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PRICING OF COMPLEMENTARY GOODS AS AN IMPLICIT FINANCIAL ARRANGEMENT*

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Abstract

This paper studies the common pricing practice of firms selling a durable good at a low price and a complementary consumable good at a high price. In our model, consumers discount future payments while firms receive a steady-state flow of revenues from selling the durable and consumable goods. As a result, there are potential gains from deferring consumers' payments to the future. We show that when firms commit to constant prices and consumer lock-in is possible, firms choose pricing consistent with the practice in monopoly and competition. Our result provides a new efficiency argument in the aftermarket literature.

Keywords: aftermarkets, complementary goods, consumer lock-in, durable goods, implicit financial arrangements
JEL Classification Codes: L10, L15, D40

I. *Introduction*

It is a widely observed pricing policy that firms selling complementary goods, one durable and the other consumable, offer the durable good at a low price (often below marginal cost) while profiting from the sale of the consumable good. Examples of complementary goods to

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which this policy is often applied are razors and blades, printers and cartridges, and cellular phones and monthly service plans. For this pricing policy to be sustainable, firms need to have some market power in the market for the consumable good so that the sale of the durable good creates sufficient demand for the consumable good they produce. One way for firms to gain market power is to use consumer lock-in. For example, razors (printers) are usually designed to work only with blades (cartridges) produced by the same firm, and wireless service providers offer cellular phones at highly discounted prices if users make a long-term (usually two-year in the United States) commitment to subscribe to service plans from the providers.

In this paper, we study why the practice of offering a durable good below marginal cost and locking in consumers of a consumable good exists, and what welfare implications it has on consumers and firm owners. We explain the practice using different perspectives of consumers and firms. When a consumer decides whether to purchase the durable good, he discounts the expenditure on the consumable good because the purchases of the consumable good are made in the future. On the contrary, a firm faces many consumers purchasing the durable good and the consumable good in each period, and thus in its steady-state profit, it does not discount the revenue from selling the consumable good. Due to this difference in the individual and aggregate perspectives of a consumer and a firm, respectively, there are potential gains from deferring consumers' payments to the future. If a firm can lock in its consumers, these gains can be realized, leading to firms lowering the price of the durable good below marginal cost and charging a high price on the consumable good. When a firm has monopoly power in the market for the durable good, it can extract some of realized gains. When firms operate in a competitive durable good market, all the gains accrue to consumers.

In our theory, the pricing practice can be interpreted as involving an implicit financial arrangement where a firm offers a loan to a consumer when the consumer purchases the durable good and receives repayments when the consumer purchases the consumable good.[1] Because consumers and firms have different relative weights on loans and repayments in our model, a firm can finance the loan it provides in a period with the repayments it receives in the same period while offering terms beneficial to consumers.[2] When firms cannot require consumers of the durable good to purchase the consumable good from the same firm, they cannot guarantee that the loan is repaid in the implicit financial arrangement, and thus the arrangement is not viable. Therefore, in our theory, consumers benefit from being locked in because lock-in allows consumers to obtain beneficial delay of payments to the future.

A similar insight to ours can be found in Kaserman (2007), who shows that the efficient contract between a buyer and a seller can require a tie-in arrangement when the buyer discounts future payoffs more than the seller does. In our model, the different relative weights on the prices of the two goods between consumers and firms are not due to different discount factors but because of the individual and aggregate perspectives of consumers and firms, respectively. Also, we analyze the two cases of monopoly and competition whereas Kaserman (2007)

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[1] By using such an implicit financial arrangement, firms can save transaction costs associated with writing an explicit financial contract. Thus, our theory will be more relevant in a situation where these transaction costs are high relative to gains. See Section VI for related discussions.

[2] In our model, the gains from the implicit financial arrangement stem from the different perspectives of consumers and firms. As the arrangement offers credit to consumers who purchase the durable good, consumers' credit constraints will be another reason for the gains especially when the purchase of the durable good is big compared to that of the consumable good (as in the printer and cellular phone examples).

focuses on the case where bargaining power is on the part of the buyer (i.e., competition). An analogy can be made with insurance. If there are two individuals with different degrees of risk aversion, they can reach a mutually beneficial insurance agreement in which the less risk averse individual takes some risk of the more risk averse individual. On the other hand, if an individual deals with many other individuals and thus can pool their risks, he can offer an insurance plan that is beneficial to risk averse individuals, regardless of his degree of risk aversion. The former scenario corresponds to Kaserman's (2007) explanation using different discount factors, while the latter is analogous to our explanation relying on the difference in the individual and aggregate perspectives.

In our discussion, consumer lock-in refers to a setting in which a consumer who has purchased the durable good needs to purchase the consumable good from the same firm in order to use the durable good. Thus, consumer lock-in can be regarded as a form of tie-in sales or tying arrangements. In early literature, different theories have provided alternative explanations for tie-in sales. A traditional theory views tie-in sales as a price discrimination device, using IBM tabulating machines and punch cards as a main example [see, for example, Burstein (1960) and Telser (1979)].[3] If the price of the consumable good is above marginal cost, the firm obtains a higher return from a consumer who uses the durable good more intensively. Thus, tie-in allows a monopolist to extract surplus from high-demand consumers. An alternative theory explains the practice using risk reduction [see Liebowitz (1983)]. If consumers are uncertain about the intensity with which they consume the consumable good per unit of the durable good, a lower price of the durable good can reduce the risk to consumers from purchasing the durable good. These arguments based on price discrimination and risk reduction do not play a role in our model because we assume that consumers having the durable good demand the consumable good up to one unit in each period, which makes the intensity of use the same among all consumers who use the durable good.

This paper deals with a strategy of selling durable goods, and there is a rich body of literature that studies various issues arising from selling durable goods. Here we review some of important contributions in durable goods theory, while a more extensive survey can be found in Waldman (2003). A first strand of literature began with Swan (1970) and studies the issue of optimal durability. Waldman (1996) develops the idea of planned obsolescence and shows that due to time inconsistency a durable good monopolist overinvests in R&D, which makes used units obsolete. Hendel and Lizzeri (1999a) show that a durable good monopolist underinvests in durability for a similar motivation to reduce the quality of used units and to increase the price of new units. A second strand of literature was inspired by Coase (1972), who argued that a durable good monopolist's price will fall immediately to marginal cost when it cannot commit to future prices or quantities. Subsequent works studied whether Coase's conjecture was correct [e.g., Gul, Sonnenschein, and Wilson (1986)] and tactics to prevent the price from falling to cost [e.g., Butz (1990)]. A third strand of literature followed the lead of Akerlof (1970), who studied asymmetric information and adverse selection in the used car market. Hendel and Lizzeri (1999b) extend Akerlof's (1970) analysis by introducing new durable goods and find a similar conclusion that adverse selection can result in too little trade. In this paper, we abstract away from these three issues by making simplifying assumptions. In particular, we assume that

[3] Telser (1979) also discusses loss leaders, the sale of a commodity below marginal cost, as an application of his analysis. The durable good in our model can be considered as a loss leader.

firms take durability as given and do not introduce new products, that firms can commit to constant prices, and that consumers know their valuations of the durable good and have no access to a secondhand market.

