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Optimal Policy for Product R&D with Endogenous Quality Ordering: Asymmetric Duopoly*

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Abstract

We examine the optimal R&D subsidy/tax policy under a vertically differentiated duopoly. In a significant departure from the existing work, we consider the case of asymmetric costs of product R&D where there is a small technology gap between firms. In our analysis, the endogeneity of quality ordering is explicitly taken into account. We show that the optimal policy is described by a firm-specific *subsidy schedule* that is contingent on firms' quality choices. The subsidy schedule not only corrects the distortion in product quality but also selects the socially preferred equilibrium. Both Bertrand and Cournot cases are analyzed.

Keywords: asymmetric duopoly; endogenous quality ordering; product R&D; R&D policy; vertical product differentiation.

JEL classification: L13, L15, L52, H25.

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1 Introduction

Research and development (R&D) activities to develop new products or to improve the quality of existing products are a crucial part of firms' activities, particularly in high-technology industries. This type of R&D is called *product R&D* and is distinguished from *process R&D*, which aims to reduce production costs.[1] Some data show that product R&D is empirically more important than process R&D (Scherer and Ross, 1990; Fritsch and Meschede, 2001).[2] As product-R&D-intensive industries tend to be highly concentrated, firms may invest strategically in product R&D. Consequently, it is unlikely that socially optimal product qualities are chosen by individual firms. Moreover, although social welfare is higher when the superior firms produce higher quality products, firms with inferior technology may strategically produce higher quality products than firms with superior technology. Thus, government intervention may be required. The important issue is to determine a policy that can induce firms to choose socially optimal product qualities.

In order to examine this issue, we make a significant departure from the existing literature on endogenous quality choice in the following two respects. First, we analyze the case of asymmetric cost of product R&D with a small technology gap between firms. Although symmetric duopoly (Aoki, 2003; Aoki and Prusa, 1997; Jinji, 2003) and asymmetric duopoly with a large technology gap (Park, 2001; Zhou *et al.*, 2002) have been

[1]Symeonidis (2003a) points out that product R&D directly affects the consumer surplus, whereas process R&D affects it only indirectly. Product R&D directly affects consumers' utility because it improves product quality. Process R&D reduces marginal production costs and hence affects consumers' utility only indirectly through an increase in output. Recent works on product R&D include Symeonidis (2003a), Bonanno and Haworth (1998), and Lin and Saggi (2002).

[2]Scherer and Ross (1990) note that about three-quarters of R&D expenditure by firms in the United States falls into the category of product R&D. Fritsch and Meschede (2001) show that the share of product R&D in all R&D expenditure in German firms is about 61%.

examined, the intermediate case has received less attention, despite its relevance in the real world.[3] The small gap case is quite different from the large gap case because in the former multiple equilibria exist in the stage of quality choice. In the latter, the equilibrium is unique. This contrast makes the optimal policy quite different. Second, we explicitly take into account the endogeneity of quality ordering. It is common in the literature to assume that the quality ordering of the two firms is exogenously given (Ronnen, 1991; Toshimitsu, 2003). That is, one firm always produces a higher quality product than the rival. However, unless there is a sufficiently large technology gap between firms, there exist two equilibria and the quality ordering is endogenously determined. We show that the endogeneity of quality ordering has important implications for the optimal R&D policy.

The model in this paper is a vertically differentiated duopoly with fixed costs of quality improvement. Our main purpose is to investigate the optimal R&D policy in a second-best environment, in the sense that the government takes the duopolistic market structure as given. We examine both Bertrand and Cournot cases in the final stage of the game.

The main contribution of this paper is to show that with a small technology gap the optimal R&D policy is characterized by firm-specific *subsidy schedules*, that is, by subsidies contingent on product qualities. Traditional flat subsidies would not work, even if they were allowed to be firm-specific. The subsidy schedules induce the firm with relatively better technology to choose the higher quality product at the socially optimal level and induce the other firm to choose the lower quality product at the socially optimal level. As there are two asymmetric equilibria in the unregulated market, the R&D policy

[3] One exception is Jinji and Toshimitsu (2004), who examine the effects of minimum quality standards under asymmetric duopoly with a small technology gap. Moreover, in the case of process R&D, Lahiri and Ono (1999) address the issue of optimal R&D policy in an asymmetric Cournot duopoly.

needs to select the socially preferred equilibrium, as well as correct distortions in product quality. In other words, the R&D subsidy/tax is used to not only correct distortions in R&D activities but also prevent a technological follower from leapfrogging and becoming a quality leader in the industry. Although the first role of R&D subsidy/tax is well-known,[4] the anti-leapfrogging nature of R&D subsidy/tax is new to the literature.

The rest of the paper is organized in the following way. Section 2 sets up the model. Section 3 examines the optimal R&D policy when firms engage in price competition at the final stage. Section 4 analyzes the case of quantity competition. Section 5 concludes.

2 The Model

The model is a version of the standard model of vertical differentiation.[5] There is a continuum of consumers indexed by θ, which is uniformly distributed on $[0, 1]$ with density one. Each consumer is assumed to either buy one unit of the vertically differentiated good or nothing. Consumer θ's (indirect) utility is given by $u = \theta q - p$ if he buys one unit of a product of quality $q \in [0, \infty)$ at price $p \in [0, \infty)$. His utility is zero if he buys nothing.

There are two firms in the market. Each firm offers a single product. The marginal and average production costs are assumed to be invariant with respect to both quality and quantity.[6] For simplicity, we let these costs be zero. The cost of product R&D is different across firms. Without loss of generality, we assume that firm 2 has lower technology so that it has to incur higher cost of product R&D than firm 1 for the same q. Let $F(q)$ be

[4] See, for example, Lambertini and Mosca (1999) and Toshimitsu (2003) for the case of product R&D and Spence (1984) and Lahiri and Ono (1999) for the case of process R&D.

[5] See, for example, Mussa and Rosen (1978), Shaked and Sutton (1982, 1983), Ronnen (1991), and Aoki (2003).

[6] This is a standard assumption in the literature. See, for example, Shaked and Sutton (1982, 1983) and Ronnen (1991).

firm 1's cost of product R&D. Firm 2's R&D cost is given by $\gamma F(q)$, where $\gamma \geq 1$. We assume $F(q) = kq^n$, where k is a positive constant and n is any integer such that $n \geq 2$.

The government implements R&D policy, which is potentially a subsidy schedule. Let $s_i < 1$ be a subsidy for firm i. A negative s_i means an R&D tax. Taking the duopolistic market structure as given, the government chooses s_i to maximize social welfare W, which is the sum of firm's profits (π^i) and consumer's surplus (CS) minus social cost of subsidy:

$$\begin{aligned} W &= \pi^1 + \pi^2 + CS - s_1 F(q_1) - s_2 \gamma F(q_2) \\ &= R^1 - F(q_1) + R^2 - \gamma F(q_2) + CS, \end{aligned} \tag{1}$$

where R^i is firm i's revenue. The government chooses R&D policy in stage 1. In stage 2 firms simultaneously choose the quality of their products. In stage 3 firms compete either in prices or in quantities. The solution is the subgame perfect Nash equilibrium (SPNE). Throughout the paper, we restrict our attention to pure-strategy equilibria.

