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NO SPECULATION IN RATIONAL EXPECTATIONS UNDER GENERALIZED INFORMATION

by

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Abstract

Let us consider a pure exchange economy under non-partitional information where the traders are assumed to have a reflexive and transitive information structure and to have strictly monotone preferences. We show the no speculation theorem: If the initial endowment is ex-ante Pareto optimal then there exists no other rational expectations equilibrium for any price with respect to which all traders are rational about expectations everywhere.

Key words: Pure exchange economy with knowledge, No speculation, Rational expectations equilibrium, Ex-ante Pareto optimum.
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1 Introduction

The purpose of this article is to show the no speculation theorem in generalized rational expectations equilibrium as follows:

Theorem 1 (No speculation theorem) *In a pure exchange economy under non-partitional information, the traders are assumed to have a reflexive and transitive information structure and to have strictly monotone preferences. If the initial endowment are ex-ante Pareto optimal then there exists no other rational expectations equilibrium for any price with respect to which all traders are rational about expectations everywhere.*

Many authors have shown that there can be no speculation in a rational expectations equilibrium for an economy under uncertainty (e.g., Kreps [1977], Milgrom and Stokey [1982], Geanakoplos [1989], Morris [1994] and others). The serious limitations of the analysis in these researches are its use of the 'partition' structure by which the traders receive information. The structure is obtained if each trader i's possibility operator $P_i : \Omega \to 2^{\Omega}$ assigning to each state ω in a state space Ω the information set $P_i(\omega)$ that i possesses in ω is reflexive, transitive and *symmetric*.

One of these requirements, symmetry, is indeed so strong that describes the hyper-rationality of traders, and thus it is particularly objectionable (Bacharach [1985]). The recent idea of 'bounded rationality' suggests dropping such assumption since real people are not complete reasoners. In this article we weaken symmetry imposing only reflexivity and transitivity. As Geanakoplos (1989) has already pointed out, this relaxation can potentially yield important results in a world with imperfectly Bayesian agents.

The idea has been performed in different settings (c.f., Geanakoplos [1989] or Fudenberg and Tirole [1991] [2]). Among other things Geanakoplos (1989) showed the no speculation theorem in the extended rational expectations equilibrium under the assumption that the information structure is reflexive, transitive and *nested* (Corollary 3.2 in Geanakoplos [1989]). The condition 'nestedness' is interpreted as a requisite on the 'memory' of the trader.

Recently, Matsuhisa and Ishikawa (2002) introduced the notion 'rationality about expectations' with respect to a price system p, interpreted as that each trader who learns from the price knows his/her expected utility. They showed the existence theorem of rational expectations equilibrium for an economy under non-nested, reflexive and transitive information structure.

[2] The references cited in Fudenberg and Tirole [1991], footnote 3, p.543

This article is in the line of Geanakoplos (1989). We shall extend the no speculation theorem in this generalized environment, which is an extension of Corollary 3.2 in Geanakoplos (1989).

This article is organized as follows: Section 2 gives an illustration of Theorem 1 by a simple example of an economy with knowledge under non-nested information structure. In Section 3 we present our model: First we recall a generalized information structure; the reflexive and transitive information structure. Secondly we introduce an economy with knowledge, which is a generalization of an economy under asymmetric information. We extend the notion of rational expectations equilibrium to an economy with knowledge. Section 4 gives the proof of Theorem 1. We conclude by giving some remarks about the assumptions in the theorem.

2 Illustrative example

Let us consider the following situation:

Two traders 1 and 2 are willing to buy and sell the tradeable emissions permits with each other. Trader 1 is interested in the global warming problem, but trader 2 is not at all. There is one commodity, and only unused allowances are transferable between two traders 1 and 2.

We shall illustrate the situation as follows: Let Ω be the state space consisting of the three states $\{\omega_1, \omega_2, \omega_3\}$: The state ω_1 represents that the temperature is higher than the normal one, the state ω_2 represents that it is the normal temperature and finally the state ω_3 represents that the temperature is lower than the normal one.

Trader 1 is sensitive to the environmental change that the temperature becomes higher or lower, and so she can know which of either ω_1, ω_2 or ω_3 is the true state when each of them occurs. Hence trader 1 has her information structure $P_1(\omega) = \{\omega\}$ for any $\omega \in \Omega$.

