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Tariffs, R&D, and Two Merger Policies

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Abstract

In an international Cournot oligopoly model, we compare two different merger policies when firms are merging endogenously and engage in research and development (R&D). In the benchmark model, countries set optimal tariff levels but do not have merger policy. If ex-ante identical firms merge internationally, they have an ex-post cost advantage over the outsiders due to tariff savings. This gives the merger an incentive to increase its R&D investment, which increases the cost dispersion further; therefore, the merger paradox, where each firm wants to be an outsider, disappears when R&D is efficient. As a result, we find different equilibrium market structures depending on the efficiency of R&D. In the second part, we compare two different merger policies, one that puts emphasis on welfare (roughly the Canadian merger policy) and another one that puts emphasis on consumer surplus (roughly the European Union’s merger policy). We show that under the “welfare-increasing” merger policy, monopoly is the equilibrium market structure when R&D is very efficient. This explains why a merger, which created a monopoly, was approved in Canada. As R&D becomes less efficient, the equilibrium market structures become less concentrated under the two different merger policies. Each merger policy can be global welfare maximizing depending on the efficiency of R&D; however, the “consumer-surplus-increasing” merger policy is optimal for a wider range of parameters.

Key Words: Competition Policy, Merger Policy, R&D, Endogenous Mergers, Tariff, Trade Policy, Cournot oligopoly, Merger Paradox.

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1 This paper has been presented at the University of Winnipeg, Osaka University-ISER, Kobe University-RIEB, “Market Quality Workshop” at Kyoto University, 2016 APTS held at National Taiwan University, and Hitotsubashi University. H. Gunay thanks Osaka University-ISER and Hitotsubashi Institute for Advanced Study for their hospitality during his visit. We are also grateful to a few colleagues. Their names will be added later.
1 Introduction

Regulators traditionally have concentrated on how mergers affect the competition in the markets; however, they have recently become interested in the research and development (R&D) aspect of these mergers. The 2010 Horizontal Merger Guideline of US specifically mentions R&D as one of the criteria in the merger approval process. Firms, on the other hand, merge to save on tariff costs, among other reasons. Therefore, to compare different merger policies, the trade policy and R&D incentives of involved firms should be studied simultaneously.

In this paper, we develop a model in which trade and merger policies are used simultaneously by each country, and firms are allowed to merge endogenously and engage in R&D. Our focus is to compare two different merger policies. The first one is similar to the Canadian merger policy, where the merger is approved by regulators provided that it is welfare increasing compared to status-quo even if it creates a monopoly. The second one is similar to the European Union’s merger policy, where the merger is approved if it is consumer-surplus-increasing compared to status-quo. In our paper, we show how these two merger policies affect the equilibrium market structure, and when they are global-welfare maximizing.

We develop a symmetric model of an oligopolistic industry with two firms in the home market and another two in the foreign market. The ex-ante identical firms produce a homogenous product. They undertake costly R&D activities to reduce their marginal cost. The firms choose to merge with other firm(s) a la Horn and Persson’s (2001a) cooperative endogenous merger formation model. If firms form an international merger, they do not incur tariff. The order of the interaction for the benchmark model is as follows. In the first stage of this game, endogenous mergers are decided (monopolies are not allowed). In the second stage, each country chooses their Nash level tariffs. In the third and fourth stage, firms choose R&D and then Cournot quantities simultaneously. To check robustness, in a variant of this model, we also allow

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2 The famous Super Propane merger was allowed in Canada even though it was creating monopolies in some local markets due to its welfare-increasing nature. (Ross and Winter 2005).
3 We do not allow monopolies in the benchmark model to be consistent with the literature (e.g. Horn and Persson 2001b and Ulus and Yildiz (2012). We allow monopoly when we introduce merger policies to the model.
4 We note that, in a complete information setting, agents rationally forecast the actions that will be taken in later stages by sequential rationality.
countries to set the optimal tariff rate for the status-quo at the beginning of the game, and hold it fixed throughout the game (i.e., tariff commitment). The equilibrium market structure (EMS) depends on the efficiency level of R&D. Asymmetric duopoly, where there is a three-firm merger and an outsider firm, is the EMS when R&D is efficient. We show that the three-firm merger, which has a marginal cost advantage due to not paying tariffs, increases its R&D investment. This increases the cost dispersion between the merger and the outsider further, especially when R&D is efficient. As a result, the merger paradox, where each firm wants to be an outsider, disappears. This makes the asymmetric international duopoly the equilibrium when R&D is efficient. When R&D is inefficient, symmetric duopoly with two international mergers is the EMS. Countries use Nash level tariffs in the off-path equilibrium so the “threat” of using the Nash level tariffs makes firms form cross-border mergers. In other words, trade liberalization occurs as an equilibrium phenomena in this model.

In the second part of the paper, we introduce merger policy to the model. As we discussed above, we compare two merger policies. The first merger policy is where the focus is on welfare. We call it a “welfare-increasing” merger policy (roughly the Canadian merger policy). The order of interaction in this game is as follows. In the first stage, firms decide whether to merge and if so with which firm(s) a la Horn and Persson. In the second stage, regulation agencies accept the merger if it is welfare-increasing compared to status-quo, which is equivalent to no merger. Firms pay a small penalty if the merger is not approved. An international merger has to be approved by both regulators, and a national merger has to be approved by only the domestic regulator. In the third stage, each country, after observing the market structure, determines its Nash level tariffs simultaneously. In the fourth stage, firms decide the level of cost-reducing R&D. In the last stage, firms produce a homogenous product and sell it in Cournot fashion. To check robustness, in a variant of the model, we allow countries to set the optimal tariff at the beginning of the game, and hold it fixed throughout the game. Our qualitative results have not changed.

5 The qualitative results are the same for both models. We also allow different order for the game; namely, R&D was chosen first, then the optimal Nash level tariffs. The qualitative results have not changed for that model neither.
6 Salant et al (1983) shows that the outsider firm is better off in Cournot markets when other firms merge due to less competition. As a result, each firm wants the other firms to merge, and hence, a free-riding incentive is present. This is called “merger paradox.”
We show that under the welfare-increasing merger policy, monopoly is the EMS when R&D is highly efficient. Per firm output is higher in monopoly compared to status-quo where there are four identical firms. Hence, the monopoly invests more on R&D than any other firm under status-quo. This limits the increase in the market price and the decrease in consumer surplus. The increase in producer surplus compensates the loss of consumer surplus and the tariff revenue. Hence, the welfare is higher when R&D is highly efficient compared to status-quo.

We also show that as R&D becomes less efficient, the EMS becomes monopoly, asymmetric duopoly, symmetric (international) duopoly, triopoly with one international merger, or the status-quo, respectively. In other words, the market becomes less concentrated as R&D becomes inefficient.

Next, we study the second merger policy where the focus is on consumer surplus. We call this a “consumer-surplus-increasing” merger policy, and this is roughly the European Union’s merger policy. The game is the same as above except that a merger is accepted by regulators only if it is consumer-surplus-increasing compared to status-quo. We show that as R&D becomes less efficient, the EMS changes from symmetric (international) duopoly to triopoly with one international merger, and then to status-quo. As expected, monopoly cannot be EMS in this case.

When one country uses the “welfare-increasing” merger policy and another uses the “consumer-surplus-increasing” merger policy, the EMS is the same as if the two countries are using the “consumer-surplus-increasing” merger policy.

The results of the paper give us a testable empirical prediction. Industries with efficient R&D must have international mergers while industries with inefficient R&D will have no (or fewer) mergers.7

We also examine global welfare in this paper. The global welfare maximizing market structure is symmetric (international) duopoly (hence free trade) when R&D is efficient, and triopoly with one international merger when R&D is inefficient (more liberal trade compared to status-quo).

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7 We assume no variable and/or fixed cost savings from mergers.
Both merger policies can be global welfare maximizing for some parameter space; however, consumer-surplus-increasing merger policy is the global welfare maximizing policy for a wider range of parameters. The interaction of the firms’ private merger incentives and the merger policy provide this seemingly surprising result. The consumer-surplus-increasing merger policy selects the market structures that increase consumer surplus compared to the status-quo. Among those market structures, firms choose the producer surplus increasing market structure given the private incentives. As a result, the welfare is higher for both consumers and producers compared to the status-quo. Therefore, it is more likely to be the global welfare increasing merger policy.

Our paper contributes to three different aspects of the existing literature. The first one is the literature combining the competition policies and trade policies (e.g. Ross 1988, Horn and Levinsohn 2001, Yano 2001). None of these papers model how firms endogenously (and voluntarily) merge, rather they allow the countries to determine the number of firms in the market as the competition policy.\(^8\) In addition, these papers do not take into account the international mergers that are observed in today’s global economies. The closest paper to ours is Horn and Levinsohn (2001) since both papers use Cournot competition, Nash level trade policies, and a partial equilibrium setting. Our model is different in the sense that we allow firms to merge endogenously and internationally, and to engage in cost reducing R&D. Our merger policies are also different since we compare two different merger policies. We show that trade liberalization occurs as an equilibrium phenomena depending on the efficiency level of R&D, among other results. In a recent paper, Breinlich et al. (2015) study how mergers in a domestic country affect the consumer surplus in the foreign (and domestic) country. They are mainly interested in whether the merger policy of a country is too lenient or too tough for the foreign consumers. They do not have R&D nor optimal tariffs. Unlike them, we allow for international mergers, and compare welfare-increasing merger policy and consumer-surplus-increasing merger policy.