3 R&D Policy under Bertrand Competition

3.1 Revenue and quality best-response

In this section, we examine the case in which firms compete in prices at the final stage. Each firm's equilibrium revenue in stage 3 is given by

$$R^i(q_1, q_2) = \begin{cases} \dfrac{4q_i^2(q_i - q_j)}{(4q_i - q_j)^2}, & \text{if } q_i > q_j, \\ \dfrac{q_j q_i (q_j - q_i)}{(4q_j - q_i)^2}, & \text{if } q_i < q_j, \end{cases} \tag{2}$$

for $i, j = 1, 2$.

Firm 1 and firm 2's profits are given by $\pi^1(q_1, q_2; s_1) = R^1(q_1, q_2) - (1 - s_1)F(q_1)$ and $\pi^2(q_1, q_2; \gamma, s_2) = R^2(q_1, q_2) - (1 - s_2)\gamma F(q_2)$, respectively. In stage 2, each firm chooses quality to maximize its own profits, given its rival's quality and subsidies. Firm 1's quality best-response, $q_1 = B^1(q_2; s_1)$, is then defined as $B^1(q_2; s_1) = q_1^H(q_2; s_1)$ if $q_2 \leq \hat{q}_2(s_1)$ and $B^1(q_2; s_1) = q_1^L(q_2; s_1)$ if $q_2 \geq \hat{q}_2(s_1)$, where each of $q_i^H(q_j; s_i)$ and $q_i^L(q_j; s_i)$ satisfies

$$\partial\pi^1(q_1, q_2; s_1)/\partial q_1 \equiv \pi_1^1(q_1, q_2; s_1) = 0, \tag{3}$$

with $q_1^L(q_2; s_1) \leq q_2 \leq q_1^H(q_2; s_1)$, and $\hat{q}_2(s_1)$ satisfies $\pi^1(q_1^H(\hat{q}_2; s_1), \hat{q}_2(s_1); s_1) = \pi^1(q_1^L(\hat{q}_2; s_1), \hat{q}_2(s_1); s_1)$. That is, firm 1 is indifferent between $q_1^H(q_2; s_1)$ and $q_1^L(q_2; s_1)$ when firm 2's product quality is $\hat{q}_2(s_1)$. Since $\partial^2 R^1/\partial q_1^2 < 0$ and $F''(q_1) > 0$, the second-order condition is satisfied for both $q_1^H(q_2; s_1)$ and $q_1^L(q_2; s_1)$. Firm 2's quality best-response, $q_2 = B^2(q_1; \gamma, s_2)$, is defined analogously.

The properties of $B^i(q_j; s_i)$ are as follows: (i) $B^i(q_j; s_i) \neq q_j, \forall q_j$; (ii) $B^i(q_j; s_i)$ is discontinuous at $q_j = \hat{q}_j(s_i)$; (iii) $dB^i(q_j; s_i)/dq_j > 0, \forall q_j \neq \hat{q}_j(s_j)$; and (iv) $dB^i(q_j; s_i)/ds_i > 0$. The third property implies that qualities are *strategic complements*.

Each firm's quality best-response curve is depicted in Figure 1. In the figure, the solid lines of B^i represent firm i's quality best-response in the unregulated market. B^1 (resp. B^2) is discontinuous at $q_2 = \hat{q}_2(0)$ (resp. $q_1 = \hat{q}_1(\gamma, 0)$).

3.2 Optimal R&D policy

Nash equilibria (NEs) in stage 2 are shown in the following lemma:

Lemma 1 *Suppose that $s_1 = s_2 = 0$. Then, if γ is small, there exist two asymmetric pure-strategy NEs in stage 2, $(q_1, q_2) = \left\{(q_{1N}^H(\gamma), q_{2N}^L(\gamma)), (q_{1N}^L(\gamma), q_{2N}^H(\gamma))\right\}$, where $q_{1N}^H(\gamma) > q_{2N}^L(\gamma)$ and $q_{2N}^H(\gamma) > q_{1N}^L(\gamma)$.*

(Proofs of lemmas and propositions are presented in the Appendix.)

When the technology gap is small, the firm with inferior technology may choose a higher quality than the firm with superior technology. As a result, there exist multiple equilibria. When the technology gap is sufficiently large, on the other hand, the inferior firm has *no* incentive to leapfrog the rival and hence the equilibrium is unique, as shown by Zhou *et al.* (2002).

In Figure 1, the two NEs in the unregulated market are given by the intersections of the solid lines of $B^1(q_2; s_1)$ and $B^2(q_1; \gamma, s_2)$ at E_1 and E_2.

We now examine the socially optimal quality pair and the R&D policy to induce firms to choose the socially optimal quality pair. Let q_{iS}^H and q_{jS}^L be qualities of the high and low quality products, respectively, that maximize social welfare when firm i (resp. firm j) produces a high (resp. low) quality product. Then, the following lemma is obtained.

Lemma 2 *For a given* γ*, (i)* $q_{iN}^H(\gamma) < q_{iS}^H(\gamma)$ *and* $q_{jN}^L(\gamma) < q_{jS}^L(\gamma)$ *for* $i, j = 1, 2$ *and* $i \neq j$*; (ii)* $q_{1S}^H(\gamma) > q_{2S}^H(\gamma)$ *and* $q_{1S}^L(\gamma) > q_{2S}^L(\gamma)$*; and (iii)* $W(q_{1S}^H(\gamma), q_{2S}^L(\gamma)) > W(q_{2S}^H(\gamma), q_{1S}^L(\gamma))$*.*

In Figure 1 the quality pairs $(q_{1S}^H(\gamma), q_{2S}^L(\gamma))$ and $(q_{2S}^H(\gamma), q_{1S}^L(\gamma))$ are depicted as S_1 and S_2. The third result in Lemma 2 implies that S_1 is socially preferred to S_2. In other words, social welfare is higher when firm 1, the firm with superior technology, produces a higher quality product. Because of this property, the following two roles are required to the government policy: The first role is to correct the distortion in firms' quality choices and the second role is to select the socially preferred equilibrium. The optimal R&D policy in the case of small technology gap is then as shown in the following proposition:

Proposition 1 *Under duopoly with price competition in stage 3, if there is a small tech-*

nology gap between firms, the optimal R&D policy is given by subsidy schedules:

$$s_1 \begin{cases} = s_1^H(\gamma) > 0, & \text{if } q_1 > q_2 \text{ and } q_2 \leq \hat{q}_2(0), \\ \leq s_1^H(\gamma), & \text{if } q_1 < q_2 \text{ and } q_2 \leq \hat{q}_2(0), \\ = \hat{s}_1, & \text{if } q_1 < q_2 \text{ and } q_2 \geq \hat{q}_2(0), \\ \leq \hat{s}_1, & \text{if } q_1 > q_2 \text{ and } q_2 \geq \hat{q}_2(0), \end{cases} \tag{4}$$

$$s_2 \begin{cases} = s_2^L(\gamma) > 0, & \text{if } q_1 > q_2 \text{ and } q_1 \geq \hat{q}_1(0,\gamma), \\ \leq s_2^L(\gamma), & \text{if } q_1 < q_2 \text{ and } q_1 \geq \hat{q}_1(0,\gamma), \\ = \hat{s}_2, & \text{if } q_1 < q_2 \text{ and } q_1 \leq \hat{q}_1(0,\gamma), \\ \leq \hat{s}_2, & \text{if } q_1 > q_2 \text{ and } q_1 \leq \hat{q}_1(0,\gamma), \end{cases} \tag{5}$$

where $s_1^H(\gamma)$ and $s_2^L(\gamma)$ are implicitly defined by $B^1(q_{2S}^L(\gamma); s_1^H(\gamma)) = q_{1S}^H(\gamma)$ and $B^2(q_{1S}^H(\gamma); s_2^L(\gamma), \gamma) = q_{2S}^L(\gamma)$, and $\hat{s}_1$ and $\hat{s}_2$ are subsidies that jointly eliminate the equilibrium where $q_1 < q_2$. There is a unique SPNE outcome, in which $q_1 > q_2$.

