DEREGULATION IN THE FORMAL CREDIT MARKET AND ITS IMPACT ON INFORMAL CREDIT

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Received June 2005; Accepted February 2006

Abstract

This paper analyses the effects of deregulation of formal interest-rate in terms of a model of strategic interaction between the formal and the informal lender, in which borrowers are differentiated in terms of their capacity to pay collateral. The formal lender is subject to interest-rate ceiling and faces the possibility of strategic default. Strategic default by the entrepreneur is however not possible in case of the informal lender, as the informal lender can fully observe the entrepreneur. It is shown that there is a range of interest-rates such that if the interest-rate ceiling lies in that range then deregulation of the formal interest-rate will cause informal lending to expand and formal lending to contract. This is in contrast with the conventional wisdom that in the face of interest rate deregulation formal lenders always gain in market share.

Key Words: Informal Credit, Segmentation, Competition, Collateral, Deregulation

JEL Classification: G21, G28, O17

I. Introduction

The past decade has witnessed a revival of the debate on state intervention versus non-intervention as the optimal credit policy, especially in the context of developing countries. Initially it was believed that state intervention through nationalisation and regulation of commercial banks would help to mitigate the financial dualism that infest the credit markets in most LDCs. This is manifested through the co-existence of formal-lenders (FLs) and informal-lenders (ILs), in the credit markets of these countries, which stands in stark contrast to the integrated, organised and efficiently functioning credit markets of the developed countries. FLs refer to the large institutional lenders, like commercial banks and other

* The author wishes to thank Professor Abhirup Sarkar for valuable comments and suggestions. I am grateful to Prof. Sugata Marjit, Prof. Sharmila Banerjee, Dr. Diganta Mukherjee and Dr. Tannoyee Chatterjee for their insightful comments. I am grateful to the seminar participants at Economic Research Unit, Indian Statistical Institute, at the Winter school in Quantitative Economics organised by The Econometric Society and IIMC and the 10th Annual Conference on Contemporary Issues In Development Economics at Jadavpur University, for their comments. The usual disclaimer applies.
government owned banks that are subject to various central bank regulations. The ILs on the other hand are a heterogeneous lot and consist of non-institutional lenders like indigenous bankers, moneylenders, traders, landlords etc., who are outside the gambit of the central bank.

Studies reveal that the formal and informal lenders have various structural differences (WBER 1990, Ray 1998) which explains their co-existence. In the LDCs not only do the FLs and ILs coexist but also, typically small borrowers (less wealthy or collateral poor) both in the agricultural sector and industrial sectors are denied access to formal credit. This segment of borrowers therefore has to rely on informal lenders for meeting most of their credit needs on extremely unfavourable terms. Typically, while offering unsecured loans, the ILs charge extremely high rates of interest which may range between 18% to 200% per annum (Aleem, 1993; Siamwala, 1990) which makes the cost of borrowing high in the informal credit market compared to the formal one. This leaves the informal borrower with little or no surplus, who are pushed to their reservation levels of utility. Curbing informal lending and expanding formal credit was thus seen as a way for providing equal access to credit to all classes of borrowers at uniform interest rates.

The policy of nationalisation and directed credit turned out to be a partial success (Basu, 1984, 90; Bardhan and Udry, 1999). This led to a shift in policy in favour of financial liberalisation in the 1980’s and 1990’s as several developing countries in Asia, Latin America and Africa, like India, Sri Lanka, Thailand, Indonesia, South Africa, Morocco, Kenya, Tanzania, Chile, Mexico, experimented by deregulating the interest rate and/or allowing free entry into formal banking sector (Williamson and Mahar, 1998). This paper attempts to model such a credit market scenario as is found in the developing economies and to examine the implications of alternative government policies of state intervention and non-intervention, for extending the reach of formal credit and mitigating the dominance of the IL vis-à-vis the small borrower.

The existing literature on financial liberalisation is largely concerned only with the formal credit market. Moreover the focus is on the impact of deregulation on aggregate savings, investment and growth, the volume of credit, interest rates, the extent of rationing, profitability and efficiency in the allocation of credit rather than on the effectiveness of financial liberalisation in channeling formal credit to the small borrower. A more complete approach to the problem of financial liberalisation in the context of developing countries would be to construct models that explicitly take financial dualism into consideration by incorporating strategic interaction between the FL and the IL. We find a flavour of this approach in the models of vertical-links developed by Hoff and Stiglitz (1998), Bose (1998a), Floro and Ray (1997). These models present an alternative to the policy of "horizontal" displacement of the ILs by FLs, developed by Bell (1990), Chakrabarty and Chaudhuri (2001) and Kochar (1991). Thus, while these models analyse the role of state intervention in curbing financial dualism, they have not addressed the issue of financial liberalisation.

This paper develops a model of formal-informal interaction for analysing the impact of financial liberalisation by focussing on only one aspect of liberalisation; that of interest rate

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1 Op cit.

For a discussion on the share and terms of formal and informal credit in small-scale industries in developing countries including India, see Timberg and Aiyar (1984), Little et al. (1987), Jain (1999), Nanda (1999). For rural indebtedness see Malik (1991), Rural Labour Enquiry (1990, 1997).

2 For an overview, see Mookherjee and Ray (2001).
deregulation. Country experiences in this regard have been varied. Specifically the effect of deregulation of formal interest rate on collateral requirement by banks, which determine the size and penetration of formal credit market, has been in both directions. While some studies identify downward movement in collateral requirement following deregulation (ILO, 2001, 1998); other studies find a strong negative impact on the access of small holders to credit (Janvry et al, 1997). In this paper we consider the issue of whether deregulation of formal interest will necessarily lead to a contraction in the size of the informal credit market. We do so in terms of a framework in which borrowers are differentiated in terms of their capacity to pay collateral, which is observable, and all loan contracts are exclusive. Exclusivity of informal contracts is a well-documented fact (Aleem, 1993; Siamwalla et al., 1990). Both exclusive and non-exclusive formal contracts are considered in the literature (Bell, 1990; Jain, 1999). We consider the case of exclusive contracts only.

