$, all firms earn the highest variable profits by integration in the North; (ii) if $\eta < \eta < \bar{\eta}$, all firms earn the highest variable profits by integration in the South (FDI); and (iii) if $0 < \eta < \tilde{\eta}$, all firms earn the highest variable profits by outsourcing in the South.$^{29}$

Figure 6 implies that if the fixed cost is the same across organizational forms, domestic outsourcing cannot arise in equilibrium because only the highest $\psi_k^N(\eta)$ is important for determining the optimal organization of production. This result is, of course, not realistic; many studies have documented the growth of outsourcing not only across but also within national borders.

Note that the figure shows another threshold, namely $\bar{\eta}$. This is the threshold at which firms earn the same variable profits from domestic integration and foreign outsourcing. Using this threshold, we next examine a simple dynamic extension of the static model under which domestic outsourcing can appear in equilibrium.

---

$^{29}$This result is similar to that in Antràs (2005). The difference lies in the firm boundaries in the North; since Antràs assumes domestic complete contracts, the choice between domestic integration and domestic outsourcing is indifferent. In contrast, Lemma 3 shows that if there is no fixed organizational cost, the northern production occurs only within firm boundaries. This is because the variable profits from foreign outsourcing is always higher than those from domestic outsourcing under assumptions (11) and (13).
3.3 Dynamics: The Product Cycle

Now, we consider the dynamic extension of the static model. The setting of the dynamics is similar to that in the closed economy, but a new viewpoint is open when international trade is allowed.

The dynamics of the elasticity of headquarter services is captured by the function $\eta = g(t)$, which has the same properties as (9). Furthermore, we assume that $\delta^\ell$ is time-invariant so that $\bar{\eta}$ and $\bar{\eta}$ are not affected by time and are determined uniquely. Under these conditions, there are unique time thresholds, $\bar{t}$ and $\tilde{t}$, respectively, which determine the highest variable profits from different organizational forms in the dynamics. That is,

$$
\max_{k \in \{V,O\}, \ell \in \{N,S\}} \psi^\ell_k(\eta) = \begin{cases} 
\psi^N_V(\eta) & \text{if } 0 \leq t < \bar{t} \\
\psi^S_V(\eta) & \text{if } \bar{t} < t < \tilde{t} \\
\psi^S_O(\eta) & \text{if } t < \tilde{t} < \infty 
\end{cases}
$$

We summarize this result in the following lemma:

**Lemma 4.** There exist unique time thresholds $\bar{t}$ and $\tilde{t}$ with $\tilde{t} > \bar{t}$ such that: (i) when $0 \leq t < \bar{t}$, all firms earn the highest variable profits by integration in the North; (ii) when $\bar{t} < t < \tilde{t}$, all firms earn the highest variable profits by integration in the South (FDI); and (iii) when $t < \tilde{t} < \infty$, all firms earn the highest variable profits by outsourcing in the South.

Note that there exist other thresholds: $\tilde{\eta}$, $\check{\eta}$, and $\hat{\eta}$. We denote their time thresholds by $\tilde{t}$, $\check{t}$, and $\hat{t}$, respectively. As in the closed economy, we assume that

$$
\psi^N_V(1) > \psi^N_V(\check{\eta}).
$$

Under these circumstances, these time thresholds $(\bar{t}, \tilde{t}, \check{t}, \hat{t})$ and the relationship in (12) lead to a new product cycle in which firm heterogeneity plays a significant role.

Consider first when $\bar{\eta} < \eta < 1$, which corresponds to $0 \leq t < \bar{t}$. In this case, Figure 6 shows that

$$
\psi^N_V(\eta) > \psi^S_V(\eta) > \psi^S_O(\eta) > \psi^N_O(\eta).
$$

Then, it follows from (16) and (12) that only the northern production occurs in equilibrium.\(^{31}\) In addition, condition (10') assures that only domestic vertical integration is prevailing in $\eta < \eta < 1$. Thus, as in the closed economy analyzed in the previous section, only domestic vertical integration

\(^{30}\)This condition ensures that only domestic integration is pervasive in early stages of the product cycle.