Among topics in durable goods theory, this paper is closely related to aftermarket monopolization or tying. In our model, the market for the complimentary consumable good can be considered as an aftermarket, and a firm uses consumer lock-in to monopolize its aftermarket. The literature on aftermarket monopolization has grown largely out of debates on the *Kodak* case,[4] and there are many theories to explain aftermarket monopolization. We follow classification of these theories by Waldman (2003). A first class of theories is based on consumer lock-in. In these theories, a firm gains market power in the aftermarket from locked-in consumers and exercises this market power to exploit these consumers.[5] Borenstein, MacKie-Mason, and Netz (2000) present a theory in this class, which uses a firm's lack of commitment to explain supracompetitive pricing in the aftermarket. A second class of theories explains aftermarket monopolization based on firms' market power in the foremarket (i.e., the market for durable goods). For example, Chen and Ross (1993) use a price discrimination argument similar to the one in the early literature, Hendel and Lizzeri (1999a) point out an incentive for a durable good monopolist to monopolize the maintenance market in order to prevent consumers from raising durability, and Morita and Waldman (2004) show that a durable good monopolist can avoid problems due to time inconsistency by monopolizing the maintenance market. A third class of theories provides efficiency rationales for aftermarket monopolization. Unlike the previous two classes of theories in which firms use aftermarket monopolization to extract consumer surplus and thereby create deadweight loss, theories in this class argue that the practice increases social and consumer welfare. Elzinga and Mills (2001), Carlton and Waldman (2010), and Morita and Waldman (2010) provide scenarios relying on fixed R&D costs and switching costs in which maintenance-market monopolization enhances efficiency. Because of the aforementioned simplifying assumptions in our model, the issues raised in the existing theories are not relevant in this paper. The main contribution of this paper to the literature on aftermarket monopolization is to provide a new efficiency argument based on "consumer credit."

The rest of the paper is organized as follows. Section II describes the model. Section III analyzes equilibrium pricing in the cases of monopoly and competition when firms can lock in their consumers, while Section IV analyzes equilibrium pricing when consumer lock-in is impossible. Section V investigates the welfare implications of lock-in and competition. Section VI provides discussions on our results, and Section VII concludes.

II. *The Model*

There are two complementary goods, a durable good (good A) and a consumable good

[4] *Eastman Kodak Co. v. Image Technical Services, Inc. et al.*, 112 S. Ct. 2072 (1992). In that case, independent service providers sued *Kodak* for refusing to sell spare parts to them thereby monopolizing the maintenance market for its copiers and micrographic equipment.

[5] A related argument is the reputation argument of Shapiro (1995), who points out that reputational considerations would limit firms' opportunistic behavior to exploit locked-in consumers. A similar argument can be made to justify our assumption that firms can commit to constant prices.

(good B). Time is discrete, and the horizon is infinite. We assume that the life span of the durable good is random while that of the consumable good is deterministic. Specifically, the durable good becomes no longer usable in the next period with probability $\alpha \in (0, 1)$, regardless of how long it has been used. Then the probability that the durable good lasts for exactly k periods is $(1-\alpha)^{k-1}\alpha$, for $k=1, 2, ...$, and the expected life span of the durable good is $1/\alpha$. The life span of the consumable good is assumed to be one period.[6]

There is a continuum of potential consumers of mass 1, indexed by i. In each period, one unit of the durable good can be used only with one unit of the consumable good (i.e., fixed intensity of use). The per-period benefit of using one unit of the durable good to consumer i is denoted by v_i, and for analytic simplicity, we assume that it is uniformly distributed on $[\underline{v}, \bar{v}]$, where $0 \leq \underline{v} < \bar{v}$. The per-period benefit to a consumer is saturated at one unit of the durable good, and thus the marginal benefit of the durable good beyond one unit is zero. There are n identical firms, indexed by j, that can produce both the durable good and the consumable good. The durable good produced by a firm can be used only with the consumable good produced by the same firm. In other words, the two goods are incompatible if they are produced by different firms. We call this compatibility restriction *lock-in*.[7] We assume that each firm j determines constant prices p_{jA} and p_{jB}, where p_{jA} and p_{jB} are the prices of the durable good and the consumable good, respectively, produced by firm j. Consumers are infinitely lived and discount future payoffs with a common discount factor $\delta \in [0, 1)$. In each period, consumers make decisions about purchasing the two goods to maximize their total discounted payoffs, taking the prices as given. We restrict the prices to be nonnegative, i.e., $p_{jA}, p_{jB} \geq 0$ for all $j=1, ..., n$. This restriction can be interpreted to be stemmed from monitoring imperfection. When firms cannot monitor the usage of the durable good or the consumption of the consumable good by consumers, consumers can purchase a good of a negative price just to make money rather than to actually use or consume it. Thus, nonnegative prices prevent firms from being exploited as a money pump.

Since prices are constant and the benefit of usage is saturated at one unit of the durable good, we can focus on the binary decision of a consumer whether to use one unit of the durable good (together with one unit of the consumable good replaced each period) or none. If consumer i purchases one unit of the durable good, the total discounted usage benefit to consumer i from that unit is given by

$$v_i + \beta v_i + \beta^2 v_i + \cdots = \frac{v_i}{1-\beta},$$

where $\beta = (1-\alpha)\delta$. The total discounted usage cost of one unit of the durable good produced by firm j is given by

$$p_{jA} + p_{jB} + \beta p_{jB} + \beta^2 p_{jB} + \cdots = p_{jA} + \frac{p_{jB}}{1-\beta}.$$

[6] Our results can be extended easily to any finite fixed life span of the consumable good, with an additional assumption that the durable good purchased in a period cannot be used with the consumable good purchased in an earlier period.

[7] We will study the case where lock-in cannot be imposed in Section IV and analyze its welfare implications in Section V.

That is, $p_{jA}+p_{jB}/(1-\beta)$ is the expected discounted payment that a consumer makes to firm j for the life span of the durable good when he purchases a unit of the durable good from firm j. Hence, consumer i receives a nonnegative net benefit of using the durable good produced by firm j if and only if $v_i \geq (1-\beta)p_{jA}+p_{jB}$. When consumer i has a unit of the durable good produced by firm j, he will purchase the consumable good if $v_i \geq p_{jB}$, assuming that the probability that the durable good becomes no longer usable in the next period is the same regardless of whether it is actually used or not in the current period. Since $p_{jA} \geq 0$, $v_i \geq (1-\beta)p_{jA}+p_{jB}$ implies $v_i \geq p_{jB}$.[8] Hence, any consumer i who chose to purchase the durable good from firm j is willing to replace the consumable good in every period while the durable good lasts. As a result, a constant fraction of consumers use the durable good, facing constant prices.

We define the *usage price*, normalized as per-period average, at the prices of the two goods (p_A, p_B) as

$$P(p_A, p_B) = (1-\beta)p_A + p_B.$$

Note that $P(p_A, p_B)$ is equal to the valuation of a consumer who is indifferent between using and not using the durable good at the prices (p_A, p_B). Because the benefit to a consumer is independent of firms producing the goods, only the firms that offer the lowest usage price can obtain a positive demand. Let $\tilde{P}$ be the lowest usage price, i.e., $\tilde{P}(\mathbf{p}_A, \mathbf{p}_B) = \min_j \{(1-\beta)p_{jA} + p_{jB}\}$, where $(\mathbf{p}_A, \mathbf{p}_B) = (p_{1A}, p_{1B}, \ldots, p_{nA}, p_{nB})$. Then consumer i uses the durable good (produced by a firm that offers the lowest usage price) if and only if $v_i \geq \tilde{P}$, and the fraction of consumers using the durable good is given by

$$q(\tilde{P}) = \left[\frac{\bar{v}-\tilde{P}}{\bar{v}-\underline{v}}\right]_0^1,$$

where $[x]_y^z = \min\{\max\{x, y\}, z\}$. The surplus of consumer i facing the lowest usage price $\tilde{P}$, measured in per-period average terms, can be expressed as

$$CS_i(\tilde{P}) = \max\{v_i - \tilde{P}, 0\}.$$

That is, the surplus of a consumer measures the per-period average net benefit of the consumer. The (total) consumer surplus can be computed as

$$CS(\tilde{P}) = \frac{1}{\bar{v}-\underline{v}} \int_{\underline{v}}^{\bar{v}} CS_i(\tilde{P}) dv_i = \begin{cases} 0, & \text{if } \tilde{P} > \bar{v}, \\ \dfrac{(\bar{v}-\tilde{P})^2}{2(\bar{v}-\underline{v})}, & \text{if } \underline{v} \leq \tilde{P} \leq \bar{v}, \\ \dfrac{\bar{v}+\underline{v}}{2} - \tilde{P}, & \text{if } \tilde{P} < \underline{v}. \end{cases} \tag{1}$$

Firms produce the durable good and the consumable good at constant marginal costs of $c_A > 0$ and $c_B > 0$, respectively. There is no fixed cost of production. Firms choose constant

[8] If the durable good always remains usable in the next period when it is not used in the current period, consumer i purchases the consumable good if $v_i \geq \alpha\delta p_{jA} + p_{jB}$, which is implied by $v_i \geq (1-\beta)p_{jA} + p_{jB}$.

prices to maximize their per-period profits in the *steady state*, where they face a constant stream of demand for each good.[9] In the steady state, a user purchases one unit of the consumable good in each period, while he purchases one unit of the durable good when his durable good fails, which occurs with probability α in each period. Hence, the per-user margin in the steady state for a firm that charges (p_A, p_B) is given by

$$m(p_A, p_B) = \alpha(p_A - c_A) + (p_B - c_B).$$

The steady-state profit of a firm is the product of its per-user margin and the fraction of consumers who use the durable good produced by the firm. We say that a firm is *active* if it supplies a positive quantity of the durable good.

