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It is well recognized that the impact of subsidization/taxation policies hinges on the market structure to which they apply. We show that different degree of efficiency gain sharply changes the comparisons of optimal subsidy, total outputs and social welfare between mixed and private duopoly. What is more, for an imposition of an optimal subsidy, welfare may increase, decrease, or remain unchanged with privatization, which depends on the level of the cost efficiency gap and the taxation burden. However, it may be possible to raise welfare through privatization as long as the efficiency gain prevails or no excess taxation burden exists. Government sets higher subsidy to stimulate firms’ production if the value of cost-differential is assured.

Keywords: Privatization; Mixed Duopoly; Cost Efficiency Gap; Subsidization; Excess Taxation Burden

JEL Classification: L13, L33, H20
such as East Europe, Latin America, and so called BRICs - Brazil, Russia, India and China. As we observed, wide features of industry involve this issue, which includes the airline, rail, telecommunications, electricity, energy and so on. For instance, China adopts privatization policy to reform its state-owned enterprises since 1978. And during the 1980s, Brazil and other Latin American countries evolve into the “Washington consensus”, and then attempt to implement couple of sophisticated anti-inflation policies with a growth strategy based on market liberalization, such as privatization.

Over the past few decades, proliferated theoretical literatures involve the exploration of privatization. De Fraja and Delbono (1989) in a mixed oligopoly model showed that the privatization of welfare-maximizing public firms may improve social welfare. Bös (1997) presented a theory of incomplete privatization contracts to postulate an elaborate system of price regulation of privatized monopolies, which typically is missing in developing countries. Matsumura (1998) and Bennett and Maw (2003) explicitly considered the possibility of partial privatization. Along with the extension of free-entry market, Matsumura and Kanda (2005) assessed the welfare implications of partial privatization in a homogeneous oligopoly. Brandão and Castro (2007) demonstrated that the presence of a public enterprise can be an alternative to direct regulation to avoid the excess entry problem. Integrating the past relevant literatures, Wang and Chen (2010) highlighted the importance of cost efficiency gap between public and private firm, and showed the relation of cost efficiency gap and foreign competition with optimal privatization at free entry market.

Recently, the World Bank has urged India to make more far-reaching reforms to raise the current low collection rate of user charges. In particular, privatization in power and subsidies adjustment must be done if it is to free itself from what is known as the “Hindu loop” (growth rates of around 5-6 percent). In fact, India requires much higher growth rates through such reforms if it is to alleviate the living conditions of some 300 million living below the poverty line. Studies on optimal subsidy in mixed oligopoly have gained attentions. White (1996) showed that the same subsidy rate yields the first-best outcome in both mixed and private oligopoly in his Cournot setting (irrelevance result). A series of papers demonstrated that there are no consequences from privatization in a mixed oligopoly when the government uses a subsidy to ensure first best. Fjell and Heywood (2004) obtained the relevance result of privatization, and depicted if instead privatization results in a public leader becoming a private leader, the optimal subsidy, output and welfare are all reduced. Other correlated papers included that Tomaru (2006) utilized partial privatization setting, Kato and Tomaru (2007) incorporated various objective functions of private firms, and Hashimzade et al. (2007) considered product differentiation. Matsumura and Tomaru (2009) considered excess tax burden in mixed duopoly and examined the optimal subsidy policy. They demonstrated that privatization affects both the optimal subsidy rate and the resulting welfare. In their paper, they ignored the cost efficiency gap between public firm and private firm in their setting. Tomaru and Saito (2010) took demonstrate that a privatized firm should always be at least as efficient as a public firm. Furthermore, many empirical observations have shown that public firms in many developing countries are run less efficiently than private firms. See such relevant discussions and surveys, e.g. Vickers and Yarrow (1991), Megginson and Netter (2001), Gouret (2007), and Bai et al. (2009).

2 See Li (1997), Jefferson and Su (2006), and Bai et al. (2009) for careful survey and demonstrations.

3 Apparently, the issue of privatization and subsidy is being discussed widely, not only in economics, but also in law. See, e.g. Diamond (2008).
account into the endogenous timing in mixed duopoly with subsidization.

