RISK PREMIUM AND THE STABILITY OF SOME ADJUSTMENT PROCESSES UNDER UNCERTAINTY

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I. Introduction

In the Neoclassical theory of a firm under perfect competition that the entrepreneur behaves as if his demand function, and factor costs are known with certainty plays an important role. Although it is recognized that the firm may be uncertain as to the form of these functions, the entrepreneur is assumed to compress his judgements about a function into a best estimate. He then behaves as if the best estimate represents the function with certainty. If the entrepreneur anticipates the market clearing demand function correctly, he should find the maximal satisfaction in the sense that the equality of the anticipated marginal risk premium and the actual marginal risk premium holds. In this situation, he has no interest in changing his expectation because his expectation is realized in a market. Namely, it is a correct expectation. Naturally we can define it as a market equilibrium. Under certain conditions, we can obtain the existence and the stability of a market equilibrium under uncertainty. The analysis of this respect is important not only because of the generalization of the traditional theory, but because it introduces additional considerations, such as attitude towards risk, that may help to better explain the observed behavior. It will appear that our results contain that of the traditional theory of a firm under perfect competition.
II. Notations, assumptions and behavior of a firm

In this section we consider the behavior of a firm that produces only one consumption good under uncertainty. It is assumed that the entrepreneur behaves as a price taker and that he also behaves as if his utility and cost function were known to him with certainty. But he cannot know the market clearing demand function with certainty. In other words, there is uncertainty about the market clearing price. Then, he must decide his production decision prior to observing the market clearing price. And he does produce some amount of consumption good by the optimal rule explained later. Next, the amount of consumption good is supplied at a market where the market clearing price is determined by the market clearing demand function. We assume further that the entrepreneur acts according to the expected utility hypothesis. Is there a correct expectation for the entrepreneur? In other words, is there a market equilibrium in this model?

In order that we answer this question appropriately, some notations and assumptions are introduced.

$p =$ the price of consumption good,
$p_t =$ the price of consumption good in the $t$-th period,
$\bar{p} =$ the expected price of anticipated price with a density function $f(p),$
$\bar{p}_t =$ the expected price of anticipated price with a density function $f_t(p)$ in the $t$-th period,
$p_t^* =$ the market clearing price in the $t$-th period,
$x =$ the output rate,
$x_t =$ the output rate in the $t$-th period,
$x_t^* =$ the optimal output rate defined later for the entrepreneur in the
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t-th period,
$p=D(x)=$ the market clearing demand function for consumption good in the market,

$\mu=$ the adaptive multiplier in price expectation,

$R=$ the risk premium rendered by the entrepreneur.

Assumption (A.1)

A utility function $U(px-C(x))$ is a twice continuously differentiable, strictly increasing, concave, bounded and cardinal function of a profit $\pi=px-C(x)$.

Assumption (A.2)

A cost function $C(x)$ has continuous second derivatives with

(a) $C'(x)>0$ for $x \geq 0$ and

(b) $\lim_{x \to \infty} C'(x) = \infty$

Assumption (A.3)

The market clearing demand function is downward sloping with respect to output rate, that is,

(a) $D'(x)<0$ for $x \geq 0$.

And further conditions

(b) $\lim_{x \to 0} D(x) = \infty$ and (c) $\lim_{x \to -\infty} D(x) = 0$

are satisfied.

Assumption (A.4)

The risk premium $R$ which will be defined later is twice continuously differentiable function of both expected price and the output rate, that is, $R=R(\tilde{p}, x)$.

Assumption (A.5)

$$\frac{\partial}{\partial x} \left[ \frac{\partial}{\partial x} R(\tilde{p}, x) + C'(x) \right] > 0.$$
Remark 1: According to K. J. Arrow [2] and J. W. Pratt [8], the concavity of a utility function implies that the entrepreneur behaves as a risk averse agent in this economy.

Remark 2: The market clearing (that is, objective) demand function is not necessarily downward sloping with respect to output rate. But in this respect we suppose A.3 for the simplicity of the exposition. In this point, see H. Nikaido [6] [7].