\(^{31}\)Using the same framework of Figure 3, we can describe (14) in ($\Theta, \pi$) space. Then, it is clear that $\pi^S_k$ always lies under $\pi^S_k$. This implies that the southern production is never chosen in equilibrium.
exists in $0 < t < \bar{t}$ and then domestic vertical integration and domestic outsourcing come to coexist in $\bar{t} < t < \tilde{t}$.

Next, we consider the case when $\bar{\eta} < \eta < \tilde{\eta}$, which corresponds to $\bar{t} < t < \tilde{t}$. It follows that

$$\psi^S_V(\eta) > \psi^N_V(\eta) > \psi^S_O(\eta) > \psi^N_O(\eta).$$

In this stage, we see that FDI comes to emerge in equilibrium: the most productive firms engage in FDI, the next productive firms integrate in the North, and the less productive firms outsource in the North.

The intuition behind this result stems from the mechanism shown by Melitz (2003). In the dynamics, we assume that the components $m$ become more important over time. Because $m$ can be produced more cheaply in the South, all firms have an incentive to change their location of production from the North to the South in $\bar{t} < t < \tilde{t}$. Due to the higher fixed cost of the South, however, if low-productivity firms shift to the South, they cannot cover this fixed cost, resulting in lower profits. As a result, they still remain in the North. Only the most productive firms can earn higher profits in the South, thereby letting them undertake FDI.

By the same argument, we have

$$\bar{\eta} < \eta < \tilde{\eta} \iff \bar{t} < t < \tilde{t} \iff \psi^S_V(\eta) > \psi^S_O(\eta) > \psi^N_V(\eta) > \psi^N_O(\eta); \quad (17a)$$

$$\eta < \eta < \bar{\eta} \iff \tilde{t} < t < \bar{t} \iff \psi^S_V(\eta) > \psi^S_O(\eta) > \psi^N_V(\eta) > \psi^N_O(\eta); \quad (17b)$$

$$0 < \eta < \eta \iff \bar{t} < t < \infty \iff \psi^S_O(\eta) > \psi^S_V(\eta) > \psi^N_O(\eta) > \psi^N_V(\eta). \quad (17c)$$

Then it is possible in (17a) that all organizational forms appear, while domestic integration disappears in (17b). In (17c), on the other hand, domestic and foreign outsourcing remains, and if the wage differential between the North and the South is sufficiently large, only foreign outsourcing prevails in the last stage.

Figure 7 summarizes this product cycle. In the figure, $(k, \ell)$ denotes organizational form.\textsuperscript{32} We can see from the figure that the ownership shifts from integration to outsourcing as in the closed economy, and the location shifts from the North to the South gradually. Note that these shifts are closely related to a firm’s productivity level: the firms’ strategy for organizational choices varies with time according to productivity.\textsuperscript{33} We summarize this result in the following proposition:

\textsuperscript{32}In each stage, organizational forms are arranged from high productivity to low productivity.

\textsuperscript{33}In the open-economy setting, the least productive firms also exit the industry due to negative profits. Since $\psi^k_V(\eta)$ are increasing in time, however, the productivity threshold in the open economy becomes small relative to that in the closed economy over time. In other words, international trade reduces the likelihood of a shakeout. This finding replicates the results of Ederington and McCalman (2007), which assume a situation where the possibility of a shakeout is derived from industrial evolution characterized by an endogeneous technology choice.
**Proposition 3.** The model displays a product cycle such that: (i) when $0 \leq t < \hat{t}$, only domestic integration exists; (ii) when $\hat{t} < t < \bar{t}$, domestic integration and domestic outsourcing coexist; (iii) when $\bar{t} < t < \check{t}$, domestic integration, and domestic outsourcing, and FDI coexist; (iv) when $\check{t} < t < \tilde{t}$, all organizational forms coexist; (v) when $\tilde{t} < t < \infty$, domestic outsourcing, FDI, and foreign outsourcing coexist; and (vi) when $\tilde{t} < t < \infty$, only foreign outsourcing exists in the last stage.$^{34}$