Our paper also falls into the literature that use endogenous mergers. Horn and Persson (2001a), endogenized the mergers as a cooperative coalition game by allowing firms to freely communicate and signing binding contracts. We follow Horn and Persson (2001a) to determine

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\(^8\) Breinlich et al. (2016), criticize the modelling of the competition policy in these papers as regulation agencies cannot determine the number of firms in a free market.
the equilibrium market structures in the first stage of our game. Horn and Persson (2001b) study an endogenous formation model when two firms in two countries face bilateral trade costs. They study neither merger policy nor optimal tariffs since they have trade costs in the model. Using Horn and Persson’s endogenous merger formation model, Ulus and Yildiz (2012) study a differentiated goods oligopoly where firms compete in prices. We study a Cournot oligopoly setup, our firms engage in R&D investment, and our focus is on the use of trade and merger policy. Qiu and Zhou (2006a) explain that endogenous mergers occur if there is cost asymmetry and negative demand shock. Our firms are ex-ante identical but the mergers cause cost-asymmetry among firms due to tariffs. This cost asymmetry affects the mergers in equilibrium.10

Our paper also contributes to the R&D literature. The relation between mergers and R&D has been studied by Stenbacka (1991), Kabiraj and Mukherjee (2000), Davidson and Ferrett (2007), and Matsushima et al. (2013). None of these papers studies the international aspect of the mergers, which bring new insights, as follows. The nature of the mergers, international or national, determines whether there will be cost dispersion among firms due to tariff.11 We show that the international merger, not facing tariff, increases its R&D investment compared to the outsider firm(s) facing tariff. One implication of this observation is on the “merger paradox” (Salant et al. (1983). The merged firm, with its increased R&D, has a cost advantage which increases its profits more than the outsider firm(s). This mitigates or eliminates the “merger paradox.”

The plan of the paper is as follows. In the second section, we describe the benchmark model with trade policy of Nash level tariffs. Then, we add the “welfare-increasing” merger policy to the model. After that, we analyze the “consumer-surplus-increasing” merger policy, and when each

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9 Qiu and Zhou (2006b) also study an endogenous merger where firms are randomly chosen and decide whether to form mergers with another firm.

10 Cabral (2005), Davidson and Mukherjee (2007), Cabral (2003), and Spector (2003), broadly speaking, analyze the effect of free entry on mergers. Saggi and Yildiz (2006) study the link between merger incentives when there are two exporting countries and one importing country. Chaudhuri and Benchekroun (2012) study the effect of bilateral tariff reduction on exogenous mergers and social welfare. None of these papers study the effect of R&D on mergers in an endogenous merger formation, nor do the countries use an active merger policy.

11 Ishida et al. (2011) study how entry of new firms to the market and cost dispersion affects R&D in a Cournot market. They do not study mergers.
country use a different merger policy. Finally, we derive the global welfare maximizing market structures. We discuss our results and some of our assumptions in the conclusion.

2 Model

Two countries are assumed to coexist, denoted by $z = F$ (foreign country), $H$ (home country). In each country, there are two firms which produce a homogenous product. The product can be exported or sold domestically. If exported, there will be a tariff equal to $t_z$ ($z = H, F$), imposed on each unit of product by the importing country. Each firm’s ($i = 1, ..., 4$) ex-ante marginal costs of production is $c$. Assume firms 1 and 2 are located in the home country, and firms 3 and 4 are located in the foreign country. The two markets are segmented and firms compete in the Cournot fashion. Entry to this industry is restricted.

The following symmetric inverse demand system is assumed.

$$p_z(q_z) = \alpha - \sum_{i=1}^{4} q_{iz}, \quad z = H, F$$  \hspace{1cm} (1)

Where $p_z$ is the product price in the home or foreign country, $q_{iz}$ is the quantity of product produced by firm $i$ demanded in country $z$, and $\alpha$ is a parameter which represents the product market size. We assume $\alpha > c$ to guarantee that firms produce positive quantities.

Each firm $i$ engages in R&D activity which reduces its marginal cost, denoted by $e_i$. Perfect appropriability and zero spillover are assumed. Thus, each firm’s effective ex post marginal cost of selling domestically is equal to $c - e_i$, while the effective ex post marginal cost of exporting is $c + t_H - e_i$ for foreign firms and $c + t_F - e_i$ for home firms. The firms are risk neutral and have zero fixed costs. Therefore, the following profit functions $\pi_i$ can be obtained.

$$\pi_i = \begin{cases} (p_H - (c - e_i)).q_{iH} + (p_F - (c - e_i) - t_F).q_{iF} - ke_i^2, & i = 1, 2 \\ (p_F - (c - e_i)).q_{iF} + (p_H - (c - e_i) - t_H).q_{iH} - ke_i^2, & i = 3, 4 \end{cases}$$  \hspace{1cm} (2)

\[12\] We follow Horn and Persson (2001b) and Ulus and Yildiz (2012) with modelling two firms in two countries.

\[13\] We assume that merger cost synergies, such as exogenous fixed cost savings as a result of mergers, are absent here. The reason is that we want to show that firms will merge even in the absence of such exogenously given synergies. The results must be robust to small cost synergies by continuity of profit functions.
In the above profit functions, a quadratic cost function, $ke_i^2$, for R&D expenditure is assumed where $k$ is a positive scalar and can be interpreted as the firm’s efficiency in conducting R&D in the production process. In order to guarantee interior solutions, we make the following assumption.

**ASSUMPTION 1**

*Firms are not too much efficient in conducting R&D, that is, $k > \bar{k}(= 2.266485386)$.*

Firms are allowed to merge nationally or internationally (a la Horn and Persson (2001a)). They do so to gain higher market concentration and in the case of cross-border merger(s) to save on tariff costs since an international merger does not pay any tariff.\(^{14}\)

The basic game is set as follows. In the first stage, firms make their merger decisions endogenously a la Horn and Persson (2001a). In the second stage, after observing the new market structure, the countries set their Nash level tariffs simultaneously. In the third stage, firms decide the level of R&D. In the fourth stage, firms produce a homogenous product and engage in a Cournot competition. With this formulation, Nash level tariffs respond to industry level changes (market structure change) but not firm level changes (such as firm’s R&D and output decisions). To check robustness, in a variant of the model, we allow countries to set the optimal Nash tariff level at the beginning, and they are not allowed to change those tariff levels (i.e., tariff commitment. Our qualitative results are the same, and we report the results of this variant of the model in various footnotes along the paper.\(^{15}\)

2.1 Subgame Perfect Nash Equilibrium

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\(^{14}\) Since goods are homogenous and marginal costs are linear, an international merger can divide its production among its locations in any way it wants. This is enough to make them avoid paying tariffs (tariff-jumping argument). R&D can be done in one location and the technology can be transferred to any location without cost. One can generalize the model with non-linear costs and there will still be tariff jumping incentives for some parameter ranges but this will only complicate the model without bringing any new insights.

\(^{15}\) It seems like there is no consensus in the literature about using pre-committed tariffs. Hence, we also allowed different order for the game; namely, R&D was chosen first, then the optimal Nash level tariffs. The qualitative results have not changed for that model neither. Given that the agents can rationally forecast the moves in the later stages, we doubt that the other changes in the timing of the game will change the qualitative results.
We start solving the game from the last stage. In the last stage, we find Cournot outputs given the market structures, tariffs, and R&D investments. Then, we derive Nash level R&D, then Nash level tariffs, and finally the equilibrium market structures. Since, we will derive equilibrium market structures, first we list all possible market structures below.

1. Status quo (no mergers): \(\{S\} = \{1,2,3,4\}\)
2. Triopoly with one national merger: \(\{N_1\} = \{12,3,4\}; \{N_2\} = \{1,2,34\}\)
3. Triopoly with one international merger: \(\{I_1\} = \{13,2,4\}; \{I_2\} = \{1,3,24\}; \{I_3\} = \{14,2,3\}; \{I_4\} = \{1,4,23\}\)
4. Symmetric national duopoly: \(\{NN\} = \{12,34\}\)
5. Symmetric international duopoly: \(\{II_1\} = \{13,24\}; \{II_2\} = \{14,23\}\)
6. Asymmetric international duopoly: \(\{IN_1\} = \{123,4\}; \{IN_2\} = \{124,3\}; \{IN_3\} = \{1,234\}; \{IN_4\} = \{124,3\}\)
7. Monopoly: \(\{M\} = \{1234\}\)

We assume that monopoly is not allowed by the antitrust regulators. This assumption is also maintained in Horn and Persson (2001b) and Ulus and Yildiz (2012).

2.2 Equilibrium output and R&D

In this subsection, we show how we derive the equilibrium output and R&D for the market structure \(\{IN_1\}\). The equilibrium for \(\{IN_2\}, \{IN_3\}, \text{ and } \{IN_4\}\) are symmetric. The calculations for the other market structures are similar and delegated to the Appendix.