Remark 3: In general, the risk premium rendered by the entrepreneur depends upon a utility function, a density function of anticipated price and output rate. But, for the sake of simplicity we assume A.4 in this note. See the example in Appendix.

Remark 4: The assumption of A.5 is a sufficient condition for the existence of the optimal output rate. This condition corresponds to the increasing marginal cost in the traditional theory of a firm under perfect competition. On the other hand, the assumption of A.6 corresponds to the non-increasing marginal revenue which is always 0 in the traditional theory of a firm under perfect competition because the marginal revenue is always $\overline{p}$ in this paper's notation.

**The optimal rule for the entrepreneur**

Since he anticipates a density function $f(p)$ of anticipated price, the expected utility of a profit is obtained as

$$EU(\pi) = \int_{0}^{\infty} f(p)U(px - C(x)) dp,$$

where $E$ is an expectation operator.

Here we introduce a concept of a risk premium $R$ rendered by the
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entrepreneur by using a certainty equivalence axiom. The risk premium $R$ is defined as such a magnitude that

$$U(E_\pi - R) = EU(\pi)$$

holds. In other words, the risk premium is defined as the maximum amount the entrepreneur would pay to avoid the risk and obtain the expected value of that risk with certainty.

Since a utility function is a strictly increasing function of a profit, the maximization of $EU(\pi)$ is equivalent to that of $E\pi - R$ with respect to output rate $x$. First and second order conditions for maximum are obtained to output as

$$-p - \frac{\partial}{\partial x} R(p, x) = C'(x)$$

and

$$-\frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} R(p, x) + C'(x) \right\} > 0.$$

By using this optimal rule the entrepreneur can find the optimal output rate.

Working of model

In the beginning of the $t$-th period, the entrepreneur knows only the past experiences with certainty, that is, the past market clearing prices, the output rates, the density functions of anticipated price, his utility and cost functions. He does produce the optimal output. The market rate $x_t*$ which is determined from above mentioned date by using the optimal rule. Then, the output rate $x_t*$ is supplied for the market. The market clearing price $p_t*$ is determined from the market clearing demand function $p_t* = D(x_t*)$. If this market clearing price is not equal to the entrepreneur's expected price $p_t$, he changes the expectation about next period's output price because his expectation appears to be false.
in the sense that the ex ante marginal risk premium is not equal to the ex post (actual) marginal risk premium. In this way he learns about the market clearing demand function by comparing the expected price and the market clearing one. The equality of both prices provides the equality of the ex ante marginal risk premium and the ex post marginal risk premium. We can call this state as a correct expectation from the point of view by the entrepreneur. In other words, this state is defined as a market equilibrium in this model.

*Remark 5*: Because we obtain from (1)

\[
\frac{R}{1 - \frac{\partial R}{\partial p}} \frac{\partial R}{\partial p} = C'(x),
\]

we could interpret that the firm under uncertainty behaves as if he were a monopolist confronted with a subjective demand function. Continuing line of reasoning, we could construct the subjective demand function as follows. Since the left-hand side of (1) is the expected marginal revenue,

\[
p_t = \bar{p}_t x_t - R(\bar{p}_t, x_t) + \text{constant}
\]

is the subjective demand function in the \( t \)-th period. Assuming that the subjective demand function is consistent expectation for the entrepreneur in the sense that if \( x_t = x_t^* \), then \( p_t = \bar{p}_t \), we have

\[
p_t = \bar{p}_t - \frac{R(\bar{p}_t, x_t) - R(\bar{p}_t, x_t^*)}{x_t}
\]

where

\[
R(\bar{p}_t, x_t^*) = \int_0^{x_t^*} \frac{\partial}{\partial x} R(\bar{p}_t, x) dx
\]

and \( x_t^* \) satisfies (1).

**Formulation of adjustment process**

The recursive adjustment process stated above can be formulated
as

(i) \( \bar{p}_t - \frac{\partial}{\partial x_t} - R(\bar{p}_t, x_t^*) = C'(x_t^*) \).