Trader 2 is less sensitive than trader 1. He knows that the temperature is normal when it is so but he is ignorant of the environmental changes. He can know that ω_2 is the true state when it occurs. However, when the temperature becomes higher or lower, he cannot understand it. Thus he cannot know which of either ω_2 or ω_3 is the true state when ω_3 occurs, and he cannot know which of either ω_1 or ω_2 is the true state when ω_1 occurs. Trader 2 has his information structure $P_2(\omega_1) = \{\omega_1, \omega_2\}$, $P_2(\omega_2) = \{\omega_2\}$ and $P_2(\omega_3) = \{\omega_2, \omega_3\}$.

Suppose that traders 1 and 2 have the initial endowments $e_1(\omega) = e_2(\omega) = 1$

ton for every $\omega \in \Omega$ and they have the risk averse utilities: $U_1(x,\omega) = U_2(x,\omega) = \sqrt{x+4}$ for every $\omega \in \Omega$. Their common prior μ is given by $\mu(\omega) = \frac{3}{7}$ for $\omega = \omega_1, \omega_3$ and $\mu(\omega_2) = \frac{1}{7}$. Then it can be plainly verified that the traders' initial endowments are ex-ante Pareto optimal, and the endowments constitute the unique rational expectations equilibrium that the traders know their expectation given by the information of the price for the economy (Theorem 1).

It should be noted that P_2 does not give a partition of Ω. Nonetheless P_2 has the properties: For any $\omega \in \Omega$, $\omega \in P_2(\omega)$ and $P_2(\xi)$ j $P_2(\omega)$ whenever $\xi \in P_2(\omega)$. However P_2 is not *nested*[3] because the three sets $P_2(\omega_1) \cap P_2(\omega_3), P_2(\omega_i) \setminus P_2(\omega_j) (i,j = 1,3)$ are all non-empty.

In this article we shall investigate the pure exchange economies under generalized information structure as like this example.

3 The Model

Let Ω be a non-empty *finite* set called a *state space*, $N = \{1, 2, \cdots, n\}$ a set of finitely many *traders*, and let 2^Ω denote the field of all subsets of Ω. Each member of 2^Ω is called an *event* and each element of Ω called a *state*.

3.1 Information structure[4]

By this we mean a class $(P_i)_{i \in N}$ of mappings of Ω into 2^Ω. The mapping $P_i : \Omega \to 2^\Omega$ is said to be *reflexive* if the following property is true:

Ref $\quad \omega \in P_i(\omega) \quad$ for every $\omega \in \Omega$,

and it is said to be *transitive* if the following property is true:

Trn $\quad \xi \in P_i(\omega)$ implies $P_i(\xi)$ j $P_i(\omega)$ for any $\xi, \omega \in \Omega$.

Given our interpretation, an trader i for whom $P_i(\omega)$ j E knows, in the state ω, that some state in the event E has occurred. The set $P_i(\omega)$ will be interpreted as the set of all the states of nature that i knows to be possible at ω.

[3] See Definition 5 in Section 3.
[2] See Bacharach (1985), Binmore (1992).

3.2 Economy with knowledge[5]

A *pure exchange economy under uncertainty* is a tuple $\langle N, \Omega, (e_i)_{i \in N}, (U_i)_{i \in N}, (\mu_i)_{i \in N} \rangle$ consisting of the following structure and interpretations: There are l commodities in each state of the state space Ω, and it is assumed that Ω is *finite* and that the consumption set of trader i is R^l_+;

- $N = \{1, 2, \cdots, n\}$ is the set of n traders;
- $e_i : \Omega \to \mathsf{R}^l_+$ is i's *initial endowment*;
- $U_i : \mathsf{R}^l_+ \times \Omega \to \mathsf{R}$ is i's von Neumann and Morgenstern utility function;
- μ_i is a subjective prior on Ω for i.

For simplicity it is assumed that (Ω, μ_i) is a finite probability space with μ_i *full support*[6] for every $i \in N$.

Definition 1 An *economy with knowledge* $\mathcal{E}^K$ is a structure $\langle \mathcal{E}, (P_i)_{i \in N} \rangle$, in which $\mathcal{E}$ is a pure exchange economy under uncertainty with a state-space Ω finite and with $(P_i)_{i \in N}$ a reflexive and transitive information structure on Ω.