The strategic interaction between the FL and the IL is modelled as a sequential move game in which the FL takes into consideration the IL’s behaviour (choice of contract) -whether to segment or to compete — while choosing its contract. The FL and the IL have been differentiated in terms of information asymmetry. The IL, unlike the FL, does not face the possibility of strategic default. The fact that the ILs enjoy informational advantage has considerable empirical support.3 The order of moves is also significant in revealing a structural difference between the FL and the IL. The FL being subject to regulatory constraints and procedural norms cannot alter his offers quickly, unlike the IL who can observe the FL and react instantaneously. Our analysis reveals that for certain ranges of interest rate ceilings deregulation of the formal interest rate may actually result in the counter intuitive situation where the size of the formal credit market shrinks and informal credit expands. Thus a policy of financial liberalisation is not necessarily effective in making the benefits of formal credit reach the poorest strata of society.

The plan of this paper is as follows. Section II.1 states the assumptions regarding the basic framework. Section II.2 develops the model and discusses the nature of equilibrium in the presence of formal lending only. This enables us to understand how the FL’s problem gets differentiated due to the presence of information asymmetry. Finally in section III the case of strategic interaction between an informed IL and an uninformed FL has been discussed. It shows how the FL’s problem gets further modified if the FL takes into account the strategic behaviour of the IL. Section IV considers the implications of interest rate deregulation in the formal sector. Finally section V presents the conclusions.

II. The Model and Preliminaries

1. Assumptions

There exist two types of agents — entrepreneurs and lenders, who are risk neutral. The entrepreneurs have access to a project, whose size is fixed at unity. The project return is characterised by a two point production function, according to which output \( q \) is realised with probability \( p \) if the project is successful, and output zero is realised with probability \( (1-p) \) if

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the project fails. Thus the uncertainty in project returns is purely exogenous. Further, the production function is the same for all entrepreneurs. There is no type or effort variety. Hence there is no adverse selection problem either.

The entrepreneurs must borrow the investment goods in order to undertake the project. The lenders are of two types — formal and informal. Unlike FLs (institutional lenders like commercial banks), the ILs (indigenous bankers, moneylenders) can observe whether the project has been successful or not. Hence the ILs do not face any possibility of strategic default by the entrepreneurs on the loans extended by them. The FLs however are subject to strategic default, as they cannot observe output.\footnote{An alternative interpretation is to conclude that the problem is that of enforcement of repayment by the FL. The IL, on the other hand, can use some sanction, social or extra-legal, that allows him to enforce repayment.}

The contracts offered by the lenders are collateralised loan contracts, which require that the borrower deposits the contracted amount of collateral $C$ with the lender. At the end of the loan period either the entrepreneurs pay $r \in (0, q]$ which is the gross interest on the loans or in case of default they part with $C$ which is the amount of collateral as specified in the contract.\footnote{Collateralised or secured loan contracts is a stylised feature of formal credit markets in many countries including India. The loans are advanced against the security of some tangible assets, which may vary in form from being goods and industrial raw materials, financial securities, real estate or gold ornaments and jewellery. A charge on any such assets offered as security is created in favour of the banker which gives the lender the right to recover his dues from the sale of these assets. For certain types of charges (lien, pledge, usufructuary mortgage), the assets must be deposited with the lender when the loan is disbursed and are released only when the loan is repaid (Varshney, 2004; Swaroop, 1999). For other types of charges, like hypothecation, where the transfer of assets to FL does not take place ex-ante, we get back to the enforcement problem in that the FL may find it difficult to seize the collateral (Besley, 1995). Collaterised contracts have also been frequently considered in the literature on banking (Freixas and Rochet, 1997). Unsecured loans are more common for the informal credit market. This follows from the equilibrium behaviour of the IL who do not face the possibility of strategic default, as is discussed below.} We consider $r > 0$, as at $r = 0$, default is not possible. The loan contracts also involve a fixed transaction cost of $T$ per borrower. To keep the notation simple we assume that $T$ includes the principal or amount of loan per borrower, which is of unit size by assumption. Thus $T > 1$. We also assume that all loan contracts are exclusive as already mentioned in the introduction. All the agents are risk neutral and are interested in maximising their expected profits.

The entrepreneurs are differentiated in terms of their capacity to pay collateral, which is uniformly distributed over the interval $[0, C]$ and is divisible. The collateral is qualitatively different from the investment good (for example family heirlooms and land). We assume that the collateral is observable and marketable. However the entrepreneur is not interested in selling the collateral and investing the proceeds for financing the project. This is because his personal valuation is larger than the market value of the collateral. Also the entrepreneur’s transaction cost of selling the collateral is higher than that of the lender.

We assume that FLs initially face an interest rate ceiling at $\bar{r}$ to examine the effect of deregulation of the formal interest rate on the size and interest rates in the formal and informal credit markets. The ILs are free to choose the interest rate.

Entry into informal lending is not free or easy due to the existence of personal knowledge about borrowers on part of the lender, large resources required for incurring screening costs, giving loans etc. The existence of personalised knowledge probably leads, most naturally, to
fragmented markets (Basu and Bell, 1991). We assume for simplicity that there exists only one IL in a locality or in other words the IL enjoys a local monopoly. With regard to the formal credit market we assume that entry is restricted by law. Again this is typical to formal credit markets in many countries including India. Usually the prior sanction by the central bank is required before setting up a new bank whether by the private sector or the public sector (Varshney, 2004). We consider the case where there is only one FL in a locality.

2. FL’s Problem in the Absence of the IL

To set the stage for the discussion on strategic interaction in the next section we begin by analysing the FL’s problem in the absence of the IL. So in this section our focus is on credit market equilibrium with an uninformed monopolist FL.

Let the formal contract be denoted by \((C_k, r_k)\). The FL has to take into account the possibility of strategic default on the loans extended by it. Given the project return function and the unit loan demand functions of the entrepreneurs, the expected profit functions may be written as:

\[
\pi_{lk} = C_k(1-p) + r_k p - T \quad C_k \geq r_k \\
= C_k - T \quad C_k < r_k \tag{1a}
\]

\[
\pi_{ek} = pq - C_k(1-p) - r_k p \quad C_k \geq r_k \\
= pq - C_k \quad C_k < r_k \tag{2a}
\]

where \(\pi_{lk}\) and \(\pi_{ek}\) represent the FL’s expected profit per borrower and the entrepreneur’s
expected profit from the project if he borrows from the FL, respectively. The total surplus from the project, $S$, thus turns out to be a constant.

$$S = \pi_{lk} \pi_{ek} = pq - T > 0$$

(3)

Let $\Pi_{lk}$ represent the aggregate expected profit of the FL. Therefore, $\Pi_{lk}$ turns out to be a constant.

$$\Pi_{lk} = \pi_{lk} \int_{C_i}^C \frac{1}{C} dt = \pi_{lk} \frac{C_k}{C}$$

(4)

The FL's equilibrium contract $(C_k\theta, r_k\theta)$ is obtained as a solution to the optimisation problem,\(^6\)

$$\begin{align*}
\max_{r, c} & \hspace{1cm} \Pi_{lk} \\
\text{s.t.} & \hspace{1cm} \pi_{ek} \geq 0 \\
\text{and} & \hspace{1cm} C_k \in [0, C], r_k \in (0, q]
\end{align*}$$

(5a)

(5b)

where (5a) represents the participation constraint of the entrepreneur and (5b) ensures feasibility.