(ii) \( \bar{p}_{t+1} = \max \{ p(\theta), \mu \bar{p}_t^* + (1 - \mu) \bar{p}_t \} \),

where \( p(\theta) = C'(0) + \theta \) and \( \theta > 0 \),

and

(iii) \( p_t^* = D(x_t^*) \).

Remark 6: The condition (ii) is an adjustment equation which determines the next period's firm behavior. For some alternative version, see the following Remark 10. The \( p(\theta) \) would be interpreted as an initial cost for entrance in the market.

III. Existence of a market equilibrium

In this section we define a market equilibrium in this model as formally and prove its existence.

Definition

A triplet \((\bar{p}_t, x_t^*, p_t^*)\) is a market equilibrium which derived from a data \((U, f_t(p), C, D)\) if the following conditions are satisfied;

\( \bar{p}_t = p_t^* = p^* = p(\theta) \) for all \( t = 0, 1, 2, \ldots \).

where

\( \bar{p}_t = \int_0^\infty \bar{p}_f(\bar{p}) d\bar{p}, \)

\( p^* = \frac{\partial}{\partial x^*} - R(p^*, x^*) = C'(x^*), \)

and

\( p^* = D(x^*). \)

Let us consider the following system (a) and (b) such that

(a) \( \bar{p} - \frac{\partial}{\partial x} R(\bar{p}, x) - C'(x) = 0 \)
(b) \( \overline{p} - D(x) = 0 \), where \( \overline{p} \geq p(\theta) \).

The existence of a unique pair \((p^*, x^*)\) that satisfies both (a) and (b) constitutes a unique market equilibrium for the model. For this system of equations we can obtain two Lemmata 1 and 2.

**Lemma 1**

For each \( \overline{p} \geq p(\theta) \), there exists a unique output rate that satisfies (a). It is a continuous and monotone increasing function of the expected price \( \overline{p} \).

(Proof) From the equation \( U(E_\pi - R) = EU(\pi) \), we have by partial differential with respect to the output rate \( x \)

\[
(\overline{p} - C'(x) - \frac{\partial}{\partial x} R(\overline{p}, x))U'(E_\pi - R)
\]

\[
=(\overline{p} - C'(x))EU'(\pi) + \text{cov}(U'(\pi), p).
\]

By A.2(b), we have

\[
\overline{p} - C'(x) - \frac{\partial}{\partial x} R(\overline{p}, x) < 0
\]

for sufficiently large output rate \( x \), because of

\[
U'(E_\pi - R) > 0, \ EU'(\pi) > 0, \ \text{cov}(U'(\pi), p) < 0 \quad \text{and} \quad \overline{p} - C'(x) < 0.
\]

On the other hand, we have for \( x = 0 \)

\[
\overline{p} - C'(0) - \frac{\partial}{\partial x} R(\overline{p}, 0) > 0,
\]

because of \( \overline{p} - C'(0) \geq \theta > 0 \) (adjustment equation (ii)),

\[
\text{cov}(U'(-C(0), p) = 0, \ U'(-C(0) - R) > 0 \quad \text{and} \quad EU'(-C(0)) > 0.
\]

Since \( F(\overline{p}, x) \), the left-hand side of (a), is continuous function of \( x \) for each fixed \( \overline{p} \) by A.2 and A.4, there exists at least one positive output rate for each fixed \( \overline{p} \) by the Intermediate Value Theorem.

Further A.5 implies the uniqueness of output rate for each fixed \( \overline{p} \) such that (a) is satisfied by the output rate. We write this relation as
(4) \( x = \varphi(\bar{p}) \).

Rewriting (a) as

\[
(a') \quad \bar{p} = \frac{\partial}{\partial x} R(\bar{p}, x) = C'(x),
\]

we find that the left-hand side of above equation (a') shifts upwards with the increase of the expected price \( \bar{p} \) by A.6. Then, \( \varphi(\bar{p}) \) is a monotone increasing function of \( \bar{p} \). The continuity property is obviously derived from A.2 and A.4. Q.E.D.