In the last stage, given that the Nash level tariff rate \(t_N\) and optimal R&D level \(e\), the profit maximization problem is:

\[
\pi_{IN} = \sum_{z=H,F} \left( \alpha - \sum_{i=IN,A} q_{iz} - (c - e_{IN}) \right) \cdot q_{INz} - ke_{IN}^2
\]

\[\tag{3}\]

\[\text{In this sense, the benchmark model can be said to have a merger policy where monopoly is not allowed. It is well-known that monopoly will be the EMS in these models if it is allowed.}\]
\[ \pi_4 = \left( \alpha - \sum_{i=IN,A} q_{IF} - (c - e_4) \right) q_{AF} + \left( \alpha - \sum_{i=IN,A} q_{IH} - (c - e_4 - \tau_H) \right) q_{AH} - ke_4^2 \] 

(4)

where subscript \( IN \) denotes the merger’s variables and subscript 4 denotes the outsider firm’s variables. Given the tariff level and the R&D investments, the first order conditions are:

\[ \begin{align*}
\frac{\partial \pi_{IN}}{\partial q_{INH}} &= \alpha - c + e_{IN} - 2q_{INH} - q_{AH} = 0 \\
\frac{\partial \pi_4}{\partial q_{AH}} &= \alpha - c + e_4 - 2q_{AH} - q_{INH} - \tau_H = 0 \\
\frac{\partial \pi_{IN}}{\partial q_{INF}} &= \alpha - c + e_{IN} - 2q_{INF} - q_{AF} = 0 \\
\frac{\partial \pi_4}{\partial q_{AF}} &= \alpha - c + e_4 - 2q_{AF} - q_{INF} = 0
\end{align*} \] 

(5)

By solving them simultaneously, we get the quantities as a function of R&D and tariff:

\[ \begin{align*}
q_{INH} &= \frac{\alpha - c + 2e_{IN} - e_4 + \tau_H}{3} \\
q_{AH} &= \frac{\alpha - c + 2e_4 - e_{IN} - 2\tau_H}{3} \\
q_{INF} &= \frac{\alpha - c + 2e_{INF} - e_4 + \tau_H}{3} \\
q_{AF} &= \frac{\alpha - c + 2e_4 - e_{INF} - 2\tau_H}{3}
\end{align*} \] 

(6)

Next step is to substitute the derived quantities above into the profit equations and derive FOCs for R&D investment levels:

\[ \begin{align*}
\frac{\partial \pi_{IN}}{\partial e_{IN}} &= \frac{8(\alpha - c - e_4) - 2(9k - 8)e_{IN} + 4\tau_H}{9} = 0 \\
\frac{\partial \pi_4}{\partial e_4} &= \frac{8(\alpha - c - e_{IN} - \tau_H) - 2(9k - 8)e_4}{9} = 0
\end{align*} \] 

(7)

By solving the equations above simultaneously, we get:

\[ \begin{align*}
e_{IN} &= \frac{2(2(3k-4)(\alpha-c)+3k\tau_H)}{(9k-4)(3k-4)} \\
e_4 &= \frac{4(3k-4)(\alpha-c-\tau_H)}{(9k-4)(3k-4)}
\end{align*} \] 

(8)

Plugging the optimal R&D investments back into the quantity and price equations, we get the optimal level of quantities, R&D, and prices for the market structure \{IN\}. We summarize this in Lemma 1 below.
**Lemma 1:** In the market structure \{IN\}, the equilibrium outcome of quantities, R&D for the asymmetric international merger and the outsider, and the prices in each market as a function of tariff are:

\[ q_{INH} = \frac{(9k(3k-4)(\alpha-c-tH)+4tH(3k+2))}{3(9k-4)(3k-4)} \]

\[ q_{INF} = \frac{(9k(3k-4)(\alpha-c) + 8tH(3k-1))}{3(9k-4)(3k-4)} \]  \hspace{1cm} (9)

\[ e_{IN} = \frac{2(3k-4)(\alpha-c)+3tH}{(9k-4)(3k-4)} \]

\[ q_{FH} = \frac{(9k(3k-4)(\alpha-c-2tH)-2tH(3k+8))}{3(9k-4)(3k-4)} \]

\[ q_{AF} = \frac{(9k(3k-4)(\alpha-c-2tH)-2tH(15k-8))}{3(9k-4)(3k-4)} \]

\[ e_{4} = \frac{4((3k-4)(\alpha-c-t)-2tH)}{(9k-4)(3k-4)} \]

### 2.3 Nash Level Tariff and Trade Policy Equilibrium

In our model, each country, simultaneously, decides on the level of tariff that maximizes its own welfare after observing the market structure.

Let us call the tariff imposed by the two governments by \( t_{z}^{m} \)

\[ z \in H, F; \quad \text{and} \quad m \in \{S, N_{1}, N_{2}, I_{1}, I_{2}, I_{3}, I_{4}, NN, IN_{1}, IN_{2}, IN_{3}, IN_{4}\} \]

The firms do not pay tariff in {II} so it is not included in the above set.

The optimization problem facing the countries is as follows.

\[ \max_{t_{z}^{m}} W_{z}^{m} = CS_{z}^{m} + PS_{z}^{m} + T_{z}^{m} \]  \hspace{1cm} (10)

Where \( CS, PS, \) and \( T \) stand for consumer surplus, producer surplus, and tariff revenue, respectively. The welfare, denoted by \( W \), is a function of \( k, c, \) and \( \alpha \). The welfare for each market structure is written in the appendix. We calculate the welfare by using the MAPLE software. In the calculations, we assumed that if two firms of country H is in a three-firm merger, then H gets two thirds of the producer surplus, and country F gets one third of the producer surplus.

In Lemma 2, we write down the optimal tariff for the market structure \{IN\}. Only the outsider firm faces tariff here.
LEMMA 2 The Nash level tariff in the market structure \{IN_1\} is:

\[ t_{IN_1}^H = \frac{108k(3k - 4)(9k^2 - 11k + 4)}{12393k^4 - 39204k^3 + 40644k^2 - 16800k + 2368}(\alpha - c) \]  (11)

We report the optimal Nash tariff levels for each market structure in the figure below. The market structure \{N\} is not symmetric so we report the country H and F’s optimal tariff rate separately, and denote them as Nh and Nf, respectively. We note that the highest optimal tariff rate is in \{S\}; therefore, the optimal tariff rate will always decrease if a merger occurs. This is in line with WTO rules.

Given that, we have calculated stage 4, 3, and 2, we can now move to stage 1 of our game.

2.4. Endogenous Merger Formation of Horn and Persson (2001a):
In Horn and Persson’s (2001a) model, firms freely communicate and sign binding contracts to form mergers. Merger formation, therefore, is modeled as an endogenous cooperative coalition game, and it follows from three basic components: (i) Decisive Owners, (ii) Dominance Relation, and (iii) Equilibrium Market Structures.

(i) Decisive Owners: Decisive firm owners, with respect to two market structures, are those whose firm’s status changes as the market moves from one structure to the other. For example, in comparing the status quo where there are four firms with the market structure under which firms 1 and 2 merge, decisive firms are only firms 1 and 2. It is as if firm 1 and firm 2 go to a room to negotiate their possible merger. Firm 3 and 4 are outside the room; hence, they cannot affect the negotiations although their profits will be affected from the merger. Therefore, they are not in the decisive group. We note that side payments are not allowed in this approach so firm 3 and firm 4 cannot alter the negotiations by paying to the negotiating parties.

Moreover, in a binary comparison of two market structures, there can be more than one decisive group. For instance, when comparing status quo and duopoly with two international mergers of say {13, 24}, there will be two separate decisive groups which are {1,3} and {2,4}.

(ii) Dominance Relation: If the summation of profits of decisive firms under one market structure is greater than that under another one, the former market structure dominates the latter one. The relation is denoted by $\text{dom}$. In the case of more than one decisive groups, the summation of firms’ profits within each decisive group must be greater for one market structure than those under the other one. Then, the former market structure dominates the latter one. All market structures are binarily compared by this dominance relation.

(iii) The Equilibrium Market Structures (EMS): The set of equilibria is defined as all the market structures which are not dominated by any other market structure. In other words, under the EMS, the summation of profits of the decisive group is not lower than that under any other market structures. For example, {NN} dominates {I} if and only if the combined profit of the decisive group i.e. {1, 2, 3, 4} is greater under {NN} than under {I}. If that is the case, {I} is not in the core because it is dominated by at least one other structure.
In completing this approach, Horn and Persson (2001a) make one more assumption. The merger participants are free to choose any division of the profits subject to the constraint that the summation of the firms’ profits is equal to the merger’s profits.

Now, we are ready to give our equilibrium outcome for the subgame perfect equilibrium, which depends on \( k \), the R&D efficiency parameter. When R&D is efficient, R&D’s cost reducing effect plays a major role along with tariff-jumping effects in the model outcome.

We make the following assumption to ease the exposition since \((\alpha-c)\) appears in almost in all equations in the cutoff values.

**Assumption 2:** \((\alpha-c)=1\).

The EMS is presented in proposition 1.

**Proposition 1** –

Let \( \bar{k}=3.877711501 \). The set of EMS is

(i) \{IN\} if \( \bar{k} < k < \bar{k} \);

(ii) \{II\} if \( k > \bar{k} \).

The equilibrium market structure is the asymmetric international duopoly, \{IN\}, when R&D is efficient. The symmetric international duopoly, \{II\}, is the equilibrium when R&D is above the cutoff level \( \bar{k} \), that is, when R&D is inefficient.

In what follows, we explain the mechanism that gives this equilibrium outcome. For this, we will look into the role of R&D on the merger paradox. It is well known in the literature that there is a free-riding effect for the outsider firm since it enjoys higher profits due to the market concentration (Salant et al. 1983). We show that the efficient R&D mitigates or eliminates this effect.

**Proposition 2** Suppose countries use Nash level tariffs.

\[17\] If the countries set the optimal tariff rate at the beginning of the game, and cannot change it throughout the game, we still find the same result in Proposition 1 except that the cutoff value \( \bar{k}=3.02205446 \).
(i) In market structure {I} compared to {S}
   a) Each outsider invests more in R&D iff $k > 3.969$
   b) Each outsider invests less in R&D iff $k < \leq 3.969$.

(ii) In market structure {IN} compared to {S}
    a) Each outsider invests more in R&D iff $k > 3.470$
    b) Each outsider invests less in R&D iff $k < \leq 3.470$.