Lemma 1: Let $r_k = (C(1-p) + T)/(2-p) \equiv (C + T)/2$. Then for any $r_k \leq r_c$, $C_k = \hat{C}(r_k) = (C(1-p) + T - r_k p)/2(1-p)$ and for $r_k > r_c$, $C_k = \min(r_k, r^*)$ is the optimal choice of collateral for the given choice of $r_k$.

Proof: See appendix.

A decrease in $C_k$, with $r_k$ remaining constant reduces the FL's expected profit per borrower, but it increases the number of borrowers eligible for loans. This trade off yields a locus of optimal values of $C_k$, as a function of $r_k$ at which the FL's aggregate profit is maximum. Lemma 1 states what this optimal locus is. In figure 1 this is given by the line segments KB, BG, GH for different ranges of interest rates as demarcated by $r_c$ (corresponding to point B) and $r^*$ (point G). For $r_k \leq r_c$ the optimal value is $\hat{C}(r_k) > r_k$ given by the line segment KB; for $r_k < r_k < r^*$ this optimal value is $r_k$, given by the line segment BG. For $r_k > r^*$, optimal $C_k = r^*$, given by the vertical line segment GH. See proposition 1 for location of G.

Lemma 2: The contract $(C_k, r_k)$ with $C_k = r_k = r_c$ is the best contract among all feasible contracts with $r_k \leq r_c$.

Proof: See appendix.

In figure 1 as we move up along the $\hat{C}(.)$ curve, not only the number of borrowers eligible for loans increase, we move to higher $\pi_{lk}$ contours as well. Thus aggregate profit of the lender will increase along $\hat{C}(.)$ (the line segment KB) as $r_k$ increases as shown in lemma 2 (see appendix). Thus, using lemma 1, point B represents the best contract in the region $JADMC$.

Below we make some assumptions about the configuration of the parameters of the model.

$$T < C(1-p) < pq - T$$

(6a)

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\(^6\) This is a static maximisation problem in which we are concerned with the maximisation of a flow of profit during a period rather than a stock carried over from the past. Thus the residual collateral is not taken into consideration in the entrepreneur's pay-off function.
Assumption (6a) says that there exist collateral rich entrepreneurs but still the lender would find increasing $r_f$ to be more attractive. The left inequality of assumption (6a) ensures that the transaction cost per loan ($T$) is less than the repayment in case of default multiplied by the probability of default for the richest borrower ($C(1-p)$), but not necessarily so for entrepreneurs with a small amount of collateral. To have a meaningful problem of borrowing and lending, it is necessary that the lenders be interested in realising the surplus from the project rather than taking the collateral. Assumption (6a) ensures this. This holds even though $C > pq$, that is the value of collateral that may be obtained from the richest segment of the entrepreneurs ($C$) exceeds the expected return from the project ($pq$). This follows from (6b).

Assumptions (6b) and (6c) together imply that while the total surplus from project is less than the highest value of collateral, it is greater than the average or expected value of collateral. The above assumptions also ensure that the $C(r_f)$ curve lies between $\pi_{rk}=0$ and $\pi_{ek}=0$ contours (see figure 1). This rules out any discontinuities in the locus of optimal choices. Assumptions (6b) and (6d) are technical assumptions useful for simplifying certain results discussed later.

We now state the following proposition.

**Proposition 1:** In the absence of the IL and interest rate ceiling the FL’s equilibrium contract $(C_{kE}, r_{kE})$ is given by $C_{kE}=r_{kE}=r^*=r_{ic}, pq$.

**Proof:** See appendix.

Along the line segment AD in figure 1 the FL’s aggregate profit is maximum at point G (corresponding to $r^*$) which lies on AD above point B (corresponding to $r_{ic}$). Thus (using lemma 2) the FL’s aggregate profit is increasing along the locus of optimal $C_k$s between points K, B and G. The aggregate profit at G is also the unconstrained maximum profit of the FL. It is to be noted that since aggregate profit remains constant along the line segment GH, we may ignore this segment of the curve and consider G to be the equilibrium contract.

Note that, if the FL is faced with a binding interest rate ceiling at $\bar{r}$ (that is $r^* > \bar{r}$), then the FL’s equilibrium contract will be given by, $(C_{kE}, r_{kE})$ such that

$$r_{kE}=\bar{r} \quad \text{and} \quad C_{kE}=\tilde{C}(r_{kE}) \quad \text{when} \quad \bar{r} \leq r_{ic}$$

$$C_{kE}=\tilde{r} \quad \text{when} \quad \bar{r} > r_{ic}$$

So far we have implicitly assumed that the FL offers a unique contract to all borrowers. Suppose that the FL offers two contracts. Now, if the contract with a lower $C_k$ also yields a higher profit to the entrepreneur, then only this contract will be chosen. Thus out of the two contracts offered by the FL only one contract will prevail. Consider the other possibility, that is the contract with a lower $C_k$ yields the same or a higher profit per borrower to the FL. Then the FL earns a higher aggregate profit by offering one contract (the contract with lower $C_k$) rather than two contracts as defined above. Thus in the given framework the FL does not find it optimal to offer multiple contracts.
III. Analysis of the Interaction Between the FL and IL

We now come to the situation where there is one FL and one IL in a locality. We model this as a sequential move game between a FL and an IL in which the FL moves first followed by the IL. The FL being subject to regulatory constraints can not alter his offers quickly unlike the IL who can observe the actions of the FL and react instantaneously. We solve this game by backward induction and the solution obtained would be a sub-game perfect Nash equilibrium (SPNE).