III. *Pricing with Lock-In*

In this section, we characterize pricing equilibria[10] depending on the number of firms. Before analyzing pricing equilibria, we state an observation on the functions P and m, which is simple but critical for our results.

Lemma 1.

$$\frac{\partial P(p_A, p_B)/\partial p_A}{\partial P(p_A, p_B)/\partial p_B} > \frac{\partial m(p_A, p_B)/\partial p_A}{\partial m(p_A, p_B)/\partial p_B} \quad \textit{for all } (p_A, p_B).$$

All the proofs are relegated to the Appendix. Lemma 1 shows that due to the different perspectives of consumers and firms, consumers put a relatively lower weight on the price of the consumable good than firms do. When a consumer purchases the durable good, he pays for the durable good in the current period while paying for the consumable good in the future periods as well as in the current period. Hence, consumers discount the future payments for the consumable good and do not discount the current payment for the durable good when making a decision of purchasing the durable good. In contrast, firms dealing with many consumers face a constant stream of aggregate demands for the durable good and the consumable good in the steady state. Thus, when computing the per-user margin, firms do not differentiate between revenues from selling the durable good and the consumable good in terms of time. As a result, the usage price reacts to the price of the durable good more sensitively relative to that of the consumable good than the per-user margin does.

1. Monopoly

Suppose that there is only one firm, i.e., $n=1$. Let (p_A, p_B) be the prices of the two goods set by the monopolist. Then the steady-state profit of the monopolist is given by $\pi(p_A, p_B) = q(P(p_A, p_B))m(p_A, p_B)$, and the monopolist's problem can be expressed as

[9] Because a firm receives the same level of profit in each period of the steady state, the discount factor of a firm is immaterial in our analysis.

[10] Formally, a pricing equilibrium is a pure-strategy Nash equilibrium of a strategic-form game where players are firms, a strategy for a firm is its price choice in $\mathbb{R}^2_+$, and the payoff of a firm is its steady-state profit.

$$\max_{p_A, p_B} \pi(p_A, p_B) \text{ subject to } p_A, p_B \geq 0. \tag{2}$$

We use Π^M to denote the maximum profit of the monopolist. For the sake of analysis, we make the following assumptions.

Assumption 1. $\bar{v} > \alpha c_A + c_B$.

Assumption 2. $2\underline{v} < \bar{v} + \alpha c_A + c_B$.

The following proposition provides monopoly pricing under the two assumptions.

Proposition 1. *Under Assumptions 1 and 2, the solution to the monopolist's problem* (2) *is given by*

$$p_A = p_A^M := 0 \text{ and } p_B = p_B^M := \frac{\bar{v} + \alpha c_A + c_B}{2}. \tag{3}$$

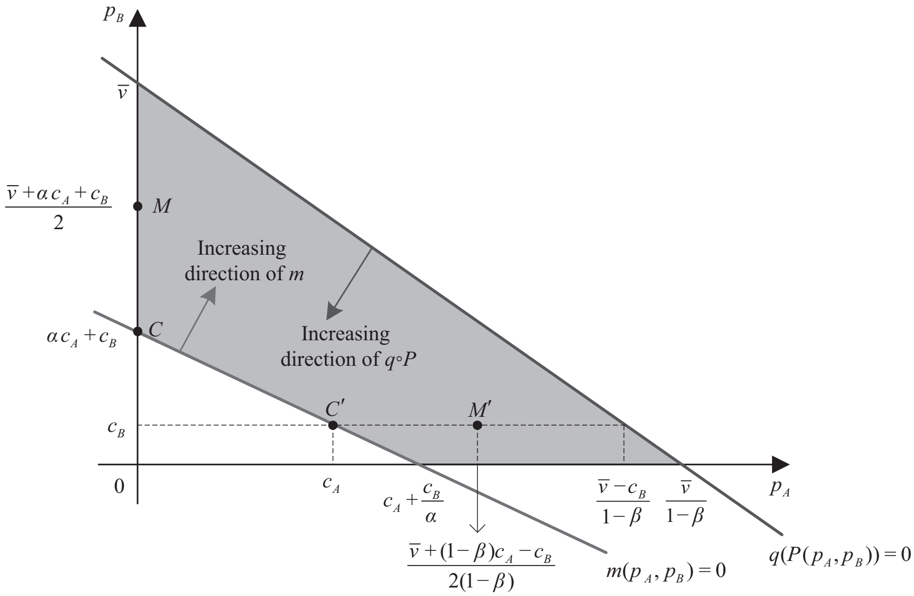
Since consumers discount payments for the consumable good while firms do not, delaying payment to the future benefits consumers and thus attracts more consumers. As a consequence, a profit-maximizing firm will set the price of the durable good as low as possible (i.e., at zero) and try to profit from the sale of the consumable good.[11] This can also be seen from Figure 1, in which it is shown that the slope of contour lines for the quantity demanded and the usage price is steeper than that of contour lines for the per-user margin. Thus, given a desired level of quantity or usage price, a firm maximizes its per-user margin by setting the price of the durable good at zero. At the same time, given a desired per-user margin, a firm minimizes the usage price and thus maximizes the quantity demanded by offering the durable good for free. Proposition 1 shows that monopoly pricing also involves the free durable good while the price of the consumable good is chosen to maximize the monopoly profit given the zero price of the durable good.

By Assumption 1, the maximum valuation $\bar{v}$ is sufficiently high to guarantee the existence of $p_A \geq 0$ and $p_B \geq 0$ such that $q(P(p_A, p_B)) > 0$ and $m(p_A, p_B) > 0$, which implies $\Pi^M > 0$. If Assumption 1 is not satisfied, the valuations of consumers are too low that no firm finds it profitable to sell the goods and thus there will be no production at equilibrium. By Assumption 2, the minimum valuation $\underline{v}$ is sufficiently low to guarantee that a consumer with the minimum valuation does not use the durable good at the monopoly prices. We impose Assumption 2 to make the expression for the monopoly price of the consumable good simpler. Without Assumption 2, we need to consider two cases depending on whether a consumer with the minimum valuation uses the durable good or not at the prices given in (3), and the monopoly solution is modified as $p_A^M = 0$ and $p_B^M = \max\{(\bar{v} + \alpha c_A + c_B)/2, \underline{v}\}$.

Note that $p_A^M < c_A$ and $p_B^M > c_B$. The monopolist sets the price of the durable good below

[11] In our model, we impose the nonnegativity constraint on the price of the durable good, motivated by monitoring imperfection. More generally, we can impose an arbitrary lower bound $\underline{p}_A$ on the price of the durable good, determined by other considerations such as government regulations. Then a profit-maximizing firm will choose the lower bound $\underline{p}_A$ as the price of the durable good. Thus, the free durable good in our results is not an essential feature of the pricing practice we want to explain in this paper, although we do observe freebie marketing occasionally. Rather, our main focus is to explain a low price of the durable good (below cost) and a high price of the consumable good (above cost).