**Lemma 2**

There exists a unique price \( p^* \) such that

\[ p^* = D(\varphi(p^*)) \]

holds when the condition (*) \( D^{-1}(p(\theta)) > \varphi(p(\theta)) \)

is satisfied in the market, where \( p(\theta) \) is the subsistence expected price of the entrepreneur which depends upon the expectation for the market clearing demand function \( D(x) \).

**Remark 7:** The condition (*) implies that the demand is relatively large enough for the subsistence expected price \( p(\theta) \) of the entrepreneur. In other words, excess demand for the consumption good prevails at \( p(\theta) \).

(Proof) Let us consider the function

\[ G(\bar{p}) = D^{-1}(\bar{p}) - \varphi(\bar{p}), \]

where \( D^{-1}(\bar{p}) \) is the inverse function of the market clearing demand function \( D(x) \). A.3 implies that \( D^{-1}(\bar{p}) \) is decreasing function of \( \bar{p} \) with following properties:

\[ \lim_{\bar{p} \to 0} D^{-1}(\bar{p}) = \infty \text{ and } \lim_{\bar{p} \to \infty} D^{-1}(\bar{p}) = 0. \]

Then we have

\[ \lim_{\bar{p} \to \infty} G(\bar{p}) < 0, \]
because of \( \lim_{p \to \infty} \varphi(p) = 0 \).

On the other hand, \( G(p) \) is positive for sufficiently close to but larger price than the subsistence expected price \( \bar{p} \) because of Lemma 1, the condition (*) of Lemma 2, and A.3. Since \( G(p) \) is continuous function of \( p \) by A.3 and Lemma 1, again, by the Intermediate Value Theorem there exists at least one positive price \( p^* \geq p(\theta) \) such that \( G(p^*) = 0 \) holds. The monotonicity of \( G(p) \) with respect to the expected price \( \bar{p} \) assures the uniqueness of \( p^* \geq p(\theta) \) such that

\[
p^* = D(\varphi(p^*))
\]

holds. Q. E. D.

From Lemmata 1 and 2, we obtain the following

**Theorem 1 (Existence of a market equilibrium)**

Under Assumptions A.1-A.6 and the condition (*) of Lemma 2, there exists a unique market equilibrium.

**Remark 8:** The uniqueness of a pair \( (p^*, x^*) \) is assured by the well-known Gale and Nikaido's Univalent Theorem in [5] because the principal minors of the Jacobian matrix for the system (a) and (b) are all positive by A. 4, 5 and A. 6, where the Jacobian matrix is

\[
J = \begin{bmatrix}
1 - \frac{\partial}{\partial p} \left\{ \frac{\partial}{\partial x} R(p, x) \right\} & -\frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} R(p, x) + C'(x) \right\} \\
-\frac{\partial}{\partial x} \left\{ \frac{\partial}{\partial x} R(p, x) + C'(x) \right\} & -D'(x)
\end{bmatrix}
\]

**Remark 9:** Though the exposition is explained about a representative firm, but existence of a market equilibrium can be proved for the model in which several firms having (same or different) production functions produce the same consumption good.

**IV. Stability of the market equilibrium**

We now consider the stability properties of adjustment process(i)-
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-(iii). We can prove the following

**Theorem 2**

Under A.1-A.6 and the condition (*) of Lemma 2, there exists an adjustment multiplier \( \mu^* (\varepsilon) > 0 \) such that the triplet \( (\bar{p}_t, x_t^*, \bar{p}_t^*) \) starting from any given \( \varepsilon \) neighbourhood of the market equilibrium \( (p^*, x^*, p^*) \) converges to the market equilibrium triplet.

(Proof) Let us consider the adjustment process (i)-(iii).

From Lemmata 1 and 2, we obtain the following relations

(6) \( p_t^* = D(\varphi(\bar{p}_t)) \) and \( x_t^* = \varphi(\bar{p}_t) \).

Then, it is sufficient to prove the existence of an adjustment multiplier such that the expected price \( p_t \) starting from any given \( \varepsilon \) neighbourhood of the market equilibrium price \( p^* \) converges to \( p^* \).