One of the most interesting points in this product cycle is that the shift to domestic outsourcing is earlier than the shift to FDI. In reality, however, there are many examples that the manufacturing is shifted to foreign subsidiaries before contracting with independent domestic firms. It can be interpreted that the model assumes relatively robust legal protection in the North, thereby enforcing almost all contracts between the northern producers. Indeed, if there is no difference in contract enforceability, this product cycle never occurs.$^{35}$ As recent empirical analysis reports, this difference has a crucial effect on international trade.$^{36}$

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$^{34}$It should be emphasized that this proposition states the possibility, not necessity, of the product cycle. In particular, if the differential across the fixed organizational cost is relatively small, the equilibrium in which all four organizations coexist may not occur.

$^{35}$The same contract enforceability means that $H$’s outside option is the same in both countries, i.e., $(\delta^N)^{\alpha} = (\delta^S)^{\alpha}$. In this case, $\check{t}$ coincides with $\tilde{t}$, letting one of equilibria described above be disappeared. See Appendix A.3 for the detailed discussion.

$^{36}$See, for example, Nunn (2007) and Levchenko (2007).
4 Concluding Remarks

In this paper, we have presented an extended product cycle model of Antràs (2005). In our model, the production of the final-good requires a combination of two inputs, headquarter services and manufactured components. Final-good producers who supply headquarter services and produce the final-good can procure components by integration or outsourcing under incomplete contracting settings. When final-good producers can choose the location of the production, intermediate inputs can be produced in the low-wage country (the South) as well as in the home country (the North).

In contrast to Antràs (2005), we consider two assumptions that are seen in recent empirical data. First, our model assumes that each firm has heterogeneous productivity of the Melitz (2003) type. Furthermore, the domestic country has incomplete contracts whose quality of contracting institution is better than foreign country. These assumptions are crucial for understanding global sourcing strategies and have a direct effect on our results.

The main result is that different organizational forms can coexist in the dynamics. In the closed economy in which firms can decide only their ownership structure, we show that a good is initially manufactured within integrated firms where product development takes place. Maturity of the good leads to the coexistence of two ownership structures: high-productivity firms keep to manufacture the good within firm boundaries, while low-productivity firms switch their manufacturing to outsourcing. This shift occurs first within low-productivity firms because they cannot cover the higher fixed cost of integration for a long time. When the good becomes sufficiently standardized, manufacturing stage of production is shifted to non-integrated firms.

We also describe the open-economy model, where firms in the North can procure intermediate inputs either from the North or from the South. In this circumstance, the ownership of production gradually shifts from integration to non-integration as in the closed economy, and the location of production gradually shifts from the North to the South. Since these two shifts in control and location occur simultaneously, different organizational forms come to coexist in equilibrium. It is important to note that the institutional difference plays a key role in this product cycle; while the southern production enables firms to enjoy the lower marginal cost, it weakens the legal protection for contractual relationships. This tradeoff generates the new product cycle mentioned above.

Although our model sheds some light on the existing literature, it is interesting to explore alternative theories to the firm for the organization of production. For example, Holmström and Milgrom’s (1991, 1994) view of the firm is applied to the international setting by Grossman and
Helpman (2004) and provides interesting results for the study of international outsourcing.\footnote{This approach, sometimes called \textit{incentive systems approach}, emphasizes the importance of balancing various incentives. The result of Grossman and Helpman (2004) is often compared with that of Antr` as and Helpman (2004). See Spencer (2005) for this detailed comparison.} For the time being, these approaches have not yet enjoyed wide circulation in the international trade literature, but we believe that it is worth exploring them in future research.\footnote{Another theory of the firm, for instance, focuses on \textit{delegation of authority} à la Aghion and Tirole (1997). Puga and Treffler (2002) and Marin and Verdier (2003) adopt this approach to the study of international organization of production. Also, Antr` as et al. (2006) apply the theory of \textit{hierarchies} à la Lucas (1978) and Rosen (1982).}