In the market structure {IN}, there is marginal cost dispersion due to tariff between the merger and the outsider firm. The merger is the cost efficient firm and hence produces more. To enjoy the benefit of cost advantage, it increases its R&D even further when R&D is efficient (See Figure 1). Then, the best response of the outsider firm is to decrease its R&D (See Figure 2). This decreases the outsider firm’s profit; hence, the outsider firm loses the free-rider effect when R&D is efficient. This is one of the main mechanisms deriving our EMS results in proposition 1.

In fact, as R&D becomes more efficient, the profit of the merger approaches to the monopoly profit.

When R&D is inefficient, the merger paradox is in effect in {IN}; hence, each firm has an incentive to leave the three-firm merger. When one firm leaves though, it has to face a tariff cost so it is profitable to merge with the outsider to save on tariff cost. Therefore, {II} becomes the EMS.

Figure 1. Merger’s R&D level in {I}, {II}, {IN}, {M}, and firm level R&D in {S}
R&D literature (e.g., Stenbacka (1991), Kabiraj and Mukherjee (2000), Ishida (2011)) do not study the effect of international mergers on R&D and our proposition 2 fills this gap.

Figure 2. Outsider’s R&D levels relative to Status Quo.
In this section, we introduce merger policy to our model; however, merger policies are not uniform across different countries. For example, Canadian Competition Act allows even monopoly if the cost savings from the merger exceeds the loss in consumer surplus. In other words, as long as welfare is greater, the merger is approved (Ross and Winter 2005). On the other hand, in the European Union, mergers should not harm consumers by lowering consumer surplus. This is one of their main criteria for approving mergers (Ross and Winter 2005). We add these two merger policies one by one to our model, and compare them.

We note that regulation agencies, such as Competition Bureau in Canada, cannot determine the optimal number of firms in the market. They only evaluate whether the merger will make things better or worse than the status-quo given the law (such as Competition Act in Canada). Hence,

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18 The famous Super Propane case in Canada was approved although the merger was creating monopoly in some local markets.
while determining the optimal number of firms in the market are useful theoretical exercises, we believe that our approach is more relevant for real-world application.\(^{19}\)

**3.1 Welfare Increasing Merger Policy (Canadian Case)**

In this subsection, we assume that regulatory agencies in each country do not authorize mergers if they decrease the country’s welfare compared to the status-quo \{S\}. In the first stage of the game, firms form mergers a la Horn and Persson (2001a). Then, regulatory agency in each country decides whether to approve the merger if the merger is international. If it is a national merger, only the regulation agency of that country makes a decision. If it is not approved, the merging firms pay a penalty. If a merger is approved, then each country chooses their Nash level tariff. The firms choose R&D investment, and then compete a la Cournot in the markets.

Firms, rationally expecting how regulation agencies will behave, never form mergers that are denied in equilibrium. Therefore, we prove in the appendix that the only acceptable market structures are \{I\}, \{II\}, \{IN\}, \{M\}, or \{S\} since firms may choose not to merge. We derive the subgame perfect equilibrium, and summarize the results in Proposition 3.

**PROPOSITION 3:** The equilibrium market structure when countries use trade and welfare-increasing merger policies is:\(^{30}\)

i. \{M\} if \(\bar{k} < k < 2.627523310\);

ii. \{IN\} if \(2.627523310 < k < 3.247600743\);

iii. \{II\} if \(3.247600743 < k < 9.532\);

iv. \{I\} if \(9.532 < k < 21.18540616\);

v. \{S\} if \(k > 21.18540616\).

When \(k\) is extremely efficient, \{M\} is the EMS. It may seem surprising that \{M\} results in higher welfare than \{S\}. The reason is that while the total industry output is higher under \{S\}, per firm output is higher under \{M\}; hence, the monopoly invests more on R&D, and becomes more cost-efficient than any other firm under \{S\} as seen in Figure 1. This limits the increase in

\(^{19}\) In Horn and Levinsohn (2001), competition policy determines the optimal number of firms.

\(^{30}\) In the variant of the model, where the optimal tariff is set at the \{S\} level throughout the game by each country, we get the same qualitative results, in the sense that only the cutoff values changes slightly except that the cutoff value for \{S\} and \{I\} change considerably; namely, \{S\} is the EMS if \(k > 10.77\) (approximately) for that model.
product price under \{M\} and the decrease in consumer surplus. As a result, the welfare is higher under \{M\} only if R&D is extremely efficient.

When \( k \) is very efficient, \{IN\} dominates the other approved market structures (except \{M\} which is welfare decreasing in this range). The tariff cost asymmetry causes the merger to increase its R&D, which in turn increases the profits. This increase in profits compensates the loss in consumer surplus due to market concentration compared to \{S\}. Therefore, \{IN\} is welfare increasing.

When \( 3.247600743 < k < 9.532 \), firms prefer \{IN\} over \{II\} for part of this range but \{IN\} is not acceptable to the country which has two firms in the merger. Hence, firms merge to form \{II\} which is welfare increasing compared to \{S\}, and approved by both countries.

When \( k \) is moderately efficient, \( 9.532 < k < 21.18540616 \), \{I\} is the EMS although \{II\} is preferred over \{I\} by the firms. However, \{II\} is welfare decreasing compared to \{S\}. In \{II\}, although firms’ profits increase and the production is more efficient due to more R&D compared to \{S\}, the loss in consumer surplus outweighs these gains.

When \( k \) is inefficient, \( k > 21.18540616 \), the other market structures are welfare decreasing compared to \{S\} except \{I\}. However, firms prefer \{S\} over \{I\} (possibly due to merger paradox); hence, \{I\} is not the EMS. The decrease in consumer surplus in the other market structures (other than \{I\}) is not compensated by profit increase and/or tariff revenue. While firms prefer \{II\} in this range, it is not approved by the competition regulators. Therefore, \{S\} is the EMS.

We note that as R&D becomes less efficient, market becomes less concentrated.

### 3.2 Consumer-surplus-increasing Merger Policy (The European Union Case)

In this subsection, the game is same as the welfare-increasing merger game except that mergers are accepted only if they do not decrease the consumer surplus.
Proposition 4: The equilibrium market structure is:\footnote{In the variant of the model, where the optimal tariff is set at the \{S\} level throughout the game by each country, we get the same qualitative results, in the sense that only the cutoff values changes slightly except that the cutoff value for \{S\} and \{I\} change considerably; namely, \{S\} is the EMS if \(k > 3.38\) (approximately) for that model.}

i. \(\{\Pi\}\) if \(\bar{k} < k < 2.830838672\);

ii. \(\{I\}\) if \(2.830838672 < k < 21.18540616\);

iii. \(\{S\}\) if \(k > 21.18540616\).

As expected, monopoly cannot be EMS since it always decreases consumer surplus.

When \(k\) is efficient, R&D is very high under \(\{\Pi\}\) and there is no tariff. Hence, the consumer surplus is higher than the one under \(\{S\}\) despite the lower market concentration.\footnote{For \(k < 2.538103665\), firms forming \(\{\Pi\}\) have an incentive to cheat on each other since each merger prefers \(\{I\}\) market structure compared to \(\{\Pi\}\). Also, each country prefers \(\{I\}\) over \(\{\Pi\}\) in terms of consumer surplus (but the other way around for welfare). Hence, even after agreeing on \(\{\Pi\}\), each pair of firms, instead of simultaneously submitting the merger approval application, would like to submit their application first. This may result in the market structure \(\{I\}\), and \(\{\Pi\}\) may not arise for this range depending on the modelling assumption. We assume that if regulation agencies receive two merger applications, it will compare them both with \(\{S\}\) and approve both if overall the consumer surplus is increasing.}

When \(k\) is moderately efficient, \(\{I\}\) is the equilibrium market structure. R&D is not that efficient so the efficiency gain of R&D only overcomes the loss in consumer surplus from having three firms under \(\{I\}\) compared to four firms under \(\{S\}\).

When \(k > 21.18540616\), \(\{I\}\) is preferred by the two countries, however, the firms do not have incentive to form one international merger due to merger paradox. Being an outsider is better when R&D is very inefficient. Therefore, \(\{S\}\) is the equilibrium.

As in the welfare-increasing merger policy case, as R&D becomes less efficient, market becomes less concentrated.

3.3 Two Types of Merger Policy Coexist

One question is that how the set of EMS looks like if home country (symmetrically foreign country) uses welfare-increasing merger policy while the foreign country (symmetrically home country) uses consumer surplus-increasing merger policy. From proposition 3, home country allows the market structures of \(\{M\}\), \(\{I\}\), \(\{\Pi\}\), \(\{IN\}\), and \(\{S\}\) to form. From proposition 4,
foreign country allows the market structures \{I\}, \{II\}, and \{S\}. The outcome of this case is presented in Proposition 5.

**Proposition 5**

If one country uses welfare-increasing merger policy and the other one uses consumer surplus-increasing merger policy, the set of EMS is

i. \( \{II\} \) if \( \bar{k} < k < 2.830838672 \);  
ii. \( \{I\} \) if \( 2.830838672 < k < 21.18540616 \);  
iii. \( \{S\} \) if \( k > 21.18540616 \).

The consumer-surplus-increasing merger policy is the binding policy when different merger policies are used. For the efficient \( k \), \{M\} and \{IN\} are the EMS in the welfare increasing merger policy case; however, these are consumer surplus decreasing, and are not approved by the foreign country.\(^{23}\) The proof is similar to the proof of Proposition 3 and 4 so it is not added.