As we observed, in the real world, public firm often undertakes the policy burden from the government, such as retaining redundant workers or providing social goods as corporate social responsibility. Lin et al. (1998) took China for example to indicate that most firms in a transition economy carry many types of policy burdens, inherited from the pre-transition system. Lin and Tan (1999) showed that the policy burden causes the loss of public firm, which results in the phenomenon of soft budget constraint (SBC). Lin and Li (2008) proposed a new explanation for the pervasive syndromes of SBC, and stated that when a public firm suffers the policy burden, privatization deteriorates SBC problem.

In this paper, we do not focus on the SBC problem. What interests us is that in the presence of excess policy burden, how does the authority make the industrial policy-subsidization/taxation? As one of essential viewpoint provided in Meade (1944), the optimal policy could be determined by the outputs equality/inequality. Notice that in mixed oligopoly, cost-differential and different objective function faced by each firm leads to output inequality while all representative consumers have the identical utility and they are risk-neutral. Therefore, in the presence of excess policy burden, it is interesting to explore how the optimal policy is affected by the cost-differential and different objective function. Due to no exploration for information sharing issue, we assume cost for any one in this game is observed. As the framework shown in Laffont and Tirole (1986), the firm knows its efficiency before contracting. After contracting, it chooses an output, which together with an efficiency-differential among each firm. Accordingly, integrating the framework of Meade (1944), Laffont and Tirole (1986), White (1996) and Wang and Chen (2010), we utilize simple mixed duopoly model with cost efficiency gap to explore optimal subsidy/tax. We show that different degree of efficiency gain sharply changes the comparisons of optimal subsidy, total outputs and social welfare between mixed and private duopoly. What is more, for an imposition of an optimal subsidy, welfare may increase, decrease, or remain unchanged with privatization, which depends on the level of the cost efficiency gap and the taxation burden. However, it may be possible to raise welfare through privatization as long as the efficiency gain prevails or no excess taxation burden exists. Government sets higher subsidy to stimulate firms’ production if the value of cost-differential is assured.

The remainder of this paper is organized as follows. We outline the basic frameworks in section II; and then compute optimal results of mixed duopoly and private duopoly in section III. Section IV is the comparisons of equilibrium outcomes and concluding remarks is provided in section V.

II. Basic Frameworks

We consider that in a closed duopoly market one public firm (firm 0) competes with one private firms (firm 1) as well as all firms produce a homogeneous good. With the utility function, $U=a(q_0+q_1)-(q_0+q_1)^2/2+I$, where $I$ is the composite good, the inverse demand function could be expressed as $P=a-Q$, where $Q=q_0+q_1$; and then total consumer surplus is given by $CS=(q_0+q_1)^2/2$. Notice that the existence of the efficiency gap between public and private firm induces to asymmetric costs. Following Wang et al. (2009) and Wang and Chen (2010), we assume that the cost function of public firm and private firm is
\( C(q_0) = f + gq_0^2/2 \) and \( C(q_1) = f + q_1^2/2 \) respectively, while the fixed cost \( f \) is assumed to be zero for simplicity. Note that the assumption \( g \geq 1 \) means that there is an efficiency gap between the public firm and private firm, namely, private firm has more cost efficiency than public firm. When \( g = 1 \), the efficiency of public firm is conformed with private one. We assume that privatization leads an efficiency gain as reduction in production cost of the public firm in mixed duopoly. Further, it is worthwhile to mentioned why we use a linearly increasing marginal cost function for a non-positive \( C'' \). As Matsumura and Kanda (2005) indicated, \( C'' > 0 \) induces the U-shaped average cost curve when fixed cost is positive. This manipulation avoids a monopoly by the public firm while the public firm is as efficient as a private firm.

The government sets \( s \) (the unit subsidy rate) for two firms. Given the cost function and unit subsidy rate, firm \( i \)'s profit is then, respectively,

\[
\pi_0 = P(Q)q_0 - \frac{1}{2}gq_0^2 + sq_0, \\
\pi_1 = P(Q)q_1 - \frac{1}{2}q_1^2 + sq_1.
\]

The government finances the specific subsidies for the two firms by taxation with excess burden; meanwhile, in this paper we assume that only public firm whose objective function is welfare-maximizing bears policy burden. Accordingly, social welfare \( W \) as in Matsumura and Tomaru (2009) is given by,

\[
W(q_0, q_1, \lambda, s) = CS + \pi_i(q_i, q_0, s) - (1 + \lambda)[sQ - \pi_0(q_0, q_1, s)],
\]

where \( \lambda \in [0, 1] \) represents the unit excess burden. The subsidy payment for the two firms is \( sQ \), but the profits of the public firm would comprise a part of this payment. Thus the social cost for financing the subsidy payment is given by \( (1 + \lambda)sQ - \pi_0 \).