We consider \( V(\bar{p}_t) = (p_t - p^*)^2 \).

\[
V(\bar{p}_{t+1}) - V(\bar{p}_t) = (\bar{p}_{t+1} - \bar{p}_t)^2 + 2(\bar{p}_{t+1} - \bar{p}_t)(\bar{p}_t - p^*).
\]

Two cases occur according to the speed of adjustment multiplier \( \mu \) because of the condition (*) of Lemma 2 and (ii), the condition of adjustment equation;

(a) \( \bar{p}_t > p(\theta) \) for all \( t \),

and

(b) there exists some integer \( t_0 \) such that

\( \bar{p}_{t_0} > p(\theta) \) and \( \bar{p}_{t_0+1} = p(\theta) \).

Case (a): By (6), \( \bar{p}_{t+1} - \bar{p}_t = \mu (D(\varphi(\bar{p}_t)) - \bar{p}_t) \), we have

\[
V(\bar{p}_{t+1}) - V(\bar{p}_t) = \mu^2 [D(\varphi(\bar{p}_t)) - \bar{p}_t]^2 + 2\mu [D(\varphi(\bar{p}_t)) - \bar{p}_t] (\bar{p}_t - p^*).
\]

We know, however, that it must be that

\( -[D(\varphi(\bar{p}_t)) - \bar{p}_t] (\bar{p}_t - p^*) > 0 \)

for all \( \bar{p}_t \neq p^* \) by Lemmata 1 and 2. Consider the ratio
where \( Z(\bar{p}) = D(\varphi(\bar{p})) - \bar{p} \). Clearly \( \mu(\bar{p}) \) is positive and, therefore, must bounded away from zero for all non-equilibrium price \( \bar{p} \neq p^* \).

Some calculations show that

\[
\mu(\bar{p}) = \frac{2}{\left\{ \frac{D(\varphi(\bar{p})) - p^*}{\bar{p} - p^*} \right\} + 1}
\]

holds for all \( \bar{p} \neq p^* \).

For all \( \bar{p} \) sufficiently close to but not equal to \( p^* \),

\[
\frac{D(\varphi(\bar{p})) - p^*}{\bar{p} - p^*}
\]

is approximately equal to the derivative of \( D(\varphi(\bar{p})) \) evaluated at \( p^* \) by definition of differentiability. So we can choose \( \mu^* > 0 \) such that \( \mu^* < \mu(\bar{p}) \) for all \( |\bar{p} - p^*| \leq \epsilon \). Then, for this \( \mu^* \) we have

\[
V(-\bar{p}_{i+1}) - V(\bar{p}_i) = \mu^* \left\{ D(\varphi(\bar{p}_i)) - \bar{p}_i \right\}^2 + 2\mu^* \left\{ D(\varphi(\bar{p}_i)) - \bar{p}_i \right\} (\bar{p}_i - p^*) \]

\[
= \mu^* (\mu^* \left\{ D(\varphi(\bar{p}_i)) - \bar{p}_i \right\}^2 < 0, \text{ because of (7).}
\]

Since \( V(\bar{p}_i) \) is a monotone decreasing sequence and bounded, \( V(\bar{p}_{i+1}) \) and \( V(\bar{p}_i) \) converge to the same value, which implies

\[
\lim_{i \to \infty} \left\{ D(\varphi(\bar{p}_i)) - \bar{p}_i \right\}^2 = 0.
\]

Case (b) : Since \( \bar{p}_{i0} > p(\theta) \), there exists some \( \mu^{**} < \mu \) such that \( \mu^{**} \bar{p}_{i0} + (1 - \mu^{**}) \bar{p}_{i0} > p(\theta) \). In this way we can reduce Case (b) to Case (a).

Q. E. D.

Remark 10 : In the case of adjustment process (I)-(III) such that

( I ) \( \frac{\partial}{\partial x^*(t)} R(\varphi(t), x^*(t)) = C'(x^*(t)) \),

( II ) \( \frac{dp^*(t)}{dt} = \max \{ p(\theta), \lambda (p^*(t) - p^*(t)) \}, \lambda > 0 \)

and

( III ) \( p^*(t) = D(x^*(t)) \),

we have the following
Theorem 2'

Under Assumptions A.1-A.6 and the condition(*) of Lemma 2, the market equilibrium is globally stable.