FIG. 1. PRICING EQUILIBRIA



Note: M: monopoly with lock-in, C: competition with lock-in, M': monopoly without lock-in, C': competition without lock-in

marginal cost while it sets the price of the consumable good above marginal cost to cover the loss from selling the durable good. At the monopoly solution, the (lowest) usage price is given by

$$\tilde{P}^M := P(p_A^M, p_B^M) = \frac{\bar{v}+\alpha c_A + c_B}{2},$$

the fraction of consumers using the durable good is

$$q^M := q(\tilde{P}^M) = \frac{\bar{v}-\alpha c_A - c_B}{2(\bar{v}-\underline{v})},$$

and the per-user margin is

$$m^M := m(p_A^M, p_B^M) = \frac{\bar{v}-\alpha c_A - c_B}{2}.$$

Assumption 1 implies that $q^M > 0$ and $m^M > 0$, while Assumption 2 implies that $q^M < 1$. The steady-state profit of the monopolist is

$$\Pi^M = q^M m^M = \frac{(\bar{v}-\alpha c_A - c_B)^2}{4(\bar{v}-\underline{v})},$$

and the consumer surplus is given by

$$CS^M := CS(\tilde{P}^M) = \frac{(\bar{v} - \alpha c_A - c_B)^2}{8(\bar{v} - \underline{v})}.$$

2. Competition

Now suppose that there is more than one firm, i.e., $n \geq 2$, and that firms choose prices to maximize their profits as in Bertrand competition. Let $q_j(\mathbf{p}_A, \mathbf{p}_B)$ be the fraction of consumers who use the durable good produced by firm j when firms choose prices $(\mathbf{p}_A, \mathbf{p}_B)$. Then we have $\sum_{j=1}^n q_j(\mathbf{p}_A, \mathbf{p}_B) = q(\tilde{P}(\mathbf{p}_A, \mathbf{p}_B))$ and $q_j(\mathbf{p}_A, \mathbf{p}_B) = 0$ for all j such that $(1-\beta)p_{jA} + p_{jB} > \tilde{P}(\mathbf{p}_A, \mathbf{p}_B)$. The tie-breaking rule in case that there are multiple firms offering the lowest usage price can be specified in an arbitrary way without affecting equilibrium prices. The steady-state profit of firm j is given by $\pi_j(\mathbf{p}_A, \mathbf{p}_B) = q_j(\mathbf{p}_A, \mathbf{p}_B) m(p_{jA}, p_{jB})$. The following proposition presents pricing equilibrium under competition.

Proposition 2. *Suppose that $n \geq 2$. Under Assumption 1, there exists at least one active firm at pricing equilibrium where all active firms choose the prices*

$$p_{jA} = p_A^C := 0 \text{ and } p_{jB} = p_B^C := \alpha c_A + c_B.$$

Moreover, there exist at least two firms that offer the lowest usage price.

As mentioned following Proposition 1, given a desired per-user margin, an active firm maximizes the quantity demanded by setting the price of the durable good at zero. Thus, profit maximization induces active firms to offer the durable good for free. At the same time, price competition drives the per-user margin to zero, which leads to the price of the consumable good equal to the overall marginal cost of producing the two goods in the steady state.

As in the monopoly pricing, we have $p_A^C < c_A$ and $p_B^C > c_B$. At the pricing equilibrium with more than one firm, the lowest usage price is given by

$$\tilde{P}^C := P(p_A^C, p_B^C) = \alpha c_A + c_B,$$

the fraction of consumers using the durable good is

$$q^C := q(\tilde{P}^C) = \frac{\bar{v} - \alpha c_A - c_B}{\bar{v} - \underline{v}},$$

and the per-user margin of active firms is

$$m^C := m(p_A^C, p_B^C) = 0.$$

Hence, the total profit of firms in the steady-state is

$$\Pi^C := q^C m^C = 0,$$

and the consumer surplus is given by

$$CS^C := CS(\tilde{P}^C) = \frac{(\bar{v} - \alpha c_A - c_B)^2}{2(\bar{v} - \underline{v})}.$$

The equilibrium analysis with identical firms can be extended to the case of firms producing homogeneous goods at different marginal costs. Let c_{jA} and c_{jB} be the marginal costs of the durable good and the consumable good, respectively, produced by firm j. Then the per-user cost of firm j in the steady-state is given by $\alpha c_{jA}+c_{jB}$. A firm with the smallest per-user cost has a cost advantage over the other firms. Suppose, without loss of generality, that the firms are numbered in ascending order of their per-user costs so that $\alpha c_{1A}+c_{1B}\leq\alpha c_{2A}+c_{2B}\leq\cdots\leq\alpha c_{nA}+c_{nB}$. If $\alpha c_{1A}+c_{1B}=\alpha c_{2A}+c_{2B}$, then we obtain a perfectly competitive outcome as in Proposition 2, and Proposition 2 generalizes to have p_B^C equal to the smallest per-user cost. If $\alpha c_{1A}+c_{1B}<\alpha c_{2A}+c_{2B}$, then firm 1 will capture the entire demand and earn a positive profit by setting $p_{1A}=0$ and p_{1B} equal to or slightly lower than $\alpha c_{2A}+c_{2B}$ at the pricing equilibrium.

We close this section with some remarks on the two important assumptions in our model: (i) a firm chooses a pair of *constant* prices, and (ii) a firm maximizes its *steady-state* profit. These two assumptions are closely related in that constant prices guarantee the existence of a steady state and that, if a firm maximizes its steady-state profit, it is without loss of generality to focus on constant prices. We discuss what happens if we relax these assumptions. First, relaxing the first assumption while keeping the second assumption would not change our results. By definition, a steady state exists only when active firms eventually choose constant prices, and the steady-state profit is independent of prices chosen in the transient phase. Hence, without loss of generality we can ignore the transient phase and focus on the eventual phase in which firms charge constant prices. Second, we can think of the possibility of relaxing the second assumption while maintaining the first assumption. One possible interpretation of maximizing steady-state profit is that if a firm somehow finds itself in the steady state (i.e., if a firm starts from the steady state) it has no incentive to change its prices. Then the remaining question is how a firm reaches the steady state. A natural approach to address this question is to imagine an initial period in which there is no consumer who already has the durable good and to assume that a firm maximizes its average discounted profit starting from the initial period. In our model, when firms set constant prices, the steady state is reached after a single period. Firm j's profit in the initial period is given by $\pi=q_j(\mathbf{p}_A, \mathbf{p}_B)[(p_{jA}-c_A)+(p_{jB}-c_B)]$, while that from the next period on is $\pi'=q_j(\mathbf{p}_A, \mathbf{p}_B)[\alpha(p_{jA}-c_A)+(p_{jB}-c_B)]$. Hence, firm j's average discounted profit is $(1-\delta_F)\pi+\delta_F\pi'=q_j(\mathbf{p}_A, \mathbf{p}_B)\{[1-\delta_F(1-\alpha)](p_{jA}-c_A)+(p_{jB}-c_B)\}$, where δ_F denotes the discount factor of firms. As long as $\delta_F>\delta$, we obtain a corner solution with $p_A=0$ as in Propositions 1 and 2 while α in the expressions of p_B^M and p_B^C is replaced by $1-\delta_F(1-\alpha)$. Thus, with this alternative assumption, our results correspond to the limiting case where δ_F approaches 1. The result with average discounted profit maximization is reminiscent of that of Kaserman (2007) who shows that, when firms have a larger discount factor than consumers, the price of the consumable good exceeds marginal cost and thus lock-in is required. In contrast to Kaserman (2007) who examines prices in the efficient contract with a competitive durable good market, we study profit-maximizing prices in both monopoly and competition scenarios. Lastly, we may relax both assumptions at the same time so that firms can choose different prices over time to maximize their average discounted profits. This makes the analysis a lot more complicated as firms have infinitely many choice variables and consumers' purchase decisions depend on their expectations on future prices. Thus, we leave analysis of this scenario for future research. By imposing the two assumptions in our model, we study a situation in which firms can make a commitment to future prices. Without commitment, a firm may be tempted to lower the price of the durable good in order to serve the residual demand (as in the Coase's conjecture) as well as to increase the price of the consumable good in order to exploit consumers who already use the durable good (as in the aftermarket literature based on consumer lock-in). These incentives in general work against firms at least in the long run, and as a result firms desire to commit to future prices if they can find a credible way of doing so.