In mixed duopoly scenario, the public firm selects its output in order to maximize social welfare, while the private firm chooses its output in order to maximize its own profit in the second stage. Otherwise, in pure duopoly, both privatized and private firm decide optimal output for maximizing its own profit. And then in the first stage, the authority levies optimal subsidy rate for welfare maximum. Note that in this paper, we assume no agency problem existing in the mixed oligopoly. Furthermore, this two-stage game leads players to obtain payoffs once. Therefore, we could ignore discount rates for simplicity but without the loss of generality. The method of backward induction is utilized to solve equilibrium outcomes.

### III. Primary Outcomes

In this section, we compute optimal results of mixed duopoly in subsection III.1, and private duopoly in subsection III.2, respectively.

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4 Couple of empirical evidences point out that public firm is more inefficient than private one. See, e.g., La Porta and Lopez-de-Silane (1999), Aizenman (2000), Warzynski (2003), and Nishiyama and Smetters (2007). It is worthwhile to mention Vickers and Yarrow (1991) who stressed that through ownership rearrangement and the change of the competitive structure of the industry from the privatization process, efficiency gap could be improved.
1. Mixed Duopoly

First, we consider mixed duopoly scenario. After observing unit subsidy rate, $s$, public firm chooses its output to maximize social welfare; and private firm chooses its output to maximize its profit. We have the following equilibrium output given $s$:

$$q_0 = \frac{(2a - s)(1 + \lambda)}{2 + 5\lambda + 3g(1 + \lambda)}$$

$$q_1 = \frac{a(g + \lambda(1 + g) + s(1 + g + 2 + g)\lambda)}{2 + 5\lambda + 3g(1 + \lambda)}.$$  \hspace{1cm} (3)

An increase in $s$ directly raises $q_1$, but it reduces $q_0$ through strategic interaction. An increase in $\lambda$ directly increases $q_1$, but decreases $q_0$ through strategic interaction as well. Not surprisingly, as in White (1996), a private firm’s output is positively related to the subsidy, whereas a public firm’s output is negatively related to the subsidy. Through indirect effect, an increase in subsidy reduces a public firm’s output while it raises private firm’s output. Turning to the optimal $s$, we substitute (3) into (2) and then maximize welfare with respect to $s$ yielding,

$$s_{MD} = \frac{a(g + g^2 - (1 + g^2)\lambda - (1 + g)(6 + 5g)\lambda^2 - (9 + g(10 + 3g))\lambda^3)}{(1 + 3\lambda)(1 + g + (2 + g)\lambda)(1 + 3\lambda + 2g(1 + \lambda))}. \hspace{1cm} (4)$$

The superscript ‘MD’ denotes the sub-game perfect Nash equilibrium (SPNE) outcome in a mixed duopoly.

**Lemma 1.** Given $s_{MD}$, the sub-game perfect Nash equilibrium (SPNE) outcomes in a mixed duopoly are, respectively,

$$q_{0_{MD}} = \frac{a(1 + \lambda)((1 + 3\lambda)^2 + g(1 + \lambda)(1 + 5\lambda))}{(1 + 3\lambda)(1 + g + (2 + g)\lambda)(1 + 3\lambda + 2g(1 + \lambda))}, \hspace{1cm} q_{1_{MD}} = \frac{ag(1 + \lambda)^2}{(1 + 3\lambda)(1 + 3\lambda + 2g(1 + \lambda))}. \hspace{1cm}$$
\[
\pi_{0}^{MD} = \frac{a^{2}g(1+\lambda)^{2}(3+(6-\lambda)\lambda+k(3-\lambda)(1+\lambda)((1+3\lambda)^{2}+g(1+\lambda)(1+5\lambda))}{2(1+3\lambda)^{2}(1+g+(2+g)\lambda)^{2}(1+3\lambda+2g(1+\lambda))^{2}},
\]

\[
\pi_{1}^{MD} = \frac{3a^{2}g^{2}(1+\lambda)^{4}}{2(1+3\lambda)^{2}(1+3\lambda+2g(1+\lambda))^{2}},
\]

\[
W^{MD} = \frac{a^{2}(1+\lambda)^{2}(1+g+(3+g)\lambda)^{2}}{2(1+3\lambda)(1+g+(2+g)\lambda)(1+3\lambda+2g(1+\lambda))^{2}}.
\]

We compare the output level of the public and private firms, and obtain \(q_{0}^{MD} - q_{1}^{MD} \geq 0\) if \(1 \leq g \leq g^{*} = \frac{3\lambda + \sqrt{4+24\lambda+45\lambda^{2}}}{2(1+\lambda)}\). Explicitly, \(g^{*}\) is monotonically increasing with \(\lambda\) for output comparison between public and private firm as indicated in Figure 1, we have the following lemma.