The proposition shows that the government can pick the socially preferred equilibrium outcome by implementing a properly designed policy schedule.[7] The optimal policy is a subsidy schedule. While the government commits to the schedule in stage 1, the actual subsidy rate to each firm is determined when firms decide their product qualities in stage 2.[8] In the policy schedules, $s_1^H(\gamma)$ induces firm 1 to choose $q_{1S}^H(\gamma)$ and $s_2^L(\gamma)$ induces firm 2 to choose $q_{2S}^L(\gamma)$. Moreover, $\hat{s}_1$ and $\hat{s}_2$ jointly eliminate the less preferred equilibrium. The other elements in the policy schedules are aimed at leaving the switching points $\hat{q}_1$ and $\hat{q}_2$ unchanged in order to make sure S_1 is the unique equilibrium in stage 2. Note

[7] This is similar to what Jinji (2003) shows in the context of strategic trade policy. In a model of third-market trade, he shows that the government of an exporting country uses a similar policy schedule to maximize its own domestic welfare.

[8] When the policy is a subsidy schedule, s_i depends on q_1 and q_2. Firms take this into account when they choose product qualities. However, since s_i does not respond to a marginal change in q_i, the standard analysis of SPNE is valid.

that for a given γ, $s_1^H(\gamma)$ and $s_2^L(\gamma)$ are uniquely determined, while $\hat{s}_1$ and $\hat{s}_2$ are not uniquely determined. Thus, while the SPNE outcome is unique, there are many SPNEs that produce the same outcome. In the unique SPNE outcome, $(q_{1S}^H(\gamma), q_{2S}^L(\gamma))$ is chosen and the government provides subsidies $s_1^H(\gamma)$ and $s_2^L(\gamma)$ to firm 1 and 2, respectively.

Figure 1 shows how the optimal R&D policy works. The dotted lines are firms' quality best-response curves with the optimal R&D policy. An R&D subsidy $s_1^H(\gamma)$ shifts up the left part of B^1 and an R&D subsidy $s_2^L(\gamma)$ shifts the left part of B^2 to the right. As for the right parts of B^1 and B^2, $\hat{s}_1$ and $\hat{s}_2$ shift them so that there is no intersection between them in $q_1 < q_2$. One example is drawn in Figure 1. Effects of the other elements in (4) and (5) are not seen in the figure, because they affect the undrawn parts of B^1 and B^2.

As a corollary of Proposition 1, the optimal R&D policies in cases of $\gamma = 1$ and sufficiently large γ are obtained.

Corollary 1 *(i) If $\gamma = 1$, the single subsidy schedule is an optimal R&D policy:*

$$s_i \begin{cases} = s_1^H(1) > 0, & \text{if } q_i > q_j \text{ and } q_j \leq \hat{q}, \\ \leq s_1^H(1), & \text{if } q_i < q_j \text{ and } q_j \leq \hat{q}, \\ = s_2^L(1) > 0, & \text{if } q_i < q_j \text{ and } q_j \geq \hat{q}, \\ \leq s_2^L(1), & \text{if } q_i > q_j \text{ and } q_j \geq \hat{q}, \end{cases} \tag{6}$$

where $s_1^H(1)$ and $s_2^L(1)$ are implicitly defined by $B^1(q_S^L; s_1^H(1)) = B^2(q_S^L; s_1^H(1), 1) = q_S^H$ and $B^1(q_S^H; s_2^L(1)) = B^2(q_S^H; s_2^L(1), 1) = q_S^L$. There are two SPNE outcomes, which are identical except for the identity of the firms.

(ii) If γ is sufficiently large, a combination of $s_1 = s_1^H(\gamma) > 0$ and $s_2 = s_2^L(\gamma) > 0$ is an optimal R&D policy. There is a unique SPNE, in which $q_1 > q_2$.

Note that for $\gamma = 1$ we have $\hat{q}_1(0, \gamma) = \hat{q}_2(0) \equiv \hat{q}$, $q^H_{1S}(\gamma) = q^H_{2S}(\gamma) \equiv q^H_S$, and $q^L_{1S}(\gamma) = q^L_{2S}(\gamma) \equiv q^L_S$. The schedule (6) is derived by substituting $\hat{s}_1 = s^L_2(1)$ in (4) and $\hat{s}_2 = s^H_1(1)$ in (5). The second part of the corollary is obtained by substituting $\hat{s}_1 = s^H_1(\gamma)$ in (4) and $\hat{s}_2 = s^L_2(\gamma)$ in (5) and choosing equalities for all elements in the schedules (4) and (5).

Corollary 1 implies that the symmetric and the large gap cases are special cases of the small gap case. If $\gamma = 1$, the optimal policy is still a subsidy schedule. However, the schedule is not necessarily firm-specific, because social welfare is independent of the identity of the firms. If γ is sufficiently large, on the other hand, the traditional flat subsidy can be optimal, while the subsidy rate has to be firm-specific. Since the unregulated NE is unique in this case, the R&D policy has only to correct the distortion in product quality.

Note that even when the NE in the unregulated market is unique, the policy specified in Corollary 1 (ii) may create an NE where $q_1 < q_2$, if the technology gap is not significantly large. In other words, the flat subsidy to each firm may cause the multiple equilibria. That case is included in the small gap case rather than the large gap case.

4 R&D Policy under Cournot Competition

We now turn to the case in which firms compete in quantities in the final stage. Each firm's equilibrium revenue in stage 3 is given by

$$R^{ci}(q_1, q_2) = \begin{cases} \dfrac{q_i(2q_i - q_j)^2}{(4q_i - q_j)^2}, & \text{if } q_i > q_j, \\ \dfrac{q_i(q_j)^2}{(4q_j - q_i)^2}, & \text{if } q_i < q_j, \end{cases} \tag{7}$$

for $i, j = 1, 2$.