(Proof) From Lemmata 1 and 2, we have
\[ p^*(t) = D(\varphi(p'(t))) \]
and
\[ x^*(t) = \varphi(p'(t)). \]
Moreover we have
\[ D(\varphi(p'(t))) - p'(t) > 0 \quad \text{for all } p(e) \leq p'(t) < p^* \]
and
\[ D(\varphi(p'(t))) - p'(t) < 0 \quad \text{for all } p'(t) > p^* \]
because of Lemmata 1 and 2. Since the market equilibrium is unique, the global stability of it is assured by above relations. \( \text{Q. E. D.} \)

Finally we have following local stability theorem under additional Assumption (A.7). Its result corresponds to the Cobweb theory of the stability under perfect competition.

Assumption (A.7)

Adjustment multiplier \( \mu \) satisfies the following condition
\[
\frac{\partial}{\partial x^*} \left\{ \frac{\partial}{\partial x^*} R(p^*, x^*) + C'(x^*) \right\} > -\mu D'(x^*) \frac{\partial}{\partial p^*} \left\{ p^* - \frac{\partial}{\partial x^*} R(p^*, x^*) \right\} \\
+ (1 - \mu) \frac{\partial}{\partial x^*} \left\{ \frac{\partial}{\partial x^*} R(p^*, x^*) + C'(x^*) \right\}
\]

Theorem 3 (Stability of the market equilibrium in the small)

Under Assumptions (A.1-A.7) and the condition (*) of Lemma 2, the adjustment process formulated by (i)-(iii) is locally stable.

(Proof) By using Taylor's expansion rule around the market equili-
brium as the approximation of the adjustment process, we obtain the following relation;

\[ x_t^* - x^* = \frac{\beta}{\alpha} (\bar{p}_t - p^*), \]

\[ \bar{p}_{t+1} - p^* = \gamma (\bar{p}_t - p^*), \]

and

\[ \bar{p}_t - p^* = \frac{\beta}{\alpha} D'(x^*) (\bar{p}_t - p^*), \]

where

\[ \alpha = \frac{\partial}{\partial x^*} \left\{ \frac{\partial}{\partial x^*} R(p^*, x^*) + C'(x^*) \right\}, \]

\[ \beta = \frac{\partial}{\partial p^*} \left\{ p^* - \frac{\partial}{\partial x^*} R(p^*, x^*) \right\} \]

and

\[ \gamma = \frac{1}{\alpha} \left\{ -\mu \beta D'(x^*) + (1-\mu) \alpha \right\}. \]

A.7 implies \(|\gamma| < 1\). Then we have from \( \lim_{t \to \infty} \bar{p}_t = p^* \) Q. E. D.

Remark 11. The assumption of A.4 includes two cases where

(1) \(-D'(x^*) \frac{\partial}{\partial p^*} \left\{ p^* - \frac{\partial}{\partial x^*} R(p^*, x^*) \right\} \) is smaller than \( \frac{\partial}{\partial x^*} \left\{ -\frac{\partial}{\partial x^*} R(p^*, x^*) + C'(x^*) \right\} \)

and

(2) the reverse relation holds.

In case of (1), the adjustment multiplier \( \mu \) is positive and \( \tilde{\mu} \) which is determined by the market clearing demand function, the cost function and the risk premium rendered by the entrepreneur, \( \tilde{\mu} \) is greater than 1. In the traditional theory of Cobweb problem, the sufficient condition for the adjustment process is

\[ C''(x) > |D'(x^*)| \] for \( \mu = 1 \).
The case is also included in A.7(1). In case of (2), the adjustment multiplier \( \mu \) is negative and larger than some \( \mu \) which is also determined by above mentioned factors.

In either cases the risk premium plays an important role in determining the adjustment multiplier for expectation of the expected price.