**Lemma 2.** \(q_{0}^{MD} - q_{1}^{MD} \geq 0\) if \(1 \leq g \leq g^{*}\).

The reason is that the objective of the public firm is welfare-enhancing. An increasing in output diminishes price, while consumer surplus raises. Note that \(\lambda = 0, g^{*} = 1\). When \(\lambda = 0\), the first-best outcome is achieved when \(P = C_{0} = C_{1}\). This implies that \(q_{0} = q_{1}\) in the case of the first-best outcome. The government chooses \(s\) to induce \(P = C_{0} = C_{1}\). When \(\lambda > 0\), this outcome is not optimal. The larger (smaller) output of firm 0 (firm 1) economizes the shadow cost of public funding and \(s\) is set to induce such an outcome.

2. **Private Duopoly**

Next, we consider the private duopoly. Firm 0 is privatized, and the efficiency of privatized firm is conformed with private one\(^5\), namely \(g = 1\). Both firms choose their outputs independently to maximize their profits, we then obtain,

\[
q_{0} = q_{1} = \frac{a+s}{4}.
\]

After privatizing, the welfare is given by,

\[
W(q_{0}, q_{1}, \lambda, s) = CS + [\pi_{0}(q_{0}, q_{1}, s) - V] + \pi_{1}(q_{1}, q_{0}, s) - (1+\lambda)[sQ - V],
\]

where \(V\) is the revenue from selling the stocks of public firm 0. While the public firm is privatized, an owner buys the public firm at price \(V\). Since \(\pi_{0}\) does not include the payment for government, \(V\), the payoff of the owner is apparently identical to the profit from the operation of the privatized firm minus the payment for government, i.e. \(\pi_{0} - V\).

Substituting (5) into (6) and maximizing it with respect to \(s\) yields,

\(^5\) This assumption is usually employed in mixed oligopoly literature to avoid a trivial solution. See also Bárcena-Ruiz and Garzón (2005). Some empirical evidences, nevertheless, indicate the superior efficiency of private firms relative to comparable publicly-owned firms (Mueller, 1989; Vining and Boardman, 1992), and the improvement in efficiency after privatization (Kikeri et al. 1992; Megginson et al. 1994).
The superscript \textquote{PD} denotes the sub-game perfect Nash equilibrium (SPNE) outcome in a private duopoly.

**Lemma 3.** Given $s^{PD}$, the sub-game perfect Nash equilibrium (SPNE) outcomes in a private duopoly are, respectively,

$$q_0^{PD} = q_1^{PD} = \frac{a(1 + \lambda)}{3 + 8\lambda}, \quad \pi_0^{PD} = \pi_1^{PD} = \frac{3a^2(1 + \lambda)^2}{2(3 + 8\lambda)},$$

$$W^{PD} = \frac{a^2(1 + \lambda)^2}{3 + 8\lambda} + \lambda V.$$  

$V = (1 - \theta)\pi_0^{PD}$, where $\theta \in [0, 1]$ is the revenue loss during the privatization process such as bureaucratic corruption and managerial inefficiency.\textsuperscript{5} For simplicity without the loss of generalization, we assume that the financial market has no agency problem, that is, $\theta = 0$. Substituting $V = \pi_0^{PD}$ and results in lemma 2 into (6) yields the following welfare,

$$W^{PD} = \frac{a^2(1 + \lambda)^2(6 + 19\lambda)}{2(3 + 8\lambda)^2} \quad (8)$$