IV. *Pricing without Lock-In*

So far we have imposed the lock-in constraint, which restricts the choice of consumers. In order to examine the welfare implications of lock-in, we relax the lock-in constraint in this section. In particular, we consider a scenario where the consumable good is supplied in a competitive market and the durable good can be used with the consumable good produced by any firm. It is easy to see that offering the durable good for free is no longer viable in such a scenario. In order for a firm offering the free durable good to avoid a loss, it has to maintain a positive margin on the consumable good, which is not sustainable in a competitive market.

1. Monopolistic Supply of the Durable Good

Suppose that there is only one firm producing the durable good and that there are many firms producing the consumable good at the marginal cost c_B. The monopolist of the durable good can also produce the consumable good at the same marginal cost. However, it does not have a control over the price of the consumable good in the competitive market, and the market price of the consumable good is given by $p_B^{M'} := c_B$. Taking the price of the consumable good as given, the monopolist solves

$$\max_{p_A \geq 0} \pi(p_A, c_B) \tag{4}$$

by choosing only the price of the durable good, and the maximum profit of the monopolist is written as $\Pi^{M'}$.

Assumption 3. $\bar{v} > (1-\beta)c_A + c_B$.

Assumption 4. $2\underline{v} < \bar{v} + [(1-\beta)c_A + c_B]$.

Note that Assumptions 3 and 4 are stronger than Assumptions 1 and 2, respectively, as $1-\beta > \alpha$.

Proposition 3. *Under Assumptions 3 and 4, the solution to the monopolist's problem without lock-in* (4) *is given by*

$$p_A = p_A^{M'} := \frac{\bar{v} + (1-\beta)c_A - c_B}{2(1-\beta)}.$$

When the consumable good is supplied in a competitive market, the only source of profit for the monopolist is the sale of the durable good. Thus, it sets the price of the durable good above marginal cost, i.e., $p_A^{M'} > c_A$, to maximize its profit. At the monopoly solution without lock-in, the usage price is given by

$$\tilde{P}^{M'} := P(p_A^{M'}, p_B^{M'}) = \frac{\bar{v} + (1-\beta)c_A + c_B}{2},$$

the fraction of consumers using the durable good is

$$q^{M'} := q(\tilde{P}^{M'}) = \frac{\bar{v} - (1-\beta)c_A - c_B}{2(\bar{v} - \underline{v})},$$

and the per-user margin is

$$m^{M'} := m(p_A^{M'}, p_B^{M'}) = \frac{\alpha[\bar{v} - (1-\beta)c_A - c_B]}{2(1-\beta)}.$$

Assumption 3 implies that $q^{M'} > 0$ and $m^{M'} > 0$, while Assumption 4 implies that $q^{M'} < 1$. The steady-state profit of the monopolist is

$$\Pi^{M'} = q^{M'} m^{M'} = \frac{\alpha[\bar{v} - (1-\beta)c_A - c_B]^2}{4(1-\beta)(\bar{v} - \underline{v})},$$

and the consumer surplus is given by

$$CS^{M'} := CS(\tilde{P}^{M'}) = \frac{[\bar{v} - (1-\beta)c_A - c_B]^2}{8(\bar{v} - \underline{v})}.$$

2. Competitive Supply of the Durable Good and the Consumable Good

Now suppose that the durable good and the consumable good are produced by many identical firms.

Assumption 5. $\bar{v} > (1-\beta)c_A + \dfrac{1-\beta}{\alpha} c_B$.

Note that Assumption 5 is stronger than Assumption 3.

Proposition 4. *Suppose that $n \geq 2$. Under Assumption 5, at pricing equilibrium without lock-in, each firm chooses $p_{jA} \geq c_A$ and $p_{jB} \geq c_B$, and there are at least two firms choosing $p_{jA} = p_A^{C'} := c_A$ and $p_{jB} = p_B^{C'} := c_B$. (The firms choosing $p_{jA} = p_A^{C'}$ are not necessarily the same as those choosing $p_{jB} = p_B^{C'}$.)*

Without lock-in, price competition occurs separately on the durable good and the consumable good, and thus each price is driven down to the corresponding marginal cost at pricing equilibrium. When firms lock in their consumers, price competition occurs on the usage price, resulting in the price of the durable good below marginal cost and that of the consumable good above marginal cost. This price configuration is no longer an equilibrium without lock-in because a firm can profitably undercut the price of the consumable good capturing the entire demand for the consumable good.

At the competitive prices without lock-in, the lowest usage price is given by

$$\tilde{P}^{C'} := P(p_A^{C'}, p_B^{C'}) = (1-\beta)c_A + c_B,$$

the fraction of consumers using the durable good is

$$q^{C'} := q(\tilde{P}^{C'}) = \frac{\bar{v} - (1-\beta)c_A - c_B}{\bar{v} - \underline{v}},$$

and the per-user margin of active firms is

$$m^{C'} := m(p_A^{C'}, p_B^{C'}) = 0.$$

Hence, the total profit of firms in the steady-state is

$$\Pi^{C'} := q^{C'} m^{C'} = 0,$$

and the consumer surplus is given by

$$CS^{C'} := CS(\tilde{P}^{C'}) = \frac{[\bar{v} - (1-\beta)c_A - c_B]^2}{2(\bar{v} - \underline{v})}.$$

As an extension, we consider a scenario where firms determine whether to make the durable good they produce compatible with the consumable good produced by other firms or not. We can model the scenario as a two-stage game in which firms make lock-in decisions in the first stage and pricing decisions in the second stage. Unfortunately, even in the duopoly case there is no pricing equilibrium with positive production in a subgame where one firm makes the durable good it produces compatible while the other firm does not. As a result, we cannot find a subgame perfect equilibrium of the two-stage game with duopoly.

Proposition 5. *Suppose that there are two firms and that the durable good produced by one firm (say firm 1) can be used only with the consumable good produced by the firm while the durable good produced by the other firm (say firm 2) can be used with the consumable good produced by either firm. Under Assumption 1, there exists no pricing equilibrium.*

In the proof of Proposition 5, we show that for every possible price configuration there is at least one firm that can profitably deviate. To illustrate this instability, consider the equilibrium outcomes of competition with and without lock-in obtained in Propositions 2 and 4. If both firms choose $(p_A^C, p_B^C) = (0, \alpha c_A + c_B)$, firm 1 can profit from lowering the price of the consumable good it produces so that consumers use the durable good produced by firm 2 and purchase the consumable good from firm 1. If both firms choose $(p_A^{C'}, p_B^{C'}) = (c_A, c_B)$, firm 1 can offer a lower usage price profitably using its ability to lock in consumers.