Firms 1 and 2's profits are given by $\pi^{c1}(q_1, q_2; s_1) = R^{c1}(q_1, q_2) - (1 - s_1)F(q_1)$, and

$\pi^{c2}(q_1, q_2; \gamma, s_2) = R^{c2}(q_1, q_2) - (1 - s_2)\gamma F(q_2)$, respectively.[9] Firm 1's quality best-response, $q_1 = \tilde{B}^1(q_2; s_1)$, is characterized by first-order condition (FOC) and given by $\tilde{B}^1(q_2; s_1) = \tilde{q}_1^H(q_2; s_1)$ if $q_2 \leq \tilde{q}_2(s_1)$ and $\tilde{B}^1(q_2; s_1) = \tilde{q}_1^L(q_2; s_1)$ if $q_2 \geq \tilde{q}_2(s_1)$, where $\tilde{q}_2(s_1)$ satisfies $\pi^{c1}(\tilde{q}_1^H(\tilde{q}_2; s_1), \tilde{q}_2(s_1); s_1) = \pi^{c1}(\tilde{q}_1^L(\tilde{q}_2; s_1), \hat{q}_2(s_1); s_1)$.[10] Firm 2's quality best-response, $q_2 = \tilde{B}^2(q_1; \gamma, s_2)$, is defined analogously. The properties of $\tilde{B}^i(q_j; s_i)$ are as follows: (i) $\tilde{B}^i(q_j; s_i) \neq q_j, \forall q_j$; (ii) $\tilde{B}^i(q_j; s_i)$ is discontinuous at $q_j = \tilde{q}_j(s_i)$; (iii) $d\tilde{B}^i(q_j; s_i)/dq_j > 0$ for $q_j \leq \tilde{q}_j(s_i)$; (iv) $d\tilde{B}^i(q_j; s_i)/dq_j < 0$ for $q_j \geq \tilde{q}_j(s_i)$; and (v) $d\tilde{B}^i(q_j; s_i)/ds_i > 0$. The third and fourth properties imply that qualities are *strategic complements* for the higher quality producer and *strategic substitutes* for the lower quality producer.

With a small technology gap, there exist two asymmetric pure-strategy NEs in stage 2, $(q_1, q_2) = \left\{(\tilde{q}_{1N}^H(\gamma), \tilde{q}_{2N}^L(\gamma)), (\tilde{q}_{1N}^L(\gamma), \tilde{q}_{2N}^H(\gamma))\right\}$. The situation is depicted in Figure 2. The solid lines of $\tilde{B}^i$ represent firm i's quality best-response in the unregulated market. It is upward sloping for $q_i > q_j$ and downward sloping for $q_i < q_j$. The two NEs are given by the intersections of $\tilde{B}^1$ and $\tilde{B}^2$ at $\tilde{E}_1$ and $\tilde{E}_2$.

The main difference from the Bertrand case is that subsidy for the firm producing a lower quality product is *negative* in this case. That is, R&D *tax* rather than subsidy should be applied to the low quality producer. The optimal subsidy for the high quality producer is again positive.[11] The optimal R&D policy is then as follows:

Proposition 2 *Under duopoly with quantity competition in stage 3, if there is a small*

[9] We assume regularity conditions on π^{ci} that guarantee concavity of the welfare function.

[10] Although the profit functions in the case of quantity competition are not locally concave for some qualities, the quality best-responses are, as Aoki (2003) shows, characterized by FOCs, rather than corner solutions.

[11] These results are qualitatively the same as what Toshimitsu (2003) has shown.

technology gap between firms, the optimal R&D policy is given by subsidy schedules:

$$s_1 \begin{cases} = \tilde{s}_1^H(\gamma) > 0, & \text{if } q_1 > q_2 \text{ and } q_2 \leq \tilde{q}_2(0), \\ \leq \tilde{s}_1^H(\gamma), & \text{if } q_1 < q_2 \text{ and } q_2 \leq \tilde{q}_2(0), \\ = \hat{s}_1', & \text{if } q_1 < q_2 \text{ and } q_2 \geq \tilde{q}_2(0), \\ \leq \hat{s}_1', & \text{if } q_1 > q_2 \text{ and } q_2 \geq \tilde{q}_2(0), \end{cases} \tag{8}$$

$$s_2 \begin{cases} = \tilde{s}_2^L(\gamma) < 0, & \text{if } q_1 > q_2 \text{ and } q_1 \geq \tilde{q}_1(0,\gamma), \\ \leq \tilde{s}_2^L(\gamma), & \text{if } q_1 < q_2 \text{ and } q_1 \geq \tilde{q}_1(0,\gamma), \\ = \hat{s}_2', & \text{if } q_1 < q_2 \text{ and } q_1 \leq \tilde{q}_1(0,\gamma), \\ \leq \hat{s}_2', & \text{if } q_1 > q_2 \text{ and } q_1 \leq \tilde{q}_1(0,\gamma), \end{cases} \tag{9}$$

where $\tilde{s}_1^H(\gamma)$ *and* $\tilde{s}_2^L(\gamma)$ *are implicitly defined by* $\tilde{B}^1(\tilde{q}_{2S}^L(\gamma); \tilde{s}_1^H(\gamma)) = \tilde{q}_{1S}^H(\gamma)$ *and* $\tilde{B}^2(\tilde{q}_{1S}^H(\gamma); \tilde{s}_2^L(\gamma), \gamma) = \tilde{q}_{2S}^L(\gamma)$ *and* $\hat{s}_1'$ *and* $\hat{s}_2'$ *jointly eliminate the equilibrium where* $q_1 < q_2$.

As in the Bertrand case, there is a unique SPNE outcome where firm 1 produces a higher quality product. In the equilibrium outcome, the government provides an R&D subsidy $\tilde{s}_1^H(\gamma)$ to firm 1 and imposes an R&D tax $\tilde{s}_2^L(\gamma)$ on firm 2.[12]

The situation is depicted in Figure 2. The dotted lines are $\tilde{B}^1$ and $\tilde{B}^2$ with the optimal R&D policy. An R&D subsidy $\tilde{s}_1^H(\gamma)$ shifts up the left part of B^1 and an R&D tax $\tilde{s}_2^L(\gamma)$ shifts the left part of B^2 to the left. As for the right parts of B^1 and B^2, $\hat{s}_1'$ and $\hat{s}_2'$ shift them so that there is no intersection between them in $q_1 < q_2$.

The result is partly similar to what Lahiri and Ono (1999) show. The government helps the firm with relatively better technology to choose a higher quality product at the socially optimal level. For the inferior firm, the government not only prevents it from

[12] As in the Bertrand case, with $\gamma = 1$ a single subsidy schedule is an optimal R&D policy and with a sufficiently large γ the firm-specific flat subsidy is an optimal R&D policy.

choosing a higher quality but also taxes its R&D even if it would choose a lower quality.

The R&D subsidy on the high quality producer and the R&D tax on the low quality producer raise the degree of product differentiation. The increased product differentiation expands the market share of the high quality producer, which is welfare-improving. This is similar to Lahiri and Ono's (1988) result in the case of heterogeneous production costs among firms. They show that a cost reduction in the firm with lower marginal cost expands its market share and reduces the market share of less efficient firms and that the shift in production from less efficient firms to the more efficient firm improves social welfare. In our case, an expansion in the market share of the high quality producer improves social welfare. Our result is also related to what Symeonidis (2003b) shows. In a mixture of horizontal and vertical differentiation, he shows that an increase in the degree of product differentiation is welfare improving.

These results contrast with the results in the Bertrand case. Recall that in the Bertrand case the government subsidizes R&D of the low quality firm as well as the high quality firm. Since products are too much differentiated in order to soften the price competition, the quality of the low quality product is too low from social point of view. Thus, increasing the quality of low quality product and reducing the degree of product differentiation improve social welfare. Unlike the Cournot case, expanding the market share of the high quality producer is not necessarily welfare-improving by itself. Note that while the incentive to soften the price competition tends to raise the quality of the high quality product, its quality level in the unregulated market is still too low from social point of view. Therefore, a subsidy on the high quality producer is also required.

5 Conclusions

In this paper, we have examined the optimal R&D policy in a duopolistic industry where goods are differentiated in quality and firms invest in R&D to improve product qualities. We have considered the optimal R&D subsidy/tax in a second-best environment where the government takes the market structure as given. Our focus has been on the case of asymmetric duopoly in the sense that the cost of product R&D is different across firms.