V. Interpretation of Theorems and concluding remarks.

As was stated briefly in section II, in this economy the entrepreneur cannot observe the exact market clearing price until his output rate is produced and provided in the market. So he must predict the market clearing price with a density function of anticipated price. He then derive a rule of the optimal output rate by using the certainty equivalence axiom of the risk premium. In doing so, he behaves as if he were a monopolist with a given demand function while he behaves, in fact, a price taking agent in the market. We can thus obtain the subjective demand function by the entrepreneur as already mentioned procedures in section II (Remark 5). When we derive it, it is important to see how the concept of a correct expectation plays a dominant role. In market equilibrium, the expectation by the entrepreneur is correct by definition. Remembering this fact in mind, we can interprete Theorems 1-3 as follows.

Theorem 1 implies that the entrepreneur finds the correct expectation under certain plausible conditions. He learns about the market clearing demand function under some adjustment process (i)-(iii) or (I)-(III). Then, Theorem 2 implies that for the appropriately chosen adjustment multiplier the entrepreneur can learn about the market clearing demand function by the extent he wishes to know. And more
learning by doing in the market provides him more satisfaction than before. While the adjustment multiplier plays an important role in the stability of the market equilibrium in the case of discrete time adjustment process, we know from Theorem 2' that the adjustment process (I)-(III) always gives the correct expectation, irrespective of the speed of the adjustment multiplier. This contrast result has been already observed by the Neoclassical theory in the stability problem and recently pointed out by Arrow and Hahn [3]. Theorem 3 implies that learning by doing brings the entrepreneur the correct expectation provided the initial expectation being close to the correct expectation. But, he can obtain some of the properties of the market clearing demand function by using the subjective demand function because the stability in the small cannot give him the whole information about the market clearing demand function. From Theorems 1-3, it is clearly seen that all we can obtain is a generalization of the traditional theory of a firm under perfect competition. While generalization is provided by introducing the risk premium rendered by the entrepreneur under uncertainty, the introduction of it is important to consider the firm behavior under uncertainty. But some alternative approaches to this problem would provide us the more desired results. Further studies are waited.

APPENDIX

We suppose that
\[ U(\pi) = -\exp(-\delta \pi), \quad \delta > 0 \]
\[ f_i(\bar{p}) = \frac{\exp\left(-\frac{p_i}{\bar{p}_i}\right)}{\bar{p}_i}, \quad \bar{p}_i \geq p(\theta), \]
\[ C'(x) = \epsilon_0 x + \epsilon_1, \quad \epsilon_0, \epsilon_1 > 0, \]
and

\[ p = \frac{\gamma}{x}, \quad \gamma > 0 \]

are a utility function, a density function of anticipated price, a marginal cost function, and a market clearing (that is, an objective) demand function, respectively.

Some calculations show that the above example satisfies all of the conditions A.1-A.6. The market equilibrium price and output rate \((p^*, x^*)\) is obtained as

\[ p^* = \frac{1}{2} \{ \epsilon_1(1 + \gamma \delta) + \sqrt{\epsilon_1^2(1 + \gamma \delta)^2 + 4 \epsilon_0 \epsilon_1(1 + \gamma \delta)} \} \]

and

\[ x^* = \frac{\gamma}{p^*} \text{ if } \epsilon_1(1 + \gamma \delta) < p(\theta) < p^*. \]

Then we have the effects upon the equilibrium price \(p^*\) and output rate \(x^*\) with respect to \(\delta, \gamma, \epsilon_0\) and \(\epsilon_1\), respectively as follows:

\[
\left( \frac{\partial p^*}{\partial \delta}, \frac{\partial p^*}{\partial \gamma}, \frac{\partial p^*}{\partial \epsilon_0}, \frac{\partial p^*}{\partial \epsilon_1} \right) = (+, +, +, +)
\]

and

\[
\left( \frac{\partial x^*}{\partial \delta}, \frac{\partial x^*}{\partial \gamma}, \frac{\partial x^*}{\partial \epsilon_0}, \frac{\partial x^*}{\partial \epsilon_1} \right) = (-, -, -, -)
\]

REFERENCES


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