We denote by $\mathcal{F}_i$ the field generated by $\{P_i(\omega) \mid \omega \in \Omega\}$ and by $\mathcal{F}$ the join of all $\mathcal{F}_i (i \in N)$; i.e. $\mathcal{F} = \vee_{i \in N} \mathcal{F}_i$. It is noted that the atoms $\{A_i(\omega) \mid \omega \in \Omega\}$ of $\mathcal{F}_i$ is the partition induced from P_i. We denote by $\{A(\omega) \mid \omega \in \Omega\ \}$ the set of all atoms $A(\omega)$ containing ω of the field $\mathcal{F} = \vee_{i \in N} \mathcal{F}_i$.

By an *allocation* we mean a profile $\mathsf{a} = (a_i)_{i \in N}$ of $\mathcal{F}_i$-measurable functions a_i from Ω into R^l_+ such that for every $\omega \in \Omega$,

$$\sum_{i \in N} a_i(\omega) \ \mathsf{5} \sum_{i \in N} e_i(\omega).$$

We denote by $\mathcal{A}$ the set of all allocations and denote by $\mathcal{A}_i$ the set of all the i'th components: $\mathcal{A} = \times_{i \in N} \mathcal{A}_i$.

We shall often refer to the following conditions: For every $i \in N$,

A-1 The function $e_i(\cdot)$ is $\mathcal{F}_i$-measurable with $\sum_{i \in N} e_i(\omega)$ ª 0 for all $\omega \in \Omega$.
A-2 For each $x \in \mathsf{R}^l_+$, the function $U_i(x, \cdot)$ is $\mathcal{F}_i$-measurable.
A-3 For each $\omega \in \Omega$, the function $U_i(\cdot, \omega)$ is strictly monotone on R^l_+.
A-4 For each $\omega \in \Omega$, the function $U_i(\cdot, \omega)$ is continuous, strictly quasi-concave and *non-saturated*[7] on R^l_+.

Here it is noted that **A-4** implies **A-3**.

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[5] See Matsuhisa and Ishikawa (2002).
[6] I.e., $\mu_i(\omega)$ ª 0 for every $\omega \in \Omega$.
[7] I.e.; For any $x \in \mathsf{R}^l_+$ there exists an $x' \in \mathsf{R}^l_+$ such that $U_i(x', \omega)$ ª $U_i(x, \omega)$.

We set by $\mathsf{E}_i[U_i(a_i)]$ the *ex-ante* expectation defined by

$$\mathsf{E}_i[U_i(a_i)] := \sum_{\omega\in\Omega} U_i(a_i(\omega),\omega)\mu_i(\omega)$$

for each $a_i \in \mathcal{A}_i$. The endowments $\mathsf{e} = (e_i)_{i\in N}$ are said to be *ex-ante Pareto-optimal* if there is no allocation $\mathsf{a} = (a_i)_{i\in N}$ such that for all $i \in N$, $\mathsf{E}_i[U_i(a_i)]$ = $\mathsf{E}_i[U_i(e_i)]$ and that for some $j \in N$, $\mathsf{E}_j[U_j(a_j)]$ ª $\mathsf{E}_j[U_j(e_j)]$.

3.3 Rationality about expectation

Let $\mathsf{E}_i[U_i(a_i)|P_i](\omega)$ denote the *interim* expectation defined by

$$\mathsf{E}_i[U_i(a_i)|P_i](\omega) := \sum_{\xi\in\Omega} U_i(a_i(\xi),\xi)\mu_i(\xi|P_i(\omega)).$$

We set the event:

$$[\mathsf{E}_i[U_i(a_i)|P_i](\omega)] := \{\xi \in \Omega \mid \mathsf{E}_i[U_i(a_i)|P_i](\xi) = \mathsf{E}_i[U_i(a_i)|P_i](\omega)\ \},$$

and the event

$$[\mathsf{E}_i[U_i(\cdot)|P_i](\omega)] := \bigcap_{a_i\in\mathcal{A}_i} [\mathsf{E}_i[U_i(a_i)|P_i](\omega)],$$

which is interpreted as the event *'i's expectation at ω'*. We denote $R_i = \{\omega \in \Omega \mid P_i(\omega)$ j $[\mathsf{E}_i[U_i(\cdot)|P_i](\omega)]\ \}$, interpreted as the event that i knows his/her interim expectation, and we denote by $R = \bigcap_{i\in N} R_i$, interpreted as the event that all traders know their interim expectations.