Proposition 3, 4 and 5 imply that we have a testable empirical prediction. As R&D becomes less efficient, market becomes less concentrated. Hence, we must observe more international mergers in the industries with efficient R&D, and no (or fewer) mergers in the industries with inefficient R&D.

**3.4 Global Welfare (Trade Policy Game)**

Now, we discuss the global welfare and analyze whether the optimal global welfare is achieved under these different models. Global welfare is calculated as the sum of the two countries’ welfare.

**PROPOSITION 6** – When countries impose Nash level tariffs, the market structure that maximizes the global welfare is:

a) \( \{II\} \) iff \( \bar{k} < k < 5.973 \)  
b) \( \{I\} \) iff \( k > 5.973 \)

\(^{23}\) \{IN\} is consumer surplus decreasing for both countries regardless of which country has the outsider. This is proven in the proof of Proposition 4.
We note that since these market structures are symmetric, they also maximize each country’s welfare.

Figure 3. Global Welfare under different market structures.

Corollary 1:

a) The welfare increasing merger policy coincides with the global welfare when
$3.247600743 < k < 5.973$ or $9.532 < k < 21.18540616$.

b) The consumer-surplus-increasing merger policy coincides with the global welfare when
$k < k < 2.830838672$ or $5.973 < k < 21.18540616$.

At first, one might think that “welfare-increasing” merger policy must be the global welfare increasing merger policy but consumer-surplus-increasing merger policy does a better job for a wider range of parameters. The consumer-surplus-increasing merger policy selects the market
structures that increase consumer surplus compared to the status-quo. Among those market structures, firms choose the producer surplus increasing market structure given the private incentives. Given this interaction of the private incentives and the restriction of the merger policy, the welfare is higher for both consumers and producers compared to the status-quo; therefore, consumer-surplus-increasing merger policy maximizes global welfare for a wider range of parameters.

Since {I} and {II} are global welfare maximizing market structures, we compare them by decomposing their welfare into consumer surplus, producer surplus, and tariff revenue as R&D efficiency varies. {I} gives higher consumer surplus compared to {II} due to less market concentration. Figure 4a shows that the difference in consumer surplus between {I} and {II} increases as R&D becomes less efficient. {II} gives higher producer surplus compared to {I} due to more market concentration. Figure 4b shows that the difference in producer surplus between {I} and {II} decreases as R&D becomes less efficient except for a small parameter range. Figure 4c shows that tariff revenue increases as R&D becomes less efficient under {I}. The reason is that the merger does not invest too much in R&D when R&D is less efficient. As a result, the outsiders have a sizeable market share, and tariff revenue is relatively high for each country. When R&D is more efficient, on the other hand, the merger increases its market share at the expense of the outsiders. This results in low tariff revenue. There is no tariff revenue under {II}.

{II} is the global welfare maximizing market structure when R&D is efficient since the producer surplus difference between {I} and {II} outweighs the consumer surplus difference and tariff revenue effect. As R&D becomes less efficient, the consumer surplus difference and tariff revenue effect increase while the producer surplus difference decreases. This makes {I} the global welfare maximizing structure.

Figure 4. Welfare Decomposition

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24 The difference increases until approximately $k=2.75$, then decreases.
Discussion and Conclusion

In this section, we will discuss some of the modelling assumptions and extensions.

Using Horn and Persson’s endogenous merger setting in the first stage of the game necessitates using two firms in each country (e.g., Horn and Persson (2001b), Ulus and Yildiz (2012)). Otherwise, the calculations would get complex. We believe that even with this setting, we get novel results and intuition such as how R&D affects mergers and the EMS.

One might add additional variations such as cost heterogeneity among firms or exogenous variable cost saving after merger. If the cost heterogeneity is small, our qualitative results will not change due to continuity.

Since there is no consensus on the timing of determining the Nash level tariffs, we checked robustness of our model, by assuming a fixed tariff rate set in the beginning of the game. This tariff rate is the optimal tariff rate for the status-quo market structure, and countries are not allowed to change this. We find that our qualitative results are same but only the cutoff values changes.

We made another robustness check with our benchmark model. We change the stage of the games in the sense that R&D is decided first, and Nash tariffs are set afterwards. This has not changed our qualitative results. We do not expect that other changes in the timing of the game will result in any significant change since agents optimally forecast the actions in the later stages.
We note that the optimal tariff levels are less than the status-quo optimal tariff level; hence, each country obeys the WTO rules when setting the optimal tariff rate in our model.

With this paper, we fill a gap in the literature since different merger policies have not been compared to our best knowledge. Given that billions of dollars of mergers occur each year, we believe that finding an optimal merger policy has practical implications that will raise the welfare of countries.

Reference


**APPENDIX**

Here, we will find the equilibrium quantities, R&D, and Nash level tariffs for each market structure as we did in the main text for the market structure \{IN\}.

**No merger \{S\}:**

Here, we show how we derive the equilibrium output and R&D for the market structure \{S\}. The calculations for the other market structures are similar and summarized.

In the last stage, given that the Nash level tariff rates $t_H$ and $t_F$, and optimal R&D level $e$, the profit maximization problem for firm 1 is:

$$\max_q \pi_1 = \left( \alpha - \sum_{i=1}^{4} q_{1H} - (c - e_1) \right) q_{1H} + \left( \alpha - \sum_{i=1}^{4} q_{1F} - (c - e_1) - t_F \right) q_{1F} - k e_1^2$$

(12)

Where subscript $i$ denotes the firm $i$’s variables. The profit functions for the other firms are similar to that of firm 1 and thus not displayed. Given the tariff levels and the R&D investments, the first order conditions are:
By solving the equation above simultaneously, we get:

For R&D investment levels:

Next step is to substitute the derived quantities

By solving them simultaneously, we get the quantities as a function of R&D and tariff:

where \( q_{1H} \) and \( q_{1F} \) are the quantity of product produced by firm \( I \) sold in country \( H \) and country \( F \) (firm \( I \)’s export), respectively.

Next step is to substitute the derived quantities above into the profit equations and derive FOCs for R&D investment levels:

By solving the equation above simultaneously, we get:

\[ e_1 = e_2 = \frac{4(25k-8)(\alpha-c)+10kt \mu-8t \mu}{(5k-8)(25k-8)} \]
\[ e_3 = e_4 = \frac{4(25k-8)(\alpha-c)+10kt \mu-8t \mu}{(5k-8)(25k-8)} \]
Plugging the optimal R&D investments back into the quantity and price equations, we get the optimal level of quantities, R&D, and prices as a function of tariff for the market structure \( \{ S \} \).

\[
q_{1H} = \frac{(25k(5k-8)(\alpha-c)+2(125k^2-120k+32)t_H-4(65k-24)t_F)}{5(5k-8)(25k-8)},
\]

\[
q_{1F} = \frac{(25k(5k-8)(\alpha-c)-375k^2-460k+96)t_F+16(15k-4)t_H}{5(5k-8)(25k-8)} \tag{19}
\]

\[
p_H = \frac{5\alpha(5k-8) + 100c + 2(25k - 4)t_H + 8t_F}{5(25k - 8)}
\]

\[
p_F = \frac{5\alpha(5k-8) + 100c + 2(25k - 4)t_F + 8t_H}{5(25k - 8)}
\]

**One National Merger \( \{ N \} \):** Two firms in the same country merge and the firms in the other country remain individual competing units. The result will be a market structure of triopoly with one national merger (i.e. \( \{ N_1 \} \) or \( \{ N_2 \} \)). The profit maximization problem of the firms under \( \{ N_1 \} \) market structure is described. By symmetry, \( \{ N_2 \} \) would result in symmetric outcomes and is therefore not displayed.

\[
\max_{q, e} \pi_N = \left( \alpha - \sum_{i=N,3,4} q_{ih} - (c - e_N) \right) \cdot q_{NH} + \left( \alpha - \sum_{i=N,3,4} q_{if} - (c - e_N) - t_F \right) \cdot q_{NF} - ke_N^2
\]

\[
\max_{q, e} \pi_j = \left( \alpha - \sum_{i=N,3,4} q_{jf} - (c - e_j) \right) \cdot q_{jF} + \left( \alpha - \sum_{i=N,3,4} q_{ih} - (c - e_j) - t_H \right) \cdot q_{jH} - ke_j^2, \quad j = 3,4 \tag{21}
\]

Where subscript \( N \) means the related variable belongs to the merger under \( \{ N \} \) market structure and variables with subscript \( j \) belongs to the outsider firms. Solving the profit maximization problem of the three firms, we obtain the following results.
One International Merger \{I\}: Consider a triopoly market structure in which two firms from different countries merge and the other firms remain outsiders. The result will be one of the structures of \{I_1\}, \{I_2\}, \ldots, \{I_4\}. The profit maximization problem of the firms under \{I_1\} market structure is described below. The other three cases are symmetric, and can be derived easily.

\[
\max_{q,e} \pi_I = \sum_{z=H,F} \left( \alpha - \sum_{l=1,2,4} q_{lz} - (c - e_l) \right) \cdot q_{iz} - ke_i^2
\]  

(23)

\[
\max_{q,e} \pi_2 = \left( \alpha - \sum_{l=1,2,4} q_{lH} - (c - e_2) \right) \cdot q_{2H} + \left( \alpha - \sum_{l=1,2,4} q_{lF} - (c - e_2) - t_F \right) \cdot q_{2F} - ke_2^2
\]  

(24)
\[
\max_{q,e} \pi_4 = \left( \alpha - \sum_{i=1,2,A} q_{iF} - (c - e_4) \right) \cdot q_{4F} \\
+ \left( \alpha - \sum_{i=1,2,A} q_{iH} - (c - e_4) - t_H \right) \cdot q_{4H} - k e_4^2 
\]