\section{Appendix}

\subsection{Proof of the Upward-Sloping of $\psi_V(\eta)$}

From (7), $\psi_V(\eta)$ is given by

$$
\psi_V(\eta) = \frac{1 - \alpha [\beta_V \eta + (1 - \beta_V)(1 - \eta)]}{[(1/\alpha)(w/\beta_V)\eta[w/(1 - \beta_V)]^{1-\eta}]^{\alpha/(1-\alpha)}}.
$$

We define

$$y(\eta) \equiv 1 - \alpha [\beta_V \eta + (1 - \beta_V)(1 - \eta)] > 0,$$

and

$$z(\eta) \equiv \left[\left(\frac{1}{\alpha}\right)\left(\frac{w}{\beta_V}\right)^{\eta}\left(\frac{w}{1 - \beta_V}\right)^{1-\eta}\right]^{\alpha/(1-\alpha)} > 0.$$

That is, $y(\eta)$ and $z(\eta)$ are the numerator and denominator of (A.1), respectively. Differentiating (A.1) with respect to $\eta$ yields

$$\frac{\partial \psi_V(\eta)}{\partial \eta} = \frac{-\alpha(2\beta_V - 1) + y(\eta)\frac{\alpha}{1-\alpha} \log \frac{\beta_V}{1-\beta_V}}{z(\eta)}.
$$

Since $\beta_V > \frac{1}{2}$ (see (5)), $y(\eta)$ is decreasing in $\eta$:

$$y'(\eta) = -\alpha(2\beta_V - 1) < 0 \quad \text{if} \quad \beta_V > \frac{1}{2}.$$

Thus if the numerator of (A.2) evaluated at $\eta = 1$ is positive, $\frac{\partial \psi_V(\eta)}{\partial \eta} > 0$ holds. From the above definition, we have

$$y(1) = 1 - \alpha \beta_V,$$

and then the numerator of (A.2) is given by

$$\zeta(\beta_V) \equiv -\alpha(2\beta_V - 1) + (1 - \alpha \beta_V)\frac{\alpha}{1 - \alpha} \log \frac{\beta_V}{1 - \beta_V}.
$$

We can easily see from (A.3) that

$$\zeta\left(\frac{1}{2}\right) = 0, \quad \zeta(1) = +\infty, \quad \text{and} \quad \zeta'(\beta_V) \geq 0,$$

which mean that the numerator of (A.2) is always positive for $\beta_V \in (\frac{1}{2}, 1)$. Therefore as long as $\beta_V > \frac{1}{2}$, $\psi_V(\eta)$ is monotonously increasing in $\eta$. \quad \blacksquare
A.2 Proof of the Downward-Sloping of \( \psi^S_\ell(\eta) \)

From (15), \( \psi^S_O(\eta) \) is given by

\[
\psi^S_O(\eta) = \frac{1 - \alpha[\beta\eta + (1 - \beta)(1 - \eta)]}{(1/\alpha)(w^N/\beta)^{\eta}[w^S/(1 - \beta)]^{1-\eta}/\alpha/(1-\alpha)}.
\]

Noticing \( \beta = \frac{1}{2} \), we define the numerator and denominator of \( \psi^S_O(\eta) \) as follows:

\[
\tilde{y}(\eta) \equiv 1 - \frac{1}{2}\alpha > 0,
\]
and

\[
\tilde{z}(\eta) \equiv \left[ \frac{2(w^N)^{\eta}(w^S)^{1-\eta}}{\alpha} \right]^{\alpha/(1-\alpha)} > 0.
\]

Then, it follows from (11) that

\[
\frac{\partial \psi^S_O(\eta)}{\partial \eta} = -\tilde{y}(\eta)\frac{\alpha}{1-\alpha} \log \frac{w^N}{w^S} < 0.
\]

Therefore, \( \psi^S_O(\eta) \) is monotonously decreasing in \( \eta \).