We have shown that the optimal R&D policy is characterized by firm-specific subsidy schedules that are contingent on firms' quality choices. There exist two asymmetric Nash equilibria and social welfare is higher in the equilibrium where the firm with superior technology produces a high quality product. Thus, the R&D policy needs to not only correct the distortion in firms' quality choices but also select the socially preferred equilibrium. The firm-specific subsidy schedules induce the superior firm to produce the high quality product at the optimal quality level and the inferior firm to produce the low quality product at the optimal quality level.

The firm-specific subsidy schedule contingent on all firms' R&D activities may seem to be less practical. In the real world, however, it is observed that research grants for product R&D are allocated to projects by different amounts, depending on the evaluation of project goals and other elements of all applications. This can be interpreted as an example of the firm-specific subsidy schedule contingent on quality choices.

The results in this paper imply that it is crucial for the design of the optimal policies in vertically differentiated industries to take into account the endogeneity of quality ordering. Although it is common in the literature of vertical differentiation to focus on one equilibrium, it may obscure some important properties of the model.

For the future research, a number of potentially interesting extensions of the analysis in this paper can be considered. First of all, it will be interesting to introduce uncertainty of the outcome of R&D activities into the model. In the real world, R&D activities are typically subject to a high degree of uncertainty. Introducing uncertainty will add some additional factors to the optimal R&D policy. It may also be interesting to incorporate the dynamic aspect of R&D activities, which is sometimes emphasized in the literature. Moreover, informational asymmetry between firms and the government is sometimes pointed out as a potential obstacle for the government to implement the optimal policy. Thus, in order to discuss further the implementation of the optimal R&D policy, it may be important to take informational asymmetry into account.

A Appendix: Proofs of Lemmas and Propositions

A.1 Proof of Lemma 1

Aoki (2003) proves the existence of two pure-strategy NEs under symmetric duopoly, that is, in the case of $\gamma = 1$. Zhou *et al.* (2002) prove that with a sufficiently large technology gap, there exist a unique NE where the firm with superior technology produces a high quality product. Since Zhou *et al.* (2002) prove their result under the assumption of a general convex cost function, their result can apply to our case as well. Since $dq_1^H(q_2)/dq_2 > 0$, $dq_2^L(q_1)/dq_1 > 0$, $dq_2^L/d\gamma < 0$, and $d\hat{q}_1/d\gamma < 0$, then together with Aoki's result there must exist an NE where $q_1 > q_2$ for $\gamma \geq 1$. As for the NE where $q_1 < q_2$, the results shown by Aoki (2003) and Zhou *et al.* (2002) imply that an NE exists for a small γ but no NE exists for a sufficiently large γ. Sufficient conditions on γ for the existence of the NE where $q_1 < q_2$ in the case of $n = 2$ are as follows. Since $q_2^H(0) = 1/8k\gamma$ and $1/18k \leq \hat{q}_2 \leq 1/12k$, then $q_2^H(0) > \hat{q}_2$ if $1/8k\gamma > 1/12k$, or $\gamma < 3/2$. It holds that $1/48k < q_1^L(1/12k) < 1/47k$. Since $1/18k\gamma \leq \hat{q}_1 \leq 1/12k\gamma$, then $q_1^L(\hat{q}_2) < \hat{q}_1$ if $1/18k\gamma > 1/47k$, or $\gamma < 47/18 \approx 2.61$. Thus, if $\gamma < 3/2$, there exists an NE where $q_1 < q_2$. $\square$

A.2 Proof of Lemma 2

(i) Consider first the case of $q_1 > q_2$. Let q_1^H (q_2^L) be quality of the high (resp. low) quality product produced by firm 1 (resp. firm 2). Then, social welfare in this case is given by $W^1(q_1^H, q_2^L) = R^H(q_1^H, q_2^L) - F(q_1^H) + R^L(q_1^H, q_2^L) - \gamma F(q_2^L) + CS(q_1^H, q_2^L)$, where $R^H(q_1^H, q_2^L)$ and $R^L(q_1^H, q_2^L)$ are revenues of the high and low quality producers, respectively. Define

$\pi^{1H}(q_1^H, q_2^L) = R^H(q_1^H, q_2^L) - F(q_1^H)$ and $\pi^{2L}(q_1^H, q_2^L) = R^L(q_1^H, q_2^L) - \gamma F(q_2^L)$.

Totally differentiate W^1 and arrange terms to yield

$$\frac{dW^1}{dq_1^H} = R_H^H - F'(q_1^H) + R_H^L + CS_H + \left(R_L^H + R_L^L - \gamma F'(q_2^L) + CS_L\right)\frac{dq_2^L}{dq_1^H}, \tag{A.1}$$

where $R_H^H \equiv \partial R^H/\partial q^H$ and so on.

Evaluate Eq. (A.1) at $(q_1^H, q_2^L) = (q_{1N}^H, q_{2N}^L)$ to obtain $dW^1/dq_1^H\big|_{(q_{1N}^H, q_{2N}^L)} = R_H^L + CS_H + \left(R_L^H + CS_L\right) dq_2^L/dq_1^H$, because $\pi_H^{1H} = R_H^H - F'(q_1^H) = 0$ and $\pi_L^{2L} = R_L^L - \gamma F'(q_2^L) = 0$. Since $R_H^L + CS_H = (2q_1^H + q_2^L)(q_1^H - q_2^L)/(4q_1^H - q_2^L)^2 > 0$, $R_L^H + CS_L = (3/2)(q_1^H)^2/(4q_1^H - q_2^L)^2 > 0$, and $dq_2^L/dq_1^H = -R_{LH}^L/\pi_{LL}^{2L} > 0$, where $R_{LH}^L \equiv \partial^2 R^L/\partial q^L \partial q^H$ and so on, then $dW^1/dq_1^H\big|_{(q_{1N}^H, q_{2N}^L)} > 0$. This implies that $q_{1N}^H < q_{1S}^H$. Similarly, we obtain $dW^1/dq_2^L\big|_{(q_{1N}^H, q_{2N}^L)} = \left(R_H^L + CS_H\right) dq_1^H/dq_2^L + R_L^H + CS_L > 0$, because $dq_1^H/dq_2^L = -R_{HL}^H/\pi_{HH}^{1H} > 0$. This implies that $q_{2N}^L < q_{2S}^L$.

It can be shown in a similar way that $q_{2N}^H < q_{2S}^H$ and $q_{1N}^L < q_{1S}^L$ in the case of $q_1 < q_2$.