**IV. Comparisons of Equilibrium Outcomes**

Comparing the optimal rates obtained in mixed and private duopoly cases, we obtain the relevance result, that is, $s^{MD} \neq s^{PD}$. Suppose $\lambda = 0$ without the unit excess burden, $s^{MD} - s^{PD} = \frac{a(g - 1)}{3 + 6g}$. As we can see that given the cost inefficiency, the subsidy rate in mixed duopoly is larger than in private duopoly, which casts doubt on the proposition 4 obtained by White (1996) who did not consider the specification of cost inefficiency, he stated that when optimal subsidies are used before and after privatization of a public firm in a mixed oligopoly, the optimal subsidy rate cannot be changed. Suppose that $\lambda > 0$, the difference of two subsidies rates is as follows,

$$s^{MD} - s^{PD} = \frac{a(1 + \lambda)(g^2(1 + \lambda)^2 - (1 + 3\lambda)^2 + g\lambda(1 + \lambda)(1 + 4\lambda))}{(1 + 3\lambda)(3 + 8\lambda)(1 + g + (2 + g)\lambda)(1 + 3\lambda + 2g(1 + \lambda))}. \quad (9)$$

Owing to equivocal result of Eq. (9), we find a critical value of $g$ that yields $s^{MD} = s^{PD}$,

$$g^* = \frac{-\lambda(1 + 4\lambda) + \sqrt{4 + \lambda(24 + 8\lambda(37 + 8\lambda(1 + 2\lambda)))}}{2(1 + \lambda)}. \quad (10)$$

\textsuperscript{5} Bjorvatn and Søreide (2005) analyzed the relation between corruption and privatization. See also, Clarke and Xu (2002) for privatization and corruption issue. On the other hand, Lee and Hwang (2003) allowed managerial inefficiency in a mixed duopoly in the presence of partial privatized firm. See also Heywood and Ye (2009) for agency problem.
Note that $\lambda = 0$, $g^{**} = 1$. $g^{**}$ is a non-monotonic function with respect to $\lambda$.
In Region 1 (2) of Figure 2, $s^{MD} - s^{PD} > (\leq) 0$. Hence, given $g$ and $\lambda$, both $s^{MD} - s^{PD} > 0$ and $s^{MD} - s^{PD} < 0$ is possible. In other words, both irrelevance result and relevance result exist depending on $g$ and $\lambda$. In private duopoly, an increasing in $s$ stimulates production by both firms; thereby government has a strong incentive for increasing $s$, and hence $s^{PD} > s^{MD}$ even large $\lambda$ exists. However, in mixed duopoly, a higher $g$ reduces the total outputs in mixed duopoly, and government will need to provide a higher subsidy rate in order to stimulate production and increase consumer surplus. Accordingly, when a public firm is privatized, government has to consider how to change the subsidy rate. When $g$ is small (large), the government should increase (decrease) the subsidy rate after privatization.\footnote{As the referee pointed out that since privatization is related to governmental reform, it might be difficult for government to increase (decrease) the subsidy rate. In Tomaru and Wang (2010), we addressed such issue concerned by the referee.}

**Proposition 1.** When cost efficiency gap is small (large), the government should increase (decrease) the subsidy rate after privatization.

Even though we now know the relation of $s^{PD}$ and $s^{MD}$, but the subsidy rate itself is still not the sufficient condition to explain the welfare effect. To explore the welfare effect, we first need to delineate the effect of subsidization policy on the output, which is crucial to the change of consumer surplus. From lemmas 1, 2, 3, and mixed oligopoly literature, we learn that the impact of the policy parameter on firms’ reaction function is different in mixed oligopoly from pure oligopoly. Wang and Chen (2010) showed that the cost efficiency gap, $g$, play a crucial role on determining optimal outcomes in mixed oligopoly. Accordingly, we start analyzing the total output in the presence of $g$ and then show the change on social welfare.

We further compare $\pi^{MD}_1$ with $\pi^{PD}_1$ and find that $\pi^{MD}_1 - \pi^{PD}_1 < 0$ if $1 \leq g \leq g^{***} = \frac{1 + 6\lambda + 9\lambda^2}{1 + 3\lambda + 2\lambda^2}$. 