(25)

Where subscript \( I \) means the related variable belongs to the merger under market structure \{1\} and variables with subscript \( j \) belongs to the outsider firms. We get the following results by solving the profit maximization problems of the three firms.

\[
e_1 = \frac{3(2k - 3)(\alpha - c) + k(t_H + t_F)}{(2k - 3)(8k - 3)} 
\]

(26)

\[
e_2 = \frac{3(2k - 3)(\alpha - c) + 2kt_H - 3(2k - 1)t_F)}{2(2k - 3)(8k - 3)} 
\]

\[
e_4 = \frac{3(2k - 3)(\alpha - c) + 2kt_F - 3(2k - 1)t_H)}{2(2k - 3)(8k - 3)} 
\]

\[
q_{iH} = \frac{(16k(2k - 3)(\alpha - c) + (32k^2 - 30k + 9)t_H + 3(10k - 3)t_F)}{8(2k - 3)(8k - 3)} 
\]

\[
q_{iF} = \frac{(16k(2k - 3)(\alpha - c) + (32k^2 - 30k + 9)t_F + 3(10k - 3)t_H)}{8(2k - 3)(8k - 3)} 
\]

\[
q_{2H} = \frac{(16k(2k - 3)(\alpha - c) + (32k^2 - 30k + 9)t_H - 3(22k - 9)t_F)}{8(2k - 3)(8k - 3)} 
\]

\[
q_{2F} = \frac{(16k(2k - 3)(\alpha - c) - 3(32k^2 - 38k + 9)t_F + 3(10k - 3)t_H)}{8(2k - 3)(8k - 3)} 
\]

\[
q_{4H} = \frac{(16k(2k - 3)(\alpha - c) - 3(32k^2 - 38k + 9)t_H + 3(10k - 3)t_F)}{8(2k - 3)(8k - 3)} 
\]
\[ q_{AF} = \frac{16k(2k-3)(\alpha - c) + (32k^2 - 30k + 9)t_F - 3(22k - 9)t_H}{8(2k - 3)(8k - 3)} \]

**Two National Mergers \{NN\}:** The profit maximization problem of the firms under duopoly market structure under which firms 1 and 2 and firms 3 and 4 merge can be written as follows.

\[
\pi_{NH} = \left( \alpha - \sum_{i=NH,NF} q_{iH} - (c - e_{NH}) \right) \cdot q_{NHH} \\
+ \left( \alpha - \sum_{i=NH,NF} q_{iF} - (c - e_{NH} - t_F) \right) \cdot q_{NFH} - k e_{NH}^2
\]

(27)

\[
\pi_{NF} = \left( \alpha - \sum_{i=NH,NF} q_{iF} - (c - e_{NF}) \right) \cdot q_{NFF} \\
+ \left( \alpha - \sum_{i=NH,NF} q_{iH} - (c - e_{NF} - t_H) \right) \cdot q_{NFH} - k e_{NF}^2
\]

(28)

Where variables with subscript \( NH \) are related to the merger between firms 1 and 2, and variables with subscript \( NF \) belong to the merger between firms 3 and 4. Solving the profit maximization problem for the two firms results in below equations for the optimum levels of product and R&D expenditure (the two firms are symmetric and thus only firm \( NH \)'s variables are displayed).

\[
e_{NH} = \frac{2(2(3k - 4)(\alpha - c) + 3kt_H - 2(3k - 2)t_F)}{(3k - 4)(9k - 4)}
\]

(29)

\[
q_{NHH} = \frac{9k(3k - 4)(\alpha - c) + (27k^2 - 24k + 8)t_H - 2(15k - 8)t_F}{3(3k - 4)(9k - 4)}
\]

\[
q_{NFH} = \frac{9k(3k - 4)(\alpha - c) - 2(27k^2 - 33k + 8)t_F + 8(3k - 1)t_H}{3(3k - 4)(9k - 4)}
\]

**Two International Mergers \{II\}:** If two multinational firms emerge as a result of merger, we face with either market structures of \{II\} or \{II\}. Let’s consider the market structure \{II\}
which is a duopoly in which firms 1 and 3 merge and firms 2 and 4 merge, and solve their profit maximization problems. The other structure will result in symmetric outcomes.

\[ \pi_i = \sum_{z=H,F} \left( \alpha - \sum_{j=I_{13}I_{24}} q_{jz} - (c - e_i) \right) q_{iz} - ke_i^2, \quad i = I_{13}I_{24} \]  

(30)

Where \( I_{13} \) stands for the merge of firms 1 are 3, and \( I_{24} \) represents the merge of firms 2 and 4.

Solving the first order conditions of the profit functions above, we get the following results.

\[ q_{iz} = \frac{3k(a-c)}{9k-4}, \quad i = I_{13}I_{24} \text{ and } z = H, F \]  

(31)

\[ e_i = \frac{4(a-c)}{9k-4}, \quad i = I_{13}I_{24} \]  

(32)

\[ p_{III} = p_{II} = \frac{3k(a+2c)-4a}{9k-4} \]  

(33)

**Proof of Lemma 2:**

The model’s variables under different market structures are derived in Lemma 1 as functions of tariffs. Plugging the optimal variables back into the welfare functions, we can derive the Nash level tariffs which maximize the countries’ welfare for each market structure. The countries optimization problem’s solution is shown for the market structure \( \{\text{IN}_1\} \). The procedure is similar to this for the other market structures and therefore they are not presented. Under \( \{\text{IN}_1\} \), only the home country can impose Nash level tariff.

The welfare functions under \( \{\text{IN}_1\} \) for the two countries are as follows:

\[ W_{HH}^{H} = \]  

\[ [k((14580k^3 - 44064k^2 + 39744k - 9216)(a - c)^2 + (10206k^3 - 28512k^2 + 25056k - 6012)k_H(a - c)) - (21141k^4 - 65448k^3 + 67428k^2 - 27744k + 3904)/54(9k - 4)^2(3k - 4)^2] \]  

(34)

\[ W_{HF}^{H} = [k((10206k^3 - 32400k^2 + 31968k - 9216)(a - c)^2 - (7290k^3 - 21060k^2 + 19008k - 5184)k_H(a - c)) + (9477k^4 - 26892k^3 + 27774k^2 - 11856k - 1760)k_H]/27(9k - 4)^2(3k - 4)^2 \]  

(35)

The first order condition for the home country is given by:

\[ \frac{\partial W_{HH}}{\partial t_H} = \frac{[k((5100k^3 - 14256k^2 + 14256k - 3456)(a - c) - (21141k^4 - 65448k^3 + 67428k^2 - 27744k^2 + 3904)/27(9k - 4)^2(3k - 4)^2]} = 0 \]  

(35)

Solving the FOC for \( t_H \), we derive the Nash level tariff for the home country under the market structure \( \{\text{IN}_1\} \).
**Lemma 3** If governments choose tariffs simultaneously in the market structures, then the Nash level tariffs would be:

\[
\begin{align*}
    t^S_x &= \frac{5k(625k^2 - 960k + 192)}{4(3125k^3 - 5100k^2 + 2080k - 256)}(\alpha - c) \\
    t^{NN}_x &= \frac{k(9k - 8)}{27k^2 - 34k + 8}(\alpha - c) \\
    t^I_x &= \frac{2k(128k^2 - 186k + 45)}{1280k^3 - 2028k^2 + 936k - 135}(\alpha - c) \\
    t^{N1}_x = t^{N2}_x &= \frac{3k(38912k^5 - 178176k^4 + 292960k^3 - 215676k^2 + 70902k - 8505)(\alpha - c)}{389120k^6 - 1849344k^5 + 3261456k^4 - 2747088k^3 + 1171584k^2 - 243972k + 19683} \\
    t^{P1}_x = t^{P2}_x &= \frac{2k(51200k^5 - 227328k^4 + 35392k^3 - 23576k^2 + 64962k - 6075)(\alpha - c)}{389120k^6 - 1849344k^5 + 3261456k^4 - 2747088k^3 + 1171584k^2 - 243972k + 19683}
\end{align*}
\]

**Proof of Lemma 3:** It is similar to proof of Lemma 2. We used MAPLE to calculate the welfare for each market structure and to calculate the Nash tariff levels. Therefore, we skip the detailed proof. MAPLE code can be requested from the authors.

**Proof of Proposition 1**

We use Horn and Persson’s endogenous model; hence, first we will calculate the equilibrium profits under each market structure by using the profits calculated for stage 3 and 4, by Lemma 2 and Lemma 3. Then, by using the *dom* relation and decisive firms, we will find the EMS.