(ii) From the FOCs for welfare maximization, $(q_{1S}^H(\gamma), q_{2S}^L(\gamma))$ satisfies

$$\frac{\partial W^1}{\partial q_1^H} = R_H^H - F'(q_1^H) + R_H^L + CS_H = 0, \tag{A.2}$$
$$\frac{\partial W^1}{\partial q_2^L} = R_L^H + R_L^L - \gamma F'(q_2^L) + CS_L = 0. \tag{A.3}$$

Similarly, $(q_{2S}^H(\gamma), q_{1S}^L(\gamma))$ satisfies

$$\frac{\partial W^2}{\partial q_2^H} = R_H^L + R_H^H - \gamma F'(q_2^H) + CS_H = 0, \tag{A.4}$$
$$\frac{\partial W^2}{\partial q_1^L} = R_L^L - F'(q_1^L) + R_L^H + CS_L = 0, \tag{A.5}$$

where $W^2(q_2^H, q_1^L) = R^L(q_2^H, q_1^L) - F(q_1^L) + R^H(q_2^H, q_1^L) - \gamma F(q_2^H) + CS(q_2^H, q_1^L)$ is social welfare in the case of $q_1 < q_2$. Evaluate (A.2) and (A.3) at $(q^H, q^L) = (q_{2S}^H(\gamma), q_{1S}^L(\gamma))$ and use (A.4) to obtain $\partial W^1/\partial q_1^H\big|_{(q_{2S}^H(\gamma), q_{1S}^L(\gamma))} = R_H^H(q_{2S}^H(\gamma), q_{1S}^L(\gamma)) - F'(q_{2S}^H(\gamma)) +$

$R_H^L(q_{2S}^H(\gamma), q_{1S}^L(\gamma)) + CS_H(q_{2S}^H(\gamma), q_{1S}^L(\gamma)) = \gamma F'(q_{2S}^H(\gamma)) - F'(q_{2S}^H(\gamma)) > 0$, because $\gamma > 1$ and $F'(\cdot) > 0$. This implies that $q_{1S}^H(\gamma) > q_{2S}^H(\gamma)$. Similarly, evaluate (A.3) at $(q^H, q^L) = (q_{2S}^H(\gamma), q_{1S}^L(\gamma))$ and use (A.5) to obtain $\partial W^1/\partial q_2^L\big|_{(q_{2S}^H(\gamma), q_{1S}^L(\gamma))} = (1-\gamma)F'(q_{1S}^L(\gamma)) < 0$, which implies that $q_{1S}^L(\gamma) > q_{2S}^L(\gamma)$.

(iii) Differentiate W^1 and W^2 with respect to γ to yield, respectively,

$$\begin{aligned} \frac{dW^1}{d\gamma} &= \left\{R_H^H - F'(q_1^H) + R_H^L + CS_H\right\}\frac{dq_1^H}{d\gamma} + \left\{R_L^H + R_L^L - \gamma F'(q_2^L) + CS_L\right\}\frac{dq_2^L}{d\gamma} \\ &\quad - F(q_2^L), \qquad \text{(A.6)} \\ \frac{dW^2}{d\gamma} &= \left\{R_L^L - F'(q_1^L) + R_L^H + CS_L\right\}\frac{dq_1^L}{d\gamma} + \left\{R_H^L + R_H^H - \gamma F'(q_2^H) + CS_H\right\}\frac{dq_2^H}{d\gamma} \\ &\quad - F(q_2^H). \qquad \text{(A.7)} \end{aligned}$$

Evaluate (A.6) at $(q_{1S}^H(\gamma), q_{2S}^L(\gamma))$ and (A.7) at $(q_{2S}^H(\gamma), q_{1S}^L(\gamma))$ and use (A.2) - (A.5) to yield $dW^1/d\gamma|_{(q_{1S}^H, q_{2S}^L)} = -F(q_2^L(\gamma)) < 0$, and $dW^2/d\gamma|_{(q_{2S}^H, q_{1S}^L)} = -F(q_2^H(\gamma)) < 0$. For a given γ we have $q_2^H(\gamma) > q_2^L(\gamma)$. We also have $F'(\cdot) > 0$. Thus, we obtain

$$\frac{dW^1}{d\gamma}\bigg|_{(q_{1S}^H, q_{2S}^L)} - \frac{dW^2}{d\gamma}\bigg|_{(q_{2S}^H, q_{1S}^L)} = F(q_2^H(\gamma)) - F(q_2^L(\gamma)) > 0, \qquad \text{(A.8)}$$

which implies that, for a given γ, a marginal increase in γ reduces W^2 more than W^1. Since $W^1(q_{1S}^H(1), q_{2S}^L(1)) = W^2(q_{2S}^H(1), q_{1S}^L(1))$, it holds that $W^1(q_{1S}^H(\gamma), q_{2S}^L(\gamma)) > W^2(q_{2S}^H(\gamma), q_{1S}^L(\gamma))$ for γ marginally higher than 1. Moreover, since (A.8) holds for any feasible $\gamma \geq 1$, it implies that $W^1(q_{1S}^H(\gamma), q_{2S}^L(\gamma)) > W^2(q_{2S}^H(\gamma), q_{1S}^L(\gamma))$ holds for any feasible $\gamma \geq 1$. □

A.3 Proof of Proposition 1

Since W is higher at S_1, the government induces firms to choose (q_{1S}^H, q_{2S}^L). The FOCs for the government to maximize W with respect to s_i are given by

$$\begin{aligned}\frac{dW}{ds_i} &= \left\{R_1^1(q_1,q_2) - F'(q_1) + R_1^2(q_1,q_2) + CS_1(q_1,q_2)\right\}\frac{dq_1}{ds_i}\\ &+ \left\{R_2^1(q_1,q_2) + R_2^2(q_1,q_2) - \gamma F'(q_2) + CS_2(q_1,q_2)\right\}\frac{dq_2}{ds_i} = 0, \end{aligned} \quad \text{(A.9)}$$

where $R_1^1(q_1,q_2) \equiv \partial R^1(q_1,q_2)/\partial q_1$ and so on. Use the FOCs for each firm to maximize its own profits to rewrite Eq. (A.9) as

$$\begin{aligned}\frac{dW}{ds_i} &= \left\{R_1^2(q_1,q_2) + CS_1(q_1,q_2) - s_1 F'(q_1)\right\}\frac{dq_1}{ds_i}\\ &+ \left\{R_2^1(q_1,q_2) - s_2\gamma F'(q_2) + CS_2(q_1,q_2)\right\}\frac{dq_2}{ds_i} = 0, \quad i = 1,2. \end{aligned} \quad \text{(A.10)}$$

Totally differentiating FOCs (Eq. (3) and a similar equation for firm 2) yields

$$\frac{dq_i}{ds_i} = -\frac{\pi_{jj}^j \Gamma_i F'(q_i)}{|D|}, \qquad \text{and} \qquad \frac{dq_j}{ds_i} = \frac{\pi_{ij}^j \Gamma_i F'(q_i)}{|D|}, \qquad i,\ j = 1,2, \quad \text{(A.11)}$$

where Γ_i is an operator such that $\Gamma_1 = 1$ and $\Gamma_2 = \gamma$ and $|D| = \pi_{11}^1\pi_{22}^2 - \pi_{12}^1\pi_{21}^2 > 0$.[13] Substituting them into (A.10) yields

$$\begin{aligned} s_1\pi_{22}^2 F'(q_1) + s_2\pi_{21}^2\gamma F'(q_2) &= (R_1^2 + CS_1)\pi_{22}^2 + (R_2^1 + CS_2)\pi_{21}^2,\\ s_1\pi_{12}^1 F'(q_1) + s_2\pi_{11}^1\gamma F'(q_2) &= (R_1^2 + CS_1)\pi_{12}^1 + (R_2^1 + CS_2)\pi_{11}^1. \end{aligned}$$

Solve the simultaneous equations to yield the locally optimal s_1 and s_2:

$$s_1^* = (R_1^2 + CS_1)/F'(q_1), \qquad \text{and} \qquad s_2^* = (R_2^1 + CS_2)/\{\gamma F'(q_2)\}. \quad \text{(A.12)}$$

[13] Since $R_{11}^1 R_{22}^2 - R_{12}^1 R_{21}^2 = 0$, then $|D|$ can be rewritten as $|D| = -R_{22}^2(1-s_1)F''(q_1) - \pi_{11}^1(1-s_2)\gamma F''(q_2)$. Since $R_{22}^2 < 0$ and $\pi_{11}^1 < 0$ for both $q_1 > q_2$ and $q_1 < q_2$ and since $F''(\cdot) > 0$ and $\gamma > 1$, then $|D| > 0$.