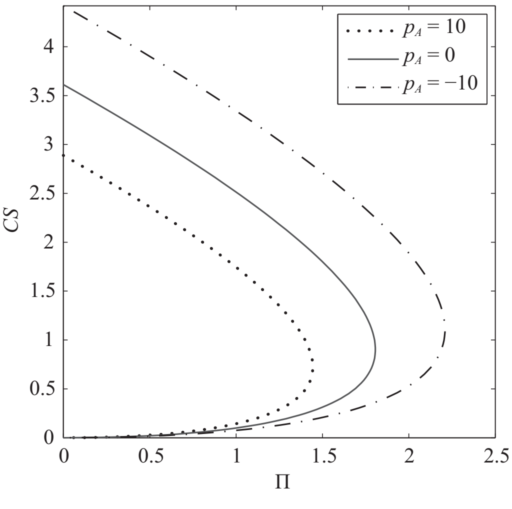
V. *Welfare Analysis*

In this section, we compare the welfare of firm owners and consumers in the four scenarios analyzed in Propositions 1 to 4. The results we have obtained in Sections III and IV are summarized in Table 1. The equilibrium prices of active firms obtained in Propositions 1 to 4 are also plotted in Figure 1. For welfare analysis, we regard firm owners and consumers as two representative agents. We measure the welfare of firm owners by the total profit of firms, Π, and that of consumers by the total consumer surplus, CS. Then the welfare possibility region

TABLE 1. SUMMARY of the RESULTS

	M (Prop. 1)	C (Prop. 2)	M' (Prop. 3)	C' (Prop. 4)
p_A	0	0	$\frac{\bar{v}+(1-\beta)c_A-c_B}{2(1-\beta)}$	c_A
p_B	$\frac{\bar{v}+\alpha c_A+c_B}{2}$	αc_A+c_B	c_B	c_B
$\tilde{P}$	$\frac{\bar{v}+\alpha c_A+c_B}{2}$	αc_A+c_B	$\frac{\bar{v}+(1-\beta)c_A+c_B}{2}$	$(1-\beta)c_A+c_B$
q	$\frac{\bar{v}-\alpha c_A-c_B}{2(\bar{v}-\underline{v})}$	$\frac{\bar{v}-\alpha c_A-c_B}{\bar{v}-\underline{v}}$	$\frac{\bar{v}-(1-\beta)c_A-c_B}{2(\bar{v}-\underline{v})}$	$\frac{\bar{v}-(1-\beta)c_A-c_B}{\bar{v}-\underline{v}}$
m	$\frac{\bar{v}-\alpha c_A-c_B}{2}$	0	$\frac{\alpha[\bar{v}-(1-\beta)c_A-c_B]}{2(1-\beta)}$	0
Π	$\frac{(\bar{v}-\alpha c_A-c_B)^2}{4(\bar{v}-\underline{v})}$	0	$\frac{\alpha[\bar{v}-(1-\beta)c_A-c_B]^2}{4(1-\beta)(\bar{v}-\underline{v})}$	0
CS	$\frac{(\bar{v}-\alpha c_A-c_B)^2}{8(\bar{v}-\underline{v})}$	$\frac{(\bar{v}-\alpha c_A-c_B)^2}{2(\bar{v}-\underline{v})}$	$\frac{[\bar{v}-(1-\beta)c_A-c_B]^2}{8(\bar{v}-\underline{v})}$	$\frac{[\bar{v}-(1-\beta)c_A-c_B]^2}{2(\bar{v}-\underline{v})}$

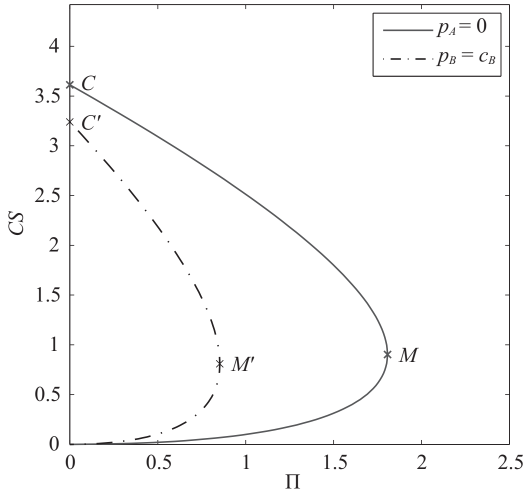
FIG. 2. WELFARE CURVES FOR $p_A=10$, 0, -10



can be obtained by evaluating Π and CS at every feasible choice of the prices (p_A, p_B) by active firms.

As can be seen from (1), the consumer surplus is determined by the prevailing usage price. For a fixed usage price, the per-user margin decreases linearly in the price of the durable good. Therefore, when the prices are restricted to be nonnegative, not only equilibrium pricing but also Pareto efficient pricing involves the free durable good. If an active firm sets $p_A>0$, then it can increase its per-user margin while offering the same usage price by decreasing p_A by Δ and increasing p_B by $(1-\beta)\Delta$. The new set of prices improves Π while CS remains the same. Figure 2 illustrates this idea using the parameter specification of $\alpha=0.1$, $\delta=0.9$, $\underline{v}=0$, $\bar{v}=10$,

FIG. 3. WELFARE COMPARISON OF PRICING EQUILIBRIA



Note: M, C, M′, and C' represent the same as in Figure 1.

$c_A=5$, and $c_B=1$. It plots Π and CS for three levels of p_A, 10, 0, and -10, while varying p_B in the region where the individual rationality of the representative firm, $\Pi \geq 0$, is satisfied. (The individual rationality of the representative consumer is captured in the usage decision of consumers.) As can be seen from Figure 2, the welfare curve moves further away from the origin as the price of the durable good becomes lower. If the nonnegative constraint on p_A is relaxed as $p_A \geq -\gamma$ for some $\gamma > 0$, the maximum achievable values of Π and CS increase without bound as γ increases to infinity. Figure 3 plots the welfare of firm owners and consumers in the pricing equilibria, using the same parameter specification. The welfare comparison is summarized in the following proposition.

Proposition 6. *Assume Assumptions 4 and 5 so that Propositions 1 to 4 hold.*

(i) (Welfare Effects of Lock-In) $CS^M > CS^{M'}$, $\Pi^M > \Pi^{M'}$, $CS^C > CS^{C'}$ *and* $\Pi^C = \Pi^{C'} = 0$. *Moreover, as* δ *goes to 1, the differences* $CS^M - CS^{M'}$, $\Pi^M - \Pi^{M'}$, *and* $CS^C - CS^{C'}$ *converge to 0.*

(ii) (Welfare Effects of Competition) $CS^C > CS^M$, $\Pi^M > \Pi^C$, $CS^{C'} > CS^{M'}$, *and* $\Pi^{M'} > \Pi^{C'}$. *Moreover,* $\Pi + CS$ *is maximized at* $(p_A^C, p_B^C) = (0, \alpha c_A + c_B)$.

We first investigate the welfare implications of lock-in. Lock-in restricts the choice of consumers in terms of the set of compatible combinations of the durable good and the consumable good. Hence, it may be claimed that lock-in limits competition among firms and thus has a negative welfare impact on consumers. In contrast to this claim, in our model, the absence of lock-in restricts the pricing choice of firms and prevents firms from offering a welfare-improving price configuration. When firms can lock in their consumers, pricing equilibria have the zero price of the durable good, which achieves the Pareto frontier given the nonnegativity constraint. Without lock-in, the price choice of firms is restricted by the competitive price of the consumable good $p_B = c_B$, which creates efficiency loss and moves the welfare curve toward the origin, as shown in Figure 3. In the monopoly scenario, lock-in allows the monopolist to create higher surplus for consumers and to achieve higher profit. In our model, the monopolist cannot use price discrimination to extract all the surplus created, and thus part of the gains from lock-in go to consumers. In the competition scenario, lock-in enables firms to offer a lower usage price and thus yield a larger consumer surplus while breaking even. When consumers' discount factor is close to 1, there is not much difference between the perspectives of consumers and firms, and thus there is little gain from delaying consumers' payments. Hence, in this case, lock-in has negligible welfare effects.

Next we investigate the welfare implications of competition. As mentioned above, firms' ability to lock in consumers induces the zero price of the durable good, whereas their inability makes them take the competitive price of the consumable good. In each scenario, monopoly yields the outcome that is most favorable to the firm while competition generates the outcome most preferred by consumers given that firms at least break even, as depicted in Figure 3. By shifting bargaining power from firms to consumers, competition lowers the usage price, which benefits consumers and hurts firms. If we treat the sum of consumer surplus and profit as social welfare, monopoly creates deadweight loss due to the monopolist's inappropriable gains, and thus social welfare is maximized at the pricing equilibrium under competition with lock-in.