Definition 2 A trader i is *rational about his expectation* at ω if $\omega \in R_i$; that is, i knows his own expectation at ω. He/she is rational *everywhere* about his expectation if $R_i = \Omega$.

Remark 1 A *partitional* information structure is an information structure $(P_i)_{i\in N}$ with the additional condition: For each $i \in N$ and every $\omega \in \Omega$

Sym $\quad \xi \in P_i(\omega)$ implies $P_i(\xi) \ni \omega$.

An economy under asymmetric information is actually an economy under partitional information structure. It is noted that every trader i in an economy under asymmetric information is always rational everywhere about expectation; i.e., $R_i = \Omega$.

3.4 Price system and rational expectations equilibrium

Let $\mathcal{E}^K = \langle N, \Omega, (e_i)_{i\in N}, (U_i)_{i\in N}, (\mu_i)_{i\in N}, (P_i)_{i\in N}\rangle$ be an economy with knowledge. A *price system* is a non-zero $\mathcal{F}$-measurable function $p : \Omega \to \mathsf{R}^l_+$. We denote by $\Delta(p)$ the set of all atoms of the smallest field $\sigma(p)$ that p is measurable, and by $\Delta(p)(\omega)$ the component containing ω. The *budget set* of a trader i at a state ω for a price system p is defined by

$$B_i(\omega, p) := \{\ a \in \mathsf{R}^l_+ \mid p(\omega)\cdot a \leqq p(\omega)\cdot e_i(\omega)\ \}.$$

Let $\Delta(p)\cap P_i : \Omega \to 2^\Omega$ be defined by $(\Delta(p)\cap P_i)(\omega) := \Delta(p)(\omega)\cap P_i(\omega)$; it is plainly observed that $\Delta(p)\cap P_i$ is a reflexive and transitive information structure of trader i. We denote by $\sigma(p)\vee\mathcal{F}_i$ the join of the fields $\sigma(p)$ and $\mathcal{F}_i$, which coincides with the field generated by $(\Delta(p)\cap P_i)$, and denote by $A_i(p)(\omega)$ the atom containing ω. On noting that P_i satisfies **Ref** and **Trn**, it can be plainly observed that

$$A_i(p)(\omega) = (\Delta(p)\cap A_i)(\omega).$$

Definition 3 A *rational expectations equilibrium* for an economy $\mathcal{E}^K$ with knowledge is a pair (p, x), in which p is a price system and $\mathsf{x} = (x_i)_{i\in N}$ is an allocation satisfying the following conditions:

RE 1 For every $i\in N$, x_i is $\sigma(p)\vee\mathcal{F}_i$-measurable.
RE 2 For every $i\in N$ and for every $\omega\in\Omega$, $x_i(\omega)\in B_i(\omega,p)$.
RE 3 For all $i\in N$, if $y_i:\Omega\to\mathsf{R}^l_+$ is $\sigma(p)\vee\mathcal{F}_i$-measurable with $y_i(\omega)\in B_i(\omega,p)$ for all $\omega\in\Omega$ then

$$\mathsf{E}_i[U_i(x_i)|\Delta(p)\cap P_i](\omega) \geqq \mathsf{E}_i[U_i(y_i)|\Delta(p)\cap P_i](\omega)$$

pointwise on Ω.
RE 4 For every $\omega\in\Omega$, $\sum_{i\in N} x_i(\omega) = \sum_{i\in N} e_i(\omega)$.

The profile $\mathsf{x} = (x_i)_{i\in N}$ is called a *rational expectations equilibrium allocation.*

3.5 Rationality with respect to price

We denote by $R_i(p)$ the event that i is rational about his expectation; i.e.,

$$R_i(p) = \{\omega\in\Omega \mid (\Delta(p)\cap P_i)(\omega) \subseteq [\mathsf{E}_i[U_i(\cdot)|\Delta(p)\cap P_i](\omega)]\ \}$$

and denote by $R(p)$ the event that all traders are rational: i.e., $R(p) = \bigcap_{i\in N} R_i(p)$. The set $R_i(p)$ is interpreted as the event that i knows his interim expectation when he receives the information of the price p, and $R(p)$ interpreted as all traders know their expectations under the information of p.