When $q_1 > q_2$, it is shown that $R_1^2 + CS_1 = (2q_1 + q_2)(q_1 - q_2)/(4q_1 - q_2)^2 > 0$ and $R_2^1 + CS_2 = 3(q_1)^2/\{2(4q_1 - q_2)^2\} > 0$. Since $F'(\cdot) > 0$ and $\gamma > 1$, for $q_1 > q_2$

$$s_1^* = (2q_1 + q_2)(q_1 - q_2)/\{(4q_1 - q_2)^2 F'(q_1)\} \equiv s_1^H(\gamma) > 0, \tag{A.13}$$

$$s_2^* = 3(q_1)^2/\{2(4q_1 - q_2)^2 \gamma F'(q_2)\} \equiv s_2^L(\gamma) > 0. \tag{A.14}$$

Since $dW/ds_i = (\partial W/\partial q_1)(dq_1/ds_i) + (\partial W/\partial q_2)(dq_2/ds_i)$, it follows that

$$\frac{d^2W}{ds_i^2} = \frac{\partial^2 W}{\partial q_1^2}\left(\frac{dq_1}{ds_i}\right)^2 + \frac{\partial W}{\partial q_1}\left(\frac{d^2q_1}{ds_i^2}\right) + 2\frac{\partial^2 W}{\partial q_1 \partial q_2}\left(\frac{dq_1}{ds_i}\right)\left(\frac{dq_2}{ds_i}\right) + \frac{\partial^2 W}{\partial q_2^2}\left(\frac{dq_2}{ds_i}\right)^2 + \frac{\partial W}{\partial q_2}\left(\frac{d^2q_2}{ds_i^2}\right).$$

Since $\partial W/\partial q_j|_{(s_1^*,s_2^*)} = 0,\ j = 1,2$, it yields that

$$\begin{aligned}
\left.\frac{d^2W}{ds_i^2}\right|_{(s_1^*,s_2^*)} &= \frac{\partial^2 W}{\partial q_1^2}\left(\frac{dq_1}{ds_i}\right)^2 + 2\frac{\partial^2 W}{\partial q_1 \partial q_2}\left(\frac{dq_1}{ds_i}\right)\left(\frac{dq_2}{ds_i}\right) + \frac{\partial^2 W}{\partial q_2^2}\left(\frac{dq_2}{ds_i}\right)^2 \\
&= \left(\frac{\partial^2 W}{\partial q_1^2}\frac{dq_1}{ds_i} + \frac{\partial^2 W}{\partial q_1 \partial q_2}\frac{dq_2}{ds_i}\right)\frac{dq_1}{ds_i} + \left(\frac{\partial^2 W}{\partial q_2^2}\frac{dq_2}{ds_i} + \frac{\partial^2 W}{\partial q_1 \partial q_2}\frac{dq_1}{ds_i}\right)\frac{dq_2}{ds_i}.
\end{aligned}$$

It is shown that $dq_1/ds_1 > 0$, $dq_2/ds_1 > 0$, $dq_1/ds_2 > 0$, and $dq_2/ds_2 > 0$. As for $\partial^2 W/(\partial q_i)^2 = R_{ii}^i + R_{ii}^j + CS_{ii} - F''(q_i)$, we have $R_{ii}^i + R_{ii}^j + CS_{ii} = -(q_j)^2(4q_i + 17q_j)/(4q_i - q_j)^4 < 0$ if $q_i > q_j$ and $R_{ii}^i + R_{ii}^j + CS_{ii} = -(q_j)^2(4q_j + 17q_i)/(4q_j - q_i)^4 < 0$ if $q_i < q_j$, $i,j = 1,2,\ i \neq j$. For $\partial^2 W/(\partial q_1 \partial q_2) = R_{12}^1 + R_{12}^2 + CS_{12}$, we have $R_{ij}^i + R_{ij}^j + CS_{ij} = q_i q_j (4q_i + 17q_j)/(4q_i - q_j)^4 > 0$ for $q_i > q_j$. We also have $\pi_{ii}^i < 0$ and $F''(q_i) > 0$, $i = 1,2$. Use these properties and Eq. (A.11) to show, after some manipulation, that

$$\frac{\partial^2 W}{\partial q_i^2}\frac{dq_i}{ds_i} + \frac{\partial^2 W}{\partial q_i \partial q_j}\frac{dq_j}{ds_i} = \frac{\Gamma_i F'(q_i)}{|D|}\left(\pi_{jj}^j F''(q_i) - \frac{(q_j)^2(4q_k + 17q_l)F''(q_j)}{(4q_k - q_l)^4}\right) < 0,$$

for $q_k > q_l$, $k,l = 1,2$, $k \neq l$ and that

$$\frac{\partial^2 W}{\partial q_j^2}\frac{dq_j}{ds_i} + \frac{\partial^2 W}{\partial q_i \partial q_j}\frac{dq_i}{ds_i} = \begin{cases} -\dfrac{3q_i q_j \Gamma_i F'(q_i) F''(q_j)}{(4q_i - q_j)^3 |D|} < 0, & \text{if } q_i > q_j, \\ -\dfrac{9q_i q_j \Gamma_i F'(q_i) F''(q_j)}{(4q_j - q_i)^3 |D|} < 0, & \text{if } q_i < q_j. \end{cases}$$

Thus, in any case it yield that $d^2W/(ds_i)^2|_{(s_1^*,s_2^*)} < 0,\ i = 1,2$.

In order to ensure that (q_{1S}^H, q_{2S}^L) is chosen in equilibrium, another NE (E_2) where $q_1 < q_2$ must be eliminated. E_2 can be eliminated by implementing R&D subsidies $s_1 = \hat{s}_1$ for $q_1 < q_2$ with $q_2 \geq \hat{q}_2$ and $s_2 = \hat{s}_2$ for $q_1 < q_2$ with $q_1 \leq \hat{q}_1$. We show an example of $(\hat{s}_1, \hat{s}_2)$ in the case of $n = 2$. Suppose that $\hat{s}_2 = 0$. Since $dB^i(q_j; s_i)/dq_j > 0$ and since $1/8k\gamma < q_2^H(q_1; 0) < 7/48k\gamma$ for $q_1 \leq 1/18k\gamma$, there is no NE where $q_1 < q_2$ if $q_1^L(1/8k\gamma; \hat{s}_1) > 1/18k\gamma$. Suppose that $\gamma = 1$. Then, it is numerically shown that with $\hat{s}_1 = 0.823$, $q_1^L(1/8k; \hat{s}_1) \approx 0.05563/k > 1/18k \approx 0.05556/k$. Since $dq_2^H/d\gamma < 0$ and $d\hat{q}_1/d\gamma < 0$, then for $\gamma > 1$, a lower $\hat{s}_1$ will work. $\square$

A.4 Proof of Proposition 2

As in the Bertrand case, Aoki (2003) proves that there exist two pure-strategy NEs in the case of $\gamma = 1$ and Zhou *et al.* (2002) prove that if the technology gap is sufficiently large, there exist a unique NE in which the firm with superior technology produces a high quality product. Since $d\tilde{q}_1^H(q_2)/dq_2 > 0$, $d\tilde{q}_2^L(q_1)/dq_1 < 0$, $d\tilde{q}_2^L/d\gamma < 0$, and $d\tilde{q}_1/d\gamma < 0$, then together with Aoki's result there must exist an NE where $q_1 > q_2$ for $\gamma \geq 1$. The results shown by Aoki and Zhou *et al.* imply that an NE where $q_1 < q_2$ exists for a small γ but no NE exists for a sufficiently large γ.