1. Specification of the Contract Space

Let \( A = \{(C, r): C \in [0, C^i], r \in (0, q]\} \) be the set of actions or contracts available to the FL and the IL. In stage 1, the FL chooses an action, which specifies a contract \((C_k, r_k) \in A\). In stage 2, the IL after observing the FL’s action chooses an action or contract. Let \( s_j = (C_j, r_j) \in A \) denote the informal contract.

Let \( \pi_{ij} \) denote the IL’s profit per borrower, when the IL chooses \((C_j, r_j)\). \( \pi_{ij} \) is given by equation (1a) with the subscript \( j \) attached to \( C \) and \( r \). Since the IL does not face the possibility of strategic default therefore equation (1b) is not relevant. Figure 2 below illustrates certain subsets \( A_{ik} \), \( i = 1, 2, 3, 4 \) of the action space \( A \), for the IL. Given any formal contract with \((C_k, r_k)\), \( C_k \) defines the vertical line above the point labelled \( C_k \) in figure 2. \( C_k \) and \( r_k \) together define the constant line \( \pi_{ik} \). These two lines specify the subsets \( A_{ik} \), \( i = 1, 2, 3, 4 \), defined below, of the action space of the IL when the formal contract is \((C_k, r_k)\).

\[
A_{1k} = \{(C_j, r_j): C_j \geq C_k \& r_j \text{ s.t. } \pi_{ij} \geq \pi_{ik}\}
\]

\[
A_{2k} = \{(C_j, r_j): C_j > C_k \& r_j \text{ s.t. } \pi_{ij} < \pi_{ik}\}
\]

\[
A_{3k} = \{(C_j, r_j): C_j < C_k \& r_j \text{ s.t. } \pi_{ij} \geq \pi_{ik}\}
\]

\[
A_{4k} = \{(C_j, r_j): C_j \leq C_k \& r_j \text{ s.t. } \pi_{ij} < \pi_{ik}\}
\]

Given \((C_k, r_k)\), let \((C_j, r_j) = (0, r_c)\) be the informal contract such that \( \pi_{ij} = \pi_{ik} \). Thus \( r_c = (C_k(1-p) + r_sp)/p \). This is denoted by the point \( Z \) in figure 2.

2. IL’s Decision Problem in Stage 2

Demand for Formal and Informal Loans

In order to obtain the SPNE of the game we begin by solving the IL’s problem in stage 2. We first consider the aggregate demand for formal and informal loans, given a formal contract \((C_k, r_k) \in A\) and an informal contract \((C_j, r_j) \in A_{ik}\), for each \( i = 1, 2, 3, 4 \). It is to be noted that for our analysis it is meaningful to consider only those contracts, which satisfy the entrepreneur’s participation constraint and for which \( \pi_{ik} \) and \( \pi_{ij} \) are positive.

Now given \((C_k, r_k)\), if the IL chooses \( s_j \in A_{ik} \), then \( \pi_{ij} \geq \pi_{ik} \). As \( \tilde{S} \) is a constant (equation (3)), we have \( \pi_{ij} \leq \pi_{ik} \). Also \( C_j \geq C_k \), as \( s_j \in A_{ik} \). With the IL choosing \( s_j \in A_{ik} \), the formal
A contract would leave the entrepreneurs at least as well off as the informal contract. It would correspond to a lower value of the collateral as well. This would imply that all the entrepreneurs who are eligible for loans from the IL, would also be eligible for loans from the FL. Therefore the demand for informal loans is zero. Thus only the FL would survive in the market giving loans to \((\bar{C} - C_k)/\bar{C}\) proportion of entrepreneurs. When the IL chooses \(s_j \in A_{3k},\) \(\pi_{lj} < \pi_{lk}.)\) Hence using constancy of \(S\) we have \(\pi_{lj} > \pi_{lk}.\) Also \(C_j > C_k\) (since, \(s_j \in A_{2k})\). Hence entrepreneurs with collateral endowment in the interval \((C_j, \bar{C})\] will borrow from the IL. The FL will give loans to entrepreneurs with collateral endowment in the interval \([C_k, C_j)\]. The market gets segmented, with the demand for informal loans being \((\bar{C} - C_j)/\bar{C}\). Entrepreneurs with collateral less than \(C_k\) do not receive any loan either from the FL or the IL.

In case the IL chooses \(s_j \in A_{3k},\) the market will get segmented again. The FL will lend to entrepreneurs with collateral endowment in the interval \([C_k, \bar{C})\]. Among the entrepreneurs with collateral endowment smaller than \(C_k,\) the IL will give loans to all entrepreneurs with collateral endowment in the interval \((C_j, C_k)\). This will happen, as the formal contract would correspond to larger collateral. It would yield a higher profit to the entrepreneurs as well. This follows from the definition of \(A_{3k}\) and the constancy of the social surplus. Alternatively the IL may choose a contract \(s_j \in A_{4k}\) competing for borrowers with the FL and drive him out of the market. This is because for all \(s_j \in A_{4k}\) the entrepreneurs earn more profit from the informal contract compared to the formal contract. This follows directly from the fact that \(\pi_{lj} \geq \pi_{lk}\) and equation (3). Moreover since \(C_j < C_k\) therefore only the IL will survive in the market giving loans to all entrepreneurs with collateral endowment in the interval \([C_j, \bar{C})\]. The demand for formal loans in this case is zero.

**Proposition 2:** Given \((C_k, r_k) ,\) (a) for each \(s_j \in A_{1k}\) and any \(s_z \in A_{1k}, s_z\) will dominate \(s_j\) and (b) all \(s_j \in A_{3k}\) are dominated by some \(s_z \in A_{4k}\).

**Proof:** See appendix.

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7 In case of identical payoff to the entrepreneur the FL is assumed to be preferred because of individual perception.
The results of this proposition follow from the preceding discussion on the demand for formal and informal loans. With the demand for informal loans being zero for $s_j \in A_{1k}$ and positive for $s_j \in A_{2k}$, part (a) of the proposition is explained. The second part of the proposition follows from the fact that the demand for informal loans if he chooses $s_j \in A_{2k}$ or $A_{4k}$ is $(\mathcal{C} - C_j) / \mathcal{C}$. Hence for any contract such as $X$ in $A_{2k}$ (see figure 2), the IL earns higher aggregate profit if he chooses a contract $Z$ in $A_{4k}$. The superiority of $Z$ follows from the fact that it lies on the left of $X$ on the same iso-profit contour. Thus while the IL’s profit per borrower remains the same, the number of borrowers increases, resulting in higher aggregate expected profits for the IL. This proves part (b). Proposition 2 implies that the IL will never choose a contract $s_j \in A_{1k} \cup A_{2k}$. He will choose a contract $s_j$ in $A_{3k}$ or $A_{4k}$.