FIG. 2. **Comparison of the Optimal Subsidy Rates**
is monotonically increasing with $\lambda$. Comparing total outputs, we have,

$$Q^{\text{up}} - Q^{\text{dp}} = \frac{-a(1+\lambda)((g^2(1+\lambda)^2-(1+3\lambda)^2)(1+4\lambda)-g\lambda(1+\lambda)(5+14\lambda))}{(1+3\lambda)(3+8\lambda)(1+g+(2+g)\lambda)(1+3\lambda+2g(1+\lambda))}. \quad (11)$$

We find that there is a critical level of $g$ that determines the change of total outputs,

$$g^{***} = \frac{5\lambda + 14\lambda^2 + \sqrt{4 + \lambda(56 + \lambda(317 + 4\lambda(203 + 193\lambda)))}}{2 + 10\lambda + 8\lambda^2}. \quad (12)$$

Should the government implement subsidization to stimulate total production in mixed and pure duopoly? We would like to use the reaction functions of the Cournot model in Figure 3 for exploring the effect of subsidy in mixed market setting. It can be checked from the first order condition: in the event of post-privatization, all firms’ reaction function is linear with a slope of $-1/3$ and an imposition of the specific subsidy shifts two reaction functions to the right, which increases total outputs, i.e. $Q^{\text{dp}}$. It is well recognized that the subsidy induces the output-effect, and reduces the under-production effect in the pure Cournot oligopoly. However, the under-production effect is weak in the mixed oligopoly vis-à-vis pure oligopoly, and the subsidy may not produce the output-effect.

In the event of pre-privatization, the public firm’s reaction function is linear with the slope of $-(1+\lambda)/(1+g)$. If $g > \lambda$, the slope of public firm’s reaction function should be flatter than the case of the pure duopoly. On the other hand, if $g < \lambda$, the slope of public firm’s reaction function is steeper. Checking from the first order condition, we see that the output subsidy shifts the private firm’s reaction function to the right, but shifts the public firm’s reaction
function to the left, which is shown in Figure 3. Bold real lines are the reaction functions of the mixed duopoly with \( g = 1 \) as well as \( \lambda = 0 \). Bold dotted lines indicate the impact of the subsidy on the reaction functions. Black circles are optimal outputs. In the case of \( g > \lambda \), after subsidy, the amount of the reduction of \( q_0 \) is less than the case of the bold lines, which is shown as black triangularities. Consequently, the total outputs increase. Analogously, in the case of \( g < \lambda \), after subsidy, the amount of the reduction of \( q_0 \) is larger than the case of the bold lines, which is shown as black stars. Thus, the total outputs declines.

Integrating the analysis provided above, we have the following welfare expression,

\[
W^{MD} - W^{PD} = \frac{1}{2} \sigma^2 (1 + \lambda)^{2} \left[ \frac{-6 - 19\lambda}{(3 + 8\lambda)} + \frac{(1 + g + (3 + g)\lambda)^2}{(1 + 3\lambda)(1 + g + (2 + g)\lambda)(1 + 3\lambda + 2g(1 + \lambda))} \right].
\] (13)

Ensuring the relationship between \( g \) and \( \lambda \) is critical for the welfare comparison, we set \( \Delta W = W^{MD} - W^{PD} \). \( \Delta W < 0 \) if \( g > \bar{g} = f(\lambda) \); and \( \Delta W \geq 0 \) if \( g \leq \bar{g} = f(\lambda) \), where

\[
\bar{g}(\lambda) = \frac{(1 + 3\lambda)\{\lambda(3 + 5\lambda) - \sqrt{(6 + 19\lambda)\{6 + \lambda[67 + \lambda(236 + 275\lambda)]}\} \}}{2(1 + \lambda)(3 + 26\lambda + 50\lambda^2)}.
\]

Differentiating \( \bar{g} \) with respect to \( \lambda \), we obtain that

\[
\frac{d\bar{g}}{d\lambda} = \frac{\Gamma_1 - \Gamma_2 \sqrt{(6 + 19\lambda)\{6 + \lambda[67 + \lambda(236 + 275\lambda)]\}}}{\Gamma_3 \sqrt{(6 + 19\lambda)\{6 + \lambda[67 + \lambda(236 + 275\lambda)]\}}},
\] (14)

where \( \Gamma_1 = 54 + 2079\lambda + 26019\lambda^2 + 153763\lambda^3 + 474995\lambda^4 + 745026\lambda^5 + 470000\lambda^6 \), and \( \Gamma_2 = 9 + \lambda\{84 + \lambda\{313 + 10\lambda(57 + 44\lambda)\}\}, \quad \Gamma_3 = 2(1 + \lambda)^2(3 + 26\lambda + 50\lambda^2)^2 \).