Definition 4 A trader i is said to be *rational* about his/her expectation *with respect to a price system* p at ω if $\omega \in R_i(p)$. And all traders are *rational everywhere* about their expectations if $R(p) = \Omega$.

In these circumstances it can be obtained that:

Proposition 1 (Matsuhisa and Ishikawa [2002], Theorem 2) *Suppose an economy with knowledge satisfies the conditions* A-1*,* A-2 *and* A-4*. The initial endowments allocation is ex-ante Pareto optimal if and only if it is a rational expectations equilibrium allocation relative to a price with respect to which the traders are rational everywhere about their expectations.*

In this generalized environment, Theorem 1 is to characterize the Pareto optimality of initial endowments under the extended notion 'rational expectations equilibrium' allocation relative to a price with respect to which the traders are rational everywhere about their expectations.

Definition 5 An information structure $(P_i)_{i\in N}$ is said to be *nested* if for each $i \in N$ and for all states ω and ξ in Ω, either $P_i(\omega) \cap P_i(\xi) = \emptyset$, or else $P_i(\omega)$ j $P_i(\xi)$ or $P_i(\omega)$ k $P_i(\xi)$.

Remark 2 Geanakoplos (1989) introduced the notion of 'nested information' above, and he established the no speculation theorem for an economy with knowledge under nested information:

The example proposed in Section 2 illustrates our version of no speculation theorem under a reflexive, transitive and *non-nested* information structure.

4 Proof of Theorem 1

We can now state Theorem 1 explicitly as follows:

Proposition 2 *Let* $\mathcal{E}^K$ *be a pure exchange economy with knowledge satisfying the conditions* A-1*,* A-2 *and* A-4*. If the initial endowments allocation* $\mathbf{e} = (e_i)_{i\in N}$ *is ex-ante Pareto optimal then it is the unique rational expectations equilibrium allocation for the economy* $\mathcal{E}^K$ *relative to some price system with respect to which all traders are rational everywhere about their expectations.*

Proof of Proposition 2: In view of Proposition 1, the initial endowments $\mathbf{e}$ is a rational expectations equilibrium allocation relative to a price with respect to which the traders are rational everywhere about their expectations. Therefore, Proposition 2 immediately follows from:

Proposition 3 *Let* $\mathcal{E}^K$ *be a pure exchange economy with knowledge satisfy-*

ing $\mathsf{A\text{-}2}$*. Suppose that the initial endowments allocation* e *is ex-ante Pareto optimal. If* x *is a rational expectations equilibrium allocation for the economy* $\mathcal{E}^K$ *relative to some price system with respect to which all traders are rational everywhere about their expectations then* $\mathsf{x} = \mathsf{e}$.

Before proceeding with the proof of Proposition 3 we need the below fundamental lemma that plays an essential role in the proof.

A decision function f of 2^Ω into a decision set Z is said to satisfy the *sure thing principle* if it is preserved under disjoint union; that is, for every pair of disjoint events S and T, $f(S \cup T) = d$ if $f(S) = f(T) = d$. The function f is said to be *preserved under difference* provided that for all events S and T with $S \subseteq T$, $f(T \setminus S) = d$ if $f(S) = f(T) = d$.

We note here that for each $a_i \in \mathcal{A}_i$, the decision function $f_i(a_i) : 2^\Omega \to [0,1]$ defined by

$$f_i(a_i)(X) := \mathsf{E}_i[U_i(a_i)|X] = \sum_{\xi \in \Omega} U_i(a_i(\xi), \xi)\mu_i(\xi|X).$$

is preserved under difference and it satisfies the sure thing principle.