The firms’ profit for all market structures after Nash tariffs are determined are as follows:

The profits of firms under no merger or \{S\} are:

\[
\pi^S_i = \frac{k(14435125k^5 - 5262500k^4 + 6942000k^3 - 4072960k^2 + 10782720k - 1024576)}{16(3125k^3 - 5100k^2 + 2080k - 256)^2}(\alpha - c)^2, \quad i = 1, 2, 3, 4
\]

(41)

The profits of merger and the outsiders under \{N_i\}, respectively, are given by:
Finally, if firms form a monopoly under \{M\}, the profit is:

$$\pi^M = \frac{k}{2k-1} (\alpha - c)^2.$$

The profits of firms if they form two national mergers \{NN\} are given by:

$$\pi_{iN}^{NN} = k(2\alpha k - 8)(1377k^4 - 3348k^3 + 3280k^2 - 102k + 12) \over (9k - 4)^2(27k^2 - 3k - 8)^2} (\alpha - c)^2, \ i = NH, NF \quad (44)$$

The merger’s and the outsiders’ profits under \{I_1\} are as follows:

$$\pi_{iI}^{I_1} = \frac{9k(6k - 8)(4k - 3)^2(32k^2 - 54k + 15)}{(2k - 3)^2(1280k^2 - 202k^2 + 936k - 135)^2} \over (\alpha - c)^2 \quad (45)$$

$$\pi_{iI}^{I_1} = k(655260k^7 - 121606246k^5 - 1802682k^4 + 14958756k^3 - 6984598k^2 + 1691280k - 16025) \over (2k - 3)^2(1280k^2 - 202k^2 + 936k - 135)^2 \over (\alpha - c)^2, j = 2, 4 \quad (46)$$

Profit for the two cross-border mergers under \{II_1\} can be written as:

$$\pi_{iI}^{II_1} = \frac{2k(9k - 8)}{(9k - 4)^2} (\alpha - c)^2, \ i = II_{13}, II_{24} \quad (47)$$

The merger’s and the outsider’s profits under \{IN_1\} are, respectively:

$$\pi_{iIN}^{IN_1} = \frac{2k(3k - 4)^2(2394765k^5 - 7765308k^4 + 9708316k^3 - 5795568k^2 + 1637328k - 175232)}{(12393k^3 - 39204k^2 + 40644k^2 - 16800 + 2368)^2} \over (\alpha - c)^2 \quad (48)$$

$$\pi_{iIN_1}^{IN_1} = \frac{2k(10924065k^7 - 7579262k^6 + 21213900k^5 - 309277448k^4 + 254114064k^3 - 117877248k^2 + 28605696k - 2803712)}{(12393k^3 - 39204k^2 + 40644k^2 - 16800 + 2368)^2} \over (\alpha - c)^2 \quad (49)$$

Finally, if firms form a monopoly under \{M\}, the profit is:

$$\pi^M = \frac{k}{2k-1} (\alpha - c)^2.$$

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After calculating the profits, we turn our attention to the \textit{dom} relation and decisive firms. First, consider part (i), and the following binary comparison of market structures. We will show that \{IN\} dominates all market structures when $k < k < 3.877711501(\alpha - c)^2$.

\{IN\}_1 \textit{dom} \{S\}: decisive group with respect to these two structures are \{1,2,3\}. From (41) and (48), $\pi^{IN_1}_i - (\pi^S_1 + \pi^S_2 + \pi^S_3) > 0$ holds for all $k > \bar{k}$.

\{IN\}_1 \textit{dom} \{N\}_1: decisive group with respect to these two structures are \{1,2,3\}. From (42), (43), and (48), $\pi^{IN_1}_i - (\pi^{N_1}_i + \pi^{N_1}_2 + \pi^{N_1}_3) > 0$ holds for all $k > \bar{k}$.

\{IN\}_1 \textit{dom} \{NN\}: decisive group with respect to these two structures comprises all owners. From (44), (48), and (49), $\pi^{IN_1}_i + \pi^{IN_4}_i - (\pi^{NN}_N + \pi^{NN}_F) > 0$ holds for all $k > \bar{k}$.

\{IN\}_1 \textit{dom} \{I\}_1: decisive group with respect to these two structures are \{1,2,3\}. From (45), (46), and (48), $\pi^{IN_1}_i - (\pi^{I_1}_1 + \pi^{I_1}_2) > 0$ holds for all $k > \bar{k}$.

\{IN\}_1 \textit{dom} \{II\}_1: decisive group with respect to these two structures comprises all owners. From (47), (48), and (49), we obtain the following conditions:

a) $\pi^{IN_1}_i + \pi^{IN_4}_i - (\pi^{II_1}_1 + \pi^{II_1}_2) > 0$ iff $k < k < 3.877711501$.

b) $\pi^{IN_1}_i + \pi^{IN_4}_i - (\pi^{II_1}_1 + \pi^{II_1}_2) < 0$ iff $k > 3.877711501$.

Therefore, \{IN\} is the EMS when $k < k < 3.877711501$.

Now consider part (ii) of the proposition. We will show that \{II\} dominates all other market structures when $k > 3.877711501$.

\{II\}_1 \textit{dom} \{S\}: decisive owners with respect to these two structures are two symmetric groups which are \{1,3\} and \{2,4\}. From (41) and (47), $\pi^{II_1}_{i13} - (\pi^S_1 + \pi^S_2 + \pi^S_3) > 0$ and $\pi^{II_4}_{i154} - (\pi^S_1 + \pi^S_2 + \pi^S_3) > 0$ holds for all $k > \bar{k}$.

\{II\}_1 \textit{dom} \{N\}_1: the decisive group comprises all firms. From (42), (43) and (47), $\pi^{II_1}_{i13} + \pi^{II_1}_{i24} - (\pi^N_1 + \pi^N_1 + \pi^N_4) > 0$ holds for all $k > \bar{k}$.
\{II_1\} \textit{dom} \{I_1\}: decisive owners with respect to these two structures are the two outsiders, \{2,4\}, under \{I_1\}. From (46) and (47), \(\pi^H_{I_{24}} - (\pi^{I_1}_{2} + \pi^{I_1}_{4}) > 0\) holds for all \(k > k_0\).

\{II_1\} \textit{dom} \{NN\}: decisive group with respect to these two structures comprises all owners. From (44) and (47), we obtain the following conditions:

a) \(\pi^H_{I_{13}} + \pi^H_{I_{24}} - (\pi^{NN}_{NH} + \pi^{NN}_{NF}) > 0\) iff \(k > 2.792836525\),

b) \(\pi^H_{I_{13}} + \pi^H_{I_{24}} - (\pi^{NN}_{NH} + \pi^{NN}_{NF}) < 0\) iff \(k < k_0 < 2.792836525\).

Notice that, from part (i) of the proposition, \{NN\} is dominated by \{IN_1\} for all \(k > 3.792836525\) and thus cannot be EMS. Therefore, and considering the \textit{dom} relation of \{II_1\} and \{IN_1\} from part (i), we can conclude that \{II_1\} is the EMS if \(k > 3.877711501\). □

PROOF OF PROPOSITION 2

Part (i): the difference in R&D investments by an outsider firm under \{I\} and the same firm under \{S\} is given by:

\[e_i^I - e_j^S = \frac{k(160000k^2 - 1069450k^4 + 211895k^6 - 1699947k^2 + 554949k^2 - 61560)(a - c)}{(2k - 3)(1280k^3 - 2028k^2 + 936k - 135)(3125k^3 - 5100k^2 + 2080k - 256)} \text{ for } i = 2,4; j = 1,2,3,4. \] (51)

a) \(e_i^I - e_j^S > 0\) iff \(k > 3.969\)

b) \(e_i^I - e_j^S < 0\) iff \(k < k < 3.969\).

Part (ii): the difference in R&D investments by the outsider of a merger under \{IN\} and the same firm under \{S\} is given by:

\[e_i^{IN} - e_j^S = \frac{k(2318625k^5 - 13751820k^4 + 24528612k^3 - 17952128k^2 + 5531200k - 593400)(a - c)}{(12393k^4 - 39204k^3 + 40644k^2 - 16800k + 2368)(3125k^3 - 5100k^2 + 2080k - 256)} \text{ for } j = 1,2,3,4; \] (52)

a) \(e_i^{IN} - e_j^S > 0\) iff \(k > 3.470\)

b) \(e_i^{IN} - e_j^S < 0\) iff \(k < 3.470\). □

PROOF OF PROPOSITION 3
We first find the set of market structures which will be approved by the merger policy, and then find the EMS among these market structures. We make binary comparisons of welfare levels between each market structure and status-quo to determine the welfare-increasing ones compared to status-quo.

(a) \{N\} and \{S\}

\[ W^{N,S} = W_z^N - W_z^S < 0 \quad (z = H, F) \]

holds for all \( k \). Therefore, \{N\} is never approved by competition regulators.\(^{25}\)

(b) \{NN\} and \{S\}

\[ W^{NN,S} = W_z^{NN} - W_z^S < 0 \quad (z = H, F) \]

holds for all \( k \). Therefore, \{NN\} is never approved by competition regulators.

(c) \{I\} and \{S\}

\[ W^{I,S} = W_z^I - W_z^S > 0 \quad (z = H, F) \]

holds for all \( k \). Therefore, \{I\} is always approved by competition regulators.

(d) \{II\} and \{S\}

\[ W^{II,S} = W_z^{II} - W_z^S \]

where

\[
W^{II,S} = \frac{-k(7421875k^6 - 9362500k^5 + 2432700k^4 - 25235840k^3 + 118471680k^2 - 25657344k + 2097152)(a - c)}{8(9k - 4)(3125k^3 - 5100k^2 + 2080k - 256)^2}
\]

\[ W^{II,S} > 0 \quad \text{iff} \quad \bar{k} < k < 9.532 \]

And thus, \{II\} would be approved by both countries’ regulators iff \( \bar{k} < k < 9.532 \). It will be declined otherwise.