From equation (13), we see that the higher the cost efficiency gap is, the more welfare loss in the mixed oligopoly; accordingly, the difference between two welfare measurements should be augmented. But the impact of taxation burden alters the difference between two welfare measurements. Due to the possible inverse impact from taxation burden, the output effect from efficiency gain may be offset. Given that \( \lambda > 0 \), the government adopts higher subsidy rate to increase consumer surplus, but the cost efficiency gap in production and higher subsidy rate lead to more loss of welfare than the increase of consumer surplus. However, the welfare is unchanged when the public firm and the private firm have the same efficiency. Given that \( g \) is fixed, \( W^{MD} < W^{PD} \) when no taxation burden is imposed, i.e. \( \lambda = 0 \); \( W^{MD} > W^{PD} \) when full taxation burden exists, i.e. \( \lambda = 1 \). We have the following corollary:

**Corollary 1.** For an imposition of an optimal subsidy rate, welfare may increase, decrease, or remain unchanged with privatization, which depends on the level of the cost efficiency gap and the taxation burden. However, it may be possible to raise welfare through privatization as long as the efficiency gain prevails or no excess taxation burden exists.

This corollary casts doubts on White (1996) who showed that the same subsidy rate yields the first-best outcome in both mixed and private oligopoly in his Cournot setting (irrelevance result). Studies on optimal subsidy in mixed oligopoly have gained attentions. A series of

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8 Because the policy implication of Eq. 14 is not significant, the figure is not displayed. However, from the numerical analysis, \( \frac{d\bar{g}}{d\lambda} > 0 \).
papers demonstrated that there are no consequences from privatization in a mixed oligopoly when the government uses a subsidy to ensure first best. Matsumura and Tomaru (2009) considered excess tax burden in mixed duopoly and examine the optimal subsidy policy. They demonstrated that privatization affects both the optimal subsidy rate and the resulting welfare. However, their study ignored the existence of a cost efficiency gap between public firm and private firm, and conjectured when incorporation of such effect, the result that privatization reduces welfare does not hold. Utilizing quadratic cost function, cost asymmetry comes from different production scale, but cost efficiency gain that may exist after privatization. We demonstrated that $W^{MD} < W^{PD}$ is still valid without policy burden, when $g > 1$ and $\lambda = 0$. In the case of $g = 1$ and $\lambda = 0$, we see that White's conclusion still holds. Moreover, this proposition is also different from Proposition 1 of Matsumura and Tomaru (2009). They showed that the irrelevance result with welfare does not hold and welfare is always reduced; that is, $W^{MD} > W^{PD}$ when $\lambda > 0$.

V. Concluding Remarks

In this paper, we assumed that privatization leads an efficiency gain as reduction in production cost of the public firm in mixed duopoly. In the presence of the excess taxation burden, we found that for an imposition of an optimal subsidy, welfare may increase, decrease, or remain unchanged with privatization, which depends on the level of the cost efficiency gap and the taxation burden. However, it may be possible to raise welfare through privatization as long as the efficiency gain prevails or no excess taxation burden exists. Government sets higher subsidy to stimulate firms’ production if the value of cost-diﬀerential is assured.

The brief review of past literature stated in the introduction pointed out that various factors result in the relevance result, including the order of move (Fjell and Heywood, 2004; Kato and Tomaru, 2007; Tomaru and Saito, 2010), partial privatized setting (Tomaru, 2006), and differentiated oligopoly (Hashimzade et al. 2007). What we argued here is that the consideration of efficiency gap and taxation burden may change the validity of their results.

Acknowledging a negative sign of subsidy represents a kind of taxation, it is worth to see that Mujumdar and Pal (1998) is highly related to White (1996). They explored the effect of indirect taxation in mixed oligopoly, but no cost-efficiency gap and taxation burden are considered. Interestingly, they had some similar results as ours, but not the same. They examined that first, total output is unaffected by the imposition of or change in either tax. However, in our paper, either cost-efficiency gap or taxation burden has influence on the firm’s reaction function. Thereby, the total output might be altered in taxation policy. Second, with an increase in tax, the less efficient (public) firm gains market share over the more efficient (private) firm. In our case, the implement of subsidy policy reduces inefficient (public) firm’s market share but raise efficient (private) firm’s. Essentially, they stated that privatization can increase both welfare and tax revenue. The result we obtained is not coincided with theirs. With the specific condition, privatization is beneficial for welfare: as long as the efficiency gain prevails or no excess taxation burden exists, it is possible to raise welfare through privatization.
REFERENCES