VI. *Discussion*

As mentioned in the introduction, we can consider firms as offering an implicit loan when they sell the durable good. If we take the competitive price of each good, $p_A = c_A$ and $p_B = c_B$, as a reference price, the difference between the price and the marginal cost of the durable good, $c_A - p_A$, can be considered as the size of the loan while that of the consumable good, $p_B - c_B$, can be regarded as the size of the repayment that a consumer makes whenever he purchases the consumable good. For example, in the competition scenario with lock-in, c_A and αc_A are the sizes of the loan and the repayment, respectively. The pricing of the complementary goods thus determines the sizes of the loan and the repayment in the implicit financial arrangement.

In our model, economic surplus increases as the size of the loan increases, or the price of the durable good decreases. When offering a loan, firms need to make sure that they receive repayments. That is, firms need to ensure that consumers purchasing the durable good purchase the consumable good from the same firm. There are two potential incentive problems for consumers in the implicit financial arrangement. First, a consumer may purchase the durable good when he does not need it. This can occur when $p_A < 0$, since consumers can make money by purchasing the durable good. Firms can prevent this by setting a non-negative price of the durable good, $p_A \geq 0$. When $p_A \geq 0$, a consumer whose valuation is lower than the usage price has no incentive to purchase the durable good, and a consumer who consumes the complementary goods has no incentive to replace the durable good before it becomes no longer usable. Second, a consumer who purchases the durable good may purchase the consumable good from a different firm that offers a lower price. Firms can prevent this by binding consumers to purchase the durable good and the consumable good from the same firm. Hence, the first incentive problem restricts the size of the loan that firms can offer, while the second incentive problem introduces lock-in as a commitment device for consumers.

Based on our theory, we can build hypotheses that can be tested empirically. First, a key element of our model is the assumption that a firm obtains a constant stream of revenues in the steady state, which leads to the different perspectives of consumers and firms. Second, regarding the problem of firms choosing between the two alternatives, a formal financial contract and an implicit arrangement underlying pricing, the transaction cost of writing a formal contract relative to its benefit will be larger when smaller purchases are involved. Thus, we can expect that the pricing practice is observed more commonly for products that have more stable demands and are relatively cheaper. Note that the classic example of freebie marketing in which razors are given away for free to increase demand for blades is consistent with these hypotheses.

VII. *Conclusion*

In this paper, we have analyzed the pricing decision of firms producing a durable good and a complementary consumable good in the cases of monopoly and competition. In our model, when firms can lock in their consumers, the durable good is offered at a price below marginal cost while the consumable good is priced above marginal cost. We have given this pricing policy an interpretation as an implicit financial arrangement, which benefits consumers by allowing them to delay their payments to the future. Our welfare analysis shows that consumers are better off when they are locked in, since lock-in makes the pricing policy sustainable. Our work contributes to the rich body of literature on tie-in sales and aftermarkets by providing a new efficiency argument based on consumer credit, and measuring the relevance of competing theories in a particular market is an empirical question to be investigated in future research.

Appendix

Proof of Lemma 1:

$\partial P(p_A, p_B)/\partial p_A = 1-\beta$ and $\partial P(p_A, p_B)/\partial p_B = 1$ for all (p_A, p_B). Also, $\partial m(p_A, p_B)/\partial p_A = \alpha$ and $\partial m(p_A, p_B)/\partial p_B = 1$ for all (p_A, p_B). Hence, it is equivalent to show that $1-\beta > \alpha$, which follows from $\delta < 1$.

Proof of Proposition 1:

Assuming that $0 < q(P(p_A, p_B)) < 1$, the monopolist solves

$$\max_{p_A, p_B \geq 0} \frac{1}{\bar{v}-\underline{v}}[\bar{v}-(1-\beta)p_A - p_B][\alpha(p_A - c_A)+(p_B - c_B)].$$

Assumption 1 implies that there is feasible (p_A, p_B) with $\pi(p_A, p_B) > 0$. We use a two-stage maximization procedure to solve the monopolist's problem. We fix the usage price at P in stage one and choose (p_A, p_B) that satisfies $P = (1-\beta)p_A + p_B$ in stage two. Given the stage-one choice P, the stage-two problem can be written as

$$\max_{p_A \geq 0} \frac{1}{\bar{v}-\underline{v}}(\bar{v}-P)\{P-[(1-\beta)-\alpha]p_A - \alpha c_A - c_B\}.$$

Since $1-\beta > \alpha$ (see Lemma 1), the stage-two optimum occurs at $p_A = 0$ and $p_B = P$. Then the stage-one maximization problem is

$$\max_{P\geq 0} \frac{1}{\bar{v}-\underline{v}}(\bar{v}-P)[P-\alpha c_A-c_B],$$

which yields the optimal usage price $P=(\bar{v}+\alpha c_A+c_B)/2$. Note that Assumptions 1 and 2 validate the initial assumption that $0<q(P(p_A, p_B))<1$.

Proof of Proposition 2:

Assumption 1 is needed to have positive production, or at least one active firm, at equilibrium. The argument of Bertrand price competition can be applied to show that the usage prices of firms go down until the per-user margin is zero. Moreover, Lemma 1 implies that a firm minimizes its usage price for a given per-user margin by setting $p_A=0$. Therefore, active firms choose (p_A^C, p_B^C) such that $p_A^C=0$ and $\alpha(p_A^C-c_A)+(p_B^C-c_B)=0$. Solving these equations together yields p_B^C in the proposition. If there is more than one active firm, then the last sentence of the proposition holds trivially. Suppose that there is only one active firm and that every other firm chooses (p_{jA}, p_{jB}) such that $(1-\beta)p_{jA}+p_{jB}>(1-\beta)p_A^C+p_B^C$. Then the active firm can increase its profit by increasing its usage price, contradicting equilibrium. Thus, there must exist a firm other than the only active firm that sets its usage price equal to that of the active firm.

Proof of Proposition 3:

Assuming that $0<q(P(p_A, c_B))<1$, the monopolist solves

$$\max_{p_A\geq 0} \frac{1}{\bar{v}-\underline{v}}[\bar{v}-(1-\beta)p_A-c_B]\alpha(p_A-c_A).$$

Since the objective function is concave, the solution $p_A^{M'}$ can be found using the first-order optimality condition. Note that Assumptions 3 and 4 validate the initial assumption. Assumption 3 guarantees the existence of feasible p_A such that $\bar{v}-(1-\beta)p_A-c_B>0$ and $p_A-c_A>0$, implying that $q^{M'}>0$. Assumption 4 implies that $q^{M'}<1$.

Proof of Proposition 4:

Without lock-in, consumers will compare the prices of each good. Let $\tilde{p}_A:=\min_j p_{jA}$ and $\tilde{p}_B:=\min_j p_{jB}$. Then the lowest usage price is given by $P(\tilde{p}_A, \tilde{p}_B)$, and the fraction of consumers using the durable good is $q(P(\tilde{p}_A, \tilde{p}_B))$. Let $q_{jA}(\mathbf{p}_A, \mathbf{p}_B)$ and $q_{jB}(\mathbf{p}_A, \mathbf{p}_B)$ be the fraction of consumers purchasing the durable good and the consumable good from firm j, respectively, when firms choose prices $(\mathbf{p}_A, \mathbf{p}_B)$. Then

$$q_{jA}(\mathbf{p}_A, \mathbf{p}_B)=\begin{cases} q(P(\tilde{p}_A, \tilde{p}_B)) & \text{if } p_{jA}<p_{j'A} \text{ for all } j'\neq j, \\ 0 & \text{if there exists some } j'\neq j \text{ such that } p_{j'A}<p_{jA}. \end{cases}$$

In case of a tie, a tie-breaking rule can be specified arbitrarily. Similarly,

$$q_{jB}(\mathbf{p}_A, \mathbf{p}_B)=\begin{cases} q(P(\tilde{p}_A, \tilde{p}_B)) & \text{if } p_{jB}<p_{j'B} \text{ for all } j'\neq j, \\ 0 & \text{if there exists some } j'\neq j \text{ such that } p_{j'B}<p_{jB}. \end{cases}$$