(e) \{IN\} and \{S\} for the country with the merger bias (two of the three merger participants is located in the country)

\[ W^{IN,S} = [ -k(1920776319140625k^{13} - 3320498277437500k^{12} + 11922460406131500k^{11} - 355794523870795200k^{10} + 68560178757428240k^9 - 903480426252666432k^8 + \]

\[ + k(1920776319140625k^{12} 3320498277437500k^{11} - 11922460406131500k^{10} + 355794523870795200k^9 - 68560178757428240k^8 + 903480426252666432k^7 + \]

\[ + k(1920776319140625k^{11} - 3320498277437500k^{10} + 11922460406131500k^9 - 355794523870795200k^8 + 68560178757428240k^7 - 903480426252666432k^6 + \]

\[ + k(1920776319140625k^{10} - 3320498277437500k^9 + 11922460406131500k^8 - 355794523870795200k^7 + 68560178757428240k^6 - 903480426252666432k^5 + \]

\[ + k(1920776319140625k^9 - 3320498277437500k^8 + 11922460406131500k^7 - 355794523870795200k^6 + 68560178757428240k^5 - 903480426252666432k^4 + \]

\[ + k(1920776319140625k^8 - 3320498277437500k^7 + 11922460406131500k^6 - 355794523870795200k^5 + 68560178757428240k^4 - 903480426252666432k^3 + \]

\[ + k(1920776319140625k^7 - 3320498277437500k^6 + 11922460406131500k^5 - 355794523870795200k^4 + 68560178757428240k^3 - 903480426252666432k^2 + \]

\[ + k(1920776319140625k^6 - 3320498277437500k^5 + 11922460406131500k^4 - 355794523870795200k^3 + 68560178757428240k^2 - 903480426252666432k + \]

\[ + k(1920776319140625k^5 - 3320498277437500k^4 + 11922460406131500k^3 - 355794523870795200k^2 + 68560178757428240k - 903480426252666432) \]

\[ \text{for all } k \].

\[ W^{IN,S} > 0 \quad \text{iff} \quad \bar{k} < k < 9.532 \]

And thus, \{IN\} would be approved by both countries’ regulators if \( \bar{k} < k < 9.532 \). It will be declined otherwise.

\(^{25}\) We calculated the welfares by using MAPLE. They are available from the authors.
Where \( \mathcal{W}_{I,N} = W_2^{IN} - W_2^S \), \( z = H, F \). The following conditions are derived:

\[
\mathcal{W}_{I,N} > 0 \iff k < k < 3.247600743
\]

And thus, \{IN\} would be approved by the regulators of the country hosting two of the three-merger participants \( \iff k < k < 3.247600743 \). It will be declined otherwise.

(f) \{IN\} and \{S\} for the country without the merger bias (one of the three merger participants is located in the country)

\[
\mathcal{W}_{I,N} = \left( k(10903305895375k^{13} + 195681414937500k^{12} - 761883297151500k^{11} + 409671401595200k^{10} - 112402203275748240k^9 + 19032539988250432k^8 - 214226443124120064k^7 + 165912436513503232k^6 - 89583888282341376k^5 + 33614809018482680k^4 - 8501845307359232k^3 + 1420192975421440k^2 - 137268933165056k + 5879810228224)(\alpha - c)^2/\left[ 24(12393k^4 - 39204k^3 + 40644k^2 - 16800k + 2368)^2(3125k^3 - 5100k^2 + 2080k - 256)^2 \right] \right); (54)
\]

Where \( \mathcal{W}_{I,N} = W_2^{IN} - W_2^S \), \( z = H, F \). \( \mathcal{W}_{I,N} > 0 \) holds for all \( k \). Thus, the merger is approved by the regulators of the country hosting one of the three merger participants. Since, the merger must be approved by both countries, and from (e-iii), \{IN\} is approved for \( k < k < 3.247600743 \).

Therefore, the set of merger applications which can be approved by the competition regulators of the two countries are \{I\}, \{II\}, \{IN_1\}, \{IN_2\}, \{IN_3\}, and \{IN_4\}.

To determine the EMS among the approved market structures, we need to find the \textit{dom} relation between \{I\} and \{S\}. This relation is as follows for, say \{I_1\}, which the decisive group is \{1,3\}. From (45) and (41), we obtain the following conditions:

i) \( I \textit{ dom} S \iff \pi^{I_1}_M - (\pi^S_1 + \pi^S_3) > 0 \iff k < k < 21.18540616 \)

ii) \( S \textit{ dom} I \pi^{I_1}_M - (\pi^S_1 + \pi^S_3) < 0 \iff k > 21.18540616 \)
Hence, \{I\} is dominated by \{S\} for very inefficient \(k\).

Given the dom relation between \{I\} and \{S\}, and the proof of Proposition 1, we can determine the EMS

a. \{M\} if \(\bar{k} < k < 2.627523310\);

b. \{IN\} if \(2.627523310 < k < 3.247600743\);

c. \{II\} if \(3.247600743 < k < 9.532\);\(^{26}\)

d. \{I\} if \(9.532 < k < 21.18540616\);

e. \{S\} if \(k > 21.18540616\). \(\Box\)

PROOF OF PROPOSITION 4: Analogous to proposition 3, first, we need to find the set of market structures which are approved according to the merger policy and then find the EMS from this set. In doing so, we make binary comparisons of consumer surplus levels between each candidate market structure and \{S\}.

(a) \{N\} and \{S\}

\[ C^{NS} = CS^N - CS^S < 0 \quad (z = H, F) \] holds for all \(k\). Therefore, \{N\} is never approved by competition regulators.

(b) \{NN\} and \{S\}

\[ C^{NNS} = CS^{NN} - CS^S < 0 \quad (z = H, F) \] holds for all \(k\). Therefore, \{NN\} is never approved by competition regulators.

(c) \{I\} and \{S\}

\[ C^{IS} = CS^I - CS^S > 0 \quad (z = H, F) \] holds for all \(k\). Therefore, \{I\} is always approved by competition regulators. However, \{I\} is dominated by \{S\} for very inefficient \(k\) (see proposition 3).

(d) \{II\} and \{S\}

\(^{26}\) If \(3.247600743 < k < 3.877711501\), \{IN\} will be denied and thus \{II\} will emerge.
\[ C^H_S = \frac{-k^2(76875k^3 - 130900k^2 + 59680k - 8192)(1875k^3 - 8500k^2 + 9760k - 2048)(\alpha - c)^2}{8(9k - 4)^2(3125k^3 - 5100k^2 + 2080k - 256)^2} \] (55)

Where \( C^H_S = CS^H - CS^S \), \( z = H, F \). The following condition is derived:

\[ C^H_S > 0 \text{ iff } k < 2.830838672 \]

And thus, \{II\} would be approved by both countries’ regulators iff \( k < 2.830838672 \).

(e) \{IN\} and \{S\}

\[ C^{IN,S} = CS^{IN} - CS^S < 0 (z = H, F) \] holds for all \( k \), regardless of the merger bias.

Therefore, \{IN\} is never approved by competition regulators.

To sum up, the set of merger applications which are approved by the competition regulators of the two countries are \{I\} and \{II\}. From proposition 1 (and its proof in the appendix), and with consumer surplus-increasing merger policy, we obtain the following set of EMS:

i. If \( k < 2.830838672 \), \{II\} is the equilibrium.

ii. If \( 2.830838672 < k < 21.18540616 \), \{I\} is the equilibrium market structure.

iii. If \( k > 21.18540616 \), \{S\} is the equilibrium market structure. ■

**PROOF OF PROPOSITION 6**

The following welfare functions are derived by summing up the welfare of the two countries after the Nash tariffs. These functions are used in constructing Figure 3 in the text and Figure 4 below. The subscript \( G \) stands for global levels, and the superscript denotes the market structure:

\[ W^G_S = \frac{k(875k^2 - 1160k + 256)(40625k^3 - 69500k^2 + 30560k - 4096)(\alpha - c)^2}{4(3125k^3 - 5100k^2 + 2080k - 256)^2} \] (56)

\[ W^G_N = \frac{k(263502954496k^{11} - 2426328317952k^{10} + 9649423908864k^9 - 21859121700864k^8 + 3133115290624k^7 - 29835864154224k^6 + 19271600175744k^5 - 8454324443688k^4 + 2474180183808k^3 - 461412291159k^2 + 49517074224k - 2324522934)(\alpha - c)^2}{2(389120k^6 - 1849344k^5 + 3261456k^4 - 2747088k^3 + 1171584k^2 - 243972k + 19683)^2} \]

\[ W^G = \frac{k(117k^2 - 144k + 32)(45k^2 - 60k + 16)(\alpha - c)^2}{(9k - 4)(27k^2 - 34k + 8)^2} \]
\[ W^I_G = \frac{k(5963776k^7 - 34947072k^6 + 83117376k^5 - 103432680k^4 + 72445752k^3 - 28526742k^2 + 5875740k - 492075)(\alpha - c)^2}{(2k - 3)^2(1280k^3 - 2028k^2 + 936k - 135)^2} \]

\[ W^H_G = \frac{8k(\alpha - c)^2}{9k - 4} \]

\[ W^M_G = \frac{k(3k - 1)(\alpha - c)^2}{(2k - 1)^2} \]

In order to prove the proposition, we need to compare the global welfare of each market structure. Figure 5 depicts that \{I\} and \{II\} are the only market structures that maximize the global welfare depending on \( k \).

Figure 5 – Global welfare

We compare the global welfare under \{I\} and \{II\}.

\[ W^{II,J}_G = \left[-k(1245184k^8 - 14958592k^7 + 63192384k^6 - 132159528k^5 + 153907992k^4 - 103540518k^3 + \ldots \right] \]
\[39641076k^2 - 7986195k + 656100)(\alpha - c)^2]/(9k - 4)(2k - 3)^2(1280k^3 - 2028k^2 + 936k - 135)^2; (57)\]

Where \(W^{II} = \sum_{z} W^{II} - \sum_{z} W^{I}, z = H, F\). The following conditions are derived:

i. \(W^{II} > 0 \text{ iff } \bar{k} < k < 5.973\)

ii. \(W^{II} < 0 \text{ iff } k > 5.973\)

This completes the proof. ■