Use the procedure that is similar to what we used in the proof of Proposition 1 to derive the locally optimal s_1 and s_2 for $q_1 > q_2$:

$$\tilde{s}_1^* = (R_1^{c2} + CS_1^c)/F'(q_1), \qquad \text{and} \qquad \tilde{s}_2^* = (R_2^{c1} + CS_2^c)/\{\gamma F'(q_2)\}. \tag{A.15}$$

When $q_1 > q_2$, it is shown that $R_1^{c2} + CS_1^c = \{2q_1(q_1 - q_2) + 2(q_1)^2 - (q_2)^2\}/\{2(4q_1 - q_2)^2\} > 0$

and $R_2^{c1} + CS_2^c = -(q_1)^2/\{2(4q_1 - q_2)^2\} < 0$. Since $F'(\cdot) > 0$ and $\gamma > 1$, then for $q_1 > q_2$

$$\tilde{s}_1^* = \frac{2q_1(q_1 - q_2) + 2(q_1)^2 - (q_2)^2}{2(4q_1 - q_2j)^2 F'(q_1)} \equiv \tilde{s}_1^H(\gamma) > 0 \tag{A.16}$$

$$\tilde{s}_2^* = -\frac{(q_1)^2}{2(4q_1 - q_2)^2 F'(q_2)} \equiv \tilde{s}_2^L(\gamma) < 0. \tag{A.17}$$

We assume $F''(\cdot) > 0$ is sufficiently large to ensure the SOCs are satisfied, i.e., $d^2W^c/(ds_i)^2|_{(\tilde{s}_1^*, \tilde{s}_2^*)} < 0$, $i = 1, 2$. In order to ensure that $(\tilde{q}_{1S}^H, \tilde{q}_{2S}^L)$ is chosen in equilibrium, the government needs to eliminate $\tilde{E}_2$. We show an example of $(\hat{s}_1', \hat{s}_2')$ in the case of $n = 2$. Suppose that $\hat{s}_1' = 0$ and that $\gamma = 1$. Since $1/8k(1 - s_2) < \tilde{q}_2^H(q_1) < 7/54k(1 - s_2)$ for $q_1 \leq 5/54k(1 - s_2)$ and since $5/54k \leq \tilde{q}_2 \leq 1/9k$, then $\tilde{E}_2$ is eliminated if $7/54k(1 - s_2) < 5/54k$, or $\hat{s}_2' < -0.4$. For $\gamma > 1$, a lower tax rate (i.e., a higher $\hat{s}_2'$) will work. □

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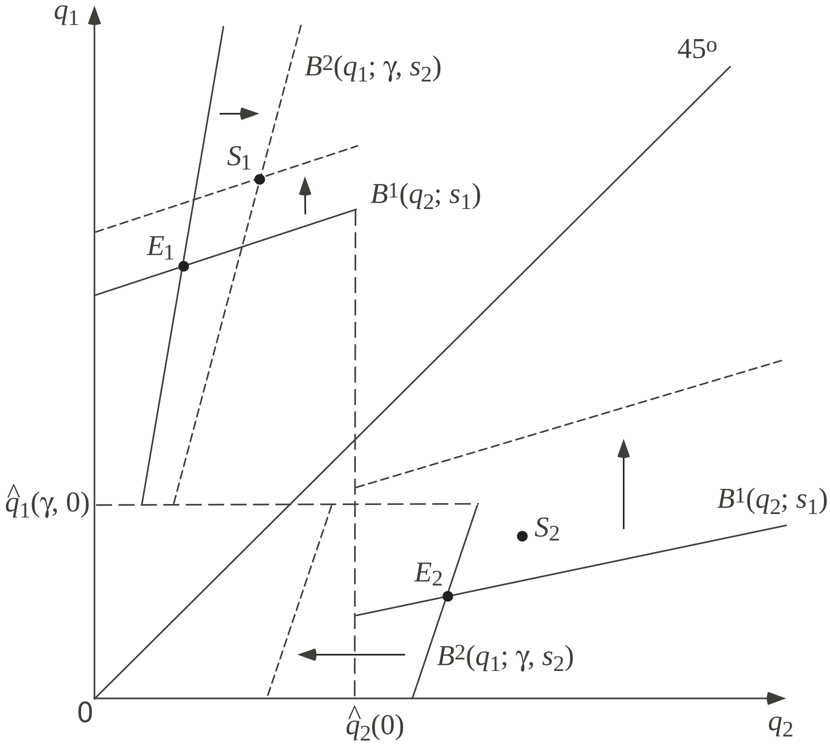

Figure 1: Nash equilibria under price competition: Small technology gap

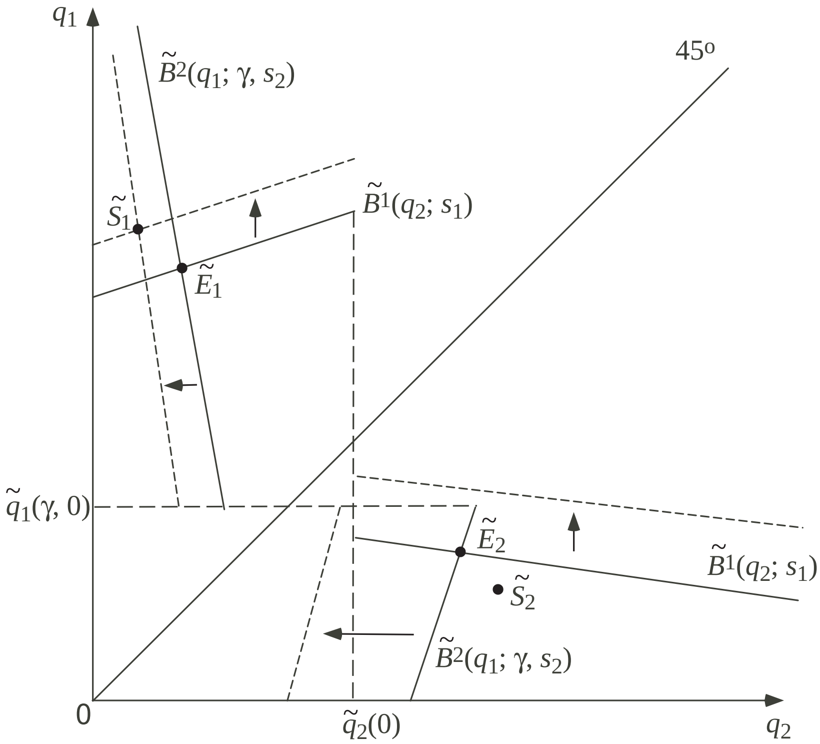

Figure 2: Nash equilibria under quantity competition: Small technology gap