The steady-state profit of firm j is given by

$$\pi_j(\mathbf{p}_A, \mathbf{p}_B)=q_{jA}(\mathbf{p}_A, \mathbf{p}_B)\alpha(p_{jA}-c_A)+q_{jB}(\mathbf{p}_A, \mathbf{p}_B)(p_{jB}-c_B).$$

Let $\mathbf{p}_{-jA}:=(p_{1A}, ..., p_{(j-1)A}, p_{(j+1)A}, ..., p_{nA})$ and $\mathbf{p}_{-jB}:=(p_{1B}, ..., p_{(j-1)B}, p_{(j+1)B}, ..., p_{nB})$. Let $\tilde{p}_{-jA}:=$

$\min_{j'\neq j} p_{j'A}$ and $\tilde{p}_{-jB} := \min_{j'\neq j} p_{j'B}$. Suppose that $q(P(\tilde{p}_{-jA}, \tilde{p}_{-jB}))=0$, i.e., there are no users when the lowest prices are given by $(\tilde{p}_{-jA}, \tilde{p}_{-jB})$. Assumption 5 guarantees that, for any $(\mathbf{p}_{-jA}, \mathbf{p}_{-jB})$ such that $q(P(\tilde{p}_{-jA}, \tilde{p}_{-jB}))=0$, there exists $(p_{jA}, p_{jB})\geq 0$ such that $\pi_j(\mathbf{p}_A, \mathbf{p}_B)>0$. Hence, there should be a positive fraction of consumers using the durable good at equilibrium.

Let $(\mathbf{p}_A, \mathbf{p}_B)$ be a pricing equilibrium and choose firm j with $q_{jA}(\mathbf{p}_A, \mathbf{p}_B)>0$. Then it must be the case that $p_{jA}=\tilde{p}_A$. Suppose that $p_{jA}>c_A$. Then there must exist another firm j' that can deviate to $p_{j'A}\in(c_A, p_{jA})$ and increase its profit. Hence, $\tilde{p}_A\leq c_A$ at equilibrium. By a similar argument, we have $\tilde{p}_B\leq c_B$ at equilibrium. Suppose that $p_{jA}<c_A$. Because choosing $(p_A, p_B)=(c_A, c_B)$ guarantees zero profit, the equilibrium profit has to be nonnegative, and it follows that $q_{jB}(\mathbf{p}_A, \mathbf{p}_B)>0$ and $p_{jB}>c_B$. However, this contradicts $\tilde{p}_B\leq c_B$. Hence, $\tilde{p}_A=c_A$ at equilibrium. Similarly, we can show that $\tilde{p}_B=c_B$ at equilibrium. It remains to show that there are at least two firms choosing $p_A=c_A$ and the same for $p_B=c_B$. Suppose that $p_{j'A}>c_A$ for all $j'\neq j$. Then firm j can increase its profit by increasing $p_{jA}\in(c_A, \tilde{p}_{-jA})$. Thus, there must exist $j'\neq j$ such that $p_{j'A}=c_A$. A similar argument can be used to show that there are at least two firms choosing $p_B=c_B$.

Proof of Proposition 5:

Suppose that there exists a pricing equilibrium. If both firms choose high prices such that there is no demand for the durable good, then firm 1 can deviate to prices that yield a positive profit under Assumption 1. Hence, there should be a positive fraction of consumers using the durable good at equilibrium.

There are three compatible combinations of the durable and consumable goods: (C1) both goods produced by firm 1, (C2) both goods produced by firm 2, and (CH) the durable good produced by firm 2 and the consumable good produced by firm 1. The usage prices of the three combinations are $(1-\beta)p_{1A}+p_{1B}$, $(1-\beta)p_{2A}+p_{2B}$, and $(1-\beta)p_{2A}+p_{1B}$, respectively. Only combinations with the lowest usage price will be used at equilibrium.

First note that equilibrium profits are nonnegative because choosing $(p_A, p_B)=(c_A, c_B)$ guarantees zero profit regardless of the prices set by the other firm.

Suppose that CH is used at equilibrium. If $p_{2A}>c_A$, then firm 1 can deviate to $p_{1A}\in(c_A, p_{2A})$ while keeping the same p_{1B} so that it can sell C1 and increase its profit. Similarly, if $p_{1B}>c_B$, then firm 2 can find a profitable deviation. Thus, it must be the case that $p_{2A}=c_A$ and $p_{1B}=c_B$. However, $p_{2A}=c_A$ and $p_{1B}=c_B$ cannot be an equilibrium either. If firm 1 chooses $p_{1A}=0$ and p_{1B} slightly less than $(1-\beta)c_A+c_B$, then it can sell C1 and earn a positive profit. Hence, CH cannot be used at equilibrium.

Suppose that C1 is used at equilibrium. If $p_{1A}>c_A$, then firm 2 can choose $p_{2A}\in(c_A, p_{1A})$ and $p_{2B}>p_{1B}$ so that CH is used and it makes a positive profit. If $p_{1A}\leq c_A$, then $p_{1B}\geq c_B$. Suppose $m(p_{1A}, p_{1B})>0$. Then firm 2 can undercut the usage price of C1 by setting $p_{2A}=p_{1A}$ and p_{2B} slightly lower than p_{1B} and make a positive profit by supplying C2. Thus, $m(p_{1A}, p_{1B})=0$ at equilibrium. Choosing (p_{1A}, p_{1B}) such that $m(p_{1A}, p_{1B})=0$ is a best response of firm 1 only if firm 2 chooses (p^C_A, p^C_B). However, given that firm 2 chooses $p^C_B>c_B$, firm 1 can deviate to $p_{1A}>p^C_A$ and $p_{1B}\in(c_B, p^C_B)$ to make a positive profit. Hence, C1 cannot be used at equilibrium.

Suppose that C2 is used at equilibrium. If $p_{2B}>c_B$, then firm 1 can choose $p_{1A}>p_{2A}$ and $p_{1B}\in(c_B, p_{2B})$ so that CH is sold and it makes a positive profit. If $p_{2B}\leq c_B$, then $p_{2A}\geq c_A$, and firm 1 can undercut the usage price of C2 by setting $p_{1A}=0$ and p_{1B} just below $(1-\beta)p_{2A}+p_{2B}$. At these prices, firm 1 makes a positive profit by supplying C1. Hence, C2 cannot be used at equilibrium.

Therefore, there is no user at equilibrium, which is a contradiction.

Proof of Proposition 6:

We use the expressions for CS and Π in the four scenarios given in the text (also replicated in Table 1).

(i) First, we have $\Pi^C=\Pi^{C'}=0$. Also, $CS^M>CS^{M'}$, $\Pi^M>\Pi^{M'}$, and $CS^C>CS^{C'}$ follow from $1-\beta>\alpha$. As δ goes to 1, $1-\beta=1-(1-\alpha)\delta$ goes to α, which induces the differences to converge to 0.

(ii) By Assumption 5, which implies Assumption 3, we have positive values of CS^M, CS^C, $CS^{M'}$, $CS^{C'}$, Π^M, and $\Pi^{M'}$. Then it follows that $CS^C>CS^M$, $\Pi^M>\Pi^C$, $CS^{C'}>CS^{M'}$, and $\Pi^{M'}>\Pi^{C'}$. In order to maximize $\Pi+CS$, we need to set $p_A=0$ because it achieves the largest per-user margin given a usage price. With $p_A=0$ imposed, firms face the demand function $\left[\frac{\bar{v}-p_B}{\bar{v}-\underline{v}}\right]_0^1$ with constant marginal cost αc_A+c_B. In this situation, social welfare is maximized when the price equals the marginal cost, i.e., $p_B=\alpha c_A+c_B$. Note that this is the pricing equilibrium under competition with lock-in. (If $\underline{v}\leq\alpha c_A+c_B$, the maximum occurs uniquely at $p_B=\alpha c_A+c_B$. Otherwise, any $p_B\in[0, \underline{v}]$ maximizes social welfare.)

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