Optimal Contract

**Proposition 3:** Given $(C_k, r_k)$, let $s_3 \in A_{3k}$ ($s_4 \in A_{4k}$) denote the IL’s best response contract in the contract sub-space $A_{3k} (A_{4k})$. Then (a) $s_3 = (0, q)$ and (b) $s_4 = (0, r), r \rightarrow r_c$ from below, where $r_c = (C_k(1 - p) + r_k p) / p$.

**Proof:** See appendix.

If the IL decides to segment the market and lend only to the entrepreneurs who get rationed by the FL, then he would choose an interest rate that would leave the entrepreneurs at their reservation payoff of zero and earn $(pq - T)$ per borrower. In that case the IL would be better off by making clean advances (offering $(0, q)$, point W in figure 1), as his aggregate expected profit would be increasing linearly in the number of loans. This therefore expounds why unsecured loans are common in the informal credit market and yet the cost of borrowing is higher in case of informal loans with informal borrowers faring worse than their formal counterparts. Given $(C_k, r_k)$ the IL’s aggregate expected payoff if he chooses $s_3$ would be,

$$ (pq - T) C_k / \mathcal{C} - \Pi_{seg} \text{ (say).} \quad (7a) $$

The corresponding payoff to the FL in this case would be

$$ \Pi_{lk}(\mathcal{C} - C_k) / \mathcal{C} \quad (7b) $$

**Remark 1:** FL’s aggregate expected profit, if the IL chooses to segment the market (equation (7b)) is the same as the FL’s aggregate expected profit in the absence of the IL (equation (4)).

If the IL decides to compete with the FL, then he should optimally choose $s_4$. In figure 2 for any contract $Z \in S_4$, the corresponding contract $V \in S_3$ dominates $Z$. Again the contract $V$ is dominated by the contract $Z'$. Intuitively the informal contract $s_4$ would leave the entrepreneurs just as well off as the formal contract $(C_k, r_k)$ and would also enable the IL to have access to the entire market. Hence the IL’s aggregate expected payoff if he chooses $s_4$ is,

$$ C_k(1 - p) + r_k p - T = \Pi_{comp} \text{ (say) } \quad (8) $$

The corresponding payoff to the FL would be zero.

We may summarise the IL’s choice problem in terms of the following remark.

**Remark 2:** Given a formal contract $(C_k, r_k)$, $C_k \geq r_k$, the IL would choose to segment (compete) with the FL according as $\Pi_{seg} > (\leq) \Pi_{comp}$. 
3. FL’s Decision Problem in Stage 1

In stage 1, when choosing its optimal contract, the FL would take into consideration the optimal response of the IL, consequent upon its actions. Since the FL earns zero expected profit if the IL chooses to compete, we have the following observation.

**Remark 3:** The FL would never choose a contract such that the IL’s optimal choice is \( s_4 \).

In order to derive the SPNE we now prove certain lemmas. Lemmas 3 and 4 are related to the above remark and analyse the optimal choices (segmentation or competition) for the IL for given choices of \((C_k, r_k)\).

**Lemma 3:** Given (6d), for formal contracts \((C_k, r_k)\) with \( C_k / c_8 (r) \), and \( r_k / c_8 (\text{ric}) \), \( s_3 \) (segmentation) is the optimal choice for the IL in stage 2, where \( C(r_k) \) and \( r_{ic} \) are as defined in lemma 2.

**Proof:** See appendix.

The above lemma establishes that segmentation is optimal for the IL if the formal interest rate is low \(( \leq r_{ic} \) ). In figure 1, for formal contracts lying on KB the IL would always segment.

**Lemma 4:** Given (6d), there exists an \( \tilde{r}_0 = \frac{CT}{(C + T - pq)} \in (T/p, r^*) \) such that for formal contracts \((C_k, r_k)\) with \( C_k = r_k \in [r_{ic}, \tilde{r}_0] \), \( s_3 \) (segmentation) is the optimal contract for the IL in stage 2. For \( C_k = r_k \in [\tilde{r}_0, pq] \), the IL’s optimal contract in stage 2 is \( s_4 \) (competition).

**Proof:** See appendix.

This is the complement to lemma 3. Consider formal interest rates above \( r_{ic} \). This lemma establishes the cut off level of \( r_k (= \tilde{r}_0) \) below (above) which segmentation (competition) is preferable to the IL. In figure 1, \( \tilde{r}_0 \) will be located on the line segment FG \((\text{as} r_{ic} < T/p < r_0 < r^*)\). These two lemmas together dictate the optimal behaviour of the IL for different levels of \( r_k \).

**Lemma 5:** The FL’s optimal choice of collateral for a given choice of \( r_k \), in the presence of the IL is given by (i) \( C_k = \hat{C}(r_k) \) for \( r_k \in (0, r_{ic}] \), (ii) \( C_k = r_k \) for \( r_k \in (r_{ic}, \tilde{r}_0] \) and (iii) \( C_k = \tilde{r}_0 \) for \( r_k > \tilde{r}_0 \).

**Proof:** See appendix.

Lemma 5 is the counterpart to Lemma 1. It completely describes the FL’s optimal choice of \( C_k \) as a function of \( r_k \) in the presence of the IL. In particular it highlights the role of segmentation (when the FL behaves as if there is no IL) and competition (when the FL is driven out of the market). Of course the collateral choice has a ceiling and due to (6b), the market of the FL becomes constant for interest rates higher than a critical value.

In section 3B, we have so far considered contracts \((C_k, r_k)\) with \( C_k \geq r_k \). Now if we allow \( C_k < r_k \), then any contract \((C_k, r_k)\) with \( C_k < r_k \) is equivalent to another contract \((C_k, \hat{r}_k)\) with \( \hat{r}_k = C_k \). So we need not consider this case separately.

Lemmas 1 and 5 imply that the FL’s locus of optimal choices of \( C_k \) as a function of \( r_k, r_k \leq r_{ic} \), remains the same in the presence and absence of the IL, given by the segment KB in fig. 1. Thus we have the following.