On the other hand, \( \psi^S_V(\eta) \) is

\[
\psi^S_V(\eta) = \frac{1 - \alpha[\beta_V^S\eta + (1 - \beta_V^S)(1 - \eta)]}{(1/\alpha)(w^N/\beta_V^S)^{\eta}[w^S/(1 - \beta_V^S)]^{1-\eta}/\alpha/(1-\alpha)},
\]
and define

\[
\bar{y}(\eta) \equiv 1 - \alpha[\beta_V^S\eta + (1 - \beta_V^S)(1 - \eta)] > 0,
\]
and

\[
\bar{z}(\eta) \equiv \left[ \frac{\alpha}{1-\alpha} \left( \frac{w^N}{\beta_V^S} \right)^{\eta} \left( \frac{w^S}{1 - \beta_V^S} \right)^{1-\eta} \right]^{\alpha/(1-\alpha)} > 0.
\]

Then, we have

\[
\frac{\partial \psi^S_V(\eta)}{\partial \eta} = -\alpha(2\beta_V^S - 1) + \bar{y}(\eta)\frac{\alpha}{1-\alpha} \left( \log \frac{\beta_V^S}{1 - \beta_V^S} - \log \frac{w^N}{w^S} \right).
\]

Thus, \( \psi^S_V(\eta) \) is downward-sloping if

\[
\frac{\beta_V^S}{1 - \beta_V^S} < \frac{w^N}{w^S}.
\]

Condition (A.4) indicates that the wage differential across countries is larger than the ratio of an ex post bargaining between \( H \) and \( M \). Therefore, we can conclude that \( \psi^S_V(\eta) \) is also monotonously decreasing in \( \eta \), as long as the wage differential is satisfied with (A.4).  ■

A.3 Discussions on the Headquarter Thresholds

In this subsection, we derive the relationship between the headquarter thresholds (\( \bar{\eta}, \eta, \bar{\eta} \)) from \( \psi^S_\ell(\eta) \). We show that these thresholds are necessarily greater than one-half under Symmetric Nash Bargaining and the threshold in the closed economy is between those in the open economy (\( \bar{\eta} < \eta < \bar{\eta} \)). To see this, we focus on \( \psi_k(\eta) \) first, and then on \( \psi^S_k(\eta) \).
In the text, if $H$ could freely choose its fraction of revenue $\beta_k$, it would choose $\beta^* \in [0, 1]$ that maximizes $\psi_k(\eta)$. This function is given by

$$
\beta^*(\eta) = \frac{\eta(\alpha\eta + 1 - \alpha) - \sqrt{\eta(1-\eta)(1-\alpha\eta)(\alpha\eta + 1 - \alpha)}}{2\eta - 1}.
$$

(A.5)

The function $\beta^*(\eta)$ is depicted by the solid curve in Figure A.1. It rises in $\eta$; $\beta^*(0) = 0$ and $\beta^*(1) = 1$. The arrows show the direction of rising profits. Then, given $\beta$ and $\beta_V$, we can find a threshold $\bar{\eta}$ such that all firms earn the same variable profits from integration and outsourcing: $\psi_V(\bar{\eta}) = \psi_O(\bar{\eta})$. Clearly, $\bar{\eta}$ is determined so that $\beta$ and $\beta_V'$ are in the same distance from $\beta^*(\eta)$ and the result here is consistent with that of Figure 2.41 Furthermore, the figure illustrates that $\bar{\eta}$ is always greater than one-half under Symmetric Nash Bargaining.