Lemma 4 (Fundamental Lemma) *Let* $Q : \Omega \to 2^\Omega$ *be a reflexive and transitive information structure on* Ω *and* $A : \Omega \to 2^\Omega$ *the partition induced from* Q *that is defined by*

$$A(\omega) := \{\xi \in \Omega \mid Q(\xi) = Q(\omega)\ \}.$$

Suppose that f *is a decision function which satisfies the sure thing principle and is preserved under difference. If* $Q(\omega) \subseteq \{\xi \in \Omega \mid f(Q(\xi)) = f(Q(\omega))\ \}$ *for an* $\omega \in \Omega$ *then we obtain that for every* $\xi \in Q(\omega)$,

$$f(A(\xi)) = f(Q(\omega)).$$

Proof: See Matsuhisa and Ishikawa (2002). □

Proof of Proposition 3: Let p be a price system. It is first noted that for all $\omega \in \Omega$ and all $a_i \in \mathcal{A}_i$,

$$\mathsf{E}_i[U_i(a_i)|(\Delta(p) \cap P_i)](\omega) = U_i(a_i(\omega), \omega). \tag{1}$$

In fact, since $\Omega = R_i(p)$ it follows from Fundamental Lemma (Lemma 4) that for all $\omega \in \Omega$, $\mathsf{E}_i[U_i(a_i)|(\Delta(p) \cap P_i)](\omega) = \mathsf{E}_i[U_i(a_i)|A_i(p)](\omega)$. In view of $\mathsf{A\text{-}2}$ it is observed that for every $\omega \in \Omega$, $\mathsf{E}_i[U_i(a_i)|A_i(p)](\omega) = U_i(a_i(\omega), \omega)$, as required.

For each $\omega \in \Omega$ we denote by $\mathcal{E}^K(\omega)$ the economy with complete information $\langle N, (e_i(\omega))_{i\in N}, (U_i(\cdot,\omega))_{i\in N}\rangle$. Let (p, x) be a rational expectations equilibrium for $\mathcal{E}^K$. We shall show that

Claim $(p(\omega), \mathsf{x}(\omega))$ is a competitive equilibrium of $\mathcal{E}^K(\omega)$ for any $\omega \in \Omega$

Proof of Claim: For each $i \in N$ and for any $y_i \in B_i(\omega, p)$, we set the $\sigma(p) \vee \mathcal{F}_i$-measurable function $z_i : \Omega \to \mathsf{R}^l_+$ by

$$z_i(\xi) = \begin{cases} y_i & \text{if} \quad \xi \in A_i(p)(\omega) \\ e_i(\xi) & \text{if} \quad \xi \notin A_i(p)(\omega), \end{cases}$$

and $z_i(\omega) \in B_i(\omega, p)$. It follows from RE 3 together with Eq.(1) that for every $\omega \in \Omega$,

$$\begin{aligned} &\mathsf{E}_i\left[U_i(x_i)|(\Delta(p) \cap P_i)\right](\omega) = U_i(x_i(\omega), \omega) \\ &= \mathsf{E}_i[U_i(z_i)|(\Delta(p) \cap P_i)](\omega) = U_i(z_i(\omega), \omega) = U_i(y_i, \omega). \end{aligned}$$

Therefore by RE 4 we have observed that $(p(\omega), \mathsf{x}(\omega))$ is a competitive equilibrium of $\mathcal{E}^K(\omega)$.

Now we shall turn to the proof of Proposition 3. Because e is ex-ante Pareto optimal, there exists $j \in N$ such that $\mathsf{E}_j[U_j(e_j)]$ ª $\mathsf{E}_j[U_j(x_j)]$. On noting that μ is full support it can plainly obtained that for some $\omega_0 \in \Omega$, $U_j(e_j(\omega_0), \omega_0)$ ª $U_j(x_j(\omega_0), \omega_0)$. On the other hand, in view of the fundamental theorem of welfare economics for $\mathcal{E}^K(\omega)$ it follows by Claim that $\mathsf{x}(\omega)$ is Pareto optimal,[8] and it is obtained that $U_j(x_j(\omega_0), \omega_0) = U_j(e_j(\omega_0), \omega_0)$, in contradiction. This completes the proof. □

5 Concluding remarks

As already has remarked, we extend the no speculation theorem to an economy under a reflexive, transitive and *non-nested* information structure. This is a generalization of Geanakoplos's version of the no speculation theorem (Corollary 3.2 in Geanakoplos [1989]).

It will well end this article in giving a remark about the ancillary assumptions A-1, A-2, A-3 and A-4. These conditions play the crucial role on the existence of rational expectations equilibrium in an economy with knowledge. The suppression of any of these assumptions renders Proposition 1 vulnerable to the discussion and the example proposed in Remarks 4.6 of Matsuhisa and

[8] This is true without A-2.

Ishikawa (2002). Therefore each of the assumptions A-1 to A-4 is also crucial in Theorem 1.

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