**Lemma 6:** The formal contract \((C_k, r_k)\) with \( C_k = r_k = r_{ic} \) dominates all other feasible contracts
with \( r_s \leq r_c \) (even in the presence of the IL).

Proof: Follows from lemmas 1, 2 and 5.

We now state the principal theorem of this section. This theorem completely describes the optimal contract of the FL and the IL. Given the assumptions (6b) and (6d) we have the following result.

**Proposition 4:** The SPNE of a sequential move game between a FL and an IL consists of a pair of contracts \((0, q)\) for the IL and \((C_{kE}, r_{kE})\) for the FL with

\[
C_{kE} = r_{kE} = \tilde{r}_0 \in (T/p, r^*).
\]

Proof: See appendix.\(^8\)

Thus we arrive at the equilibrium pair of contracts of the sequential move game for both parties using the method of backward induction. The solution consists of a pair of contracts, \((0, q)\) for the IL (point W) and \((C_{kE}, r_{kE})\) for the FL (which lies on the line segment FG). This would constitute a SPNE of the above game. So in this situation the FL optimally chooses a collateralised contract and the IL does not find it optimal to do so.\(^9\) Note that any contract \((C_k, r_k)\) with \(C_k = \tilde{r}_0\) and \(r_k > C_k\) also yield the same aggregate profit as the equilibrium formal contract stated in the proposition. Hence, all these choices by the FL, with the IL choosing \((0, q)\), would also constitute alternative SPNEs for this game. But, as these choices are not very meaningful, we do not discuss them in detail.

Our analysis is based on the implicit assumption that the IL offers a single contract to all borrowers. However allowing for multiple informal contracts does not offer additional insight, as effectively the IL would behave like the FL in the FL’s segment of the market. We would observe the same two contracts, as were offered by the FL and the IL in case of unique contracts. Thus analytically nothing changes.

**IV. Effect of Interest Rate Deregulation**

Now let us look at the effects of deregulation of formal interest rate on the market size for the FL. As \( r^* < (C(1-p) + T)/(2(1-p)) \), point G will lie to the left of point K. This implies that the equilibrium contract of the FL, \(C_{kE} = r_{kE} = \tilde{r}_0 < r^*\), will also lie to the left of K in figure 1. Dropping a perpendicular from \((\tilde{r}_0, \tilde{r}_0)\) on the x-axis and noting the point of intersection with the segment KB, yields a rate of interest given by \(C^{-1}(\tilde{r}_0)\). This implies two ranges of interest rates. If the interest rate ceiling lies in the lower range, \((0, C^{-1}(\tilde{r}_0))\), then deregulation causes formal lending to expand. On the other hand, if the interest rate ceiling lies in the upper range \([C^{-1}(\tilde{r}_0), \tilde{r}_0)\), then deregulation will cause informal lending to expand and formal lending to contract.\(^{10}\)

---

\(^8\) If (6b) is violated then the equilibrium rate of interest might occur at \(r^*\) or some \(r > \tilde{r}_0\), depending on different parametric configuration. But as long as the equilibrium rate lies to the left of point K in figure 1, our later conclusions are qualitatively unchanged.

\(^9\) Note that, even in the absence of FL, the IL would find it optimal to choose \((0, q)\).

\(^{10}\) The results discussed above are based on assumption (6d). However with the inequality reversed, the result would still hold under certain technical conditions not very interesting or easily interpretable.
Suppose initially the financial repression is very severe (\( \bar{r} \) very low) with FL’s lending only to the very rich (\( C_k \) very high). Then deregulation will cause formal lending to increase and informal lending to decrease. This is because deregulation will cause the formal interest rate and hence profit earned per borrower to increase. This in turn will enable the FL to ask for lower collateral without either cutting down on its profit or jeopardising the incentive compatibility constraint. On the other hand, suppose initially the financial repression is not very severe (\( \bar{r}/c\bar{r} < 1(\bar{r}_0), \bar{r}_0 = 0 < C_k < C_k' \)) and the size of the formal sector is not very small. Then as the formal interest rate increases following deregulation the FL must ask for a larger collateral, in order to avoid strategic default. A larger collateral means FLs must restrict their lending only to the very rich. The FL however can afford to do so and hence have fewer borrowers as the profit per borrower is not very low initially and increases further with a rise in formal interest rate and collateral. Hence in this case the size of the formal credit market contracts if interest rate is deregulated. So we have the following result:

**Proposition 5:** Deregulation causes formal lending to contract (expand) and informal lending to expand (contract) if the financial repression is not severe (severe).

### V. Conclusion

In this paper an attempt has been made to analyse the impact of deregulation of the formal interest rate on the size of the informal credit market. This is significant in the context of the policy changes and revived debate on liberalisation versus state intervention in credit markets as the optimal credit policy, especially in the developing countries. The governments in these countries are not concerned only with ensuring a greater mobilisation of savings and an efficient allocation of credit. Another objective has been to ensure equity, by making sure that small borrowers have access to the required institutional credit as an alternative to informal credit (which may be exploitative).

Here, we have explicitly recognised the joint existence of formal and informal lender in the market. Thus, the strategic interaction between these two types of agents is a crucial element of the current paper. Modelling this interaction in the form of a sequential game helps us to analyse it more completely.

The conclusion is reached that given the acute information asymmetry faced by the FL, deregulation need not necessarily curb informal lending. The effectiveness of the policy will depend on the degree of financial repression. Specifically it is shown that there is a range of interest rates such that if the interest rate ceiling lies in that range then deregulation of the formal interest rate will cause informal lending to increase and formal lending to contract. This result therefore qualifies the conventional argument in favour of deregulation.

### Appendix

**Proof of Lemma 1:**

Let \( \hat{C}(r_k) \) be such that the derivative of the lender’s aggregate profit function corresponding to (1a) is equal to zero i.e. \( \partial \Pi_h/\partial C_k = 0 \) at \( C_k = \hat{C}(r_k) \). Then given \( r_k \), the lender’s optimal
choice of \( C_k \) subject to \( C_k \geq r_k \) is given by \( \max\{r_k, \hat{C}(r_k)\} \), where \( \hat{C}(r_k) = (\hat{C}(1-p) + T - r_k \hat{C})/\{2(1-p)\} \). Note that the second order condition for a maximum is satisfied by \( \hat{C}(r_k) \), since for \( \Pi_{lk} \) corresponding to (1a) \( \partial^2 \Pi_{lk}/\partial C_k^2 = -2(1-p)/\hat{C} < 0 \).