The same arguments are true for $\psi_k^*(\eta)$: if $H$ could freely choose its fraction of revenue $\beta_k$, the function that maximizes $\psi_k^*(\eta)$ is the same as (A.5). Since the outside option of $H$ differs among countries, however, the ex post bargaining depends on the location of $M$ as well as the ownership structure.

Figure A.2 shows the function $\beta^*(\eta)$, which is represented by the solid curve. As before, the profits rise when $H$’s share shifts vertical toward $\beta^*(\eta)$. The difference from Figure A.1 is that reflecting the relationship in (13), there are three ex post bargaining for $H$: $\beta_N^V$, $\beta_S^N$, and $\beta$. Then, it is immediately seen that there are two new thresholds, namely $\bar{\eta}$ and $\bar{\eta}$ with $\bar{\eta} > \eta$ such that $\psi_V^N(\bar{\eta}) = \psi_O^S(\bar{\eta})$ and $\psi_V^S(\bar{\eta}) = \psi_O^S(\bar{\eta})$.42 Note that since $\bar{\eta}$ is determined so that $\beta$ and $\beta_N^V$ are in the same distance from $\beta^*(\eta)$, $\bar{\eta}$ necessarily locates between $\bar{\eta}$ and $\bar{\eta}$; $\eta < \bar{\eta} < \bar{\eta}$. In addition, if both countries have the same contract enforceability, i.e., $\delta^N = \delta^S$, it is immediate from the figure that $\bar{\eta} = \bar{\eta} < \bar{\eta}$.43

### A.4 Discussions on the Outside Option

In the text, $H$’s outside option $\delta^\alpha$ is exogenously given and is assumed to be time-invariant. This assumption lets $\bar{\eta}$ be unaffected by time, but it is rather arbitrary; since the outside option is defined as payoff when the parties fail to reach an agreement, it is more natural under (9) that the outside option is also decreasing in time:

$$
\delta'(t) < 0, \quad \delta(0) = 1, \quad \text{and} \quad \lim_{t \to \infty} \delta(t) = 0.
$$

(A.6)

It is easy to see that if $\delta^\alpha$ is decreasing in $t$, $\bar{\eta}$ becomes also time-variant. From (A.6) and (5), $\beta_V = 1$ when $t = 0$ and $\beta_V = 1/2$ when $t = \infty$. Then, the range of $\bar{\eta}$ is given by $\bar{\eta} \in (\frac{1}{2}, \bar{\eta}_{\max})$, where $\bar{\eta}_{\max}(< 1)$ is $\bar{\eta}$ when $\beta_V = 1$. On the other hand, the range of $\eta$ is $\eta \in (0, 1)$. Thus, under assumptions (9) and (A.6), both $\eta$ and $\bar{\eta}$ decline over time but its speed is higher in $\eta$ than in $\bar{\eta}$; $\eta$ is initially large so that $\eta > \bar{\eta}$, but $\eta$ becomes smaller than $\bar{\eta}$ when sufficient time has passed. From Proposition 1, we know that when $\eta > \bar{\eta}$ both integration and outsourcing exist, while when $\eta < \bar{\eta}$ only outsourcing appear in equilibrium.

It should be noted that when $t = 0$, i.e., $\beta_V = 1$, $H$ can receive all of the joint profits under integration. This reflects the fact that the components $m$ are initially useless in production and $H$ has no incentive to choose outsourcing. As a result, only integration is pervasive in early stages with $\eta > \bar{\eta}$. Therefore, as long as (A.6) holds, we can realize the same result as in the text without assumption (10).
\[ \beta^* (\eta) = \frac{1}{2} (1 + \delta^*) \]

\[ \beta^N = \frac{1}{2} [1 + (\delta^N)^\alpha] \]

\[ \beta^S = \frac{1}{2} [1 + (\delta^S)^\alpha] \]

\[ \beta = \frac{1}{2} \]

**Figure A.1**: Distribution of \( \psi_k(\eta) \)

**Figure A.2**: Distribution of \( \psi^\ell_k(\eta) \)
References