Setting the derivative of the lender’s aggregate profit function corresponding to (1b), \( \partial \Pi_{lk}/\partial C_k = 0 \) yields \( C_k = r^* \). Hence the lender’s optimal choice of \( C_k \) for a given choice of \( r_k \), subject to \( C_k < r_k \), is given by \( \min\{r_k, \hat{C}(r_k)\} \) (as a corner solution). Note that the second order condition for a maximum is satisfied by \( C_k = r^* \), since for \( \Pi_{lk} \) corresponding to (1b) \( \partial^2 \Pi_{lk}/\partial C_k^2 = -2/\hat{C} < 0 \). Solving \( \hat{C}(r_k) = r_k \), for \( r_k \) yields \( r_{lk} \) such that \( \hat{C}(r_k) = r_k \) according as \( r_k < r_{lk} \) and \( r_k > r_{lk} \). Also, since \( T < \hat{C} \) we have \( r_a < r^* \).

We now consider the following cases.

Firstly, given an \( r_k \leq r_{lk} \), \( \max\{r_k, \hat{C}(r_k)\} = \hat{C}(r_k) \) (since \( \hat{C}(r_k) \geq r_k \) for \( r_k \leq r_{lk} \)) and \( \min\{r_k, r^*\} = r_k \) (since \( r_k < r_{lk} < r^* \)). Since \( \Pi_{lk} \) corresponding to (1a) and (1b) are equal at \( C_k = r_k \) and \( \hat{C}(r_k) \) maximises \( \Pi_{lk} \) subject to \( C_k \leq r_k \), therefore, the lender’s unconstrained optimal choice of \( C_k \) is given by \( C_k \rightarrow r_k \) for a given \( r_k \leq r_{lk} \), is \( \hat{C}(r_k) \).

Secondly, given an \( r_k \) such that \( r_k < r_{lk} \leq r^* \), \( \max\{r_k, \hat{C}(r_k)\} = r_k = \min\{r_k, r^*\} \). Thus given an \( r_k \) such that \( r_a < r_{lk} \leq r^* \), the unconstrained maximum of \( \Pi_{lk}(C_k \rightarrow r_k) \) occurs at \( C_k = r_k \).

Finally, given an \( r_k > r^* \), \( \max\{r_k, \hat{C}(r_k)\} = r_k \) and \( \min\{r_k, r^*\} = r^* \). Since \( \Pi_{lk} \) corresponding to (1a) and (1b) are equal at \( C_k = r_k \) and \( r^* \) maximises \( \Pi_{lk} \) subject to \( C_k < r_k \), therefore given \( r_k > r^* \), the unconstrained maximum of \( \Pi_{lk} \) occurs at \( C_k = r^* \).

Hence given an \( r_k > r_{lc} \), the lender’s optimal choice of collateral is given by \( C_k = r_k \) if \( r_{lc} < r_k < r^* \) and \( C_k = r^* \) if \( r_{lc} < r^* < r_k \) i.e. \( C_k = \min\{r_k, r^*\} \).

**Proof of Lemma 2:**

Let \( \Pi_{lk}^* \) be the FL’s aggregate expected profit, after choosing \( C_k \) optimally for a given choice of \( r_k \), when \( C_k \geq r_k \). That is

\[
\Pi_{lk}^* = (\hat{C}(1-p) + r_k p - T) (\hat{C} - \hat{C}(r_k)) / (2\hat{C}) \quad (A1)
\]

Differentiating this with respect to \( r_k \) one can check that \( \Pi_{lk}^* \) is increasing in \( r_k \). Hence using lemma 1, result follows.

**Proof of Proposition 1:**

\( C_k = r_k = r^* \) maximises \( \Pi_{lk} = (r_k - T)(\hat{C} - \hat{C}(r_k))/\hat{C} \) among all contracts for which \( r_k = C_k \). We have, \( r^* \geq (T, \hat{C}) \). (6c) implies that \( r^* < pq \). Since \( T < \hat{C} \) we have,

\[
r^* > r_{lc} \quad (A2)
\]

From (A2), it follows that the contract \( (C_k, r_k) \) with \( C_k = r_k = r^* \) yields higher \( \Pi_{lk} \) compared to the contract with \( C_k = r_k = r_{lc} \). Hence using lemma 2, it follows that contract with \( C_k = r_k = r^* \) yields higher \( \Pi_{lk} \) compared to all feasible contracts with \( r_k \leq r_{lc} \). Lemma 1 implies that the contract with \( C_k = r_k = r^* \) yields higher \( \Pi_{lk} \) compared to all feasible contracts with \( r_k > r_{lc} \). Therefore FL’s equilibrium contract is given by \( C_{kE} = r_{kE} = r^* \).

**Proof of Proposition 2:**
Given \((C_k, r_k)\), if the IL chooses \(s_j \in A_{1k}\), then the aggregate demand for informal loans is zero. Hence the IL’s aggregate profit is zero \(\forall s_j \in A_{1k}\). Again for any \(s_j \in A_{1k}\), the aggregate demand for loans is always positive. Hence for any \(s_j \in A_{1k}\), the IL’s aggregate profit is positive \(\forall s_j \in A_{1k}\) (if \(\pi_q > 0\) and \(\pi_l > 0\) i.e. when the analysis is meaningful). This proves part (a) of the proposition.

We next consider the subset \(A_{2k}\). Given \((C_k, r_k)\), consider \(s_j = (C_z, r_z) \in A_{2k}\) such that \(\pi_l = \pi_q\) and \(C_z < C_k\) where \(s_j \in A_{2k}\). That is \(s_j \in A_{2k}\). Now for \(s_j \in A_{2k}\) the aggregate demand for informal loans is \(\frac{C - C_z}{C}\) and for \(s_j \in A_{2k}\) it is \(\frac{C - C_j}{C}\). Since \(s_j \in A_{2k}\), therefore \(C_j > C_k > C_z\). This implies that \(\pi_l(C - C_z)/C > \pi_l(C - C_j)/C\), that is the IL earns higher aggregate expected profit if he chooses \(s_j \in A_{2k}\) than if he chooses \(s_j \in A_{2k}\). This proves part (b) of the proposition.

**Proof of proposition 3:**

Given \((C_k, r_k)\) with \(C_k > 0\) \((C_k = 0 \Rightarrow r_k = 0)\), consider any \(s_j \in A_{3k}\) and a contract \(s_m\) such that \(s_m = (0, r_m)\) \((r_m \leq q\) and \(\pi_l = \pi_q\). Then \(s_m \in A_{3k}\) as \(\pi_l = \pi_q\) and \(\pi_l > \pi_l\) and the value of collateral \(-0 < C_k\). Now, for any \(s_j\) belonging to \(A_{3k}\), the demand for informal loans is \((C_k - C_j)/C\). Hence comparing the aggregate profits of the IL from \(s_j\) and \(s_m\), we have \(\pi_l s_j(C - C_j)/C > \pi_l s_m(C - C_j)/C\). We next consider the contract \(s_j = (0, q)\). \(\forall r_m < q, \pi_m s_j(C - C_j)/C > \pi_m s_m(C - C_j)/C\). Hence it follows that \(s_j \in A_{4k}\) denotes the IL’s best response in \(A_{4k}\). This proves part (a) of the proposition.

Proceeding similarly we can prove that for any \(s_j \in A_{4k}\), \(C_j > 0\), there exists a contract \(s_a = (0, r_a) \in A_{4k}\), \(r_a \leq r_c = (C_k(1 - p) + r_k p)/p\), such that \(s_a\) dominates \(s_j\). Note that for any \(s_k \in A_{4k}\), the demand for informal loans is \((C - C_z)/C\). We next consider the contract \(s_a = (0, r), r \rightarrow r_c\) from below (denoted as \(r_c\)). Then as \(r \rightarrow r_c\), \(\pi_l s_{a/c} \rightarrow \pi_l s_{a/k}\). We have, \(\lim \pi_l s_{a/c} > \pi_l s_{a/k}\) aggregate profit is higher from \(s_a \forall r_a < r_c\). Hence it follows that \(s_a \in A_{4k}\) denotes the IL’s best response in \(A_{4k}\). This proves part (b) of proposition 3.

**Proof of Lemma 3:**

Given the formal contract \((C(r_k), r_k)\), \(\Pi_{seg} \supset \Pi_{comp}\) according as,

\[
(p q - T) \left[ C(1 - p) - r_k p + T \right] \supset \frac{\pi_l}{\pi_l} C(1 - p) \left[ C(1 - p) + r_k p - T \right]
\]  \(\text{(A3)}\)

The LHS of inequality \((A3)\) is decreasing in \(r_k\) while the RHS is increasing in \(r_k\). Further for \(r_k = 0\), the value of LHS is greater than the value of RHS as \(C(1 - p) < (p q - T)\) by assumption. Moreover the horizontal axis intercept of the function on the LHS, is given by \((C(1 - p) + T)/p\). Thus there exists an \(r_0 < (C(1 - p) + T)/p\) such that for \(r_k < r_0\), segmentation (choosing \(s_3\)) is better for the IL in stage 2.

Given \((6a), (A3)\) implies that a sufficient condition for \(\Pi_{seg} \supset \Pi_{comp}\) is that, \(C(1 - p) - r_k p + T > C(1 - p) + r_k - T \Rightarrow r_k \leq T/p\). Hence it follows that the critical value \(r_0 > T/p\). (6d) implies \(r_a < T/p\). Therefore \(r_0 > r_c\). Thus for formal contracts with \(C_k = C(r_k)\) and \(r_k < r_c\), the IL’s optimal choice is \(s_3\).

**Proof of Lemma 4:**

Given the formal contract \((C_k, r_k)\) with \(C_k = r_k\), the IL’s \(\Pi_{seg} \supset \Pi_{comp}\) according as, \((p q -
Both the LHS and the RHS of the above inequality are increasing in $r_k$. However, the function on the LHS is flatter than that on the RHS as $(pq - T)/C < 1$ (using assumption (6b)) and intersects the function on the RHS from above.

Thus there exists an $\tilde{r}_0 = \frac{CT}{C + T - pq}$ such that for formal contracts $(C_k, r_k)$ with $C_k = r_k < \tilde{r}_0$, segmentation (choosing $s_1$) is better for the IL in stage 2. Given assumptions (6a-d), one can check that $\tilde{r}_0 \in (T/p, r^*)$.

**Proof of Lemma 5:**

Consider the formal contract $(C_k, r_k), r_k \leq r_w, C_k \neq \hat{C}(r_k)$. Given the FL’s choice in stage 1, the IL may choose either $s_3$ or $s_4$. If the IL chooses $s_3$ (segment), then using lemma 3, remark 1 and lemma 1, for any choice of $r_k \leq r_w$, the formal contract $(C_k, r_k)$ with $C_k = \hat{C}(r_k)$ dominates all other feasible contracts (i.e. $C_k \neq \hat{C}(r_k)$). Otherwise, if the IL chooses $s_4$ (compete), the FL’s expected profit is zero for any contract.

Now, for $r_k \in (r_w, \tilde{r}_0]$, using similar reasoning and given the assumption (6b), we can establish the optimality of contracts with $C_k = r_k$.

For any formal contract $(C_k, r_k), r_k > \tilde{r}_0$, comparing IL’s profit from segmentation and competition, we determine two values for $C_k, C_0(r_k) = \frac{(pq - T)}{p - (1 - p)} C$ and $\tilde{r}_0$, such that competition (segmentation) is more profitable for IL if $C_k \in [\tilde{r}_0, C_0(r_k)]$ ($C_k \in [\tilde{r}_0, C_0(r_k)]$).

Given (6b) and $r_k > \tilde{r}_0$ (as the IL segments in this range), the aggregate profit of the FL from $(C_0(r_k), r_k)$ is decreasing in $r_k$. Hence, the optimal choice of $C_k$ here is given by $\tilde{r}_0$ (the minimum admissible value).

**Proof of Proposition 4:**

Given lemmas 5 and 6, we know that formal contracts $(C_k, r_k), r_k \in [r_w, \tilde{r}_0]$ and $C_k = r_k$ dominates all other feasible contracts.

Now, given (6b), $\tilde{r}_0 < r^*$. Also, from proposition 2 we know that among formal contracts $(C_k, r_k)$ with $C_k = r_k$, the choice of $C_k = r_k = r^*$ yields the highest profit in the absence of IL. Hence, in the presence of the IL, an optimal contract would be given by $C_k = r_k = \tilde{r}_0$, as the profit is non-increasing beyond $\tilde{r}_0$.

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